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A Decision Support Tool for Planning the Sampling of Tank 19 (U)

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A Decision Support Tool for Planning the Sampling of Tank 19 (U)

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July 23, 2001

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INTRODUCTION

The Statistical Consulting Section (SCS) of the Savannah River Technology Center (SRTC) was asked to develop a decision support system (DSS) to aid in the planning of the process to characterize the heel of Tank 19. This characterization is to be based on samples of the tank heel, and the DSS is to be used to help determine the number of samples that might be needed to provide a meaningful characterization based upon assumptions for relevant variations and volumes.

The objective of this report is to describe the framework used to develop the DSS and to document its calculations. The DSS was developed as a Microsoft Excel spreadsheet. Detailed input and output for two test cases are provided in the appendix as part of the documentation of this spreadsheet.

DISCUSSION

Two different units of measure are considered in the framing of this analysis: grams of analyte per gram of solids and micro-Curies, μCi, of analyte per gram of solids. The equations discussed below handle both of these situations as long as care is taken with the units of measure being used.

An equation for estimating the concentration (in grams of analyte per gram of solids or μCi per gram of solids) in the Tank 19 heel may be written as

$$\mathbf{A} = \frac{\sum_{\mathbf{i=1}}^{\mathbf{p}} \left(\bar{\mathbf{a}}_{\mathbf{i}} \cdot \bar{\mathbf{f}}_{\mathbf{i}} \cdot \bar{\rho}_{\mathbf{i}} \cdot \mathbf{V}_{\mathbf{i}} \right)}{\sum_{\mathbf{i=1}}^{\mathbf{p}} \left(\bar{\mathbf{f}}_{\mathbf{i}} \cdot \bar{\rho}_{\mathbf{i}} \cdot \mathbf{V}_{\mathbf{i}} \right)} \tag{1}$$

where A is the estimate of the grams (or μCi) of analyte per gram of solids in the heel, i is used as the index for the "phases" of material in the tank heel (up to p=4 phases are considered in this analysis), each $\bar{\mathbf{a}}_{\mathbf{i}}$ represents the average weight fraction or μCi per gram of solids (over the n_i samples) of the analyte in the solids fraction for the i^{th} phase, each $\bar{\mathbf{f}}_{\mathbf{i}}$ represents the average weight fraction (over n_i samples) of the solids for the i^{th} phase, $\bar{\rho}_{\mathbf{i}}$ represents the average density (over n_i samples) of the material making up the i^{th} phase, and $\mathbf{V}_{\mathbf{i}}$ represents the volume of this phase in the heel of Tank 19. It is assumed that the values used for the $\mathbf{V}_{\mathbf{i}}$'s are bounding volumes for the p phases.

The material constituting one phase in the heel of Tank 19 may be quite different from the material in another phase. It was this possibility that prompted the use of p phases in the development of the DSS. How to take samples in order to identify these phases and to obtain representative material from each one is beyond the scope of this report. Such efforts would rely on knowledge of tank history and of the relevant chemistry and on aids such as videos of the tank heel. To address all of the issues associated with the characterization of a tank heel may require a phased approach to the sampling (i.e., one or more samples are taken and evaluated, and then based upon the results, one or more additional samples are taken). The purpose of the DSS described in this report is to estimate the uncertainty of results calculated using equation (1).

One other variation in the use of equation (1) is considered in the following discussion. Recall that the volume estimates utilized in equation (1) are bounding values that induce no error propagation. If the solids contents of the material phases can be bounded, then their values too would induce no error propagation in equation (1). This variation is considered in a later section.

Uncertainty analysis as presented in [1] is used to develop the decision support system to aid in planning the sampling of the Tank 19 heel. The exact calculation of the variance of a (nonlinear) function of variables that are subject to variation {such as that given in equation (1)} is a problem of considerable mathematical complexity. Fortunately, an approximate estimate derived from a linearized approach (based on the used of a Taylor series expansion of the function with only the linear terms retained) is adequate in most applications [1]. And such an approach is used in this report to estimate the variance of A, **var(A)**.

Estimating the Variance of A

Defining some notation will help to facilitate the development of estimates for the variance and, subsequently, the uncertainty of the derived value, A, given by equation (1). Let the relative variance of the random measurement errors associated with $\bar{\mathbf{a}}_{\mathbf{i}}$ (based upon $\mathbf{n_i}$ samples) be estimated by

$$\mathbf{S}^2_{\bar{\mathbf{a}}_{\mathbf{i}}} = \frac{\left(\mathbf{S}^2_{\mathbf{t}_{\mathbf{a}_\mathbf{i}}} + \mathbf{S}^2_{\mathbf{m}_{\mathbf{a}_\mathbf{i}}} \Big/ \mathbf{n}_{\mathbf{m}_{\mathbf{a}_\mathbf{i}}} \right)}{\mathbf{n}_{\mathbf{i}}} \tag{2}$$

where $\mathbf{S}^2_{\mathbf{t}_{\mathbf{a}_\mathbf{i}}}$ represents the relative variance reflecting tank homogeneity for analyte a in phase i,

$\mathbf{S}^2_{\mathbf{m}_{\mathbf{a}_\mathbf{i}}}$ represents the relative variance of the analytical procedures used to measure for concentrations of the given analyte in the solids (as a weight fraction or as μCi per gram of solids) of the ith phase, and

$\mathbf{n}_{\mathbf{m}_{\mathbf{a}_\mathbf{i}}}$ represents the number of such analyses performed on each sample of phase i from the tank heel.

The relative variance of the random measurement errors associated with $\bar{\mathbf{f}}_\mathbf{i}$ (based upon n_i samples) is estimated by

$$\mathbf{s}^2_{\bar{\mathbf{f}}_\mathbf{i}} = \frac{\left(\mathbf{S}^2_{\mathbf{t}_{\mathbf{f}_\mathbf{i}}} + \mathbf{S}^2_{\mathbf{m}_{\mathbf{f}_\mathbf{i}}} \Big/ \mathbf{n}_{\mathbf{m}_{\mathbf{f}_\mathbf{i}}}\right)}{\mathbf{n}_\mathbf{i}} \tag{3}$$

where $\mathbf{S}^2_{\mathbf{t}_{\mathbf{f}_\mathbf{i}}}$ represents the relative variance reflecting tank homogeneity for solids in phase i,

$\mathbf{S}^2_{\mathbf{m}_{\mathbf{f}_\mathbf{i}}}$ represents the relative variance of the analytical procedures used to measure for the solids of the i^th^ phase, and

$\mathbf{n}_{\mathbf{m}_{\mathbf{f}_\mathbf{i}}}$ represents the number of such analyses performed on each sample of phase i from the tank heel.

The relative variance of the random measurement errors associated with $\bar{\rho}_\mathbf{i}$ (based upon n_i samples) is estimated by

$$\mathbf{s}^2_{\bar{\rho}_\mathbf{i}} = \frac{\left(\mathbf{S}^2_{\mathbf{t}_{\rho_\mathbf{i}}} + \mathbf{S}^2_{\mathbf{m}_{\rho_\mathbf{i}}} \Big/ \mathbf{n}_{\mathbf{m}_{\rho_\mathbf{i}}}\right)}{\mathbf{n}_\mathbf{i}} \tag{4}$$

where $\mathbf{S}^2_{\mathbf{t}_{\rho_\mathbf{i}}}$ represents the relative variance reflecting tank homogeneity for the density of phase i,

$\mathbf{S}^2_{\mathbf{m}_{\rho_\mathbf{i}}}$ represents the relative variance of the analytical procedures used to measure density for the ith phase, and

$\mathbf{n}_{\mathbf{m}_{\rho_\mathbf{i}}}$ represents the number of such analyses performed on each sample of phase i from the tank heel.

Note that the relative variances estimated by equations (2), (3), and (4) for each phase of material in the Tank 19 heel depend on the number of samples, $\mathbf{n_i}$, of that phase.

The Taylor's series approach [1] leads to an estimate of the variance of A, **var(A)** given by equation (5)

$$\mathbf{var}(\mathbf{A}) = \sum_{\mathbf{i}=1}^{\mathbf{p}} \left(\begin{array}{l} \left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_\mathbf{i}}\right)^2 \bar{\mathbf{a}}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\mathbf{a}}_\mathbf{i}} + \left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)^2 \bar{\mathbf{f}}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\mathbf{f}}_\mathbf{i}} + \left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)^2 \bar{\rho}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\rho}_\mathbf{i}} + 2\bar{\mathbf{a}}_\mathbf{i}\bar{\mathbf{f}}_\mathbf{i} \left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)\mathbf{cor}(\bar{\mathbf{a}}_\mathbf{i}, \bar{\mathbf{f}}_\mathbf{i})\mathbf{S}_{\bar{\mathbf{a}}_\mathbf{i}}\mathbf{S}_{\bar{\mathbf{f}}_\mathbf{i}} \\ + 2\bar{\mathbf{a}}_\mathbf{i}\bar{\rho}_\mathbf{i} \left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)\mathbf{cor}(\bar{\mathbf{a}}_\mathbf{i}, \bar{\rho}_\mathbf{i})\mathbf{S}_{\bar{\mathbf{a}}_\mathbf{i}}\mathbf{S}_{\bar{\rho}_\mathbf{i}} + 2\bar{\mathbf{f}}_\mathbf{i}\bar{\rho}_\mathbf{i} \left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)\mathbf{cor}(\bar{\mathbf{f}}_\mathbf{i}, \bar{\rho}_\mathbf{i})\mathbf{S}_{\bar{\mathbf{f}}_\mathbf{i}}\mathbf{S}_{\bar{\rho}_\mathbf{i}} \end{array} \right) \quad (5)$$

This equation introduces the possibility of correlation (cor) between the variation affecting pairs of measurements derived from the samples from a particular phase. This equation also indicates an assumption of no correlation among samples from different phases of material.

Since the exact forms of these correlations are not readily known, an approach will be used that conservatively estimates (intentionally increases) the variance of A. This will be accomplished by letting all correlations take the value of +1 and by taking the absolute value of the products of the corresponding pairs of partial derivatives in equation (5). Using this approach equation (5) may be rewritten as

$$\mathbf{var}(\mathbf{A}) = \sum_{\mathbf{i}=1}^{\mathbf{p}} \left(\begin{array}{l} \left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_\mathbf{i}}\right)^2 \bar{\mathbf{a}}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\mathbf{a}}_\mathbf{i}} + \left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)^2 \bar{\mathbf{f}}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\mathbf{f}}_\mathbf{i}} + \left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)^2 \bar{\rho}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\rho}_\mathbf{i}} + 2\bar{\mathbf{a}}_\mathbf{i}\bar{\mathbf{f}}_\mathbf{i} \left|\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)\right|\mathbf{S}_{\bar{\mathbf{a}}_\mathbf{i}}\mathbf{S}_{\bar{\mathbf{f}}_\mathbf{i}} \\ + 2\bar{\mathbf{a}}_\mathbf{i}\bar{\rho}_\mathbf{i} \left|\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)\right|\mathbf{S}_{\bar{\mathbf{a}}_\mathbf{i}}\mathbf{S}_{\bar{\rho}_\mathbf{i}} + 2\bar{\mathbf{f}}_\mathbf{i}\bar{\rho}_\mathbf{i} \left|\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)\right|\mathbf{S}_{\bar{\mathbf{f}}_\mathbf{i}}\mathbf{S}_{\bar{\rho}_\mathbf{i}} \end{array} \right) \quad (6)$$

Next, take the partial derivatives of A with respect to each measurement of interest in equation (6). This leads to the following equation for **var(A)**:

$$\mathbf{var}(\mathbf{A}) = \sum_{\mathbf{i}=1}^{\mathbf{p}} \left(\begin{array}{l} \left(\frac{\bar{\mathbf{f}}_\mathbf{i}\bar{\rho}_\mathbf{i}\mathbf{V}_\mathbf{i}}{\sum_{\mathbf{j}=1}^{\mathbf{p}}(\bar{\mathbf{f}}_\mathbf{j}\bar{\rho}_\mathbf{j}\mathbf{V}_\mathbf{j})}\right)^2 \bar{\mathbf{a}}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\mathbf{a}}_\mathbf{i}} + \left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)^2 \bar{\mathbf{f}}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\mathbf{f}}_\mathbf{i}} + \left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)^2 \bar{\rho}_\mathbf{i}^2 \mathbf{S}^2_{\bar{\rho}_\mathbf{i}} + 2\bar{\mathbf{a}}_\mathbf{i}\bar{\mathbf{f}}_\mathbf{i} \left|\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)\right|\mathbf{S}_{\bar{\mathbf{a}}_\mathbf{i}}\mathbf{S}_{\bar{\mathbf{f}}_\mathbf{i}} \\ + 2\bar{\mathbf{a}}_\mathbf{i}\bar{\rho}_\mathbf{i} \left|\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)\right|\mathbf{S}_{\bar{\mathbf{a}}_\mathbf{i}}\mathbf{S}_{\bar{\rho}_\mathbf{i}} + 2\bar{\mathbf{f}}_\mathbf{i}\bar{\rho}_\mathbf{i} \left|\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_\mathbf{i}}\right)\left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_\mathbf{i}}\right)\right|\mathbf{S}_{\bar{\mathbf{f}}_\mathbf{i}}\mathbf{S}_{\bar{\rho}_\mathbf{i}} \end{array} \right) \quad (7)$$

Estimating the Uncertainty of A

From [1], an estimate of the uncertainty of (1) with 95% coverage may be expressed as

$$\mathbf{Uncertainty}_\mathbf{A}(\mathbf{95\%}) = \sqrt{\mathbf{B}_\mathbf{A}^\mathbf{2} + \mathbf{t}^2_{0.05,\nu_\mathbf{A}}\ \mathbf{var}(\mathbf{A})} \quad (8)$$

where	$\mathbf{var(A)}$	is given by equation (7),
	$\mathbf{t}_{0.05,\nu_{\mathbf{A}}}$	is the upper 5%-tail of the Student t distribution with $\nu_{\mathbf{A}}$ degrees of freedom (the value for the degrees of freedom is discussed below), and
	$\mathbf{B}_{\mathbf{A}}^{2}$	represents the potential bias associated with the value of A derived using equation (1). For this analysis, it is assumed that there is no appreciable bias (i.e., $\mathbf{B}_{\mathbf{A}} = 0$).

The degrees of freedom, $\nu_{\mathbf{A}}$, is estimated by the Welch-Satterthwaite formula, as described in [1], given by

$$\nu_{\mathbf{A}} = \frac{\left[\sum_{\mathbf{i}=1}^{\mathbf{p}}\left(\left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_{\mathbf{i}}}\right)\bar{\mathbf{a}}_{\mathbf{i}}\mathbf{S}_{\bar{\mathbf{a}}_{\mathbf{i}}}\right)^{2} + \left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_{\mathbf{i}}}\right)\bar{\mathbf{f}}_{\mathbf{i}}\mathbf{S}_{\bar{\mathbf{f}}_{\mathbf{i}}}\right)^{2} + \left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_{\mathbf{i}}}\right)\bar{\rho}_{\mathbf{i}}\mathbf{S}_{\bar{\rho}_{\mathbf{i}}}\right)^{2}\right)\right]^{2}}{\sum_{\mathbf{i}=1}^{\mathbf{p}}\left(\frac{\left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_{\mathbf{i}}}\right)\bar{\mathbf{a}}_{\mathbf{i}}\mathbf{S}_{\bar{\mathbf{a}}_{\mathbf{i}}}\right)^{4}}{(\mathbf{n}_{\mathbf{i}}-1)} + \frac{\left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{f}}_{\mathbf{i}}}\right)\bar{\mathbf{f}}_{\mathbf{i}}\mathbf{S}_{\bar{\mathbf{f}}_{\mathbf{i}}}\right)^{4}}{(\mathbf{n}_{\mathbf{i}}-1)} + \frac{\left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\rho}_{\mathbf{i}}}\right)\bar{\rho}_{\mathbf{i}}\mathbf{S}_{\bar{\rho}_{\mathbf{i}}}\right)^{4}}{(\mathbf{n}_{\mathbf{i}}-1)}\right)} \tag{9}$$

where the degrees of freedom for the each of the relevant variances is $(\mathbf{n}_{\mathbf{i}} - 1)$.

Thus, the uncertainty of equation (1), with 95% coverage, simplifies to

$$\mathbf{Uncertainty}_{\mathbf{A}}\mathbf{(95\%)} = \sqrt{\mathbf{t}_{0.05,\nu_{\mathbf{A}}}^{2}\ \mathbf{var(A)}} \tag{10}$$

and the upper bound defined by

$$\mathbf{Upper\ Bound\ on\ A\ (at\ 95\%\ confidence)} = \mathbf{A} + \sqrt{\mathbf{t}_{0.05,\nu_{\mathbf{A}}}^{2}\ \mathbf{var(A)}} \tag{11}$$

One Additional Variation

One other variation in the use of equation (1) is considered in this section. Recall that the volume estimates utilized in equation (1) are bounding values that induce no error propagation. If the solids contents of the material phases can be bounded, then they too would induce no error propagation in equation (1), and equation (1) simplifies to equation (12):

$$\mathbf{A'} = \frac{\sum_{\mathbf{i=1}}^{\mathbf{p}} \left(\bar{\mathbf{a}}_{\mathbf{i}} \cdot \mathbf{c}_{\mathbf{i}} \cdot \mathbf{V}_{\mathbf{i}} \right)}{\sum_{\mathbf{i=1}}^{\mathbf{p}} \left(\mathbf{c}_{\mathbf{i}} \cdot \mathbf{V}_{\mathbf{i}} \right)} \tag{12}$$

where $\mathbf{A'}$ is the estimate of the grams (or µCi) of analyte per gram of solids in the heel, i is used as the index for the "phases" of material in the tank heel (up to p=4 phases are considered in this analysis), each $\bar{\mathbf{a}}_{\mathbf{i}}$ represents the average weight fraction or µCi per gram of solids (over the n_i samples) of the analyte in the solids fraction for the ith phase, each $\mathbf{c}_{\mathbf{i}}$ represents a bounding value for the solids content (e.g., as grams per liter) in this phase, and $\mathbf{V}_{\mathbf{i}}$ represents a bounding value for the volume of this phase in the heel of Tank 19.

Note that in equation (12) only the $\bar{\mathbf{a}}_{\mathbf{i}}$ values have variation that must be propagated into the estimate for the uncertainty of $\mathbf{A'}$. Using the notation introduced above, the Taylor's series approach is greatly simplified and leads to an estimate of the variance of $\mathbf{A'}$, **var(** $\mathbf{A'}$ **)** given by

$$\mathbf{var}\left(\mathbf{A'}\right) = \sum_{\mathbf{i=1}}^{\mathbf{p}} \left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_{\mathbf{i}}} \right)^{2} \bar{\mathbf{a}}_{\mathbf{i}}^{2} \mathbf{S}_{\bar{\mathbf{a}}_{\mathbf{i}}}^{2} \right) \tag{13}$$

Next, take the partial derivatives of $\mathbf{A'}$ with respect to each $\bar{\mathbf{a}}_{\mathbf{i}}$ in equation (13). This leads to the following equations for **var(** $\mathbf{A'}$ **)** and for the upper bound on $\mathbf{A'}$ with 95% confidence:

$$\mathbf{var}\left(\mathbf{A'}\right) = \sum_{\mathbf{i=1}}^{\mathbf{p}} \left(\left(\frac{\mathbf{c}_{\mathbf{i}} \mathbf{V}_{\mathbf{i}}}{\sum_{\mathbf{j=1}}^{\mathbf{p}} \left(\mathbf{c}_{\mathbf{j}} \mathbf{V}_{\mathbf{j}} \right)} \right)^{2} \bar{\mathbf{a}}_{\mathbf{i}}^{2} \mathbf{S}_{\bar{\mathbf{a}}_{\mathbf{i}}}^{2} \right) \tag{14}$$

$$\textbf{Upper Bound on } \mathbf{A'} \textbf{ (at 95\% confidence)} = \mathbf{A'} + \sqrt{\mathbf{t}_{0.05,\nu_{\mathbf{A'}}}^{2} \ \mathbf{var(A')}} \tag{15}$$

The degrees of freedom, $\nu_{\mathbf{A'}}$, is estimated as above by the Welch-Satterthwaite formula given by

$$\nu_{\mathbf{A}} = \frac{\left[\sum_{\mathbf{i}=1}^{\mathbf{p}}\left(\left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_{\mathbf{i}}}\right)_{\bar{\mathbf{a}}_{\mathbf{i}}} \mathbf{S}_{\bar{\mathbf{a}}_{\mathbf{i}}}\right)^{2}\right)\right]^{2}}{\sum_{\mathbf{i}=1}^{\mathbf{p}}\left(\frac{\left(\left(\frac{\partial \mathbf{A}}{\partial \bar{\mathbf{a}}_{\mathbf{i}}}\right)_{\bar{\mathbf{a}}_{\mathbf{i}}} \mathbf{S}_{\bar{\mathbf{a}}_{\mathbf{i}}}\right)^{4}}{(\mathbf{n}_{\mathbf{i}}-1)}\right)} \tag{16}$$

where the degrees of freedom for the each of the relevant variances is $(\mathbf{n}_{\mathbf{i}}-1)$.

Microsoft Excel Spreadsheet

Appendix A provides the equations used in carrying out the calculations presented above for the three different spreadsheets of the DSS. The inputs to the spreadsheets are given in Table 1 (for the first two spreadsheets) and in Table 2 (for the spreadsheet that allows a bounding value for the solids contents of the material phases).

Table 1: Outline of Inputs to the "Grams of Analyte" and "Micro-Curies of Analyte" Spreadsheets

Cell Location	Cell Description	Report Notation
A6, A12, A18, A24	Number of Samples for Phases 1,2,3,4	n_1, n_2, n_3, n_4
B6, B12, B18, B24	Average wt fraction Average wt fraction or µCi/g of A in Phase 1, 2, 3, 4 samples	$\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4$
C6, C12, C18, C24	Number of Analyses for Analyte A per Sample for Phases 1,2,3,4	$n_{m_{\rho_1}}, n_{m_{\rho_2}}, n_{m_{\rho_3}}, n_{m_{\rho_4}}$
D6, D12, D18, D24	% Relative Standard Deviation for Sample to Sample Variation (tank homogeneity for A) for Phases 1, 2, 3, 4	$S^2_{t_{a_1}}, S^2_{t_{a_2}}, S^2_{t_{a_3}}, S^2_{t_{a_4}}$
E6, E12, E18, E24	% Relative Standard Deviation for Analytical Variation for A in samples from Phases 1, 2, 3, 4	$S^2_{m_{a_1}}, S^2_{m_{a_2}}, S^2_{m_{a_3}}, S^2_{m_{a_4}}$
F6, F12, F18, F24	Average wt fraction of solids in Phase 1, 2, 3, 4 samples	$\bar{f}_1, \bar{f}_2, \bar{f}_3, \bar{f}_4$
G6, G12, G18, G24	Number of Analyses for Solids per Sample for Phases 1,2,3,4	$n_{m_{f_1}}, n_{m_{f_2}}, n_{m_{f_3}}, n_{m_{f_4}}$
H6, H12, H18, H24	% Relative Standard Deviation for Sample to Sample Variation (tank homogeneity for solids) for Phases 1, 2, 3, 4	$S^2_{t_{f_1}}, S^2_{t_{f_2}}, S^2_{t_{f_3}}, S^2_{t_{f_4}}$
I6, I12, I18, I24	% Relative Standard Deviation for Analytical Variation for solids in samples from Phases 1, 2, 3, 4	$S^2_{m_{f_1}}, S^2_{m_{f_2}}, S^2_{m_{f_3}}, S^2_{m_{f_4}}$
J6, J12, J18, J24	Average density in Phase 1, 2, 3, 4 samples	$\bar{\rho}_1, \bar{\rho}_2, \bar{\rho}_3, \bar{\rho}_4$
K6, K12, K18, K24	Number of Analyses of density per Sample for Phases 1,2,3,4	$n_{m_{\rho_1}}, n_{m_{\rho_2}}, n_{m_{\rho_3}}, n_{m_{\rho_4}}$
L6, L12, L18, L24	% Relative Standard Deviation for Sample to Sample Variation (tank homogeneity for density) for Phases 1, 2, 3, 4	$S^2_{t_{\rho_1}}, S^2_{t_{\rho_2}}, S^2_{t_{\rho_3}}, S^2_{t_{\rho_4}}$
M6, M12, M18, M24	% Relative Standard Deviation for Analytical Variation for densities in samples from Phases 1, 2, 3, 4	$S^2_{m_{\rho_1}}, S^2_{m_{\rho_2}}, S^2_{m_{\rho_3}}, S^2_{m_{\rho_4}}$
N6, N12, N18, N24	Bounding volumes for Phases 1, 2, 3, 4	V_1, V_2, V_3, V_4

Table 2: Outline of Inputs to the "mCi of Analyte-Bounded Solids" Spreadsheet

Cell Location	Cell Description	Report Notation
B6, B12, B18, B24	Number of Samples for Phases 1,2,3,4	n_1, n_2, n_3, n_4
C6, C12, C18, C24	Average wt fraction or µCi/g of A in Phase 1, 2, 3, 4 samples	$\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4$
D6, D12, D18, D24	Number of Analyses for Analyte A per Sample for Phases 1,2,3,4	$n_{m_{\rho_1}}, n_{m_{\rho_2}}, n_{m_{\rho_3}}, n_{m_{\rho_4}}$
E6, E12, E18, E24	% Relative Standard Deviation for Sample to Sample Variation (tank homogeneity for A wt) for Phases 1, 2, 3, 4	$S^2_{t_{a_1}}, S^2_{t_{a_2}}, S^2_{t_{a_3}}, S^2_{t_{a_4}}$
F6, F12, F18, F24	% Relative Standard Deviation for Analytical Variation for A in samples from Phases 1, 2, 3, 4	$S^2_{m_{a_1}}, S^2_{m_{a_2}}, S^2_{m_{a_3}}, S^2_{m_{a_4}}$
G6, G12, G18, G24	Bounding Solids content for Phase 1, 2, 3, 4	c_1, c_2, c_3, c_4
H6, H12, H18, H24	Bounding volumes for Phases 1, 2, 3, 4	V_1, V_2, V_3, V_4

Test Cases

Appendix B provides three test cases showing the inputs and primary outputs from the Excel spreadsheet. These can be used to confirm that the version of the spreadsheet being used is the same as that described in this report. Some of the information used to develop the test cases came from measurements of previous samples of the Tank 19 heel ([2] & [3]). Specifically, the data for Al, Tc^{99}, and Pu^{238} from 1986, 1996, and 2000 in Table 3 were gleaned from these earlier reports. Summary statistics for these analytes are also shown in the table. For Tc^{99} and Pu^{238}, the statistics are calculated in the original units (μCi/g) and in natural logarithms.

Table 3: Historical Data and Summary Statistics

	1986	1996	2000	Average	Std Dev	% Relative Std Dev
Solids (wt%)	81	75	-	78	4.2426	5.4%
Al wt%	0.229	0.148	0.153	0.1767	0.0454	25.7%
Tc^{99} (μCi/g)	-	0.12	0.0205	0.07025	0.0704	100.2%
$\ln(Tc^{99})$		-2.1203	-3.8873	-3.0038	1.2495	
Pu^{238} (μCi/g)	-	3.1	0.505	1.8025	1.8349	101.8%
$\ln(Pu^{238})$		1.1314	-0.6832	0.2241	1.2831	

The % relative standard deviations of Table 3 suggest estimates for sample to sample variation for use in the spreadsheets. For Tc^{99} and Pu^{238}, the standard deviation of the log values suggest estimates of ~130% for the % relative standard deviation for these analytes.

In addition to the data in Table 3, estimates of a % relative standard deviation for the analytical errors for each of the analytes – Al (2.1%), Tc^{99} (12%), and Pu^{238} (43%) – are available from [2]. These estimates are inflated slightly to Al (5%), Tc^{99} (15%), and Pu^{238} (45%) for use in the spreadsheets.

The test case for Al illustrates the “Grams of Analyte” spreadsheet with all four phases of the tank heel involved. The test case for Tc^{99} uses the “Micro-Curies of Analyte” spreadsheet with all phases in the tank heel active except the supernate.

Finally, the test case for Pu^{238} uses the “mCi of Analyte-Bounding Solids” spreadsheet with only one phase of material utilized. The bounding value for the solids content was taken from the discussion in [4].

Other information needed to complete the spreadsheet calculations is not necessarily representative of the tank heel but is entered for demonstration purposes only. The three spreadsheets (one for Al, one for Tc^{99}, and one for Pu^{238} are provided in Appendix B.

CONCLUDING COMMENTS

This report details the development of Excel spreadsheets that are to serve as a decision support system in the planning of the sampling of the Tank 19 heel. The system allows the user to input estimates for the relevant variations and volumes and values for the numbers of samples to be taken. From this information (and using an uncertainty analysis approach presented in this report), the system propagates this information into an uncertainty for the analyte concentration of interest.

REFERENCES

[1] Coleman, H. W. and W. G. Steele, Jr., **Experimentation and Uncertainty Analysis for Engineers**, John Wiley & Sons, Inc., New York, 1989.

[2] Hay, M. S., “Characterization of Tank 19F Samples in Support of Tank Closure (U),” WSRC-R-97-074, March 6, 1997.

[3] Swingle, R. F., N. E. Bibler, A. A. Ekechukwu, “Data Report: Tank 19F NE Riser Zeolite Mound Sample Analysis,” WSRC-RP-2001-00410, April 17, 2001.

[4] Landon, L. F. and T. T. Thompson, “Technical Data Summary for the Defense Waste Processing Facility, Stage 2,” DPSTD-80-39-2, December 1980.

Appendices:

Appendix A:

This appendix provides the formulas utilized in the calculations with the cell identifiers for the spreadsheets.

Appendix B:

Appendix B presents three test cases that provide the “look and feel” of the spreadsheets and that may be used to validate that the spreadsheet is working properly.

Appendix A.

Cell Identifiers and Formulas for "Grams of Analyte" and "Micro-Curies of Analyte" Spreadsheets

The cell identifiers and corresponding formulas in this section are from the "Grams of Analyte" and "Micro-Curies of Analyte" spreadsheets. They equations for these two spreadsheets are identical, only the units of measure are different (as indicated by the names of the spreadsheets).

Q6:
=(F6*J6*N6)/((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))

R6:
=((((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))*(B6*J6*N6))-(((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))*J6*N6))/(((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))^2)

S6:
=((((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))*(B6*F6*N6))-(((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))*F6*N6))/(((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))^2)

T6:
=IF(A6=0,0,((Q6^2*B6^2*(D6^2+(E6^2/C6))/A6)+(R6^2*F6^2*(H6^2+(I6^2/G6))/A6)+(S6^2*J6^2*(L6^2+(M6^2/K6))/A6)+2*B6*F6*SQRT(((D6^2+(E6^2/C6))/A6)*((H6^2+(I6^2/G6))/A6))*ABS(Q6*R6)+2*B6*J6*SQRT(((D6^2+(E6^2/C6))/A6)*((L6^2+(M6^2/K6))/A6))*ABS(Q6*S6)+2*F6*J6*SQRT(((H6^2+(I6^2/G6))/A6)*((L6^2+(M6^2/K6))/A6))*ABS(R6*S6)))

Q7:
=(F12*J12*N12)/((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))

R7:
=((((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))*(B12*J12*N12))-(((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))*J12*N12))/(((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))^2)

S7:
=((((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))*(B12*F12*N12))-(((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))*F12*N12))/(((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))^2)

Appendix A.

Cell Identifiers and Formulas for “Grams of Analyte” and “Micro-Curies of Analyte” Spreadsheets *(continued)*

T7:

```
=(IF(A12=0,0,((Q7^2*B12^2*(D12^2+(E12^2/C12))/A12)+(R7^2*F12^2*(H12^2+(I12
^2/G12))/A12)+(S7^2*J12^2*(L12^2+(M12^2/K12))/A12)+2*B12*F12*SQRT(((D12^2
+(E12^2/C12))/A12)*((H12^2+(I12^2/G12))/A12))*ABS(Q7*R7)+2*B12*J12*
SQRT(((D12^2+(E12^2/C12))/A12)*((L12^2+(M12^2/K12))/A12))*ABS(Q7*S7)+2*
F12*J12*SQRT(((H12^2+(I12^2/G12))/A12)*((L12^2+(M12^2/K12))/A12))*ABS(R7*
S7))))
```

Q8:

```
=(F18*J18*N18)/((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))
```

R8:

```
=((((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))*(B18*J18*N18))-
(((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))*
J18*N18))/(((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))^2)
```

S8:

```
=((((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))*(B18*F18*N18))-
(((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))*
F18*N18))/(((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))^2)
```

T8:

```
=(IF(A18=0,0,((Q8^2*B18^2*(D18^2+(E18^2/C18))/A18)+(R8^2*F18^2*(H18^2+
(I18^2/G18))/A18)+(S8^2*J18^2*(L18^2+(M18^2/K18))/A18)+2*B18*F18*
SQRT(((D18^2+(E18^2/C18))/A18)*((H18^2+(I18^2/G18))/A18))*ABS(Q8*R8)+
2*B18*J18*SQRT(((D18^2+(E18^2/C18))/A18)*((L18^2+(M18^2/K18))/A18))*
ABS(Q8*S8)+2*F18*J18*SQRT(((H18^2+(I18^2/G18))/A18)*((L18^2+(M18^2/K18))/
A18))*ABS(R8*S8))))
```

Q9:

```
=(F24*J24*N24)/((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))
```

R9:

```
=((((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))*(B24*J24*N24))-
(((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))*
J24*N24))/(((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))^2)
```

Appendix A.

<u>Cell Identifiers and Formulas for</u>
<u>"Grams of Analyte" and "Micro-Curies of Analyte" Spreadsheets *(continued)*</u>

S9:

```
=((((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))*(B24*F24*N24))-(((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))*F24*N24))/(((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))^2)
```

T9:

```
=(IF(A24=0,0,((Q9^2*B24^2*(D24^2+(E24^2/C24))/A24)+(R9^2*F24^2*(H24^2+(I24^2/G24))/A24)+(S9^2*J24^2*(L24^2+(M24^2/K24))/A24)+2*B24*F24*SQRT(((D24^2+(E24^2/C24))/A24)*((H24^2+(I24^2/G24))/A24))*ABS(Q9*R9)+2*B24*J24*SQRT(((D24^2+(E24^2/C24))/A24)*((L24^2+(M24^2/K24))/A24))*ABS(Q9*S9)+2*F24*J24*SQRT(((H24^2+(I24^2/G24))/A24)*((L24^2+(M24^2/K24))/A24))*ABS(R9*S9))))
```

S11

```
=((IF(A6=0,0,(Q6^2*B6^2*(D6^2+(E6^2/C6))/A6)+(R6^2*F6^2*(H6^2+(I6^2/G6))/A6)+(S6^2*J6^2*(L6^2+(M6^2/K6))/A6)))+(IF(A12=0,0,(Q7^2*B12^2*(D12^2+(E12^2/C12))/A12)+(R7^2*F12^2*(H12^2+(I12^2/G12))/A12)+(S7^2*J12^2*(L12^2+(M12^2/K12))/A12)))+(IF(A18=0,0,(Q8^2*B18^2*(D18^2+(E18^2/C18))/A18)+(R8^2*F18^2*(H18^2+(I18^2/G18))/A18)+(S8^2*J18^2*(L18^2+(M18^2/K18))/A18)))+(IF(A24=0,0,(Q9^2*B24^2*(D24^2+(E24^2/C24))/A24)+(R9^2*F24^2*(H24^2+(I24^2/G24))/A24)+(S9^2*J24^2*(L24^2+(M24^2/K24))/A24))))^2
```

S12:

```
=((IF(A6=0,0,(((Q6^2*B6^2*(D6^2+(E6^2/C6))/A6)^2+(R6^2*F6^2*(H6^2+(I6^2/G6))/A6)^2+(S6^2*J6^2*(L6^2+(M6^2/K6))/A6)^2)/(A6-1))))+(IF(A12=0,0,(((Q7^2*B12^2*(D12^2+(E12^2/C12))/A12)^2+(R7^2*F12^2*(H12^2+(I12^2/G12))/A12)^2+(S7^2*J12^2*(L12^2+(M12^2/K12))/A12)^2)/(A12-1))))+(IF(A18=0,0,(((Q8^2*B18^2*(D18^2+(E18^2/C18))/A18)^2+(R8^2*F18^2*(H18^2+(I18^2/G18))/A18)^2+(S8^2*J18^2*(L18^2+(M18^2/K18))/A18)^2)/(A18-1))))+(IF(A24=0,0,(((Q9^2*B24^2*(D24^2+(E24^2/C24))/A24)^2+(R9^2*F24^2*(H24^2+(I24^2/G24))/A24)^2+(S9^2*J24^2*(L24^2+(M24^2/K24))/A24)^2)/(A24-1)))))
```

R12:

```
=FLOOR(S11/S12,1)
```

Appendix A.

Cell Identifiers and Formulas for "Grams of Analyte" and "Micro-Curies of Analyte" Spreadsheets *(continued)*

M27:
=((B6*F6*J6*N6)+(B12*F12*J12*N12)+(B18*F18*J18*N18)+(B24*F24*J24*N24))/((F6*J6*N6)+(F12*J12*N12)+(F18*J18*N18)+(F24*J24*N24))

M28:
=SQRT(T6+T7+T8+T9)/M27

M29:
=TINV(0.05,R12)*M28

M30:
=M27+M29*M27

Appendix A.

Cell Identifiers and Formulas for "mCi of Analyte – Bounded Solids" Spreadsheet

The cell identifiers and corresponding formulas in this section are from the "mCi of Analyte – Bounded Solids" spreadsheet.

P6:
=(G6*H6)/((G6*H6)+(G12*H12)+(G18*H18)+(G24*H24))

Q6:
=IF(B6=0,0,((P6^2*C6^2*(E6^2+(F6^2/D6))/B6)))

P7:
=(G12*H12)/((G6*H6)+(G12*H12)+(G18*H18)+(G24*H24))

Q7:
=IF(B12=0,0,((P7^2*C12^2*(E12^2+(F12^2/D12))/B12)))

P8:
=(G18*H18)/((G6*H6)+(G12*H12)+(G18*H18)+(G24*H24))

Q8:
=IF(B18=0,0,((P8^2*C18^2*(E18^2+(F18^2/D18))/B18)))

P9:
=(G24*H24)/((G6*H6)+(G12*H12)+(G18*H18)+(G24*H24))

Q9:
=IF(B24=0,0,((P9^2*C24^2*(E24^2+(F24^2/D24))/B24)))

Q11:
=((IF(B6=0,0,(P6^2*C6^2*(E6^2+(F6^2/D6))/B6)))+(IF(B12=0,0,(P7^2*C12^2*(E12^2+(F12^2/D12))/B12)))+(IF(B18=0,0,(P8^2*C18^2*(E18^2+(F18^2/D18))/B18)))+(IF(B24=0,0,(P9^2*C24^2*(E24^2+(F24^2/D24))/B24))))^2

Q12:
=((IF(B6=0,0,(((P6^2*C6^2*(E6^2+(F6^2/D6))/B6)^2)/(B6-1))))+(IF(B12=0,0,(((P7^2*C12^2*(E12^2+(F12^2/D12))/B12)^2)/(B12-1))))+(IF(B18=0,0,(((P8^2*C18^2*(E18^2+(F18^2/D18))/B18)^2)/(B18-1))))+(IF(B24=0,0,(((P9^2*C24^2*(E24^2+(F24^2/D24))/B24)^2)/(B24-1)))))

Appendix A.

Cell Identifiers and Formulas for "mCi of Analyte – Bounded Solids" Spreadsheet *(continued)*

P12:
=FLOOR(Q11/Q12,1)

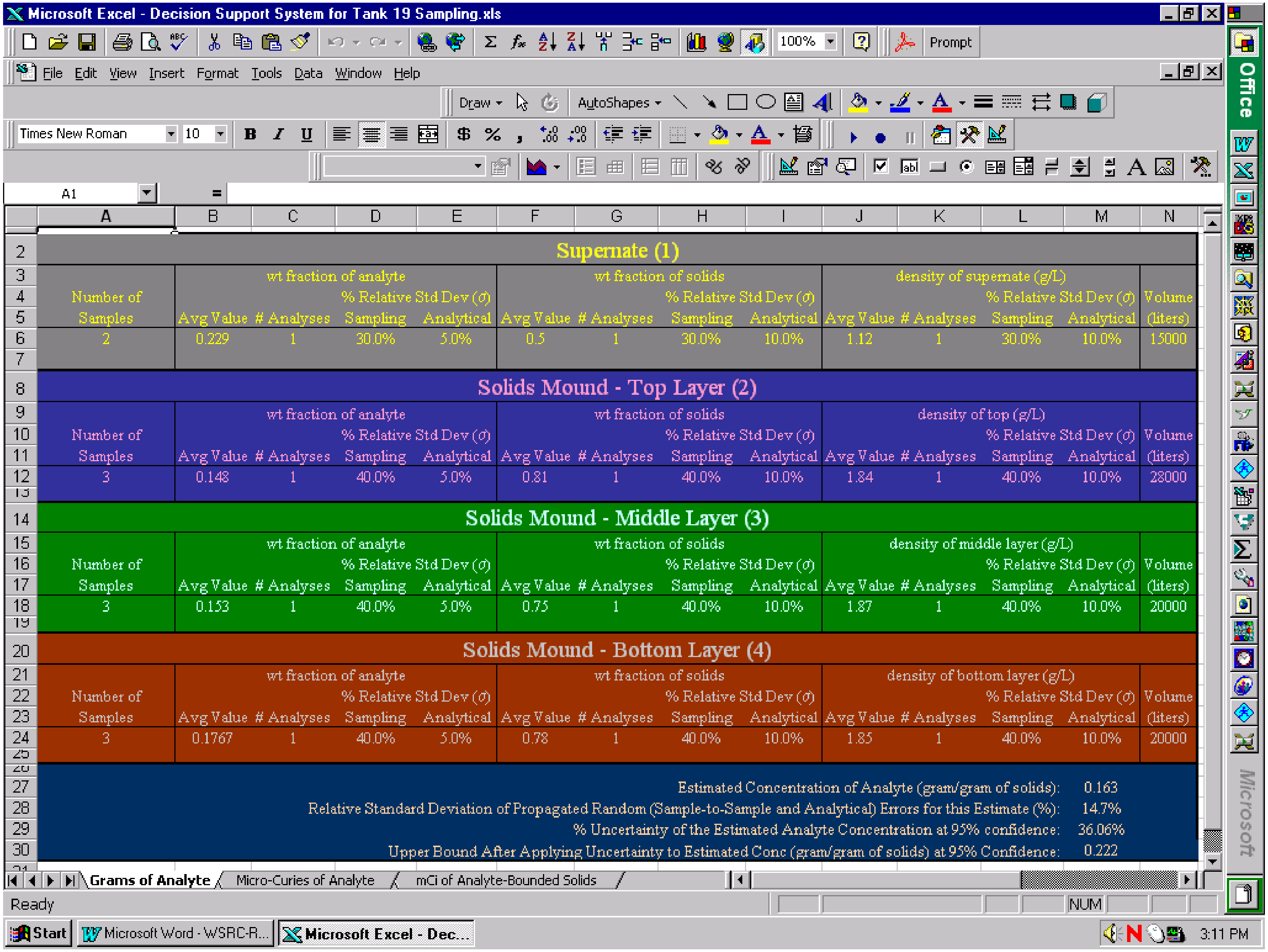
H27:
=((C6*G6*H6)+(C12*G12*H12)+(C18*G18*H18)+(C24*G24*H24))/((G6*H6)+(G12*H12)+(G18*H18)+(G24*H24))

H28:
=SQRT(Q6+Q7+Q8+Q9)/H27

H29:
=TINV(0.05,P12)*H28

H30:
=H27+H29*H27

Appendix B.



Supernate (1)

	wt fraction of analyte				wt fraction of solids				density of supernate (g/L)				
Number of Samples			% Relative Std Dev (σ)				% Relative Std Dev (σ)				% Relative Std Dev (σ)		Volume (liters)
	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	
2	0.229	1	30.0%	5.0%	0.5	1	30.0%	10.0%	1.12	1	30.0%	10.0%	15000

Solids Mound - Top Layer (2)

	wt fraction of analyte				wt fraction of solids				density of top (g/L)				
Number of Samples			% Relative Std Dev (σ)				% Relative Std Dev (σ)				% Relative Std Dev (σ)		Volume (liters)
	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	
3	0.148	1	40.0%	5.0%	0.81	1	40.0%	10.0%	1.84	1	40.0%	10.0%	28000

Solids Mound - Middle Layer (3)

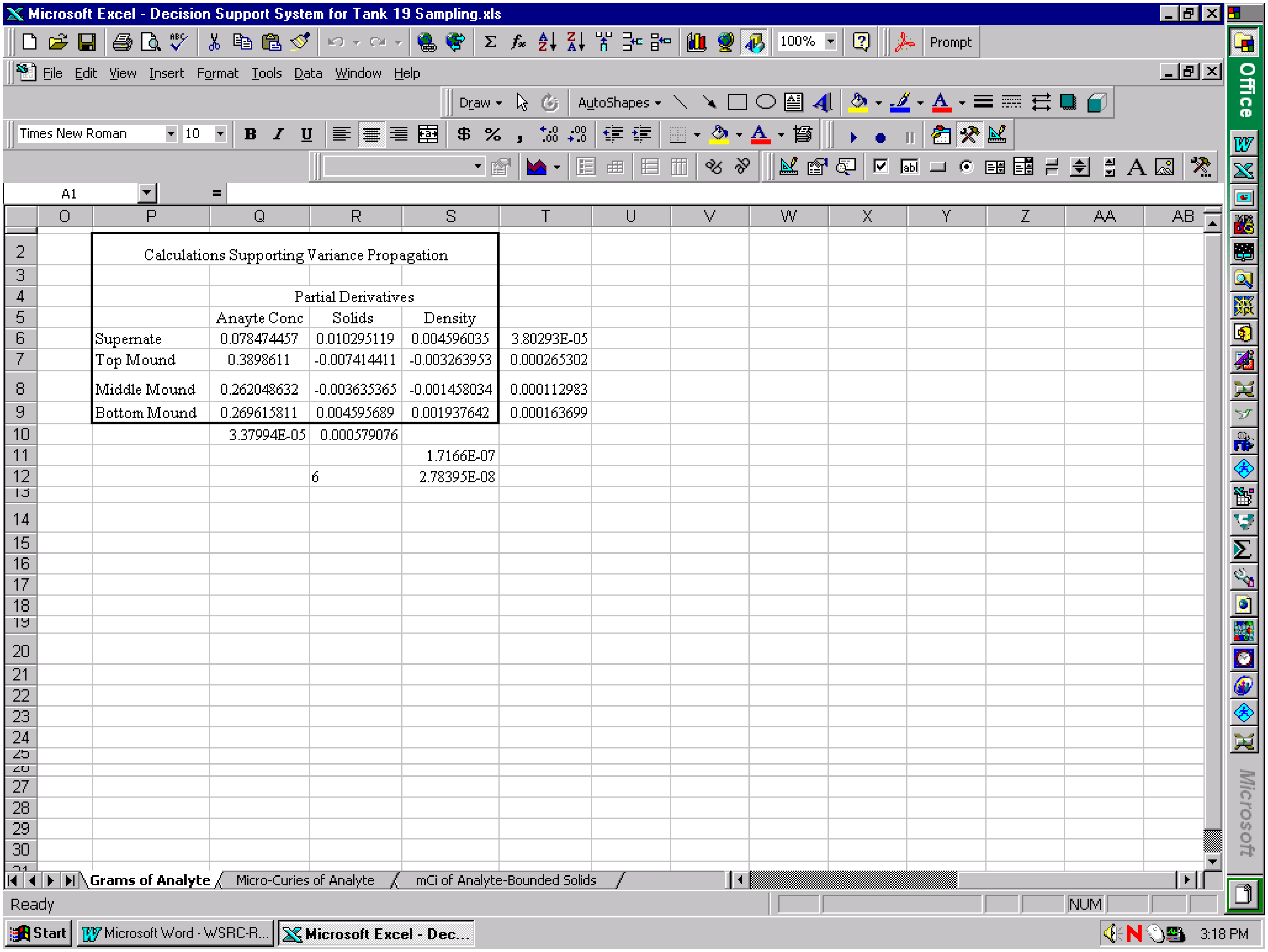
	wt fraction of analyte				wt fraction of solids				density of middle layer (g/L)				
Number of Samples			% Relative Std Dev (σ)				% Relative Std Dev (σ)				% Relative Std Dev (σ)		Volume (liters)
	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	
3	0.153	1	40.0%	5.0%	0.75	1	40.0%	10.0%	1.87	1	40.0%	10.0%	20000

Solids Mound - Bottom Layer (4)

	wt fraction of analyte				wt fraction of solids				density of bottom layer (g/L)				
Number of Samples			% Relative Std Dev (σ)				% Relative Std Dev (σ)				% Relative Std Dev (σ)		Volume (liters)
	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	
3	0.1767	1	40.0%	5.0%	0.78	1	40.0%	10.0%	1.85	1	40.0%	10.0%	20000

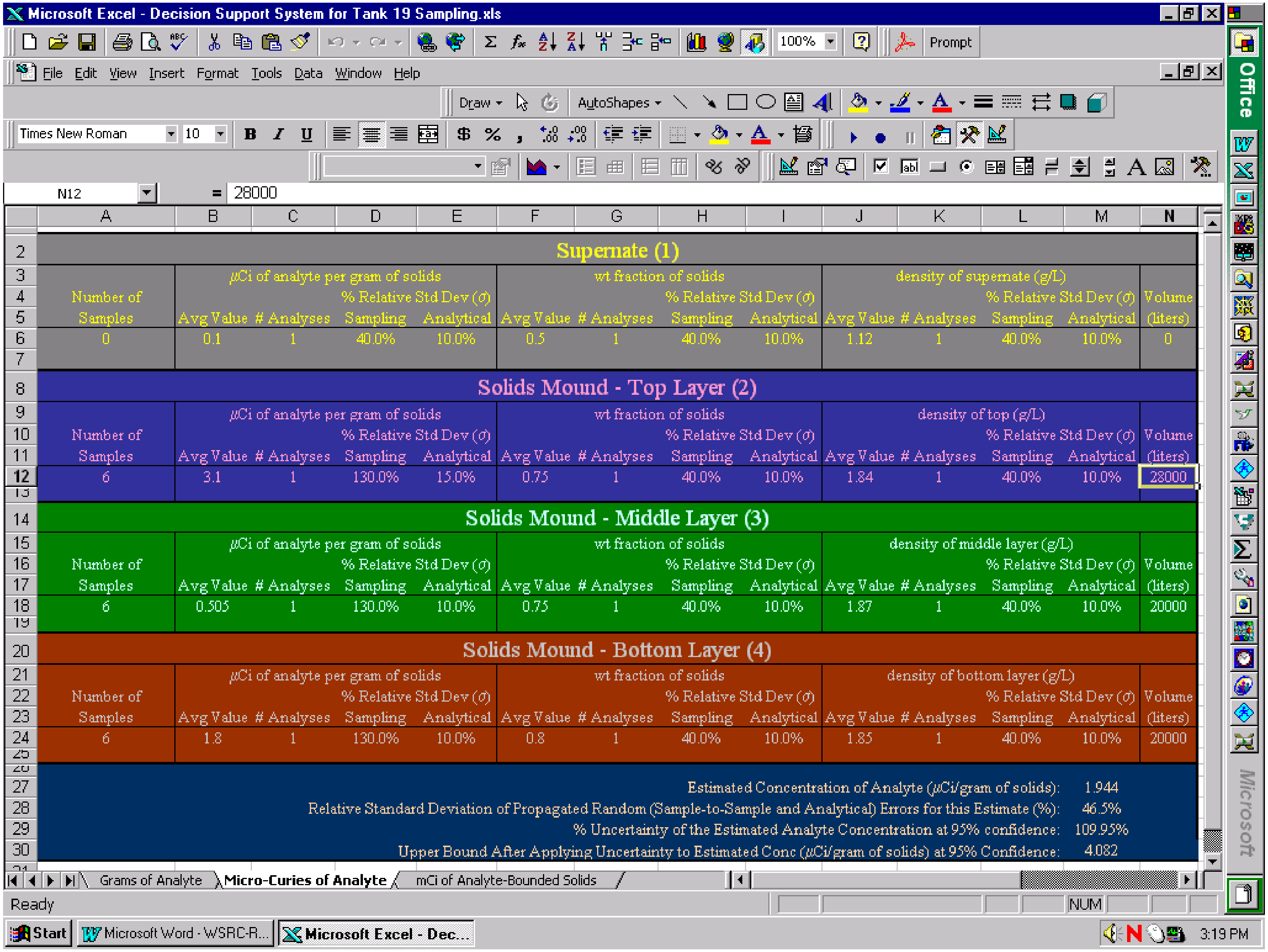
Estimated Concentration of Analyte (gram/gram of solids):	0.163
Relative Standard Deviation of Propagated Random (Sample-to-Sample and Analytical) Errors for this Estimate (%):	14.7%
% Uncertainty of the Estimated Analyte Concentration at 95% confidence:	36.06%
Upper Bound After Applying Uncertainty to Estimated Conc (gram/gram of solids) at 95% Confidence:	0.222

Appendix B.



	P	Q	R	S	T
2	Calculations Supporting Variance Propagation				
3					
4		Partial Derivatives			
5		Anayte Conc	Solids	Density	
6	Supernate	0.078474457	0.010295119	0.004596035	3.80293E-05
7	Top Mound	0.3898611	-0.007414411	-0.003263953	0.000265302
8	Middle Mound	0.262048632	-0.003635365	-0.001458034	0.000112983
9	Bottom Mound	0.269615811	0.004595689	0.001937642	0.000163699
10		3.37994E-05	0.000579076		
11				1.7166E-07	
12			6	2.78395E-08	

Appendix B.



N12 = 28000

Supernate (1)

Number of Samples	µCi of analyte per gram of solids				wt fraction of solids				density of supernate (g/L)				Volume (liters)
			% Relative Std Dev (σ)				% Relative Std Dev (σ)				% Relative Std Dev (σ)		
	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	
0	0.1	1	40.0%	10.0%	0.5	1	40.0%	10.0%	1.12	1	40.0%	10.0%	0

Solids Mound - Top Layer (2)

Number of Samples	µCi of analyte per gram of solids				wt fraction of solids				density of top (g/L)				Volume (liters)
			% Relative Std Dev (σ)				% Relative Std Dev (σ)				% Relative Std Dev (σ)		
	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	
6	3.1	1	130.0%	15.0%	0.75	1	40.0%	10.0%	1.84	1	40.0%	10.0%	28000

Solids Mound - Middle Layer (3)

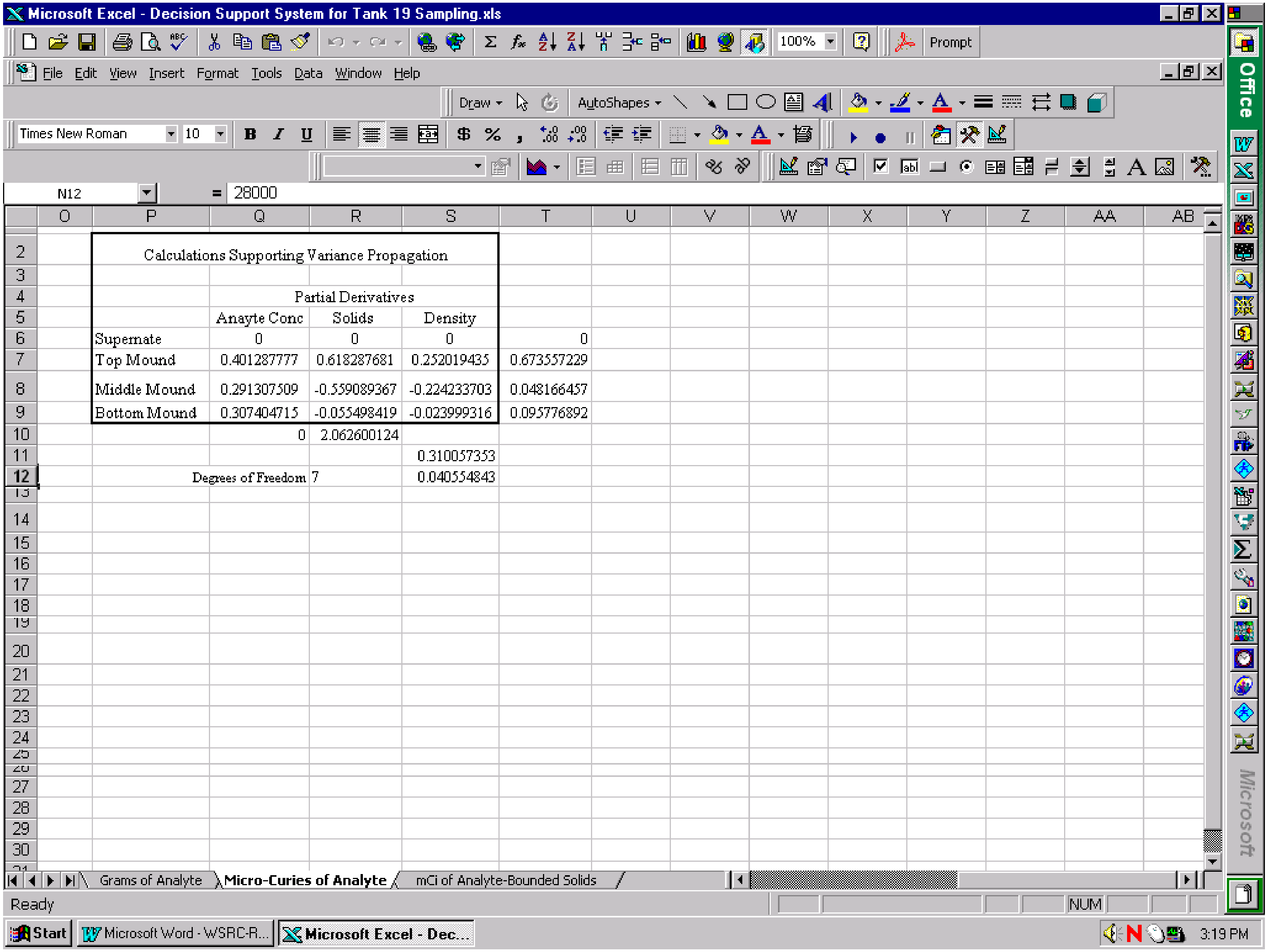
Number of Samples	µCi of analyte per gram of solids				wt fraction of solids				density of middle layer (g/L)				Volume (liters)
			% Relative Std Dev (σ)				% Relative Std Dev (σ)				% Relative Std Dev (σ)		
	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	
6	0.505	1	130.0%	10.0%	0.75	1	40.0%	10.0%	1.87	1	40.0%	10.0%	20000

Solids Mound - Bottom Layer (4)

Number of Samples	µCi of analyte per gram of solids				wt fraction of solids				density of bottom layer (g/L)				Volume (liters)
			% Relative Std Dev (σ)				% Relative Std Dev (σ)				% Relative Std Dev (σ)		
	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	Avg Value	# Analyses	Sampling	Analytical	
6	1.8	1	130.0%	10.0%	0.8	1	40.0%	10.0%	1.85	1	40.0%	10.0%	20000

Estimated Concentration of Analyte (µCi/gram of solids):	1.944
Relative Standard Deviation of Propagated Random (Sample-to-Sample and Analytical) Errors for this Estimate (%):	46.5%
% Uncertainty of the Estimated Analyte Concentration at 95% confidence:	109.95%
Upper Bound After Applying Uncertainty to Estimated Conc (µCi/gram of solids) at 95% Confidence:	4.082

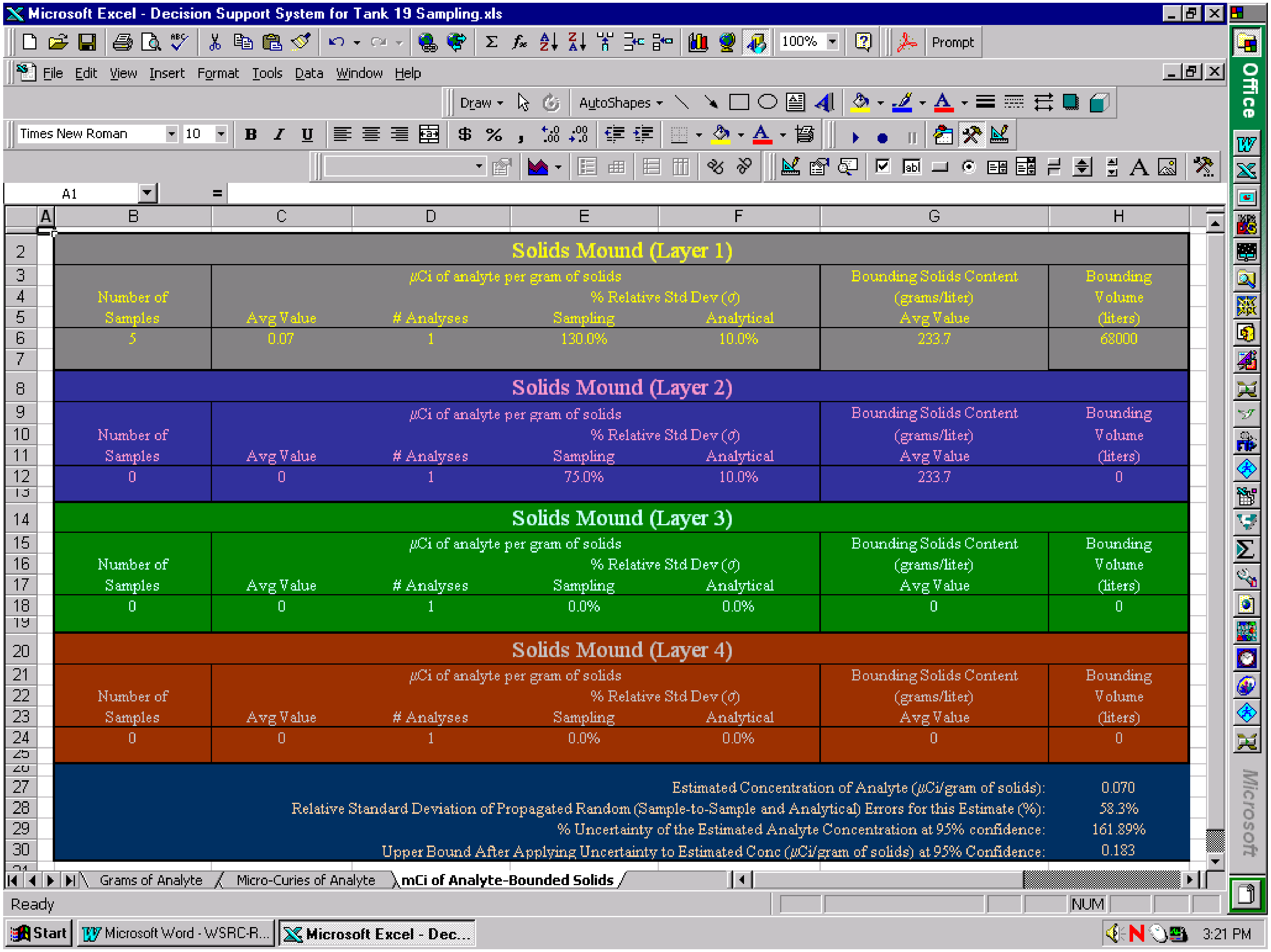
Appendix B.



N12 = 28000

	P	Q	R	S	T
2	Calculations Supporting Variance Propagation				
3					
4		Partial Derivatives			
5		Anayte Conc	Solids	Density	
6	Supernate	0	0	0	0
7	Top Mound	0.401287777	0.618287681	0.252019435	0.673557229
8	Middle Mound	0.291307509	-0.559089367	-0.224233703	0.048166457
9	Bottom Mound	0.307404715	-0.055498419	-0.023999316	0.095776892
10		0	2.062600124		
11				0.310057353	
12	Degrees of Freedom	7		0.040554843	

Appendix B.



Solids Mound (Layer 1)

Number of Samples	µCi of analyte per gram of solids: Avg Value	# Analyses	% Relative Std Dev (σ): Sampling	% Relative Std Dev (σ): Analytical	Bounding Solids Content (grams/liter) Avg Value	Bounding Volume (liters)
5	0.07	1	130.0%	10.0%	233.7	68000

Solids Mound (Layer 2)

Number of Samples	µCi of analyte per gram of solids: Avg Value	# Analyses	% Relative Std Dev (σ): Sampling	% Relative Std Dev (σ): Analytical	Bounding Solids Content (grams/liter) Avg Value	Bounding Volume (liters)
0	0	1	75.0%	10.0%	233.7	0

Solids Mound (Layer 3)

Number of Samples	µCi of analyte per gram of solids: Avg Value	# Analyses	% Relative Std Dev (σ): Sampling	% Relative Std Dev (σ): Analytical	Bounding Solids Content (grams/liter) Avg Value	Bounding Volume (liters)
0	0	1	0.0%	0.0%	0	0

Solids Mound (Layer 4)

Number of Samples	µCi of analyte per gram of solids: Avg Value	# Analyses	% Relative Std Dev (σ): Sampling	% Relative Std Dev (σ): Analytical	Bounding Solids Content (grams/liter) Avg Value	Bounding Volume (liters)
0	0	1	0.0%	0.0%	0	0

Estimated Concentration of Analyte (µCi/gram of solids):	0.070
Relative Standard Deviation of Propagated Random (Sample-to-Sample and Analytical) Errors for this Estimate (%):	58.3%
% Uncertainty of the Estimated Analyte Concentration at 95% confidence:	161.89%
Upper Bound After Applying Uncertainty to Estimated Conc (µCi/gram of solids) at 95% Confidence:	0.183