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MODEL FOR RESIDUAL SATURATIONS AND CAPILLARY IMBIBITION AND DRAINAGE PRESSURES IN GRANULAR MATERIALS

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Abstract

A pore saturation model expresses the capillary pressure as a function of a characteristic pore pressure and the wetting phase saturation. Singularity analyses of the total energies of the wetting and nonwetting phases give the residual saturations for the two phases. The total energy consists of a potential term and a work term associated with the effective pressure gradient for each phase. The derived residual wetting saturation is 0.236, and the derived residual nonwetting saturation is 0.884.

The model includes separate pressures for imbibition and drainage to account for capillary hysteresis. In the model, the pressure gradient for the wetting phase defines the imbibition pressure, and the nonwetting phase pressure gradient defines the drainage pressure. At the residual nonwetting saturation, the two pressures differ by the characteristic pore pressure. The two pressures coincide at a critical minimum saturation of 0.301. The model also includes an entry head to account for the minimum force required for drainage to begin. The model utilizes a single fitting parameter, a characteristic pore pressure, which can be related to a characteristic pore diameter.

The model successfully correlates a selected set of laboratory imbibition and drainage data for a uniform sand. The predicted residual wetting saturation and critical minimum saturation agree with measurements. The characteristic pore pressure used to fit the capillary pressures corresponds to a pore diameter approximately equal to the mean particle diameter. The use of a characteristic pore pressure in the capillary pressure model restricts its applicability to granular materials with open, approximately uniform pore structures.

Nomenclature

A	interfacial area in the Gibbs' adsorption theorem calculation
d_p	particle diameter for porous material
F	Helmholz free energy for the Gibbs' adsorption theorem
G_s	potential energy associated with saturation of the porous medium
H	molar enthalpy
k	Darcy coefficient for permeability
k_n	relative permeability to the nonwetting phase
k_w	relative permeability to the wetting phase
n_i	number of moles of the ith solute in the Gibbs' adsorption theorem calculation
P_a	average pressure in the pores of the porous medium
P_c	intrinsic capillary pore pressure, equal to the gas-liquid pressure difference
$P_{c,d}$	capillary pressure measured for drainage of the wetting phase
$P_{c,i}$	capillary pressure measured for imbibition of the wetting phase
$P_{c,i,0}$	minimum capillary pressure required to imbibe the wetting phase at the residual nonwetting phase saturation
P_{flow}	characteristic pressure for flow in capillary pores
P_g	gas phase pressure in capillary pores
P_l	liquid phase pressure in capillary pores
P_{pot}	potential pressure associated with partial saturation of capillary pores
$P_{v,i}$	vapor pressure of the ith solute in the Gibbs' adsorption theorem calculation
P_o	ambient pressure for liquid in contact with the porous medium
R_g	universal gas law constant
s	relative saturation with the wetting phase
s_{cr}	critical saturation where the capillary pressure for imbibition equals the capillary pressure for drainage
s_{max}	maximum saturation for a fissured porous material
s_{min}	minimum saturation for a fissured porous material
s_{rn}	residual nonwetting phase saturation
s_{rw}	residual wetting phase saturation
S_s	entropy associated with partial saturation

T	absolute temperature
v_n	velocity of the nonwetting phase
v_w	velocity of the wetting phase
V	molar volume
W	work required for flow through the porous medium
W_g	work required for flow of gas through the porous medium
W_l	work required for flow of liquid through the porous medium
z	distance in the direction of flow
ε	porosity
μ_n	dynamic viscosity of the gas or nonwetting fluid
μ_w	dynamic viscosity of the liquid or wetting fluid
σ	air-water interfacial tension

1. Introduction

Capillary pressure represents the pressure required to displace a given volume of one of the phases from a two-phase mixture in a porous medium. The capillary pressure is typically measured either by imbibition or by drainage. Imbibition pressure measurements begin with the porous medium, typically a soil, saturated with the nonwetting phase, which would be the gas in a gas-liquid mixture. (For convenience, this paper will refer to the terms porous medium and soil, wetting phase and liquid, and nonwetting phase and gas interchangeably.) The soil is connected to a reservoir containing the liquid, and pressure differences are recorded as the medium absorbs measured volumes of this phase. Drainage pressure measurements start with the soil completely saturated with the wetting phase. For gas-liquid mixtures, this is often accomplished by using a soluble gas, such as carbon dioxide, that can be purged completely from the soil. One end of a section of the soil is attached to a pressurized reservoir of gas, and the pressure required to displace a given fraction of the liquid is measured.

Capillary measurements commonly are correlated using the Leverett (1941) J function. The J function is used to scale capillary pressures with the Darcy permeability of the soil, the interfacial tension between the wetting and nonwetting phases, and the contact angle between the wetting phase and the soil. The J function exists because the capillary pressure is a characteristic function of the relative saturation of the two phases in the soil.

Capillary pressure measurements for soils with uniform pore size distributions spike at an irreducible fractional wetting phase saturation, s_{rw}. At this residual wetting saturation, which is the same for imbibition and drainage, no additional amount of liquid can be displaced, no matter how much pressure is applied. Conversely, at least in theory, there is no pressure that can be applied to prevent the soil from beginning to absorb liquid. As the liquid saturation increases, the imbibition pressure decreases. The pressure drops to zero, and absorption of liquid stops, at a maximum fractional wetting phase saturation, s_{rn}, called the residual nonwetting saturation. Significant drainage does not occur until a threshold pressure referred to as the displacement pressure or the entry head is exceeded. It has been conjectured that the displacement pressure represents the pressure required to dislodge the gas from the capillary pores so that it will flow. The drainage pressure then increases as the liquid saturation decreases. At any given saturation, the drainage capillary pressure exceeds the imbibition capillary pressure. The hysteresis between the two pressures is greatest at the residual nonwetting saturation.

The relationship between the capillary pressure and the saturation changes for material with large pores, such as glass beads, loose sand, or gravel, and for consolidated materials with monolithic structures, such as many rocks. For materials with large pores, the most notable difference is that the residual wetting saturation decreases and may disappear entirely, so that the material drains completely. For highly consolidated materials, some pores may not be accessible to flow, so the overall residual saturations may be higher than the residual saturations in the porous fraction of the material. This study focuses on fine granular materials with an open pore structure, which exhibit a limiting value for the residual wetting and nonwetting saturations.

Historically, the capillary pressure has been correlated as a function of the liquid saturation, normalized with respect to the difference between the residual nonwetting and wetting saturations. Most capillary pressure models utilize two parameters to relate the pressure to the relative saturation. The relationship typically takes the form of a power law relation (Brooks and Corey, 1964; van Genuchten, 1980), an exponential equation (Kosugi, 1994, 1996), or a combination of these two forms applied over different liquid saturation ranges (Rossi and Nimmo, 1994). Brooks and Corey (1964) assigned the entry head, defined as the minimum capillary pressure during drainage, as one parameter and used the other parameter to characterize the pore size distribution. Several other models have followed this convention.

Parker and Lenhard (1987) addressed the hysteresis between drainage and imbibition. They developed a model that includes effects of gas entrapment during imbibition. Their model defines limiting maximum capillary heads for drainage and minimum capillary heads for imbibition. They state that the actual pressure must lie at some value between these limits that depends on the flow history. A simple method of applying this model is to define separate limiting capillary pressures for imbibition and drainage (White and Oostrom, 1996).

Another complicating factor in modeling the capillary pressure is the existence of fractures through which flow preferentially occurs. To model fractured media, Klavetter and Peters (1986) and Nitao (1988) used dual porosity functions that assign different capillary pressure-saturation relations to the fractures and to the low permeability solid matrix. Their models equated the fracture and solid matrix pressures.

Most recent attempts to model capillary pressure stem from a network resistance analog developed by Fatt (1956). Network models use Monte Carlo methods to calculate pressures and flows in a network of capillary pores with a specified coordination number, loosely defined as the number of neighboring pores connected to each pore. Dodd and Kiel (1959) added the concept of dead end pores to the network model. This feature allowed them to calculate the residual wetting saturation as a function of the coordination number and fluid properties. Other notable modifications to the network modeling approach include the use of volume-average equations of motion (Lin and Slattery, 1982) and the use of percolation theory to describe the pore structure (Larson, Scriven, and Davis, (Sahimi et al., 1986)).

This paper presents a departure from both traditional analytic and network models. A probabilistic pore pressure model and global energy balances replace the semi-empirical approach of analytic models and the detailed pore structure and flow equations and the Monte Carlo approach used by the network models. The energy balances are solved for singularities in the driving forces for flow of the wetting and nonwetting phases, to obtain the residual wetting and nonwetting saturations, respectively. The pore pressure models are then used to calculate the capillary pressure as a function of saturation. Results are compared to selected measurements of residual saturations and capillary pressures.

2. Derivation of Energy Balance for Capillary Pressure Model

The singularity analyses for the residual saturations are based on comparisons of the potential energy associated with the difference between the pressures of the two phases in the soil and the work required for flow of each phase. The analysis is loosely analogous to the Gibbs' adsorption theorem, which relates work done by surface forces to the chemical potential of a system (Koenig and Swain, 1933; Guggenheim, 1936; Morrow, 1970). In its simplest form, Gibbs' adsorption theorem states that the interfacial tension σ of a system equals the change in the Helmholz free energy F per unit interfacial area A at constant temperature and volume:

$$\sigma = \left(\frac{\Delta F}{\Delta A}\right)_{T,V} \tag{1}$$

Gibbs' adsorption theorem can be recast as an energy balance, with the product $\sigma \Delta A$ representing the work required to extend a surface and the term ΔF representing the accompanying change in the chemical potential energy of the system. The potential energy is defined in terms of chemical activities, which for an ideal solution at equilibrium, are the vapor pressures. Thus, for Gibbs' adsorption theorem,

$$\Delta F = -\sum_i n_i R_g T \Delta\, ln\left(P_{v,i}\right) \tag{2}$$

where R_g is the gas law constant, n_i is the number of moles of solute i adsorbed on the liquid interface, and $P_{v,i}$ is the vapor pressure of solute i.

The capillary pressure model presented in this paper extends this representation from a surface to a volume of soil. The model replaces the chemical potential with a mechanical potential based on the stored interfacial energy and substitutes a flow work term for the work needed to extend an interface. Figure 1 presents a simple schematic representation of the model. As this figure shows, the model is applied to an adiabatic control volume within a larger volume of soil, from which the only energy transfer occurs by flow of gas and liquid in or out. The model evaluates changes of the potential energy within the control volume and changes in the amount of flow work performed by the control volume on its surroundings.

The energy balance for the control volume takes the form

$$\Delta H = \Delta G_s + \Delta W \tag{3}$$

where H is the enthalpy, G_s is the mechanical potential associated with the interfacial energy, and W is the flow, or pressure-volume, work term. The energy balance is formulated in terms of the enthalpy because it is an open system in which liquid and gas may cross the volume boundaries.

Since the model assumes that the soil control volume is adiabatic, the enthalpy term is zero, and

$$\Delta G_s = -\Delta W \tag{4}$$

The mechanical potential and flow work terms can be expressed in terms of pressures much as the chemical potential in the Gibbs' adsorption equation. For a control volume initially comprised of one mole of gas, these expressions are

$$\Delta G_s = -R_g T \Delta\, ln\left(P_{pot}\right) \tag{5}$$

and

$$\Delta W = -R_g T \Delta\left(-\,ln\left(P_{flow}\right)\right) \tag{6}$$

These expressions differ from the Gibbs' adsorption energy in that the energies are applied to a fixed volume rather than a fixed mass. The volume is defined as fixed because the capillary pressure model applies to interfacial and solid surface forces rather than to forces tied to the mass of gas or liquid present within the control volume, and will be used to derive capillary pressure and residual saturation expressions in terms of saturation volume fractions rather than masses. Nevertheless, it is convenient to define the size of the control volume such that the potential energy and flow work terms apply to molar quantities, as is done in Equations 5 and 6.

The potential energy also can be interpreted as an entropy akin to the entropy of mixing. According to this interpretation, the entropy associated with partial saturation is

$$\Delta S_s = -R_g \Delta\, ln\left(P_{pot}\right) \tag{7}$$

The model employs different flow pressures for imbibition and drainage. The flow pressure for imbibition is based on liquid phase flow, while the flow pressure for drainage is based on gas phase flow.

3. Derivation of Pore Pressure Model

To derive the pressures for the potential energy and the flow work, a model for the pore pressure as a function of saturation must be developed. This model is based on a simple statistical interpretation of the distribution of wetting and nonwetting phases in the pores. The model assumes that each pore is filled with either a liquid wetting or a gaseous nonwetting phase.

The capillary pore pressure is distributed by gas-liquid interfaces between the pores. The distribution of gas-and liquid-filled pores and the connections between pores are assumed to be random. The difference between the average pore pressure and the liquid pressure, according to this model, is equal to the product of the capillary pore pressure and the probability that a liquid-filled pore is in contact with a gas-filled pore and not with liquid in another pore. This probability, in turn, is the gas saturation. Thus, the difference between the average pore pressure and the liquid pressure is given by

$$P_a - P_l = P_c(1-s) \tag{8}$$

Because the liquid wets the soil, the liquid pressure is lower than the average pressure and this pressure difference is positive. Figure 2 depicts the forces acting on the liquid in the pores in two dimensions.

Likewise, the difference between the gas pressure and the average pore pressure is equal to the product of the capillary pressure and the liquid saturation, or,

$$P_g - P_a = P_c s \tag{9}$$

This pressure difference is also positive.

It may be noted that the gas and liquid pressure differences cancel each other, as they must, so that the volume average of the two pressures equals the average pressure:

$$P_a = P_l s + P_g(1-s) \tag{10}$$

The difference between the gas and liquid pressures is just the capillary pore pressure, as one should expect:

$$P_g - P_l = P_c \tag{11}$$

Confirmation of this model is provided by a derivation of residual gas and liquid saturations. This derivation follows.

The potential energy is calculated from the difference between the average pressure in the soil, P_a, and the reference pressure for flow through the soil, P_o. For an influx of liquid into the soil (imbibition), the reference pressure is the liquid phase pressure, and

$$P_a - P_o = P_c(1-s) \tag{12}$$

For flow of gas (drainage), the reference pressure equals the gas phase pressure, so that

$$P_a - P_o = -P_c s \tag{13}$$

These pressure differences apply only to the fractions of the pore volume occupied by liquid and gas, respectively. Consequently, for either liquid or gas flow, the potential energy is based on a pressure differential given by

$$P_{pot} = P_c s(1-s) \tag{14}$$

The pressure differential is defined to be positive for both imbibition and drainage. Substitution of this pore pressure in the equation for the potential energy yields

$$\Delta G_s = -R_g T \Delta\, ln(P_c s(1-s)) \tag{15}$$

The delta sign indicates that this equation gives the change in the potential energy for small changes in the saturation. Because this is a differential analysis, the value of G_s at the reference state can be arbitrary.

4. Analysis of Residual Wetting Saturation

At the residual wetting phase saturation, s_{rw}, the resistance to flow of the liquid becomes infinite, and, in an overall energy balance, the resistance to flow of the gas can be ignored. It may be stated, then, that for a given change in the saturation the change in the potential energy counteracts the change in the work needed to cause the liquid to flow. In other words, at $s = s_{rw}$,

$$\frac{dG_s}{ds} = -\frac{dW_l}{ds} \tag{16}$$

where W_l is the work term for liquid flow.

The liquid work function is derived from the Darcy equation for flow of two immiscible phases. The Darcy equation for the wetting phase is:

$$\varepsilon s v_w = -\frac{kk_w}{\mu_w}\frac{dP_l}{dz} = -\frac{kk_w}{\mu_w} P_c \frac{ds}{dz} \tag{17}$$

or

$$v_w = -\frac{kk_w}{\mu_w \varepsilon} P_c \frac{d\, ln(s)}{dz} \tag{18}$$

From this expression the pressure acting on a given volume of liquid is seen to be $P_c\, ln(s)$. The change in the corresponding work function is given by

$$\Delta W_l = -R_g T \Delta\, ln\left(-P_c\, ln(s)\right) \tag{19}$$

As with the potential energy, this expression gives the change in the work function for small changes in the saturation.

Substitution of this work term and the change in the potential energy in the energy balance yields

$$-R_g T \frac{d\, ln\left(P_c s(1-s)\right)}{ds} = -R_g T \frac{d\, ln\left(-P_c\, ln(s)\right)}{ds} \tag{20}$$

or

$$ln(s) = \frac{s-1}{1-2s} \tag{21}$$

This equation is satisfied for

$$s_{rw} = 0.236 \tag{22}$$

5. Analysis of Residual Nonwetting Saturation

A similar line of reasoning is used to calculate the residual saturation of the gas, s_{rn}. Here, the resistance to the gas flow is infinite, and the resistance to flow of the liquid can be ignored. Thus,

$$\frac{dG_s}{ds} = -\frac{dW_g}{ds} \tag{23}$$

where W_g is the work term for gas flow.

The Darcy equation for the gas phase is:

$$\varepsilon(1-s)v_n = -\frac{kk_n}{\mu_n}\frac{dP_g}{dz} = -\frac{kk_n}{\mu_n} P_c \frac{ds}{dz} \tag{24}$$

or

$$v_n = -\frac{kk_n}{\mu_n \varepsilon}\left(-P_c \frac{d\, ln(1-s)}{dz}\right) \tag{25}$$

From this expression the pressure acting on a given volume of gas is seen to be $-P_c\, ln(1-s)$. From this pressure must be subtracted the capillary pressure. The capillary pressure must be supplied to the gas to convert it from a stationary state in which it is in contact with a pore wall to a mobile state in which it is surrounded by liquid. The total pressure supplied to the gas phase is therefore, $-P_c\left(1+ln(1-s)\right)$, and the corresponding change in the work function is:

$$\Delta W_g = -R_g T \Delta\, ln\left(-P_c\left(1+ln(1-s)\right)\right) \tag{26}$$

Again, this expression gives the change in the work function for small changes in the saturation.

Substitution of this work term and the change in the potential energy in the energy balance gives

$$-R_g T \frac{d\, ln\left(P_c s(1-s)\right)}{ds} = -R_g T \frac{d\, ln\left(-P_c\left(1+ln(1-s)\right)\right)}{ds} \tag{27}$$

or

$$ln(1-s) = \frac{3s-1}{1-2s} \tag{28}$$

This equality is satisfied for

$$s_{rn} = 0.884 \tag{29}$$

Both the residual nonwetting saturation, 0.884, and the residual wetting saturation, 0.236, agree well with limiting saturations shown in generalized capillary and relative permeability plots (Bear, 1972; Scheidegger, 1957; Corey, 1977). The measured residual saturations for a uniform sand (Morell-Seytoux and Khanji, 1975; Morel-Seytoux et al., 1973; Vauclin, 1971) agree almost exactly with the derived values; the measured wetting saturation, estimated by graphical interpolation, was 0.230 and the measure wetting saturation was 0.884. Other measured residual wetting saturations for air-water and several aqueous-organic liquid-liquid mixtures in unconsolidated sand ranged from about 0.07 for air and water to about 0.27 for water and butyl acetate, which has a particularly low capillary pressure (Brooks and Corey, 1964; Demond and Roberts, 1991). Residual nonwetting saturations for these same systems ranged from about 0.65 for trichloroethane and water to about 0.88 for air and water.

6. Calculation of Capillary Pressure for a Laboratory Test

Measured capillary pressures depend on the characteristics of the porous material in which the measurements are made. Simple pressure models such as the one described in this work do not describe actual capillary behavior adequately largely due to the presence of inhomogeneities, such as fissures in an otherwise mostly impermeable material. Other factors that simple models typically do not address are the presence of more than two phases or condensable vapors. For these reasons, a fair evaluation of a simple capillary pressure model requires the use of data generated under carefully controlled conditions, preferably in a laboratory, using a material with a homogeneous pore distribution.

Smiles et al. (1971) (also see Vachaud and Thony, (1971)) conducted one such series of tests in a soil column filled with uniformly packed sand. The sand was screened through a 2-mm sieve to obtain a narrow particle size distribution, with 60 per cent of the material smaller than 0.3 mm and 15 per cent smaller than 0.15 mm. The porosity of the sand was measured by water displacement to be 0.41. The column was aerated so that the air remained at atmospheric pressure throughout the packed bed. Smiles et al. measured local moisture content by gamma ray attenuation, using an Am-241 source, and measured the capillary pressure using a tensiometer.

Smiles et al. depict the variations of imbibition and drainage pressures with absolute water saturation in terms of volume fraction of the bulk material. The capillary pressures measured by Smiles et al. exhibited hysteresis, with less suction required for imbibition of water than was measured during drainage. After the initial imbibition into a dry bed, subsequent imbibition and drainage cycles terminated at a maximum water saturation where no additional air was displaced and at a minimum saturation where the drainage pressure coincided with the imbibition pressure. Imbibition pressures were completely reproducible from cycle to cycle, while drainage suction pressures declined slightly but remained distinctly greater than the imbibition pressures.

The following section describes a capillary pressure model for these experiments, based on the wetting and nonwetting pressure gradient derived in the previous section. As stated previously, measured capillary pressures differ for imbibition and drainage of the wetting phase. For imbibition, the wetting phase, or liquid, supplies the motive force for displacement of the nonwetting, or gas, phase. Consequently, it may be argued that the effective capillary pressure within the porous medium is the integral of the pressure gradient for the liquid phase. In the Smiles et al. tests, the force to displace the gas was applied over the entire surface of the soil bed, but the bed was aerated so that the air within the capillary pores remained at atmospheric pressure. A hydraulic advantage accompanies the force applied to the liquid in the pores, then, in inverse proportion to the liquid saturation. It follows that the effective liquid pressure measured by the tensiometer is the integral of the liquid phase pressure gradient, divided by the saturation. This gives for the measured capillary pressure for imbibition:

$$\frac{P_{c,i}}{P_c} = \frac{P_{c,i,0}}{P_c} - \frac{ln(s)}{s} \tag{30}$$

where $P_{c,i,0}$ represents the minimum pressure required for imbibition of liquid into a saturated soil containing nondisplaceable gas.

During drainage, both liquid and gas displace the liquid that leaves the soil. Hence it may be argued that the effective capillary pressure is the volume average of the integrals of the pressure gradients for the gas and the liquid. Again, because the air remained at atmospheric pressure during drainage, there is a hydraulic advantage applied to the measured pressure, so the effective pressure is this volume-average pressure integral, divided by the saturation.

At the maximum residual nonwetting phase saturation, the measured suction pressure dropped by a step change from its value for drainage to its value for imbibition. This pressure change can be attributed to the change from a continuous gas phase at atmospheric pressure to a continuous liquid phase where, at static equilibrium, the outside pressure equals the liquid phase pressure. Since the liquid phase pressure is less than the gas phase pressure by the characteristic capillary pore pressure, the suction pressure for imbibition must be less than the pressure for drainage by just this pressure, divided by the saturation to account for the effective hydraulic advantage. These considerations, with adjustments to account for differences between the integration constants for the liquid and volume-average pressure gradients, give, for the measured capillary pressure for drainage,

$$\frac{P_{c,d}}{P_c} = \frac{P_{c,i,0}}{P_c} + \frac{1}{s_{rn}} - \frac{ln(s_{rn})}{s_{rn}} + \frac{(1-s)[1+ln(1-s)] - s\,ln(s)}{s} - \frac{(1-s_{rn})[1+ln(1-s_{rn})] - s_{rn}\,ln(s_{rn})}{s_{rn}} \tag{31}$$

The minimum suction pressure required for imbibition into a liquid-saturated soil, i.e., the entry head, most likely is a function of the capillary pore pressure. The most plausible explanation that fits the measured data is that an excess capillary force is required to displace air from an array of pores located on the surface in all three directions (one perpendicular to the surface and two in transverse directions). The required force is the three-dimensional vector sum of the forces required for unidirectional displacement, one of which corresponds to the pressure. According to this interpretation, the minimum capillary pressure is the pore pressure multiplied by the square root of three:

$$\frac{P_{c,i,0}}{P_c} = \sqrt{3} \tag{32}$$

This entry head is applied at the outer surface of the soil, so this pressure difference is not normalized with respect to the liquid saturation.

A combination of the capillary pressure equations for imbibition and drainage yields a critical saturation where these two pressures coincide:

$$\frac{1-(1-s_{rn})[1+ln(s_{rn})+ln(1-s_{rn})]}{s_{rn}} = -\frac{(1-s_{cr})[1+ln(s_{cr})+ln(1-s_{cr})]}{s_{cr}} \tag{33}$$

Substitution of s_{rn} in this expression gives, for this critical saturation,

$$s_{cr} = 0.301 \tag{34}$$

Below this saturation, the suction pressure that develops during drainage is less than the suction pressure that accompanies imbibition. The logical conclusion is that the soil will not drain to a saturation below this critical value, unless the source of liquid is removed from contact with the soil. The fact that the Smiles et al. tests terminate close to this critical saturation support this conclusion.

Figure 3 compares imbibition and drainage pressures measured by Smiles et al. with the predictions of the capillary pressure model just described. This figure plots measured suction pressures for one drainage cycle and one imbibition cycle. The drainage data represent the second overall drainage cycle and the first after the soil bed was initially and completely saturated with water. Subsequent drainages gave slightly lower suction pressures, but these pressures maintained about 80 per cent of the difference from imbibition pressures shown in the figure. The imbibition data are representative of all imbibitions. Measured capillary suction pressures are normalized with respect to the pressure difference between drainage and imbibition at the residual gas saturation. This pressure difference was estimated to be 11.95 cm water.

As the model predicts, the data exhibit both a residual nonwetting saturation and a critical saturation where the imbibition and drainage capillary pressures are equal. The model gives a remarkably accurate forecast of both of these saturations, as well as the magnitudes of the capillary pressures.

The pore size for the capillary pore pressure and the average particle diameter for the Smiles et al. tests should be of comparable magnitude. A comparison between the reported particle diameter and a pore diameter calculated from the Laplace capillary pressure relation,

$$d_p = \frac{4s}{P_c} \tag{35}$$

verifies that these two diameters are roughly equal. The calculated pore diameter is 0.244 mm. This diameter approximately equals the sand particle sieve size of 0.2 mm and the measured particle diameters, which were smaller than 0.15 mm for 10 per cent of the particles and smaller than 0.30 mm for 60 per cent of the particles by volume.

7. Conclusions

A model has been developed that uses a characteristic pore pressure to predict residual wetting and nonwetting saturations and capillary imbibition and drainage pressures. The model is based on an energy balance that equates changes in potential surface energy to changes in pressure-volume work. In the model, a simple probabilistic distribution expresses capillary forces as a function of the relative saturation and the characteristic pore pressure. Pressure gradients are derived from Darcy's law. Limiting imbibition pressures are based on flow of the wetting, or liquid, phase, and limiting drainage pressures are based on flow of the nonwetting, or gas, phase. The model implicitly assumes that the soil is homogeneous and isotropic.

The model predicts a residual wetting saturation, s_{rw}, of 0.236 and a residual nonwetting saturation, s_{rn}, of 0.884. The derived expressions for the limiting capillary pressures for imbibition, $P_{c,i}$, and drainage, $P_{c,d}$, are:

$$\frac{P_{c,i}}{P_c} = \frac{P_{c,i,0}}{P_c} - \frac{ln(s)}{s} \tag{30}$$

and

$$\begin{aligned}\frac{P_{c,d}}{P_c} = \frac{P_{c,i,0}}{P_c} + \frac{1}{s_{rn}} - \frac{ln(s_{rn})}{s_{rn}} + \frac{(1-s)[1+ln(1-s)] - s\,ln(s)}{s} \\ - \frac{(1-s_{rn})[1+ln(1-s_{rn})] - s_{rn}\,ln(s_{rn})}{s_{rn}}\end{aligned} \tag{31}$$

where P_c is the characteristic pore pressure and $P_{c,i,0}$ is the entry head required to initiate imbibition into the soil. It is argued that the entry head equals the vector sum of the characteristic pore pressure acting in all three directions at the soil surface. According to this argument,

$$\frac{P_{c,i,0}}{P_c} = \sqrt{3} \tag{32}$$

The model defined in the preceding equations accurately correlates limiting imbibition and drainage pressures in a selected laboratory test that used a uniformly packed, sieved sand (Smiles et al., 1971; Vachaud and Thony, 1971). In particular, the model predicts that there is a critical saturation, s_{cr}, of 0.301, below which the imbibition pressure exceeds the drainage pressure. Logic dictates that a soil will not drain to any lower saturation, provided that soil remains in contact with liquid. Indeed, measured saturations for the cited imbibition and drainage tests did not fall below this critical saturation.

Predicted residual saturations are in almost exact agreement with measurements made in a similarly uniform sand (Morell-Seytoux and Khanji, 1975; Morel-Seytoux et al., 1973; Vauclin, 1971). Residual saturations reported elsewhere do not agree as closely with the model predictions. Most notably, reported residual nonwetting saturations range from

0.07 to 0.27. One can speculate that the lower reported saturations might result from anisotropies in the porous materials or from gravity drainage. A study benchmarking the model with a wider selection of capillary pressure data is planned.

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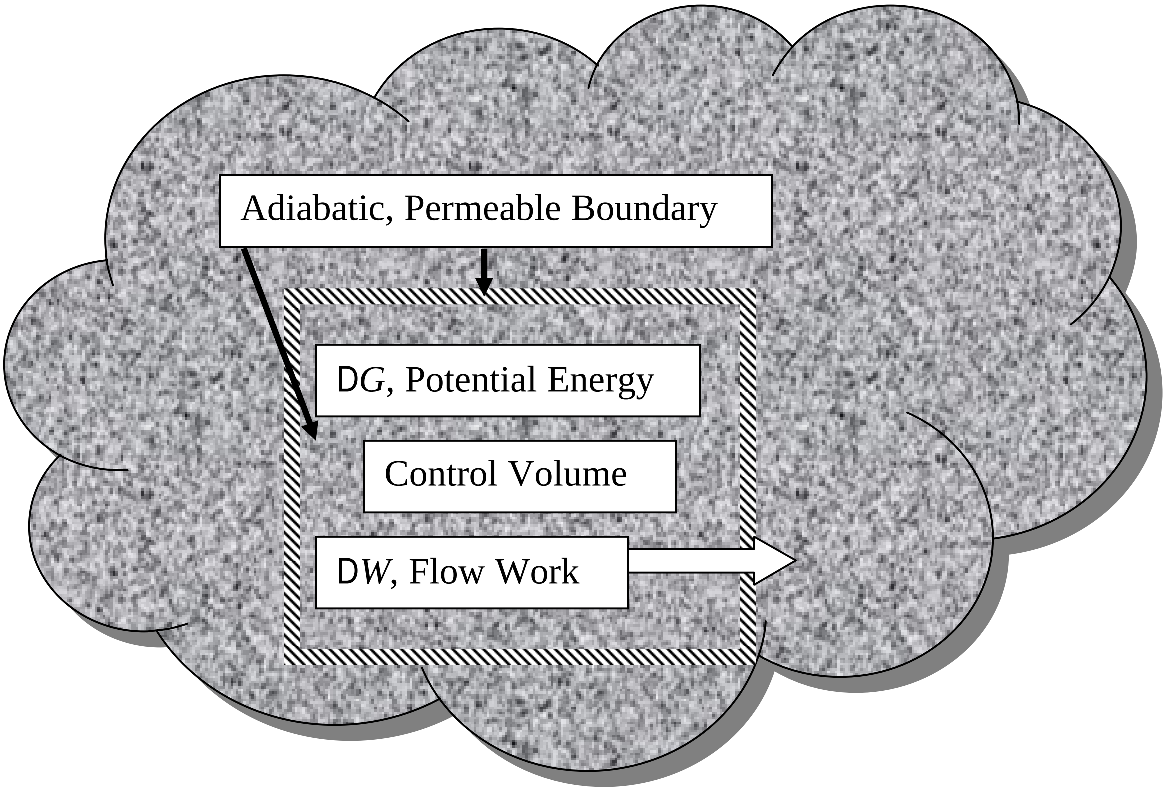

Shaded area represents a volume of soil.

Figure 1. Schematic Representation of Energy Balance Control Volume

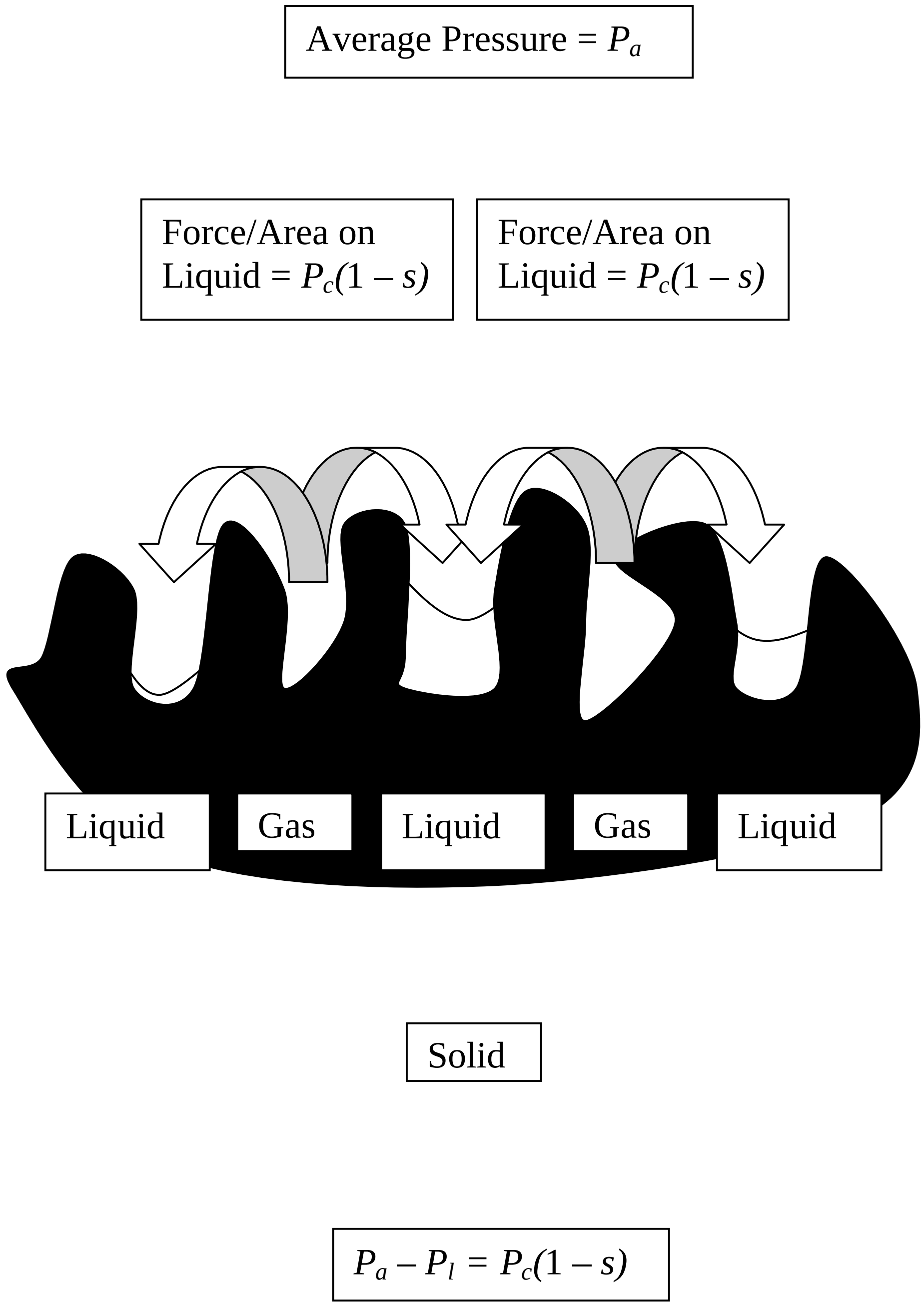

Figure 2. Capillary Forces on Liquid-Filled Pores

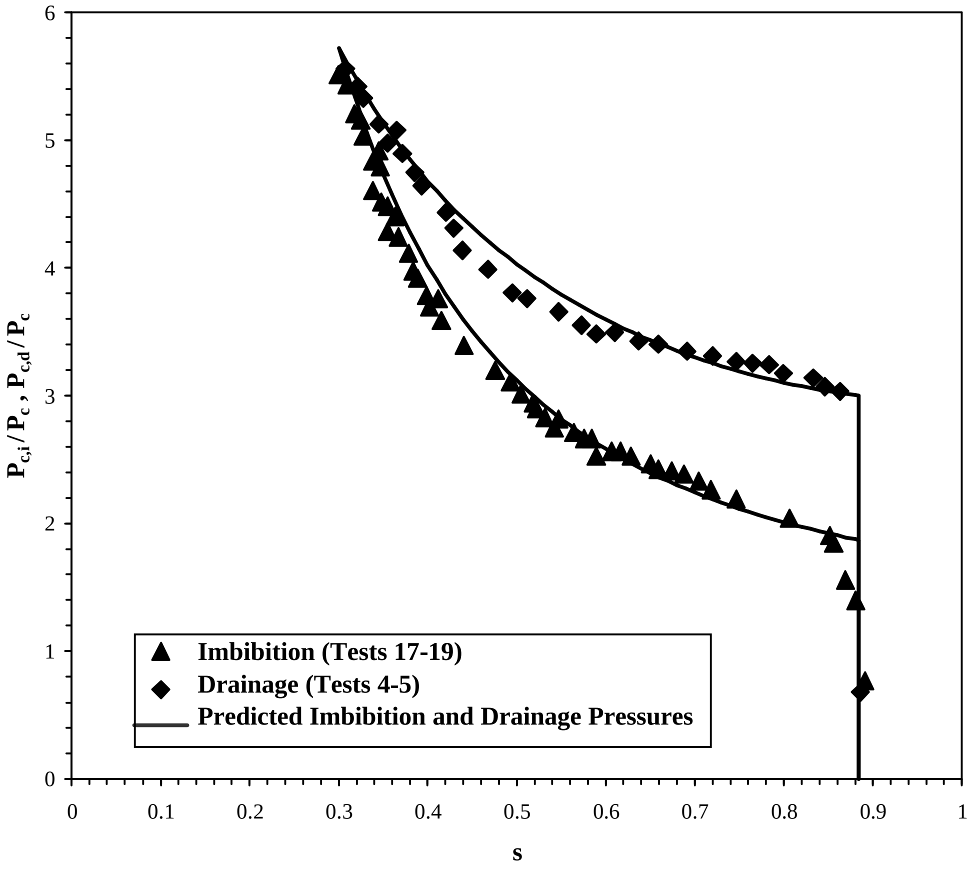

Figure 3. Comparison of Smiles et al. Measured Capillary Pressures for Imbibition and Drainage with Predicted Pressures