This document was prepared in conjunction with work accomplished under Contract No.
DE-AC09-76SR00001 with the U.S. Department of Energy.

OPST-84-751

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DPST-84-751

# INTERFACIAL REFRACTION THROUGH CURVED AND PLANE-LAYERED MEDIA

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DPST-84-751

SEPTEMBER 6, 1984

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# INTERFACIAL REFRACTION THROUGH CURVED AND PLANE-LAYERED MEDIA

## I. INTRODUCTION

In 1983, the Heat Transfer Laboratory of the Savannah River Laboratory (SRL) acquired a Laser Doppler Anemometer (LDA) for use in measuring complicated flows. One of the first applications of the LDA at SRL is to measure the velocity field in the Mark 22 bottom fitting insert.

Index of refraction changes associated with plexiglas, air, and water interfaces can alter a laser beam path. In order to obtain accurate velocity measurements with the LDA, it must be determined by how much these index of refraction changes will alter a laser beam path. This problem has been addressed by others, but their analysis is insufficient for the geometry of the Mark 22 bottom fitting insert [1, 2, 3, 4]. This report documents an analysis which determines how index of refraction changes associated with plane and cylindrical interfaces will alter a laser beam path.

## II. SUMMARY

Two laser beam tracing codes, AXIAL and CYLINDER, have been written to determine a laser beam path through plane and cylindrical interfaces. For cylindrical interfaces, an equation set was derived which describes the path of the laser beam. For plane interfaces, it was not possible to derive a single equation set. Instead, it was necessary to divide the domain up into small elements or regions. The laser beam path was then determined by calculating the path of the laser beam through each region. AXIAL and CYLINDER can be used to determine where an LDA should be positioned so that velocity measurements can be made at a specified point.

## III. BACKGROUND

### III.1 THE FRINGE MODEL

The operation of the LDA is explained through the use of a fringe model. When two coherent light beams of equal intensity intersect, constructive and destructive electromagnetic interference takes place. This interference causes the formation of alternating regions of bright light and darkness. The regions of bright light are known as fringes, and the volume in space defined by the two intersecting beams is known as the probe volume. Figure 1 is a simplified schematic illustrating these concepts.

As a particle in the fluid enters the probe volume, it scatters light as it passes through a fringe. When the particle goes through a region of darkness, it does not scatter light. This series of intermittent bursts of light will continue for as long as the particle is moving through the probe volume.

The time elapsed between each burst of light is the time it takes the particle to travel from one fringe to another. If the distance between each fringe is known, and if the time between two bursts of light is known, the velocity of the particle can be computed. Assuming the particle travels at the same velocity as the fluid, the velocity of the fluid can be determined.

The distance between each fringe is calculated from the following formula

$$(1) \quad d_f = \frac{\lambda}{2n \sin \frac{\theta}{2}},$$

where,

$d_f$ = distance between each one of the fringes

$\lambda$ = wavelength of the incident laser beams in a vacuum

$n$ = index of refraction of the material in which the probe volume is located and

$\frac{\theta}{2}$ = half angle of the incident laser beams.

Because the time between each burst of light is very short and the intensity of each burst is very low, a photomultiplier tube is used to determine the time between each burst of light.

## III.2 OPERATION OF THE LDA

One of the requirements for using an LDA is that a clear optical path to the flow field must exist. Many potential users of a LDA are hampered with obstructions which hide the flow area to be analyzed. One way to avoid this obstacle is to build plexiglas models. Another way is to use fiber optical cables.

While using plexiglas models does permit flow observation, plexiglas models can create their own problems. Index of refraction changes associated with water, plexiglas, and air interfaces alter the beam path. These changes to the laser beam path can be determined by using Snell's law

$$(2) \quad n_1 \sin\theta_1 = n_2 \sin\theta_2,$$

where,

$n_1$ = index of refraction in medium 1,

$n_2$ = index of refraction in medium 2,

$\theta_1$ = angle between the normal to the interface and the laser beam itself in medium 1 and

$\theta_2$ = angle between the normal to the interface and the laser beam itself in medium 2.

Figure 2 illustrates how index of refraction changes alter not only the half angle between two laser beams, but also the location at which the two beams intersect. For this particular example, Snell's law and simple geometrical laws were used to determine the change in the half angle and the change in the location of the probe volume. It was determined that the half angle changed by 1.367° and that the probe volume was shifted by 3.504 inches.

In addition to altering the half angle and the location of the probe volume, index of refraction changes can also alter the direction of the velocity vector. As shown in Figure 1, the velocity vector is oriented in a direction perpendicular to the bisector of the two incident laser beams. Altering the half angle between the two incident laser beams may also alter the bisector between the two beams and, hence, the direction of the velocity vector. This effect is shown qualitatively in Figure 3 for velocity measurements in two concentric annuli.

The objective of this study is to determine how index of refraction changes alter the following:

(1) The half angle between two laser beams,

(2) The direction of the velocity vector and

(3) The point at which velocity measurements are being made.

A computer graphics routine is used to allow a quick visual check of the results. Both straight and circular interfaces are analyzed.

## IV. ANALYSIS OF PLANE-LAYERED INTERFACES

### IV.1 PROBLEM DESCRIPTION

Laser Doppler Anemometers may be used to measure the flow in plexiglas models where the walls of the plexiglas model appear to be plane interfaces. These plane walls may be at an angle with respect to the vertical, and they may have different indices of refraction. Figure 4 illustrates a possible configuration of water, plexiglas, and air interfaces.

The shape of each medium, the index of refraction of each medium, and the half angle of the laser beams in air will determine the path that the beams will take through the plexiglas model shown in Figure 4. If the LDA could be arbitrarily positioned it would be a simple matter to apply Snell's law and calculate the path of the laser beams from the laser through the plexiglas test section. With the beam paths determined, the location of the probe volume and the direction of the velocity vector could be calculated. The laser beam half angle in the fluid medium could then be determined.

The above problem is usually the inverse problem that an LDA user faces. Most LDA users do not arbitrarily position an LDA and then try to determine the location of the probe volume. Usually they specify the location of the probe volume and then try to determine where the laser should be placed.

The problem of where to position the LDA in order to obtain measurements at a specified point is complicated. For discontinuous, plane-layered media, it is difficult to formulate a single set of equations relating the laser beam path to the specified measuring point. These equations depend upon the geometrical configuration of the various media which compose the test section. The equations also depend upon the angle that the laser beam makes with respect to the horizontal at the measuring point (herein referred to as the direction angle).

Moving the specified measurement point from one part of the test section to another part might result in a different geometrical configuration of the various media through which the laser beam would travel. This different geometrical configuration would require a second set of equations to describe the path of the laser beam; a third location might require a third set of equations.

For example, Figure 5a shows a plexiglas test section consisting of discontinuous sections of plexiglas and water. For point P1, one set of equations could be written which would describe the path of the laser beam through the test section. For point P2, a second set of equations would be necessary. This is because point P2 "sees" a configuration of plexiglas and water different from point P1. For point P3, a third set of equations would be necessary.

Rather than write numerous equations with each equation valid only for a small portion of the test section, the concept of using small, discrete regions to determine a laser beam path is introduced. The concept of solving a continuum problem on a discrete basis has been in existence for a number of years. The most obvious example of solving a continuum problem on a discrete basis is the application of finite elements to solve problems in structural and fluid mechanics [5]. The term "finite regions" is used to describe the technique developed in this analysis. While this technique solves a continuum problem on a discrete basis, it does not use any of the sophisticated mathematics associated with finite elements.

The best way to determine the path of the laser beam through the plexiglas test section shown in Figure 5 is to discretize the test section into small regions (or elements) as shown in Figure 5b. Snell's law can then be applied at the interface of each region. By applying Snell's law on this discontinuous basis, the path of a laser beam through the plexiglas test section can be determined.

## IV.2 PROBLEM SOLUTION

A simple example is used to explain how finite regions can be used to determine a laser beam path through a plexiglas test section. In Figure 5c, the discretized test section of Figure 5b is shown with a measurement point specified. It can be determined from equations derived in the next section that, for an LDA system with a half angle of 5.5°, the direction angle of the laser beam in the water section is 4.1325°.

A linear equation describes the path of a laser beam through transparent materials

$$y = mx + b, \tag{3}$$

where,

m = slope of the laser beam (equal to tan $\theta$, where $\theta$ is the direction angle),

b = y axis intercept of the equation,

y = y coordinate of a point on the laser beam path and

x = x coordinate of a point on the laser beam path.

Once the measurement point has been selected, and the direction angle in the region containing the measurement point has been determined, m and b in equation (3) can be calculated.

For the portion of the laser beam in region 10 as shown in Figure 5c, m is equal to 0.072251 and b is equal to 0.063875.

The next step is to find the point at which the laser beam exits region 10. This can be accomplished by simply determining the intersection of equation (3) with the region boundary. In this example, the portion of the laser beam in region 10 exits the boundary at the point x = 1.0, y = 0.13612.

Once the exit point for the region containing the specified measurement point has been calculated, the next step is to determine the next region that the laser beam will enter. By visual inspection of Figure 5c, it can be determined that the ray will enter region 9.

After determining the next region that the laser beam has entered, Snell's law must be applied at the region interface to determine the direction angle in the new region. Applying Snell's law here at the region interface results in a direction angle of 3.688° for the laser beam in the new region.

The point at which the laser beam enters region 9, and the direction angle while the beam is in region 9 have already been determined. Thus m and b in equation (3) can be recalculated for the portion of the laser beam passing through region 9. The exit point of the beam for region 9 could then be determined, and the third region into which the beam is entering could also be determined.

This process is repeated until the path of the laser beam through the remaining regions has been determined. Algorithm I describes the process by which finite regions can be used to determine the path of a laser beam through a plexiglas model. This algorithm is valid only for plexiglas models with vertical plexiglas and water sections.

## ALGORITHM I

1. The plexiglas model is discretized into rectangular regions.
2. A point in a specified region is selected as the point where velocity measurements will be made.
3. The direction angle in the specified water region is determined assuming that no angled surfaces are present.
4. With the direction angle known, an equation for the beam is formulated. The point at which the beam exits the region is determined from the intersection of the laser beam equation with the region boundary.

5. With the exit point known, a search of all regions adjacent to the current one is performed to determine the next region that the beam has entered.

6. The direction angle for the new region is calculated using Snell's law.

7. Steps 4-6 are repeated until the beam exits the test section.

The algorithm can be used to analyze a laser beam path through plexiglas models which are much more complicated than the one shown in Figures 5a, 5b, and 5c.

A modified version of Algorithm I is in Appendix A which allows complicated plexiglas models to be analyzed. This modified algorithm includes the effect that one angled plexiglas-water interface may have on a laser beam as it passes through a plexiglas model.

## IV.3 DETERMINATION OF THE DIRECTION ANGLE

Using finite regions to determine a laser beam path through a plexiglas model can be done only if the direction angle of the laser beam in the region containing the measurement point is known. This section outlines some assumptions that are made which allow the direction angle in the region containing the measurement point to be determined. It should be noted that the forthcoming discussion assumes that the LDA system is placed to the right of the plexiglas model shown in Figure 4.

The half angle of an LDA in air is a parameter of each particular system. Applying Snell's law at the air-plexiglas interface shown in Figure 4 yields the following result for the direction angle in the first plexiglas section

$$(4) \qquad \theta_2 = \text{INVSIN}\ [\frac{n_1}{n_2} \sin\theta_1],$$

where,

$\theta_1$ = direction angle in the air section (equal to the LDA half angle in air for this particular example),

$\theta_2$ = direction angle in the first plexiglas section,

$n_1$ = index of refraction for air and

$n_2$ = index of refraction for first plexiglas section.

Applying Snell's law to the first plexiglas-water interface yields

$$(5) \quad \theta_3 = \text{INVSIN } [\frac{n_2}{n_3} \sin \theta_2],$$

where,

$\theta_3$ = direction angle in the first water section and

$n_3$ = index of refraction for the first water section.

Applying Snell's law to the second water-plexiglas interface yields,

$$(6) \quad \theta_4 = \text{INVSIN } [\frac{n_3}{n_4} \sin\theta_3],$$

where,

$\theta_4$ = direction angle in the second plexiglas section and

$n_4$ = index of refraction for the second plexiglas section.

Applying Snell's law to the third plexiglas-water interface yields,

$$(7) \quad \theta_5 = \text{INVSIN } [\frac{n_4}{n_5} \sin \theta_4],$$

where,

$\theta_5$ = direction angle while it is in the second water section and

$n_5$ = index of refraction for the second water section.

Successive substitutions can be made for $\theta_4$ in equation (7) by using equations (4), (5), and (6) until an expression for $\theta_5$ in terms of $\theta_1$ is obtained. The result is

$$(8) \quad \theta_5 = \text{INVSIN } [\frac{n_1}{n_5} \sin \theta_1].$$

Thus the direction angle in the second water section is independent of the indices of refraction in the two plexiglas sections and the water section. It should be noted that the independence of this angle with the above mentioned indices of refraction is a function of the geometry. For other geometrical configurations, the direction angle may be dependent upon the indices of refraction of the intermediate materials.

Substituting in for $\theta_2$ in equation (5) by using equation (4) results in

$$(9) \qquad \theta_3 = \text{INVSIN}\ [\frac{n_1}{n_3} \sin \theta_1].$$

Again, it should be noted that the independence of $\theta_3$ with $n_2$ is because of this particular geometrical configuration.

If it is assumed that the index of refraction for the two plexiglas sections are the same, then $\theta_5$ equals $\theta_3$. This means that the direction angle for a laser beam in water sections sandwiched between vertical plexiglas sections is the same in all the water sections.

Not all plexiglas models will have vertical plexiglas-water interfaces. A plexiglas model with a plexiglas-water interface at an angle to the vertical is illustrated in Figure 6.

Figure 7 is a close up of the angled plexiglas-water interface in Figure 6. Applying Snell's law and geometrical laws to the figure, the following relationship for the direction angle $\theta_4$ in the water section is obtained

$$(10) \qquad \theta_4 = \text{INVSIN}\ [\frac{n_2}{n_3} \sin\ (\theta_3 + 90 - \alpha)] + \alpha - 90,$$

where,

$n_2$ = index of refraction for plexiglas,

$n_3$ = index of refraction for water

$\theta_3$ = direction angle for the laser beam in the plexiglas section to the right of the water-plexiglas interface,

$\theta_4$ = direction angle for the laser beam in the water section to the left of the water-plexiglas interface and

$\alpha$ = angle that water/plexiglas interface makes with the horizontal.

If the water-plexiglas interface shown in Figure 6 was reversed, (i.e., the plexiglas section was on the left and the water section was on the right) the direction angle for water sections to the left of the interface would have to be calculated in several steps. First equation (8) would be used to calculate the direction angle in the water section to the right of the angled plexiglas-water interface

$$\theta_4 = \text{INVSIN}\left[\frac{n_1}{n_5}\sin\theta_1\right], \tag{11}$$

where,

$\theta_4$ = direction angle of the laser beam in the water section to the right of the plexiglas-water interface,

$\theta_1$ = half angle of the LDA system,

$n_1$ = index of refraction for air and

$n_5$ = index of refraction for water.

Next a slightly altered version of equation (10) would be used to calculate the direction angle of the laser beam in the plexiglas section. This equation is

$$\theta_p = \text{INVSIN}\left[\frac{n_3}{n_2}\sin(\theta_4 + 90 - \alpha)\right] + \alpha - 90, \tag{12}$$

where,

$\theta_p$ = direction angle of the laser beam in plexiglas sections to the left of the angled plexiglas-water interface,

$\theta_4$ = direction angle of the laser beam in water sections to the right of the plexiglas-water interface,

$\alpha$ = angle that plexiglas-water interface makes with the horizontal,

$n_3$ = index of refraction for water and

$n_2$ = index of refraction for plexiglas.

Finally, Snell's law would be used at the vertical water-plexiglas interface to calculate the direction angle for the water section to the left of the angled plexiglas-water interface. The result is

$$(13) \qquad \theta_w = \text{INVSIN} \left[\frac{n_2}{n_3} \theta_p\right],$$

where,

$\theta_w$ = direction angle in water sections to the left of the angled plexiglas-water interface,

$\theta_p$ = direction angle in plexiglas sections to the left of the plexiglas-water interface,

$n_3$ = index of refraction for water and

$n_2$ = index of refraction for plexiglas.

## IV.4 PLACEMENT OF THE LDA

Algorithm I determines the path of two laser beams through a plexiglas test section. This information, along with the knowledge of where the two laser beams exit the focusing lens of a LDA, is enough to determine where the LDA should be placed in order to make velocity measurements at a specified point.

However, because of the curvature of the lens, it is difficult to determine exactly where the two laser beams will exit the LDA. Because of this, the concept of a reference point is introduced. The reference point concept allows the LDA user to avoid any errors that lens curvature may introduce.

Use of a reference point in conjunction with Algorithm I to determine LDA placement is described in Algorithm II.

### ALGORITHM II

(1) A point with a well known physical location (such as the corner of a wall) is chosen as the reference point.

(2) The measurement point is selected.

(3) Using the Algorithm I, the two laser beam paths from the reference point through the test section are calculated.

(4) Using the Algorithm I, the two laser beam paths from the measurement point through the test section are computed.

(5) Equations specifying how far the LDA system must be moved in the horizontal and vertical directions so that the probe volume moves from the reference point to the measurement point are shown in Figure 8.

(6) The LDA system is aligned so that the probe volume is located at the reference point, then the LDA is moved the horizontal and vertical distances specified by the equations shown in Figure 8.

## IV.5 DETERMINATION OF LDA OPTICAL PARAMETERS

Equations relating the LDA half angle in the fluid medium and the direction of the velocity vector to the two direction angles are shown in Figure 9.

## IV.6 COMPUTERIZATION OF THE CALCULATIONAL PROCESS

The computer program AXIAL was written to mechanize the calculations and algorithms outlined in the previous four sections. The structure of the program is fairly basic. The code first reads in the input data which describes the plexiglas model which will be analyzed. Next the code reads in the location of the reference point and the location of the measurement point. The code then calculates the path of the two reference beams using the method of finite regions as outlined in Appendix A. Next the code calculates the path of the two measurement point beams using the method outlined in Appendix A.

Once the paths of the two reference beams and the two measurement point beams have been determined, the code uses the method outlined in Algorithm II to position the LDA. After determining the placement of the LDA system, the code then determines the magnitude of the half angle and the direction of the velocity vector through the use of the equations shown in Figure 9. At this point all calculations are complete.

Figures 10 through 13 are flow charts for the program AXIAL and some of its subroutines. The flow charts shown here are rather brief; their purpose is to show the overall structure of the program. A brief description of the code AXIAL as well as its FORTRAN listing is included in Appendix B.

# V. ANALYSIS OF CYLINDRICALLY LAYERED INTERFACES

## V.1 PROBLEM DESCRIPTION

The radii of each tube, the index of refraction of the water, the index of refraction of each tube, and the half angle of the laser beams in air determine the path that the laser beams take through the water and the plexiglas tubes.

The analysis of cylindrically layered interfaces is somewhat easier than the analysis of plane-layered interfaces. Unlike plane-layered interfaces, analytical equations can be derived to describe the path of a laser beam through cylindrically layered interfaces.

### V.2 GOVERNING EQUATIONS

The equations which describe the path of a laser beam through nested, cylindrical tubes situated in a plexiglas box are shown in Table I. Appendix C contains the derivation of the equations. Figure 15 shows the corresponding geometrical configuration. The variables are defined in Table II.

Indicial notation is used to show the repetitive nature of the equations. The addition of extra cylinders is not troublesome and results only in additional equation segments.

### V.3 COMPUTERIZATION OF THE CALCULATIONAL PROCESS

The computer program CYLINDER was written to solve the equations shown in Table I and determine where the LDA system should be placed. A linear interpolation technique is used to solve the equations in Table I for the laser beam path. After both laser beam paths have been determined, the code uses Algorithm II to determine the LDA placement. The code then uses the equations shown in Figure 9 to determine the half angle and the direction of the velocity vector.

Figure 14 is a flow chart for the program CYLINDER. The FORTRAN listing of CYLINDER is included in Appendix D.

## VI. DISCUSSION OF RESULTS

Several test runs were made with both AXIAL and CYLINDER. In all the figures, the plexiglas regions appear as shaded sections and the water regions appear as white sections. The laser beams appear as dark black lines.

Because of the low resolution of the graphics printer, the laser beams appear to be slightly staggered. Some of the figures have inserts included with them. These inserts are close-ups of the region around the measurement point.

## VI.1 PLANE-LAYERED MEDIA

Figures 16, 17, and 18 are cross sectional views of the bottom portion of a Mark 22 assembly. When trying to measure axial velocities in fuel assemblies, the cylinders appear as plane interfaces. Thus, the code AXIAL was used to determine where an LDA should be placed in order to make measurements at specified points. The input data file specifying the geometry is in Appendix E.

Figure 16 shows the path that the laser beams would take if LDA velocity measurements were made at the specified point. As the results show, the direction of the velocity vector is unaffected; it is still oriented at 90°. This is because both laser beams travel through vertical interfaces. Index of refraction changes in these vertical interfaces affect both direction angles equally. Hence, the direction of the velocity vector is unaffected.

Figure 17 shows the path that the two laser beams would take if LDA velocity measurements were made to the left of the angled surface. As can be seen, the direction of the velocity vector is altered by 4.1°.

Figure 18 shows the path that the laser beams would take if LDA velocity measurements were made in the lower left hand corner of the test section. The gently sloping plexiglas section alters the direction of the velocity vector only slightly.

## VI.2 CYLINDRICALLY LAYERED MEDIA

Figures 19 through 26 are cross sectional views of 4 concentric cylinders located inside a plexiglas box. Figure 27 has the same geometrical configuration as Figures 19 through 26 except that the three inner cylinders have been removed. The input data file for Figures 19 through 26 is in Appendix F.

Figures 19 through 22 show the paths that the laser beams would take for a measurement point with the same radius located in four different quandrants. Figures 19 and 20 show that curvature effects alter the direction of the velocity vector by the same magnitude but in different directions. Figures 21 and 22 show the same phenomenon. Also, these figures show that for concentric geometries, there will not be any "blind spots". Curvature effects from the cylinders will cause the beams to bend upwards toward the boundaries of the cylinders.

Figures 23 through 26 are included to illustrate how moving the measurement point around inside the inner cylinder reduces the curvature effects of the cylinder walls. As the curvature effect is reduced, the direction of the velocity vector comes closer to 90°.

Figure 27 is included to illustrate what effect removing the three inner cylinders has on the direction of the velocity vector. The measurement point in this figure is the same as the one in Figure 22. In Figure 22, the direction of the velocity vector is 85.471°. In this figure, the direction of the velocity vector is 89.91°.

ABK:cjl

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## TABLE I
## EQUATIONS WHICH DESCRIBE THE PATH OF A LASER BEAM THROUGH CYLINDRICALLY LAYERED MEDIA

(14) $\beta = \mathrm{INVTAN}\ (y/x)$

(15) $R_p = x/\cos(\beta)$

(16) $\partial_{1,1,1} = \mathrm{INVSIN}\ [\frac{R_p}{R_{1,1}} \sin\ [\ \beta - \mathrm{ALPH4G}]]$

(17) $\partial_{1,1,2} = \mathrm{INVSIN}\ [\frac{n_{water}}{n_{plex}} \sin\ (\partial_{1,1,1})]$

(18) $\partial_{1,2,1} = \mathrm{INVSIN}\ [\frac{R_{1,1}}{R_{1,2}} \sin\ (\partial_{1,1,2})]$

(19) $\partial_{1,2,2} = \mathrm{INVSIN}\ [\frac{n_{plex}}{n_{water}} \sin\ (\partial_{1,2,1})]$

(20) $\partial_{2,1,1} = \mathrm{INVSIN}\ [\frac{R_{1,2}}{R_{2,1}} \sin\ (\partial_{1,2,2})]$

(21) $\partial_{2,1,2} = \mathrm{INVSIN}\ [\frac{n_{water}}{n_{plex}} \sin\ (\partial_{2,1,1})]$

(22) $\partial_{2,2,1} = \mathrm{INVSIN}\ [\frac{R_{2,1}}{R_{2,2}} \sin\ (\partial_{2,1,2})]$

(23) $\partial_{2,2,2} = \mathrm{INVSIN}\ [\frac{n_{plex}}{n_{water}} \sin\ (\partial_{2,2,2})]$

$$\partial_{i,1,1} = \mathrm{INVSIN}\ [\frac{R(i-1),2}{Ri,1} \sin(\partial_{(i-1),2,2})]$$

$$\partial_{i,1,2} = \mathrm{INVSIN}\ [\frac{n_{water}}{n_{plex}} \sin(\partial_{i,1,1})]$$

$$\partial_{i,2,1} = \mathrm{INVSIN}\ [\frac{R_{i,1}}{R_{i,2}} \sin\ (\partial_{i,1,2})]$$

$$\partial_{i,2,2} = \mathrm{INVSIN}\ [\frac{n_{plex}}{n_{water}} \sin\ (\partial_{i,2,1})]$$

TABLE I (cont'd)

EQUATIONS WHICH DESCRIBE THE PATH OF A
LASER BEAM THROUGH CYLINDRICALLY LAYERED MEDIA

(24) $$\begin{aligned} ALPHW = ALPH4G &+ \partial_{1,1,1} - \partial_{1,1,2} + \partial_{1,2,1} - \partial_{1,2,2} \\ &+ \partial_{2,1,1} + \partial_{2,1,2} + \partial_{2,2,1} - \partial_{2,2,2} \\ &\ldots\ldots\ldots + \partial_{i,1,1} - \partial_{i,1,2} + \partial_{i,2,1} - \partial_{i,2,2} \end{aligned}$$

(25) $$ALPHP = INVSIN\ [\frac{n_{water}}{n_{plex}} \sin\ (ALPHW)]$$

(26) $$ALPHGH = INVSIN\ [\frac{n_{plex}}{n_{air}} \sin\ (ALPHP)]$$

(27) $$ANGLE = ALPHW + \partial_{i,2,2}$$

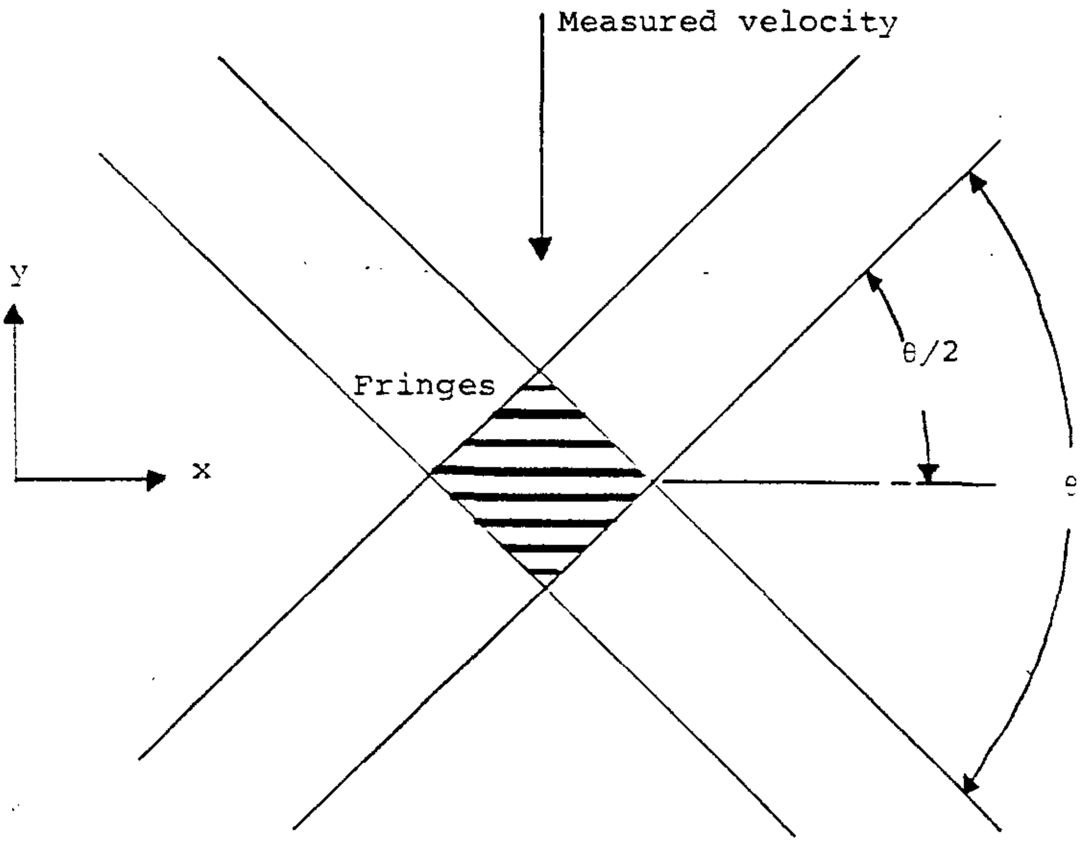
(28) $$\begin{aligned} Y1 = R_{i,2} * \sin\ (ANGLE) + [D - R_{i,2} \cos\ (ANGLE)] * [\tan(ALPHW)] \\ + t * \tan\ (ALPHP) \end{aligned}$$

# TABLE II

## DEFINITION OF NOMENCLATURE

| | | |
|---|---|---|
| x | = | x coordinate of laser beam crossing point |
| y | = | y coordinate of laser beam crossing point |
| $R_p$ | = | radius of laser beam crossing point |
| $R_{i,j}$ | = | radius of a particular cylinder. i indicates the cylinder (from center outwards). j indicates which radius (1 means inner radius, 2 means outer radius) |
| $\alpha_{i,j,k}$ | = | angle that the laser beam makes with respect to the normal at the cylinder-water interface. i indicates the cylinder (from center outwards). j indicates inner or outer radius (1 means inner radius, 2 means outer radius). k indicates the angle (1 means inner angle, 2 means outer angle). |
| ALPHW | = | direction angle for the laser beam in the water between the outer cylinder and the plexiglas box |
| ALPHP | = | direction angle for the laser beam in the plexiglas box |
| ALPH4G | = | direction angle of the laser beam in the fluid at the beam crossing. |
| ALPHGH | = | Half angle of the LDA in air. |
| Y1 | = | y coordinate of the point at which the laser beam exits the plexiglas box. |
| D | = | distance between the origin and the inside of the plexiglas wall. |
| t | = | thickness of the plexiglas box |
| ANGLE | = | angle between the line from the origin through the point where the laser beam exits the outermost cylinder, and the horizontal. |
| i | = | number of cylidners. |

THE FRINGE MODEL




Fringes are parallel to the bisector of the two laser beams. Thus, the measured velocity has a direction perpendicular to the bisector.

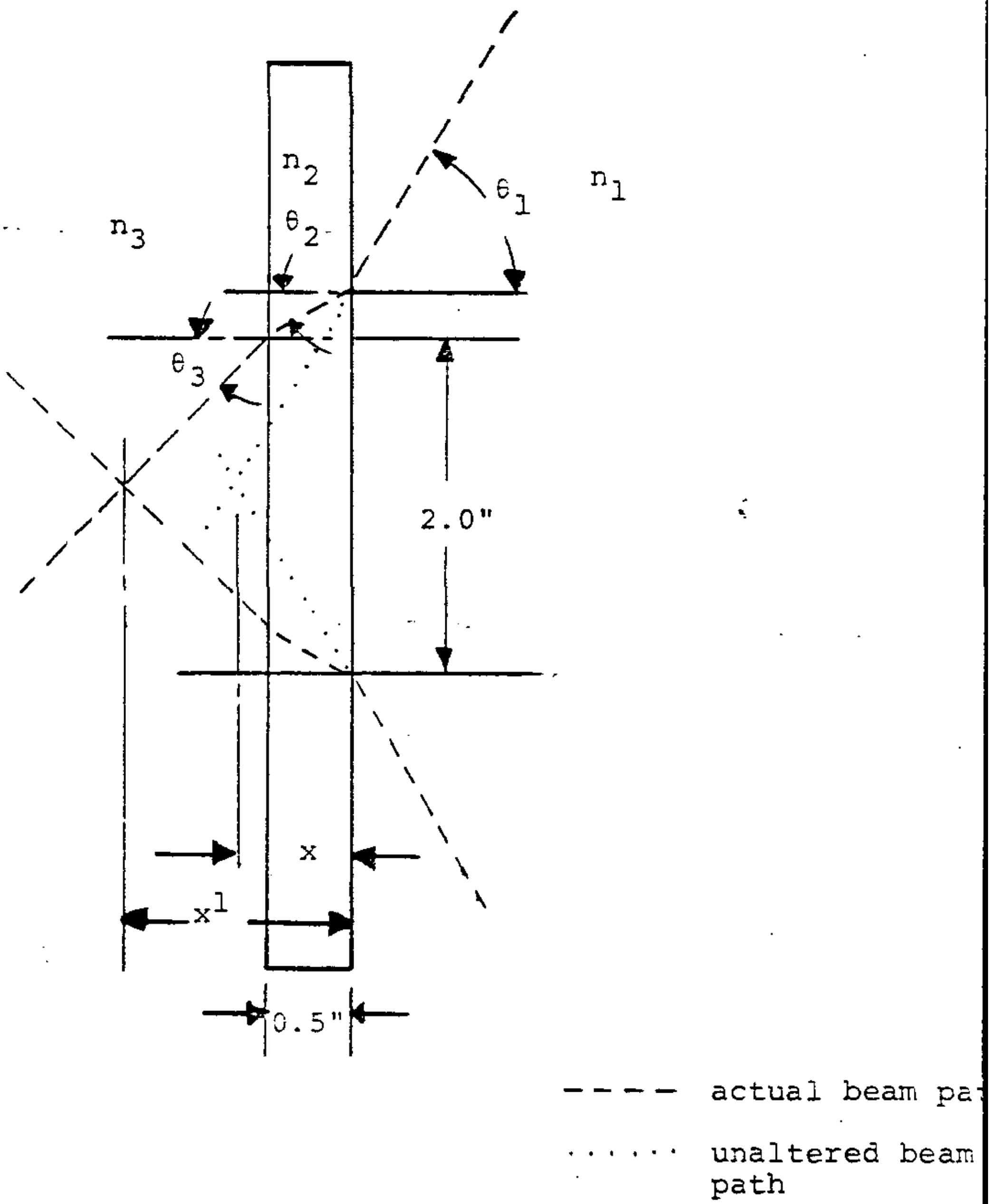
FIGURE 1

INDEX OF REFRACTION CHANGES

$\theta = 5.5^{\circ}$
$n_1 = 1.00$
$n_2 = 1.490$
$n_3 = 1.330$

Applying Snell's law results in

$\theta_2 = 3.688^{\circ}$
$\theta_3 = 4.1326^{\circ}$
$x = 10.39"$
$x^1 = 13.89"$

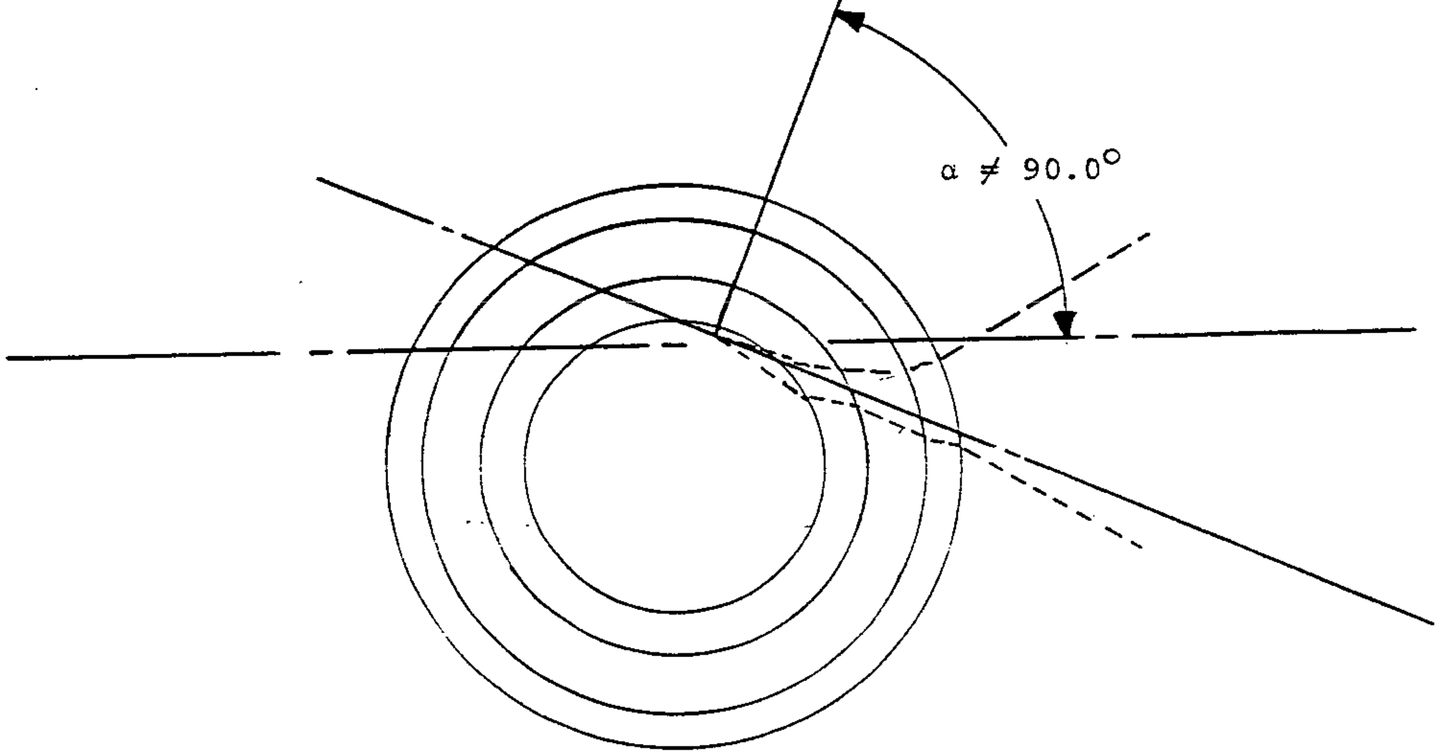



NOTE: Figure is not to scale. Dimensions have been altered to exaggerate changes in the angles.

FIGURE 2

FIGURE 3

ORIENTATION OF THE VELOCITY VECTOR

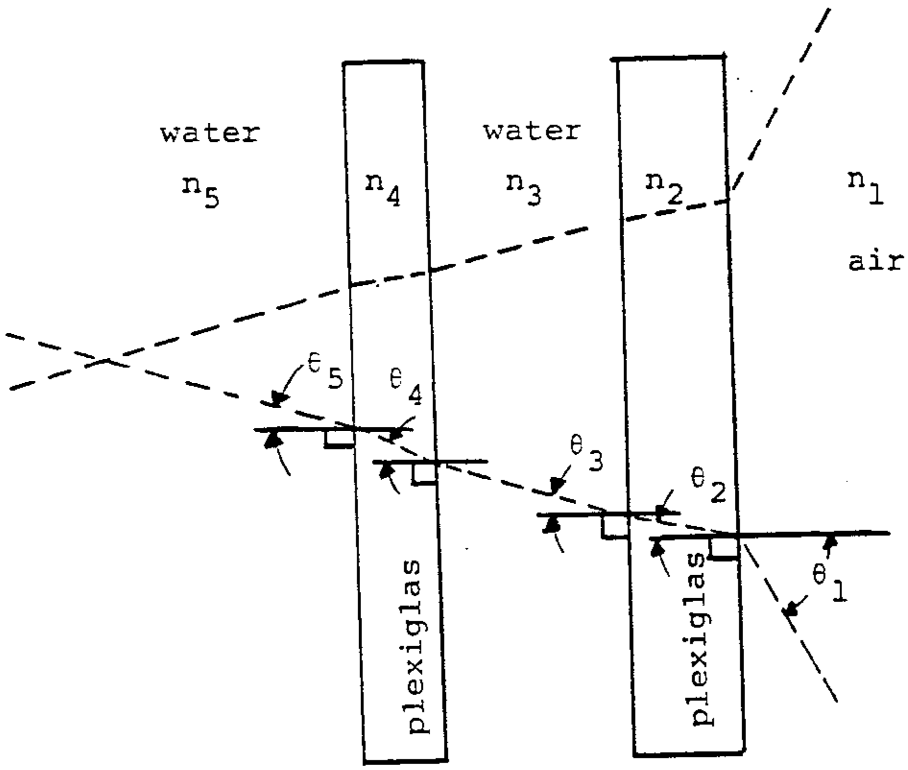



BECAUSE OF INDEX OF REFRACTION CHANGES

$\alpha \neq 90^{\circ}$

FIGURE 4

SOLVING FOR THE DIRECTION ANGLE



A PLEXIGLAS MODEL

WITH AN IRREGULAR GEOMETRY

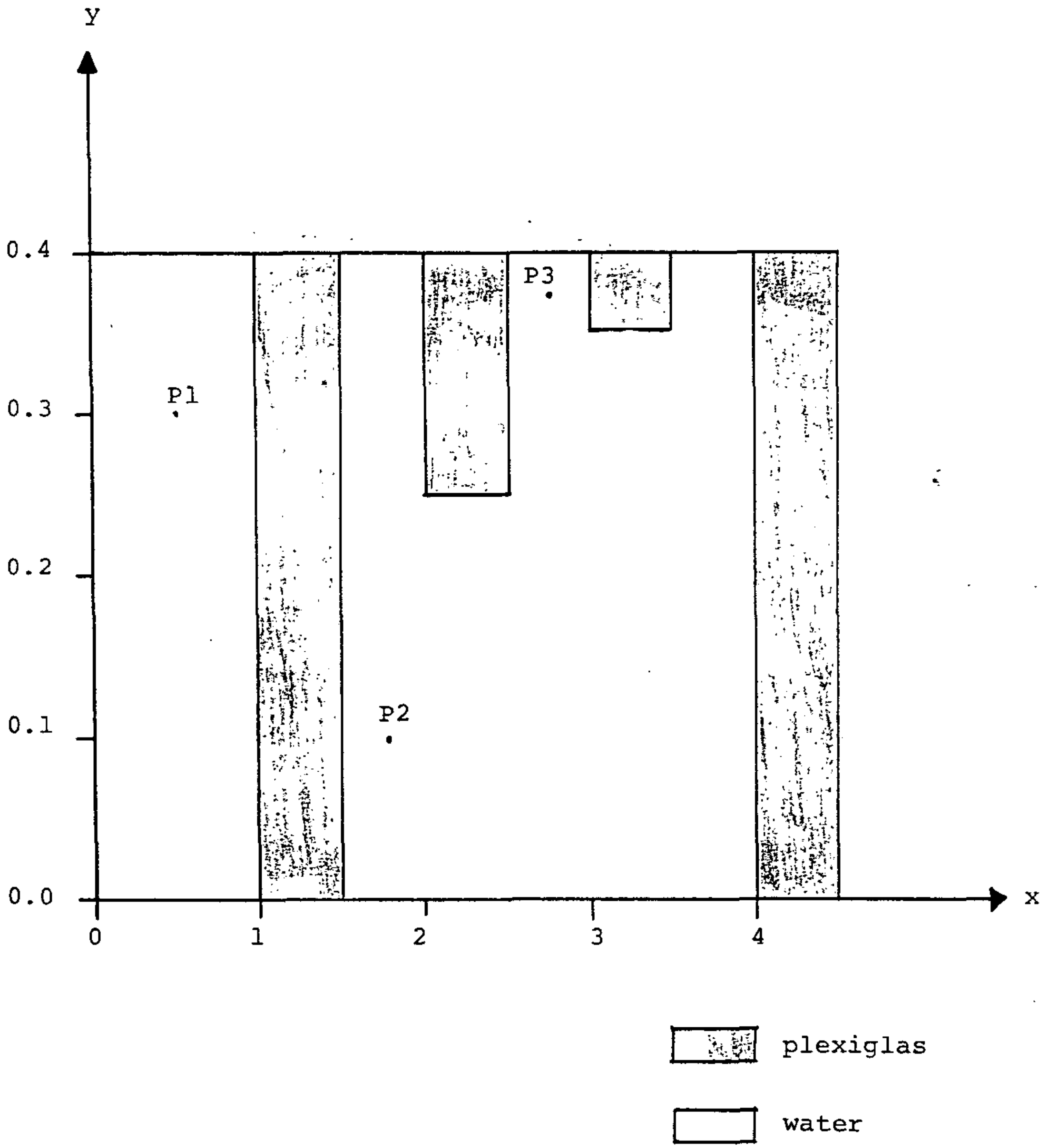



FIGURE 5a

THE DISCRETIZED PLEXIGLAS MODEL

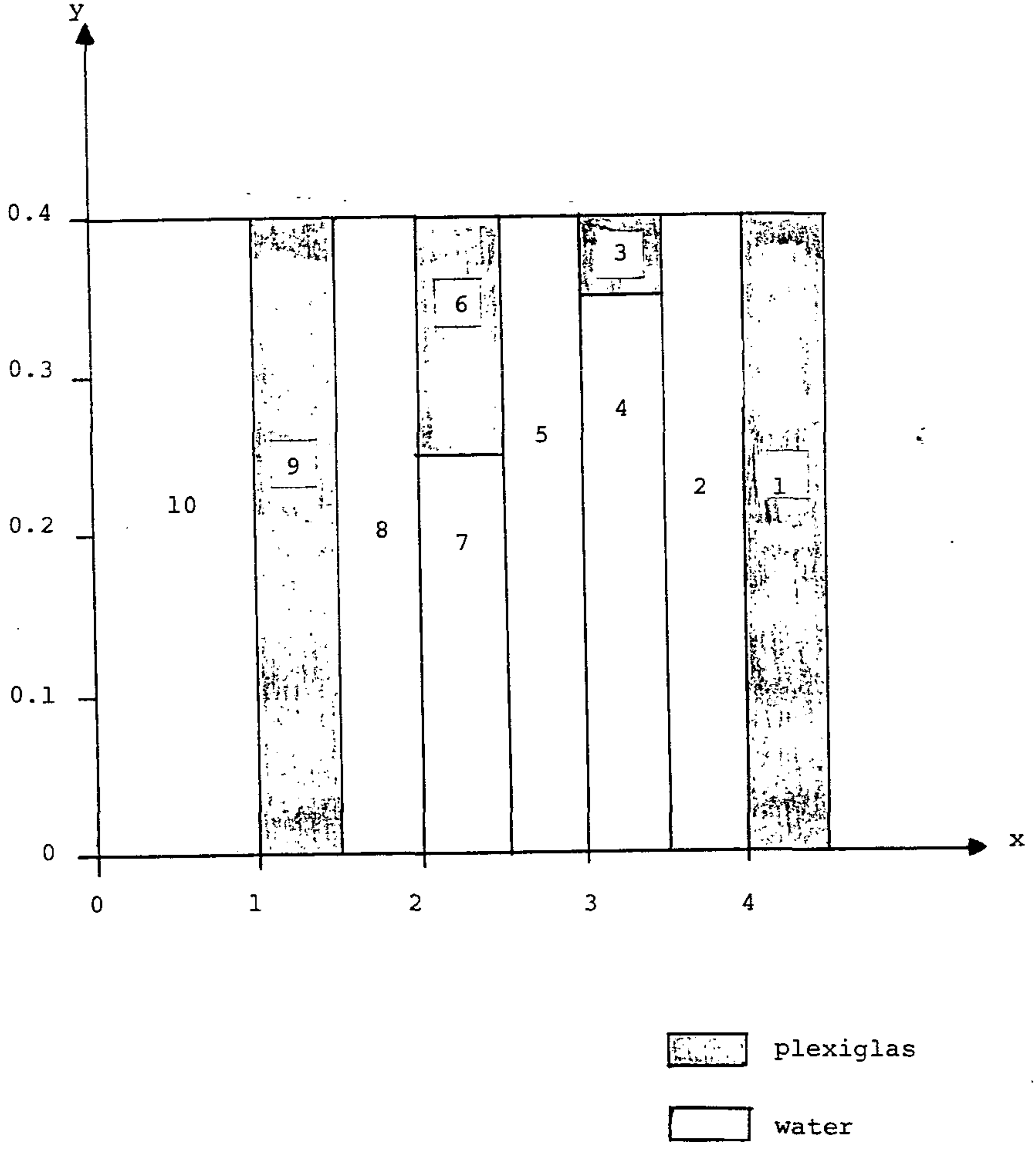



FIGURE 5b

THE LASER BEAM PATH THROUGH

THE DISCRETIZED MODEL

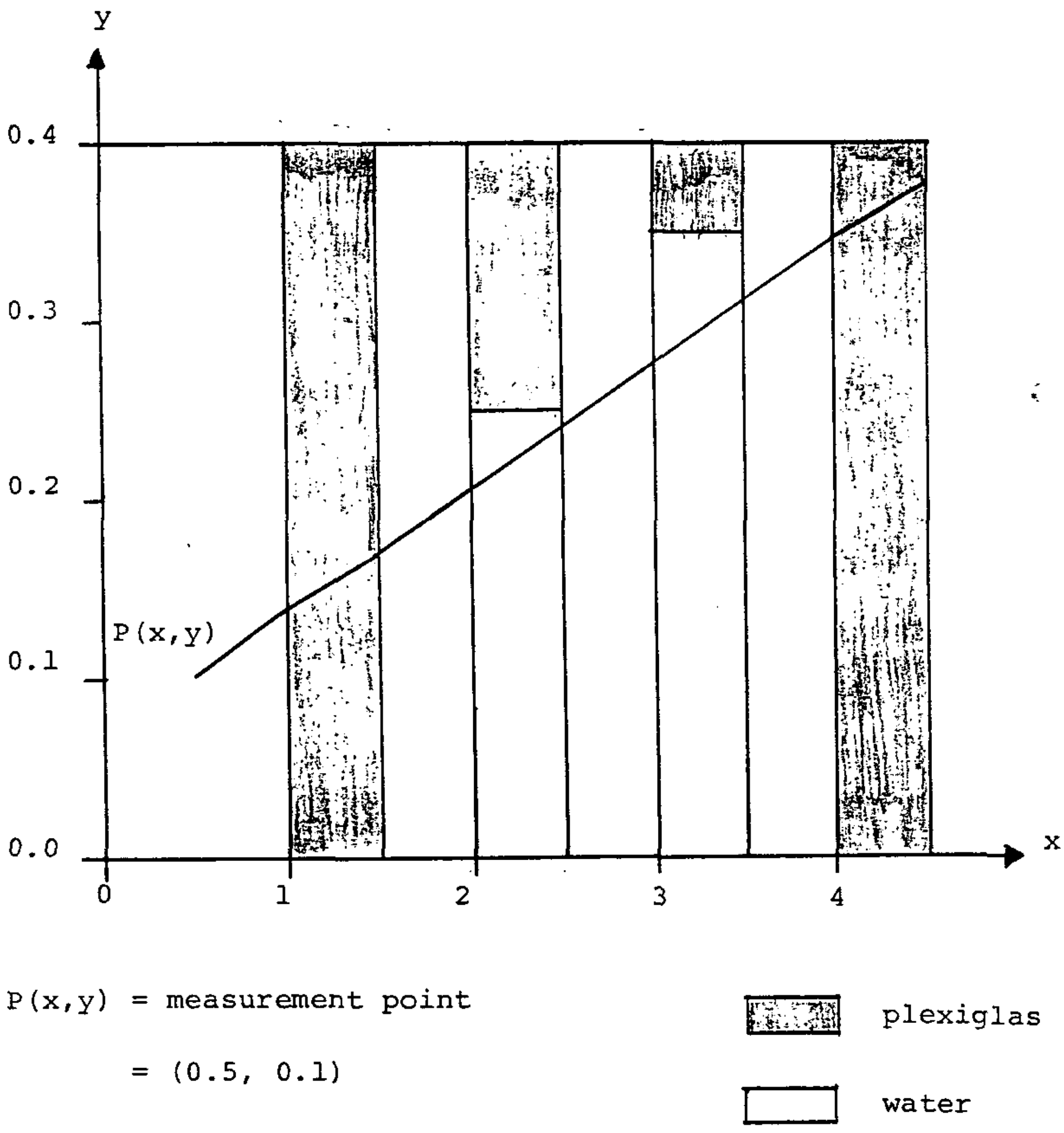



FIGURE 5c

FIGURE 6

A PLEXIGLAS MODEL WITH AN ANGLED SURFACE

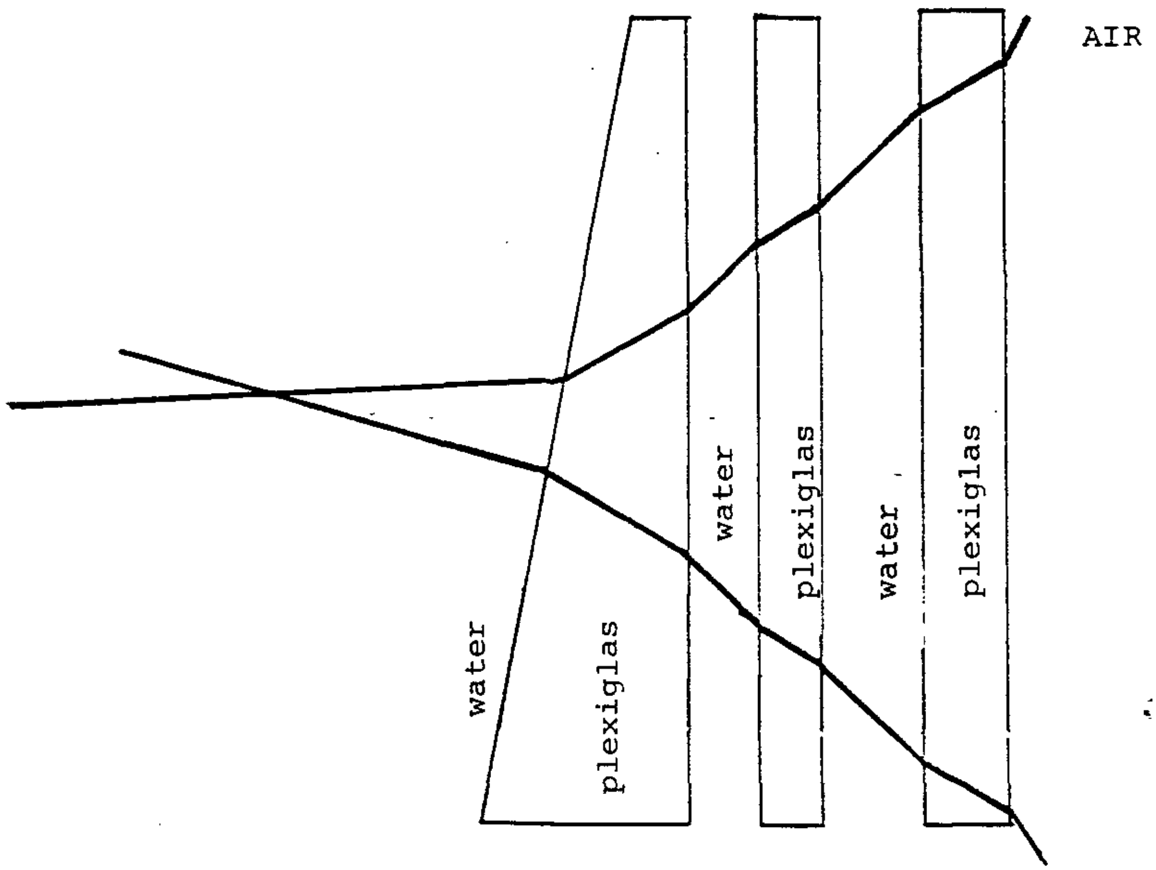



FIGURE 7

CLOSE UP OF THE PLEXIGLAS MODEL

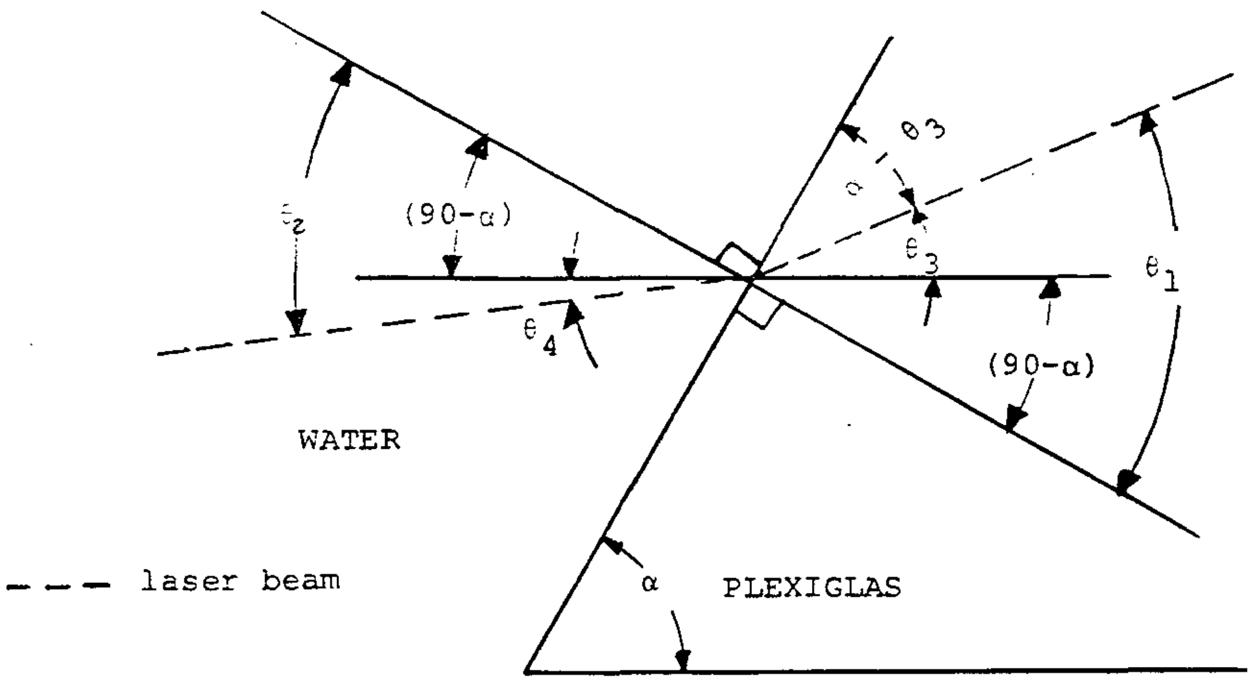


FIGURE 8

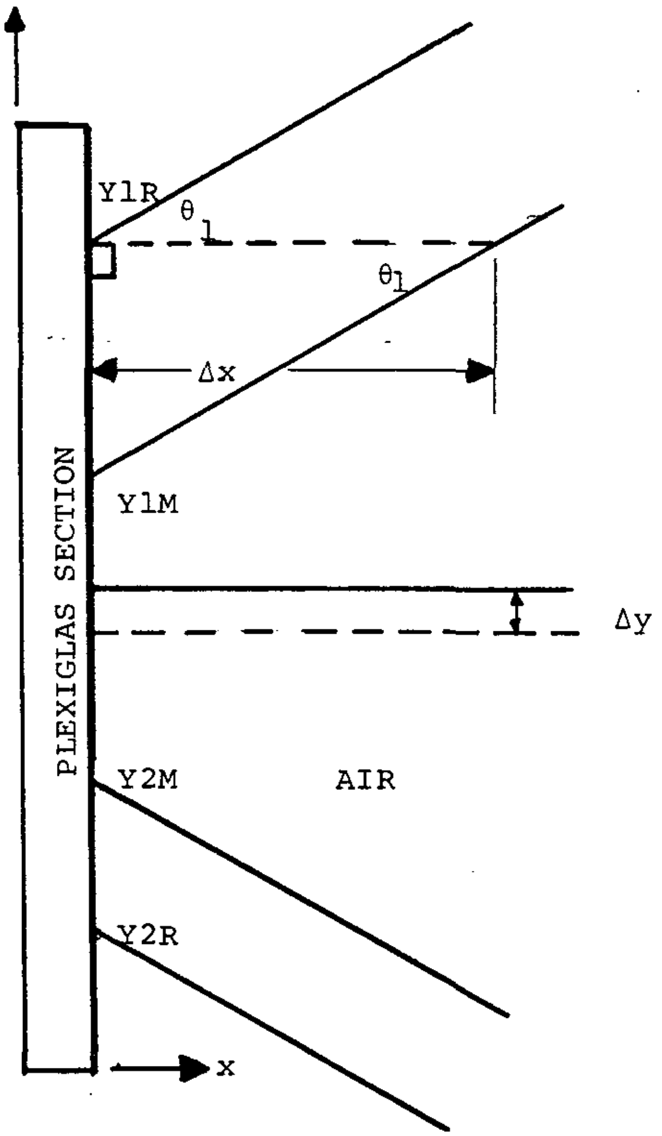
$\theta_1$ = Half angle of the laser in air

Y1R = Y coordinate of the first reference beam

Y2R = Y coordinate of the second reference beam

Y1M = Y coordinate of the first measurement point beam

Y2M = Y coordinate of the second measurement point beam.

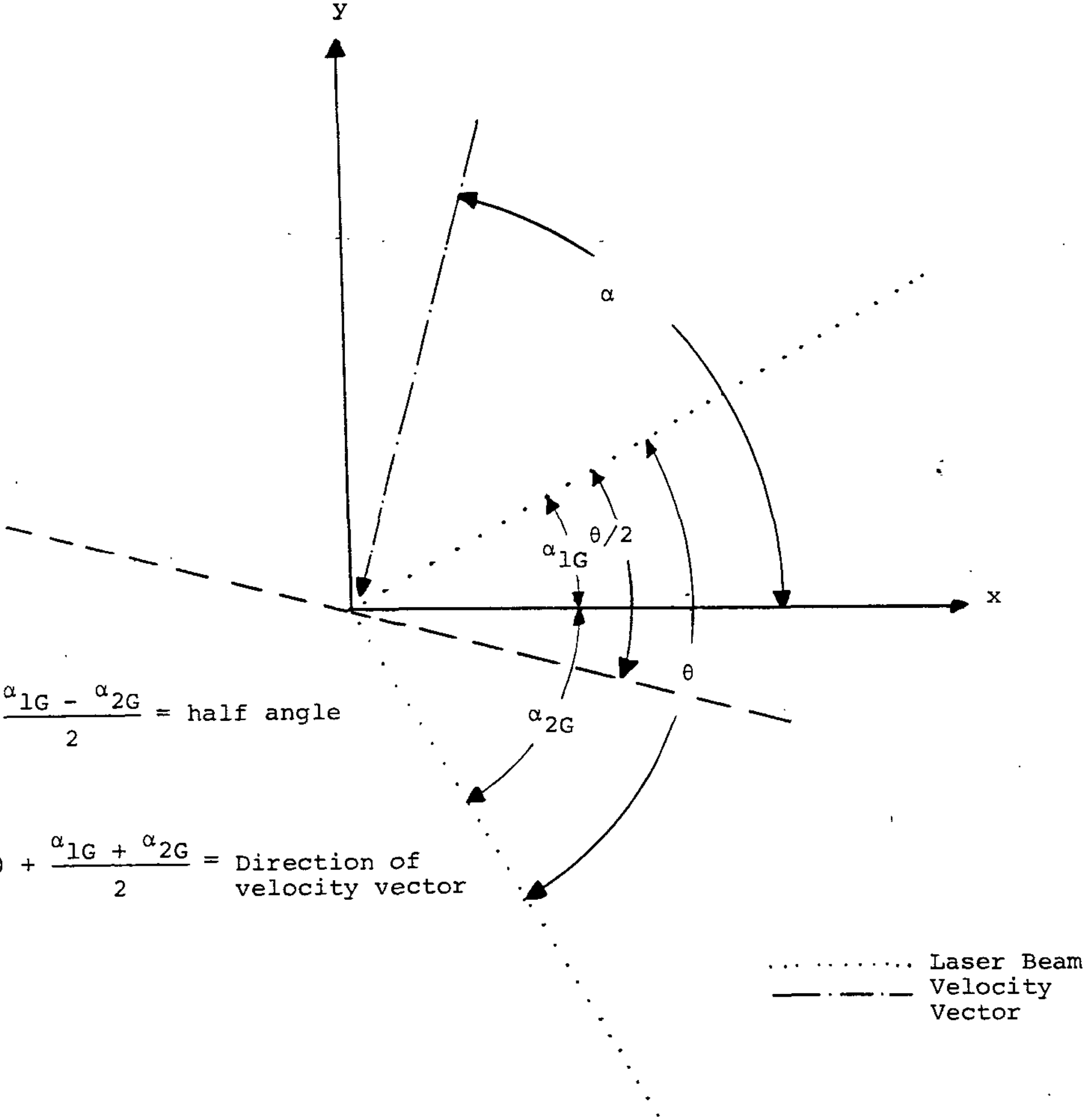



$$\Delta x = \frac{\dfrac{(Y1R - Y2R)}{2} - \dfrac{(Y1M - Y2M)}{2}}{\tan \theta_1}$$

$$\Delta y = \frac{(Y1M + Y2M)}{2} - \frac{(Y1R + Y2R)}{2}$$

FIGURE 9

LDA OPTICAL PARAMETERS




$$\frac{\theta}{2} = \frac{\alpha_{1G} - \alpha_{2G}}{2} = \text{half angle}$$

$$\alpha = 90 + \frac{\alpha_{1G} + \alpha_{2G}}{2} = \text{Direction of velocity vector}$$

FLOW CHART FOR PROGRAM AXIAL

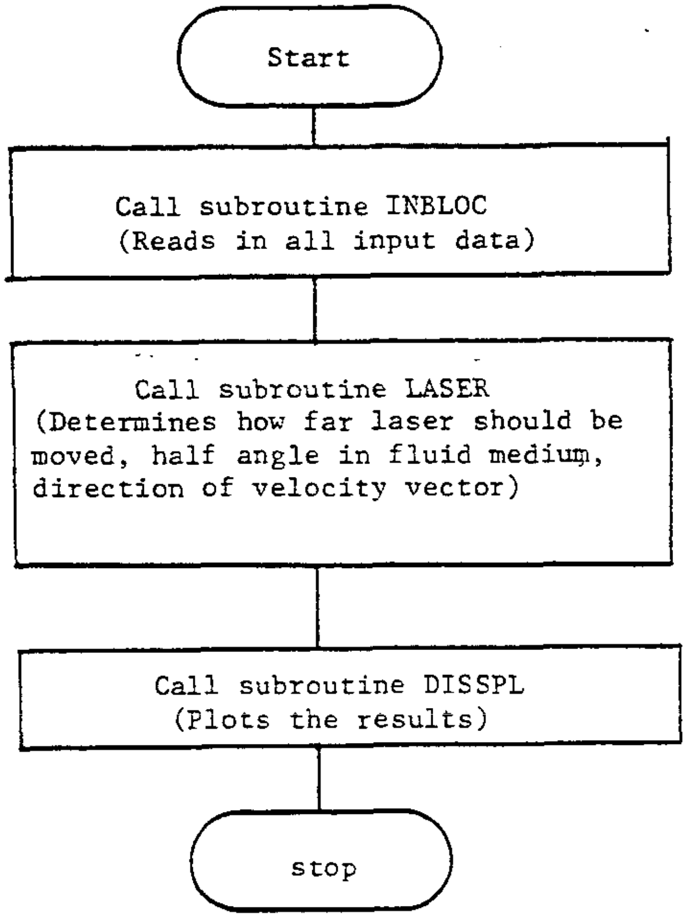



FIGURE 10

FLOW CHART FOR SUBROUTINE LASER

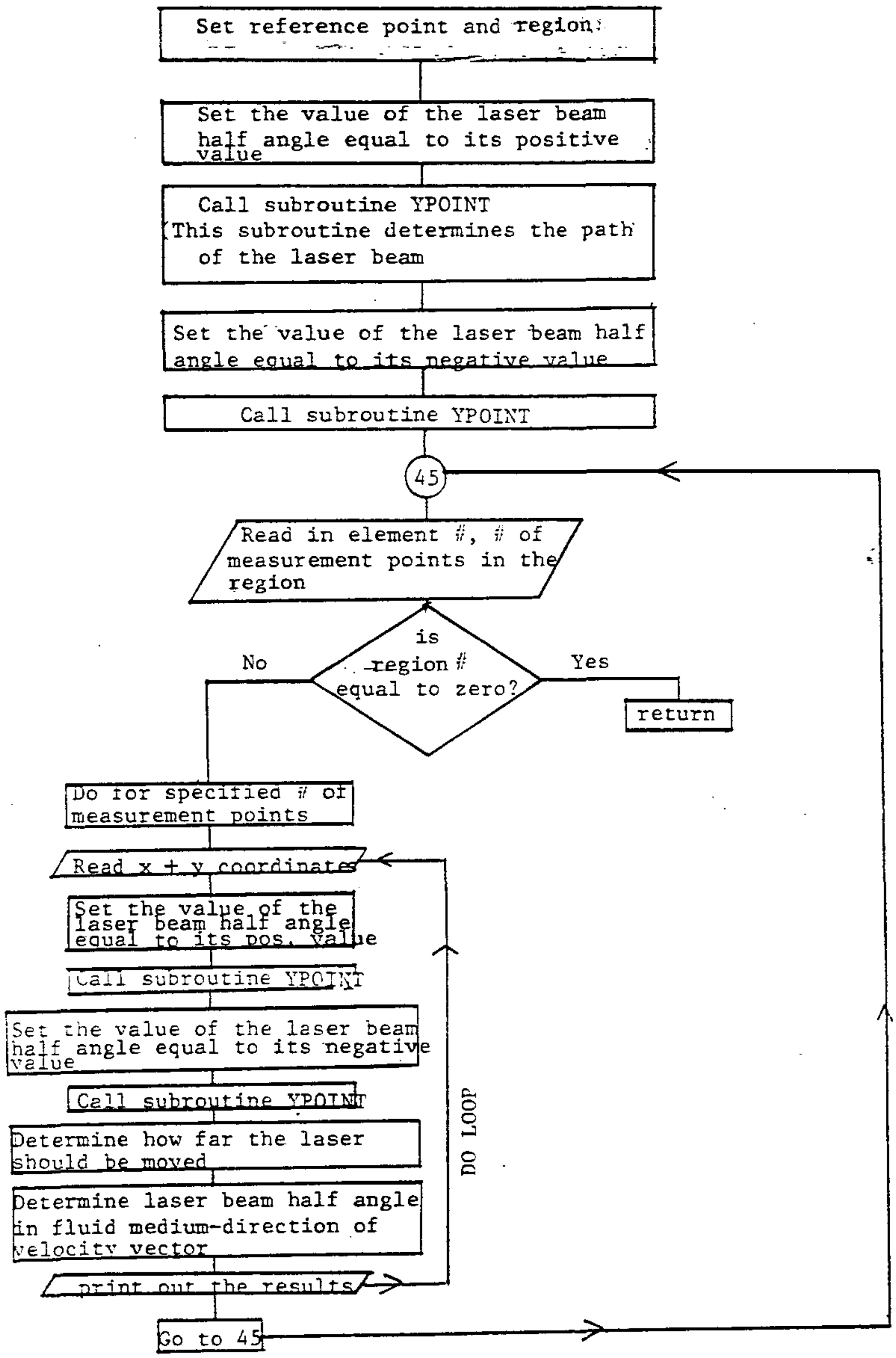



FIGURE 11

FLOW CHART FOR SUBROUTINE Y POINT

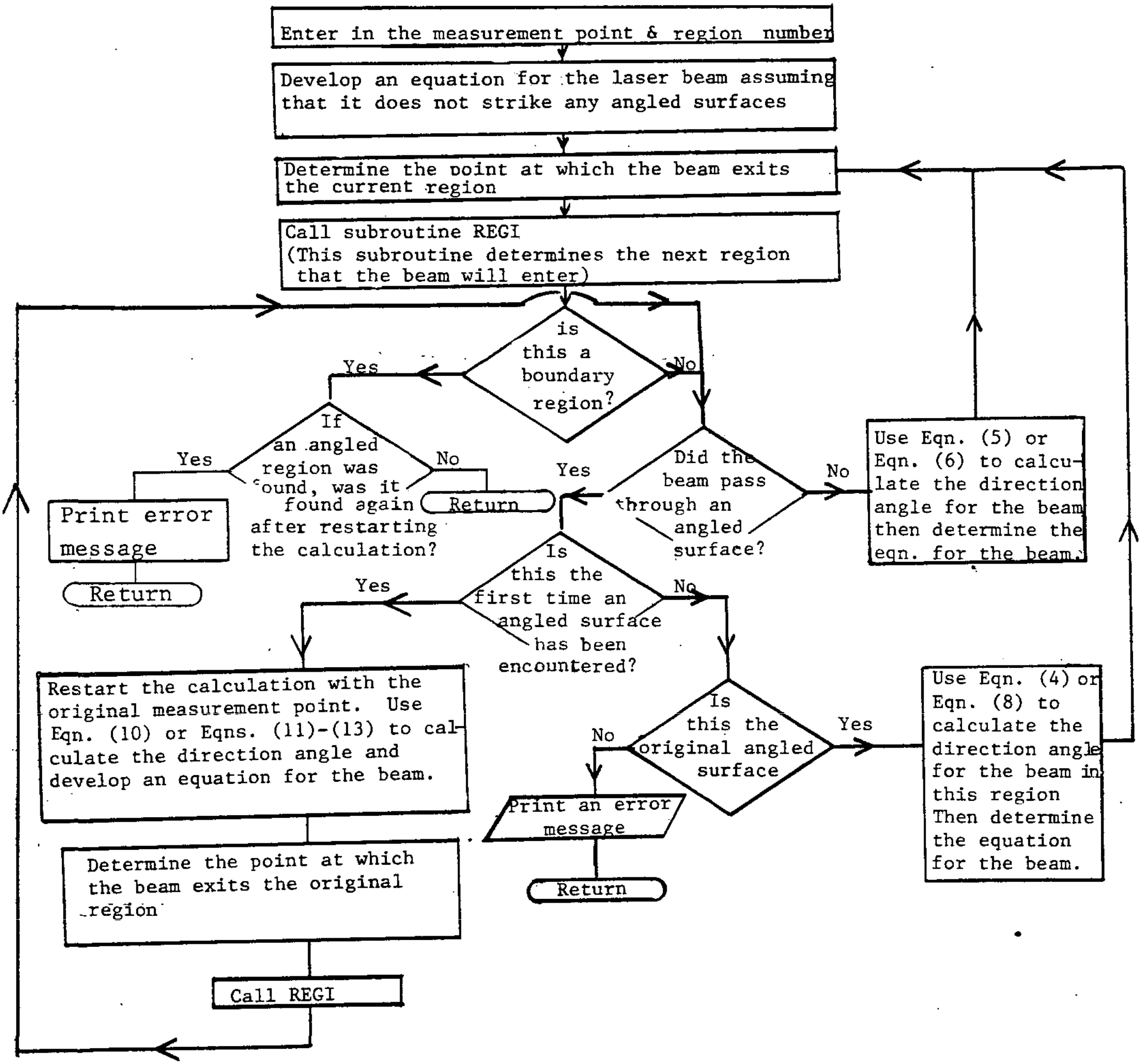



FIGURE 12

FLOW CHART FOR SUBROUTINE REGI

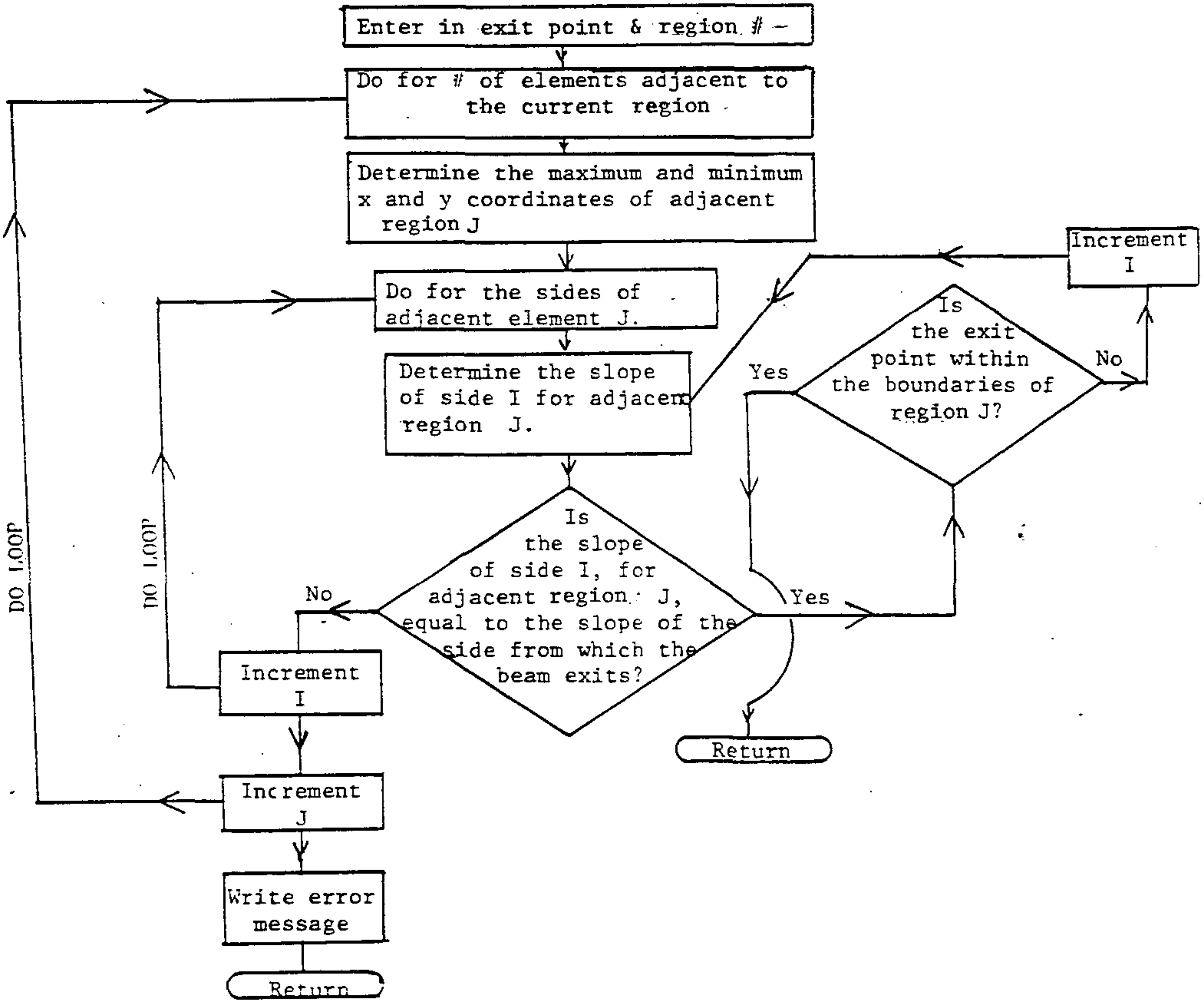
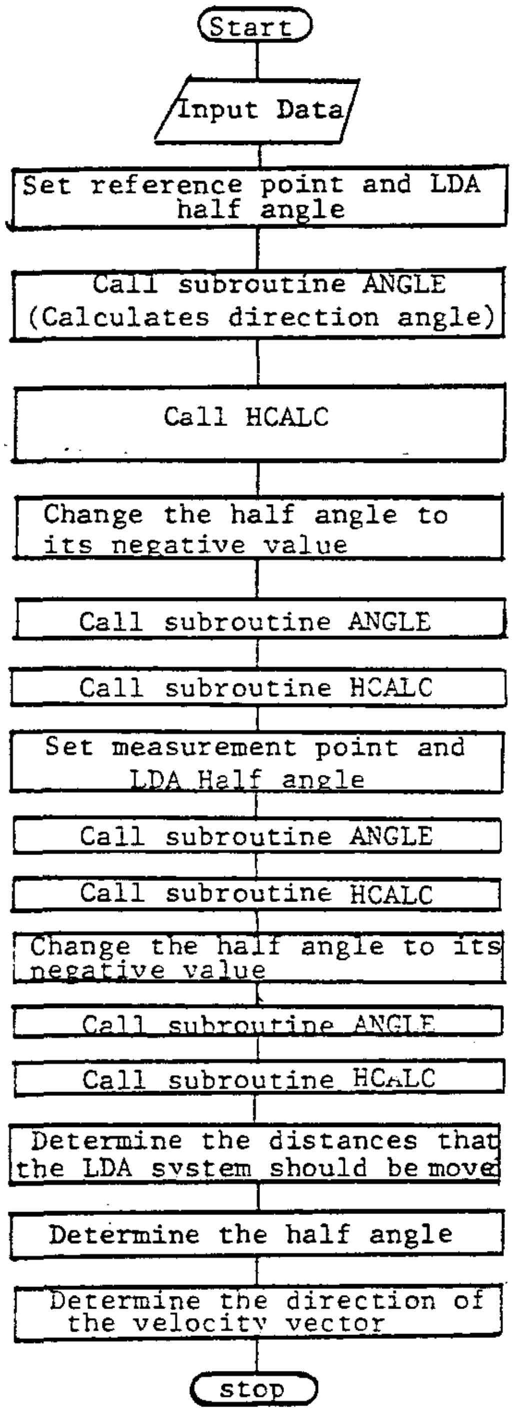



FIGURE 13

FLOW CHART FOR PROGRAM CYLINDER




Subroutine ANGLE is a subroutine which uses a linear interpolation technique to solve equations (14)-(26). Subroutine HCALC is a subroutine which solves equation (28).

FIGURE 14

CYLINDRICALLY LAYERED INTERFACES

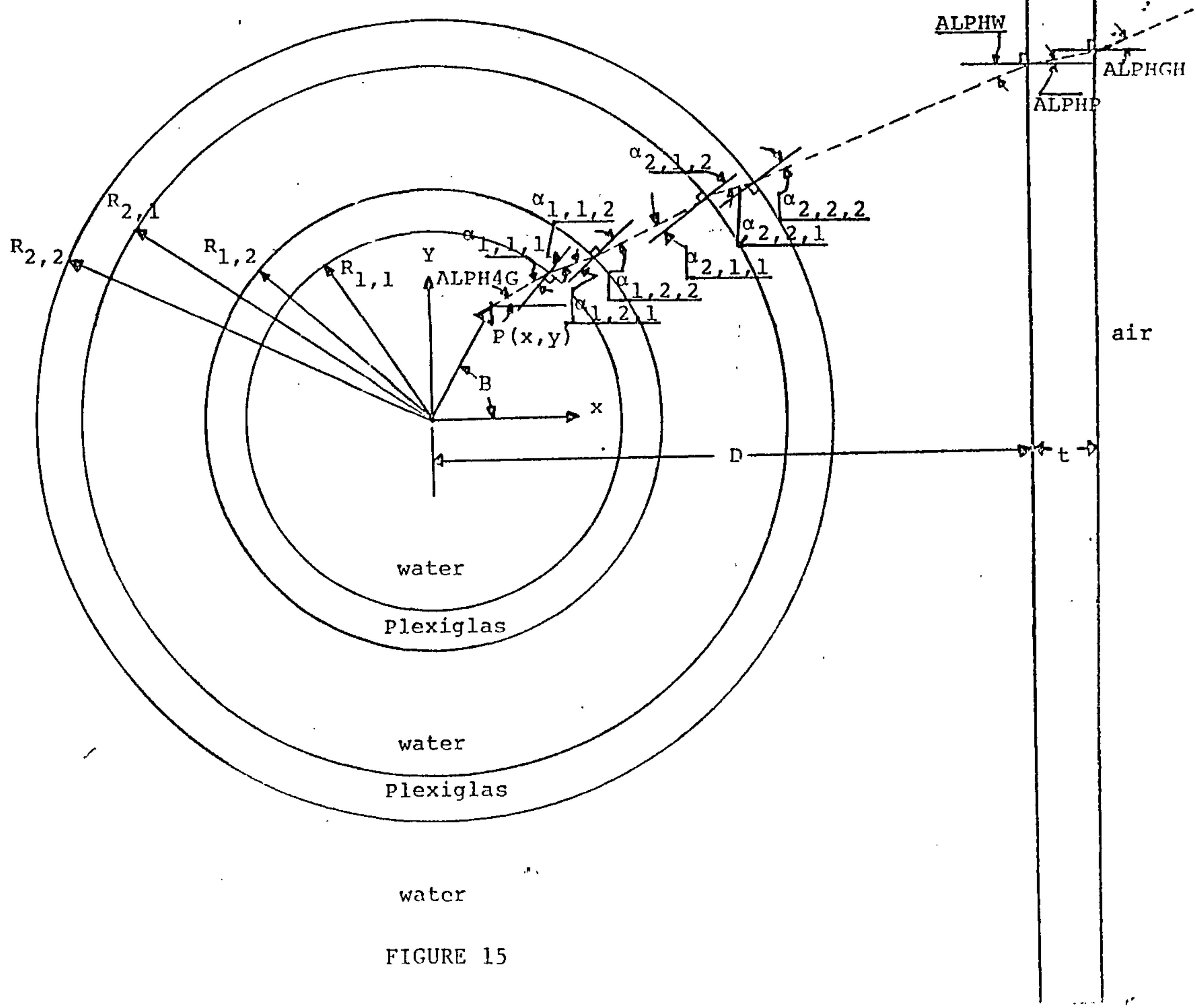



FIGURE 15

MARK 22 BOTTOM FITTING INSERT

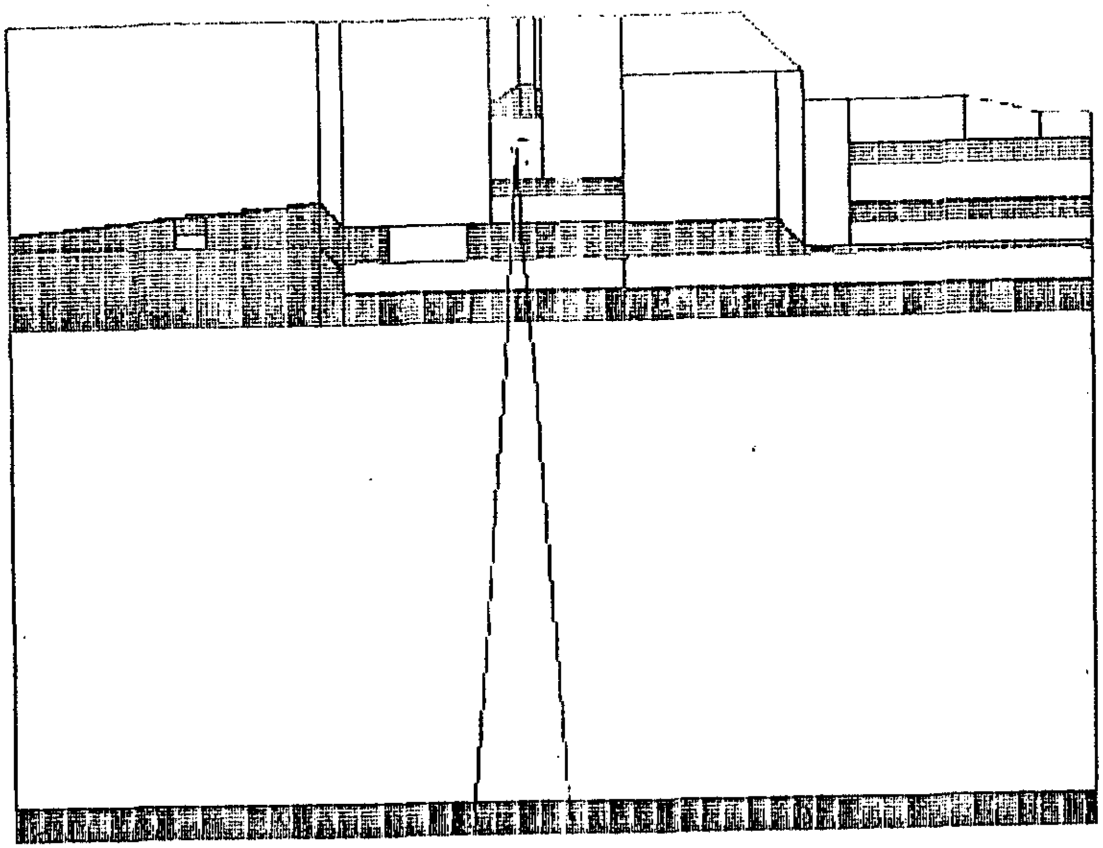


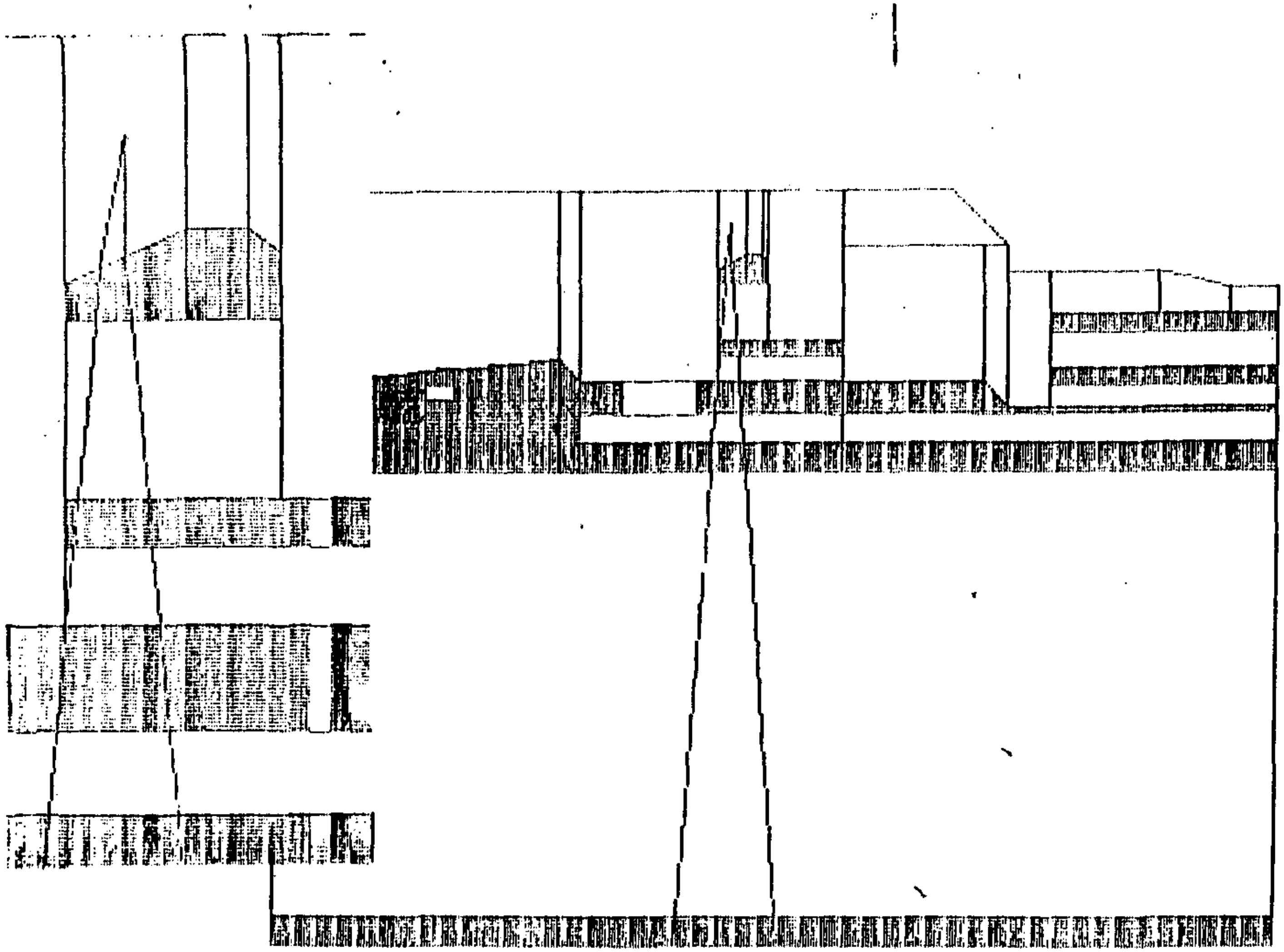
FIGURE 16

INDEX OF REFRACTION FOR WATER= 0.13300D+01 INDEX OF REFRACTION FOR PLEXIGLASS= 0.14900D+01
REFERENCE POINT IS X= 0.0 Y= 0.31600D+01
HALF ANGLE OF THE BEAM IN WATER WITH NO ANGLED SURFACES = 0.41326D+01

ALTERED HALF ANGLE = 0.41326D+01 ANGLE OF VELOCITY VECTOR(MEASURED FROM THE HORIZONTAL)= 0.90000D+02
HALF ANGLE OF THE LASER IN AIR 0.55000D+01
MEASUREMENT POINT IS X = 0.98550D+00 Y = 0.39450D+01
DELTA Y = 0.78500 DELTA X = 0.77251

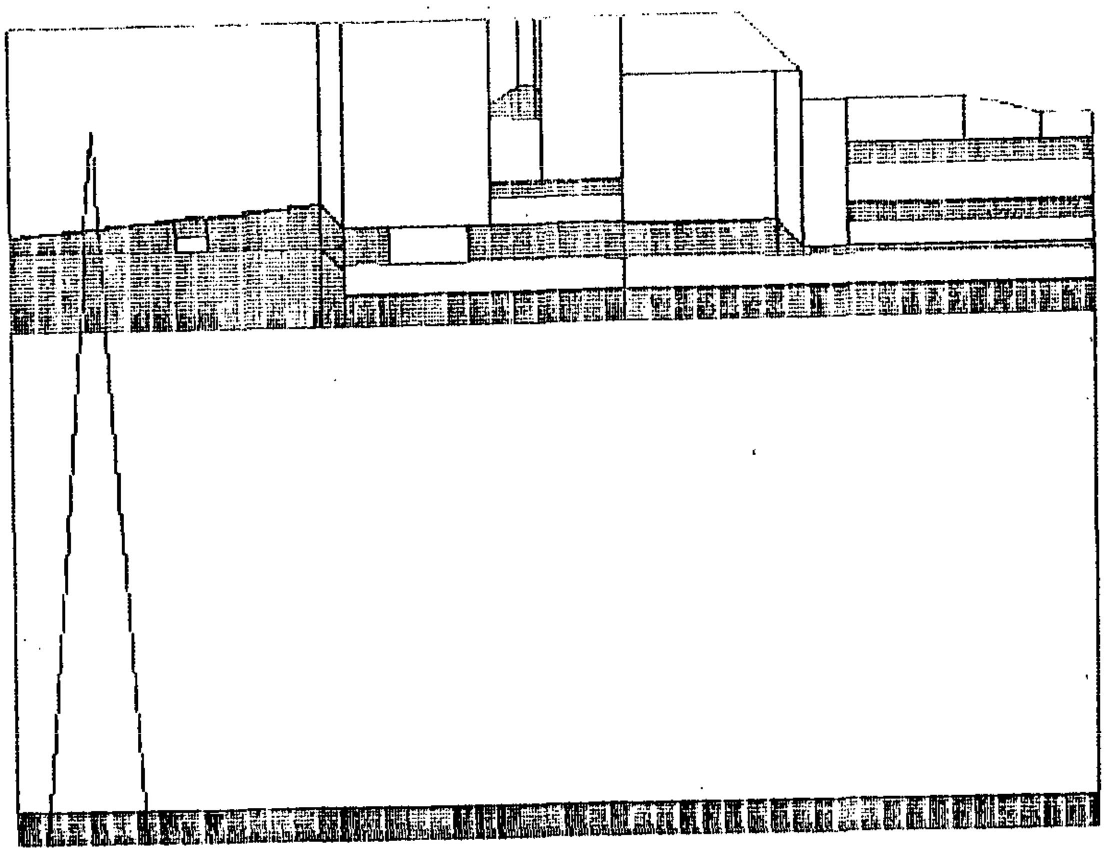
FIGURE 17

MARK 22 BOTTOM FITTING INSERT



INDEX OF REFRACTION FOR WATER= 0.13300D+01 INDEX OF REFRACTION FOR PLEXIGLASS= 0.14900D+01
REFERENCE POINT IS X= 0.0 Y= 0.31600D+01
HALF ANGLE OF THE BEAM IN WATER WITH NO ANGLED SURFACES = 0.41326D+01

ALTERED HALF ANGLE = 0.43222D+01 ANGLE OF VELOCITY VECTOR(MEASURED FROM THE HORIZONTAL)= 0.85904D+02
HALF ANGLE OF THE LASER IN AIR 0.55000D+01
MEASUREMENT POINT IS X = 0.25500D+00 Y = 0.38500D+01
DELTA Y = 0.66529 DELTA X = 0.21206



MARK 22 SECTION FITTING INSERT

FIGURE 18

INDEX OF REFRACTION FOR WATER= 0.13300D+01 INDEX OF REFRACTION FOR PLEXIGLASS= 0.14900D+01
REFERENCE POINT IS X= 0.0 Y= 0.31600D+01
HALF ANGLE OF THE BEAM IN WATER WITH NO ANGLED SURFACES = 0.41326D+01

ALTERED HALF ANGLE = 0.41366D+01 ANGLE OF VELOCITY VECTOR(MEASURED FROM THE HORIZONTAL)= 0.89394D+02
HALF ANGLE OF THE LASER IN AIR 0.55000D+01
MEASUREMENT POINT IS X = 0.79260D+00 Y = 0.63375D+00
DELTA Y = -2.53414 DELTA X = 0.63663

# FIGURE 19

```
REFE RENCE POINT: XO=      0.70000D+00      YO=    0.00000D+00
DATA POINT : X=      0.30000D+00 Y=      0.72000D+00
CYLINDER       1  INNER RADIUS=      0.80000D+00  OUTER RADIUS=      0.10000D+01
CYLINDER       2  INNER RADIUS=      0.17000D+01  OUTER RADIUS=      0.20000D+01
CYLINDER       3  INNER RADIUS=      0.27000D+01  OUTER RADIUS=      0.30000D+01
CYLINDER       4  INNER RADIUS=      0.37000D+01  OUTER RADIUS=      0.40000D+01
LASER MUST BE MOVED     -0.33026D+00 IN THE HORIZONTAL DIRECTION
 LASER MUST BE MOVED      0.75058D+00 IN THE VERTICAL DIRECTION
HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM     0.56435D+01
DIRECTION OF THE VELOCITY VECTOR      0.82702D+02
```

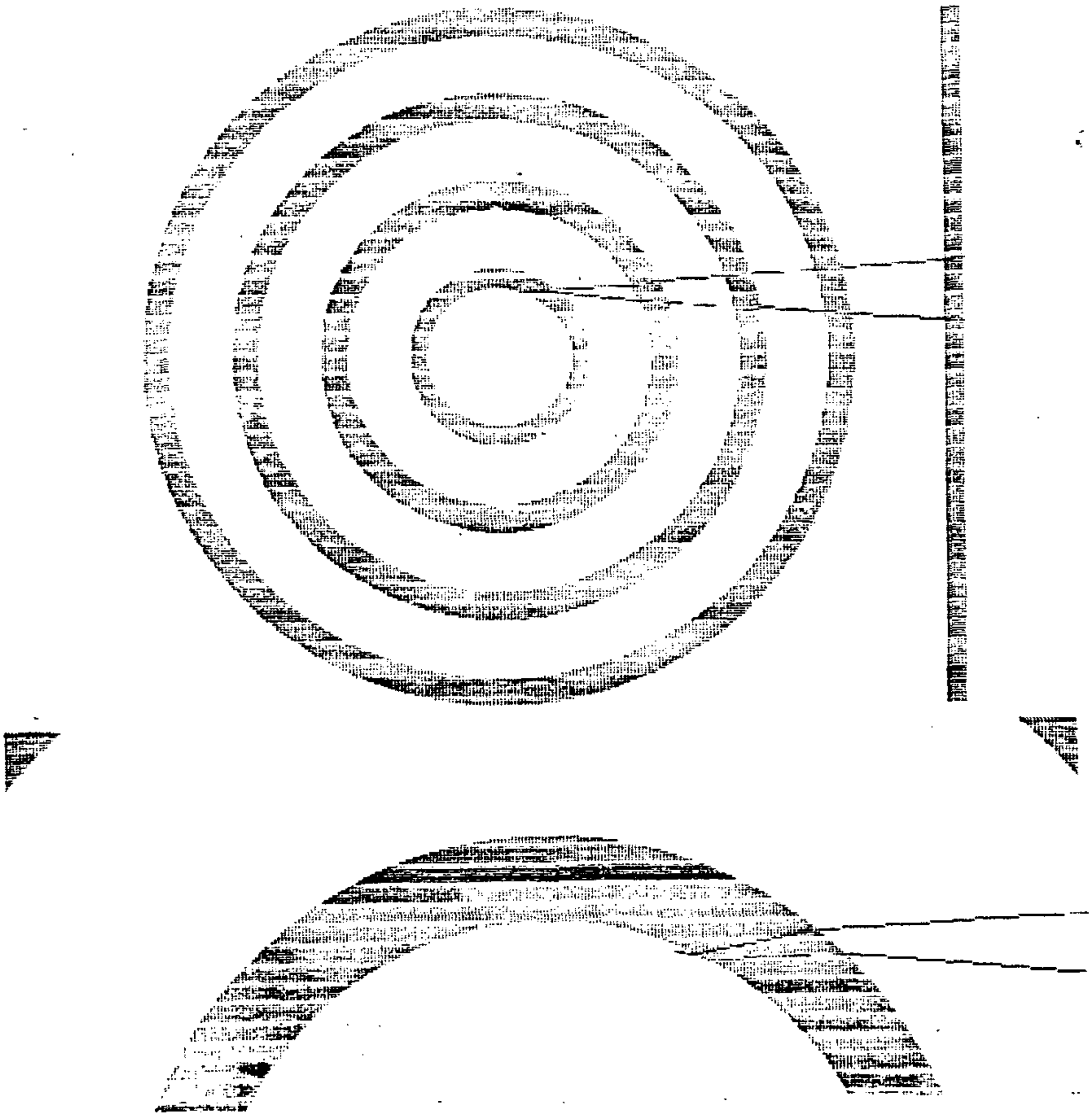


FIGURE 20

CYLINDRICAL TEST CASE

REFE RENCE POINT: XO= 0.70000D+00 YO= 0.0
DATA POINT : X= 0.30000D+00 Y= -0.72000D+00
CYLINDER 1 INNER RADIUS= 0.80000D+00 OUTER RADIUS= 0.10000D+01
CYLINDER 2 INNER RADIUS= 0.17000D+01 OUTER RADIUS= 0.20000D+01
CYLINDER 3 INNER RADIUS= 0.27000D+01 OUTER RADIUS= 0.30000D+01
CYLINDER 4 INNER RADIUS= 0.37000D+01 OUTER RADIUS= 0.40000D+01
LASER MUST BE MOVED -0.33026D+00 IN THE HORIZONTAL DIRECTION
LASER MUST BE MOVED -0.75058D+00 IN THE VERTICAL DIRECTION
HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM 0.56435D+01
DIRECTION OF THE VELOCITY VECTOR 0.97297D+02

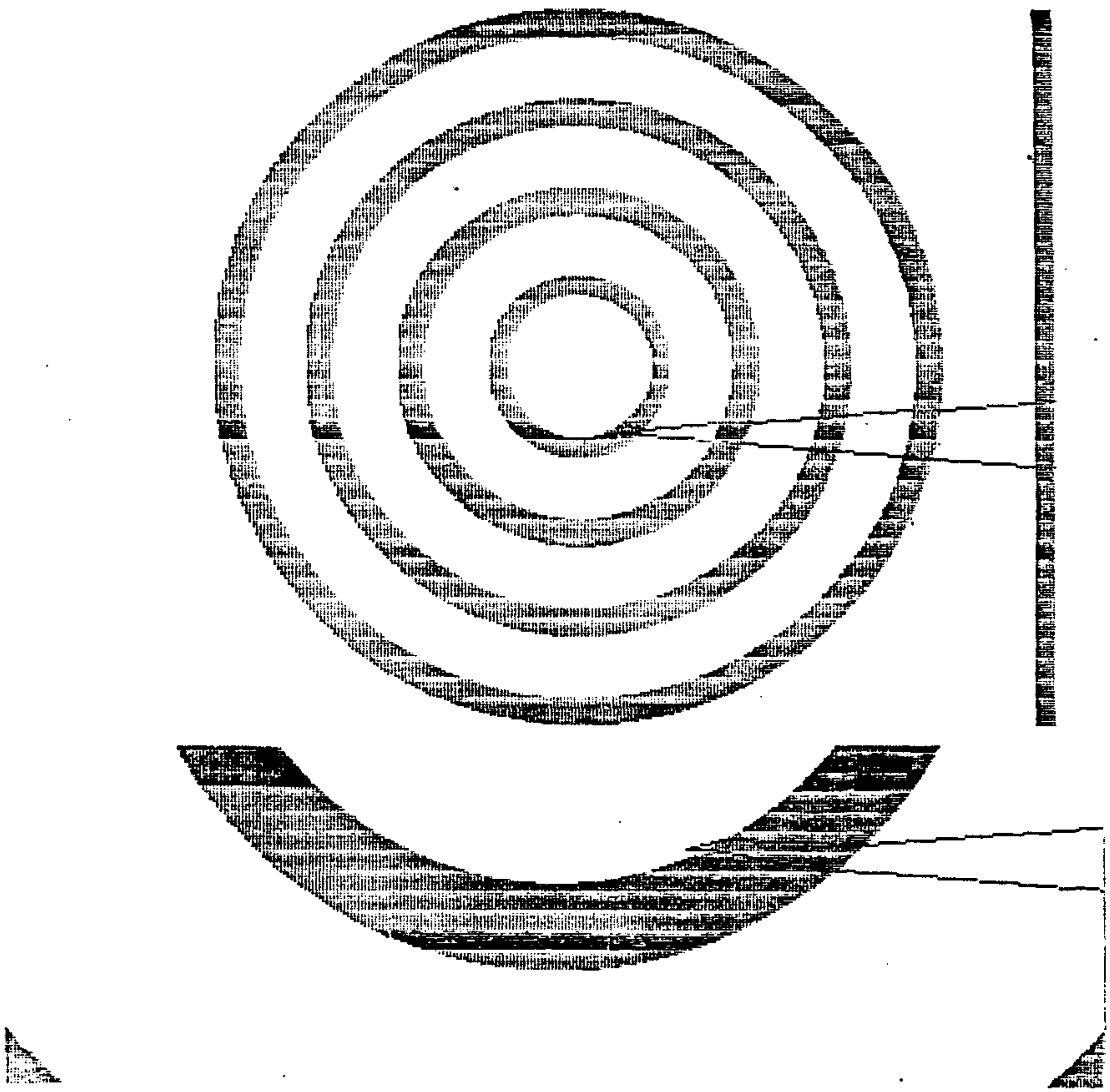
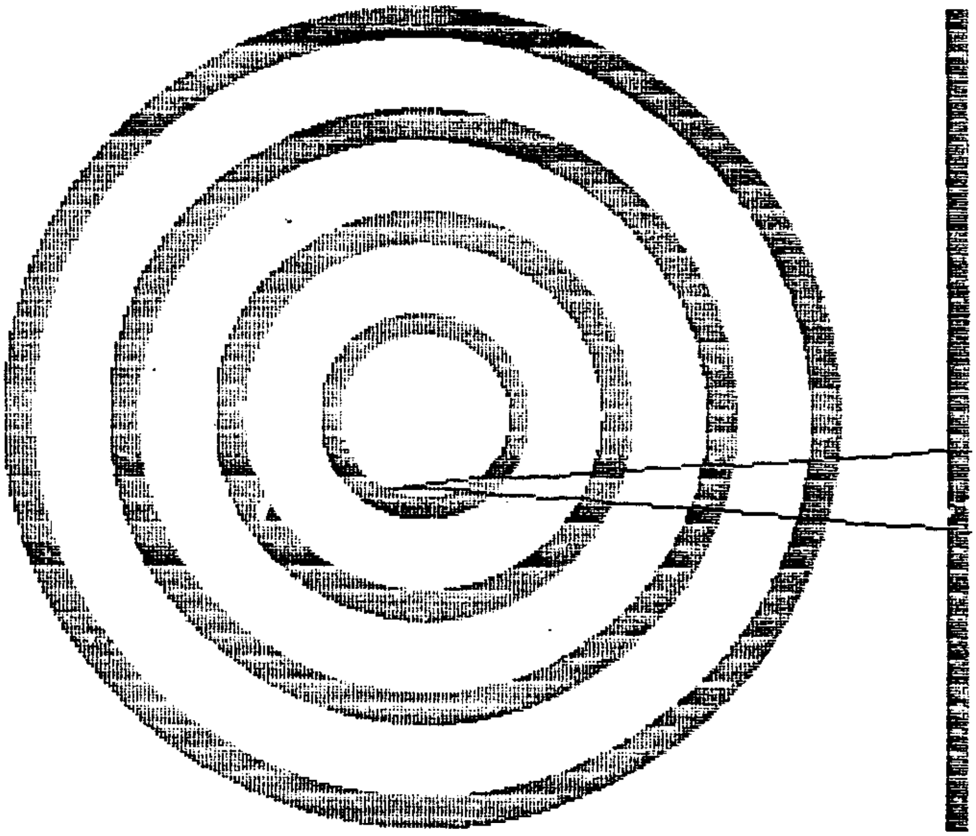


FIGURE 21

CYLINDRICAL TEST CASE

```
REFE RENCE POINT: X0=     0.70000D+00        Y0=    0.0
DATA POINT : X=     -0.30000D+00 Y=     -0.72000D+00
CYLINDER       1  INNER RADIUS=      0.80000D+00  OUTER RADIUS=      0.10000D+01
CYLINDER       2  INNER RADIUS=      0.17000D+01  OUTER RADIUS=      0.20000D+01
CYLINDER       3  INNER RADIUS=      0.27000D+01  OUTER RADIUS=      0.30000D+01
CYLINDER       4  INNER RADIUS=      0.37000D+01  OUTER RADIUS=      0.40000D+01
LASER MUST BE MOVED     -0.76652D+00 IN THE HORIZONTAL DIRECTION
 LASER MUST BE MOVED     -0.69459D+00 IN THE VERTICAL DIRECTION
HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM     0.34834D+01
DIRECTION OF THE VELOCITY VECTOR      0.94529D+02
```



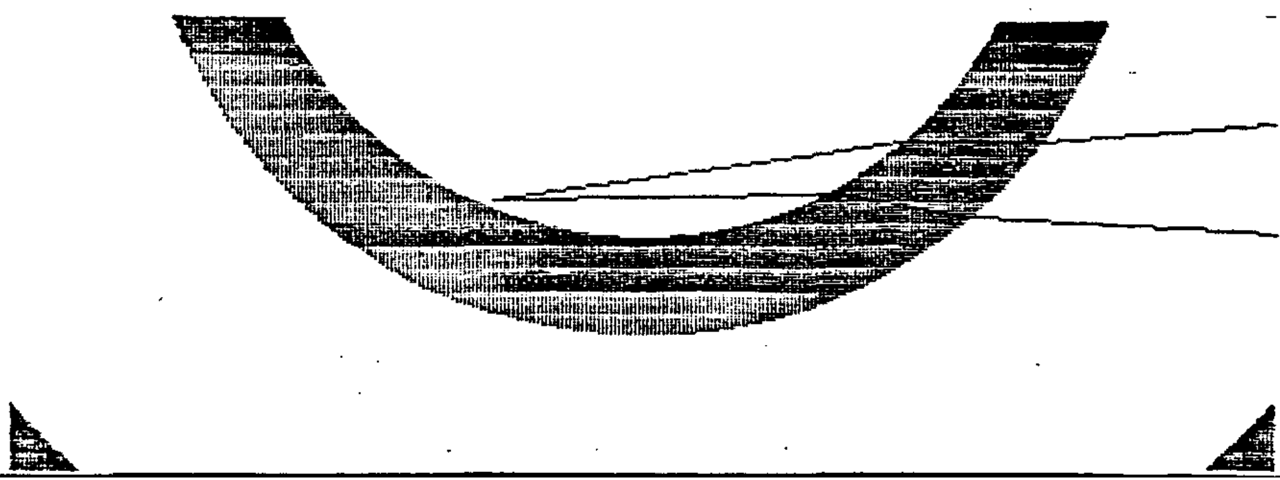


FIGURE 22

CYLINDRICAL TEST CASE

```
REFERENCE POINT: XO=     0.70000D+00      YO=    0.0
DATA POINT : X=    -0.30000D+00 Y=     0.72000D+00
CYLINDER      1  INNER RADIUS=      0.80000D+00  OUTER RADIUS=      0.10000D+01
CYLINDER      2  INNER RADIUS=      0.17000D+01  OUTER RADIUS=      0.20000D+01
CYLINDER      3  INNER RADIUS=      0.27000D+01  OUTER RADIUS=      0.30000D+01
CYLINDER      4  INNER RADIUS=      0.37000D+01  OUTER RADIUS=      0.40000D+01
LASER MUST BE MOVED    -0.76652D+00 IN THE HORIZONTAL DIRECTION
 LASER MUST BE MOVED     0.69459D+00 IN THE VERTICAL DIRECTION
HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM     0.34834D+01
DIRECTION OF THE VELOCITY VECTOR     0.85471D+02
```

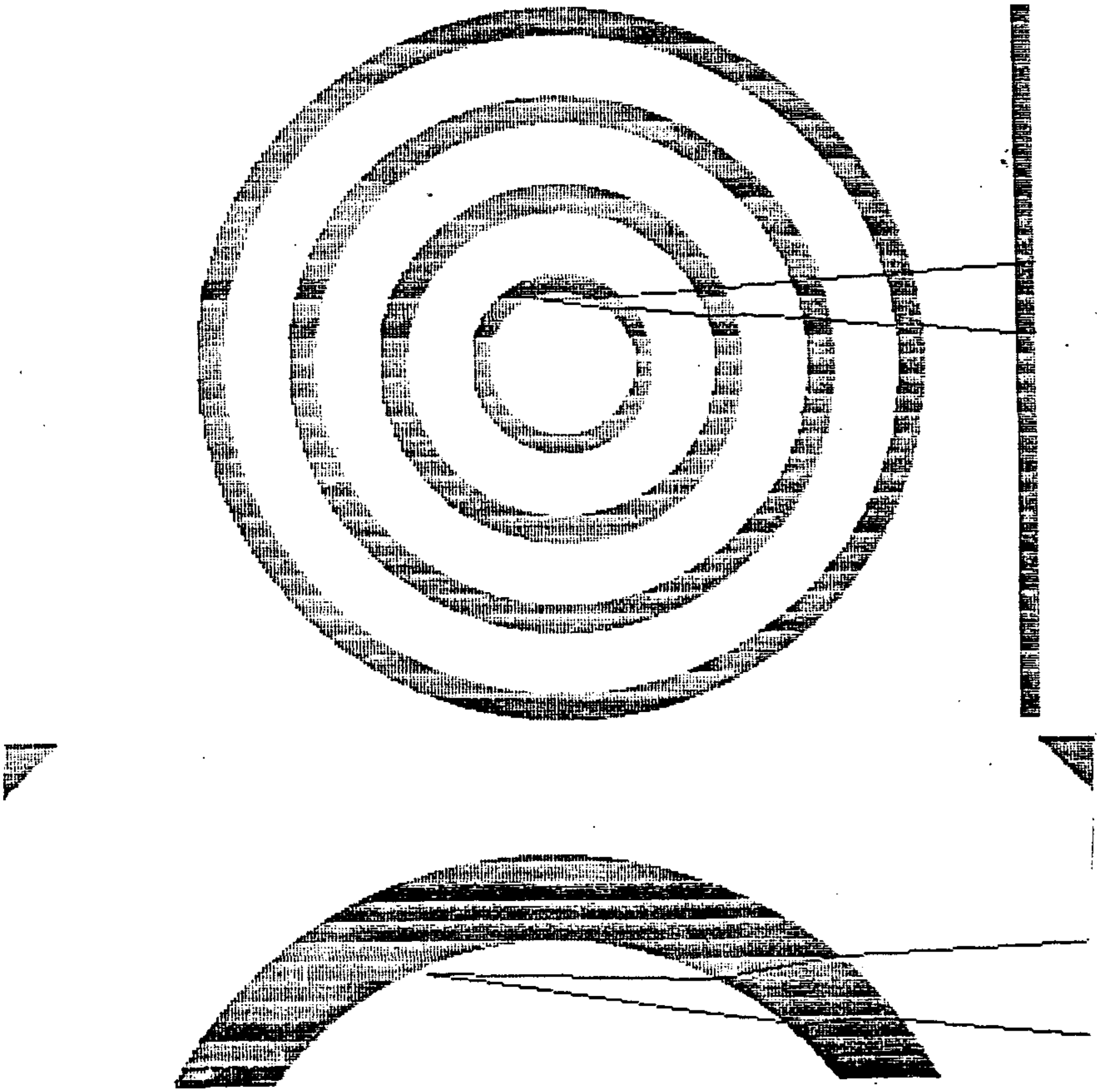


FIGURE 23

CYLINDRICAL TEST CASE

```
REFERENCE POINT: XO=     0.70000D+00       YO=    0.00000D+00
DATA POINT : X=     0.00000D+00 Y=     0.75000D+00
CYLINDER      1  INNER RADIUS=      0.80000D+00  OUTER RADIUS=     0.10000D+01
CYLINDER      2  INNER RADIUS=      0.17000D+01  OUTER RADIUS=     0.20000D+01
CYLINDER      3  INNER RADIUS=      0.27000D+01  OUTER RADIUS=     0.30000D+01
CYLINDER      4  INNER RADIUS=      0.37000D+01  OUTER RADIUS=     0.40000D+01
LASER MUST BE MOVED    -0.60073D+00 IN THE HORIZONTAL DIRECTION
 LASER MUST BE MOVED     0.74530D+00 IN THE VERTICAL DIRECTION
HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM    0.37691D+01
DIRECTION OF THE VELOCITY VECTOR     0.83360D+02
```

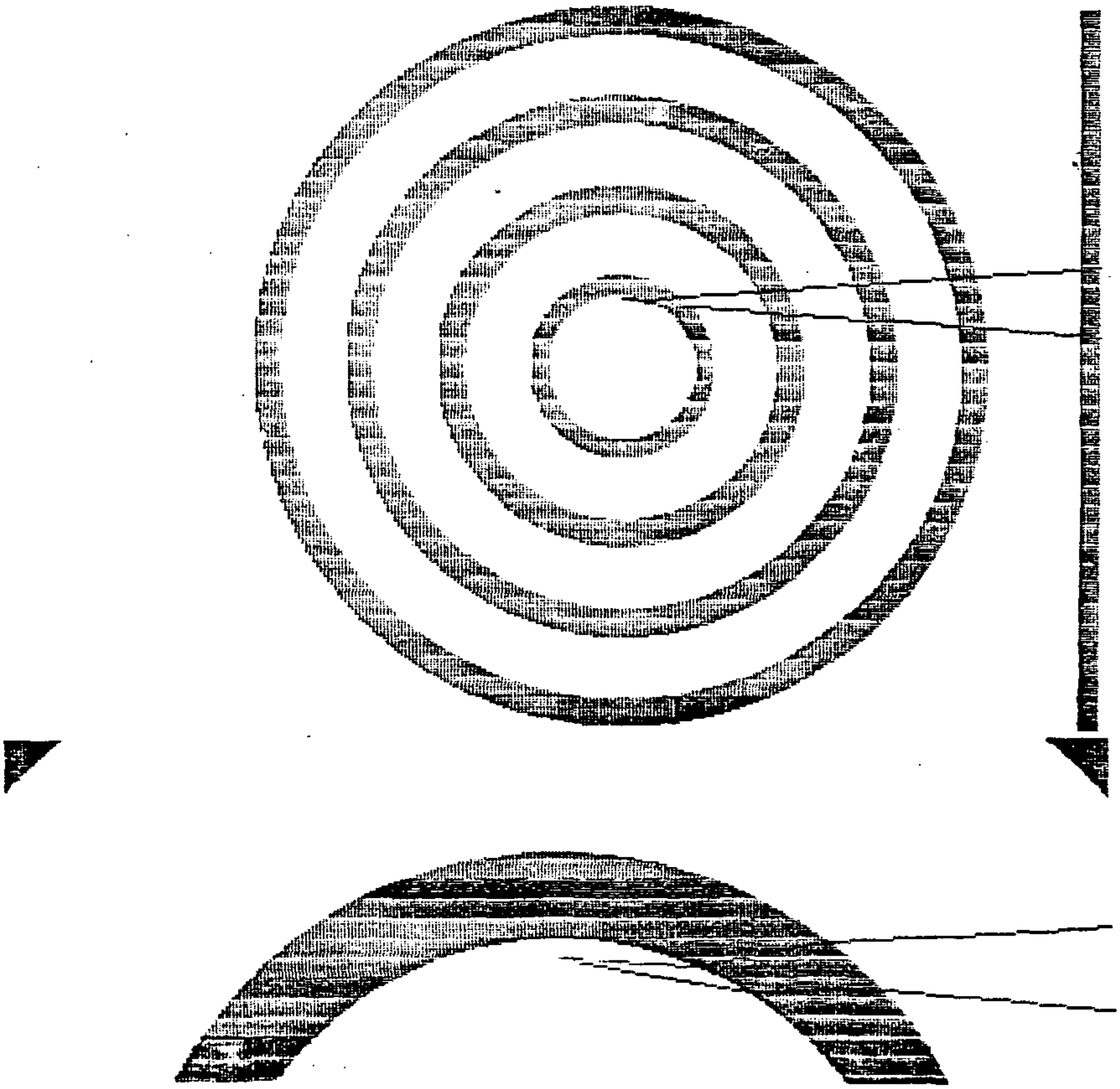


FIGURE 24

# CYLINDRICAL TEST CASE

REFE RENCE POINT: XO= 0.70000D+00 YO= 0.00000D+00

DATA POINT : X= 0.20000D+00 Y= 0.60000D+00

CYLINDER 1 INNER RADIUS= 0.80000D+00 OUTER RADIUS= 0.10000D+01

CYLINDER 2 INNER RADIUS= 0.17000D+01 OUTER RADIUS= 0.20000D+01

CYLINDER 3 INNER RADIUS= 0.27000D+01 OUTER RADIUS= 0.30000D+01

CYLINDER 4 INNER RADIUS= 0.37000D+01 OUTER RADIUS= 0.40000D+01

LASER MUST BE MOVED -0.40815D+00 IN THE HORIZONTAL DIRECTION

LASER MUST BE MOVED 0.60887D+00 IN THE VERTICAL DIRECTION

HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM 0.43006D+01

DIRECTION OF THE VELOCITY VECTOR 0.87220D+02

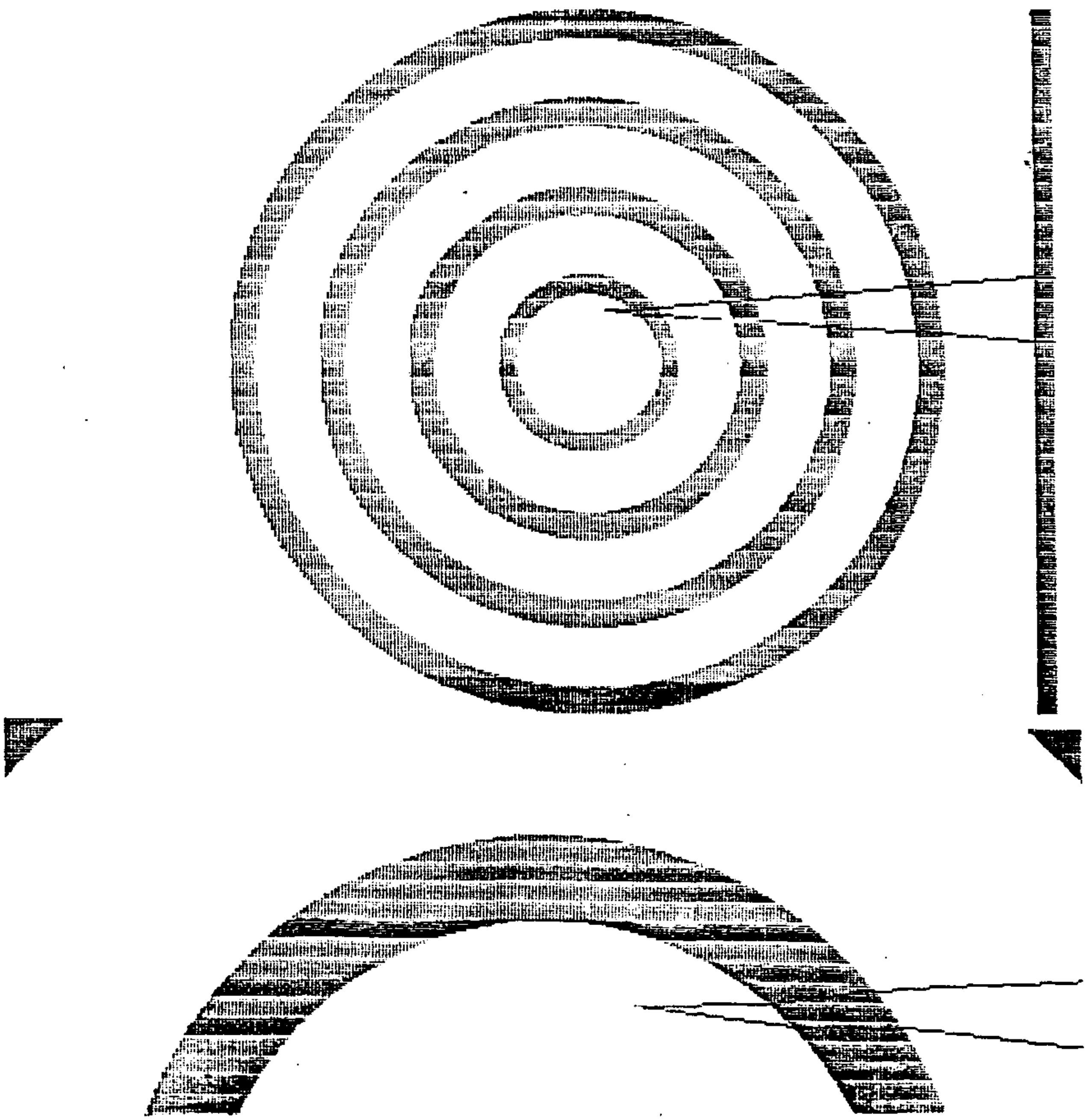
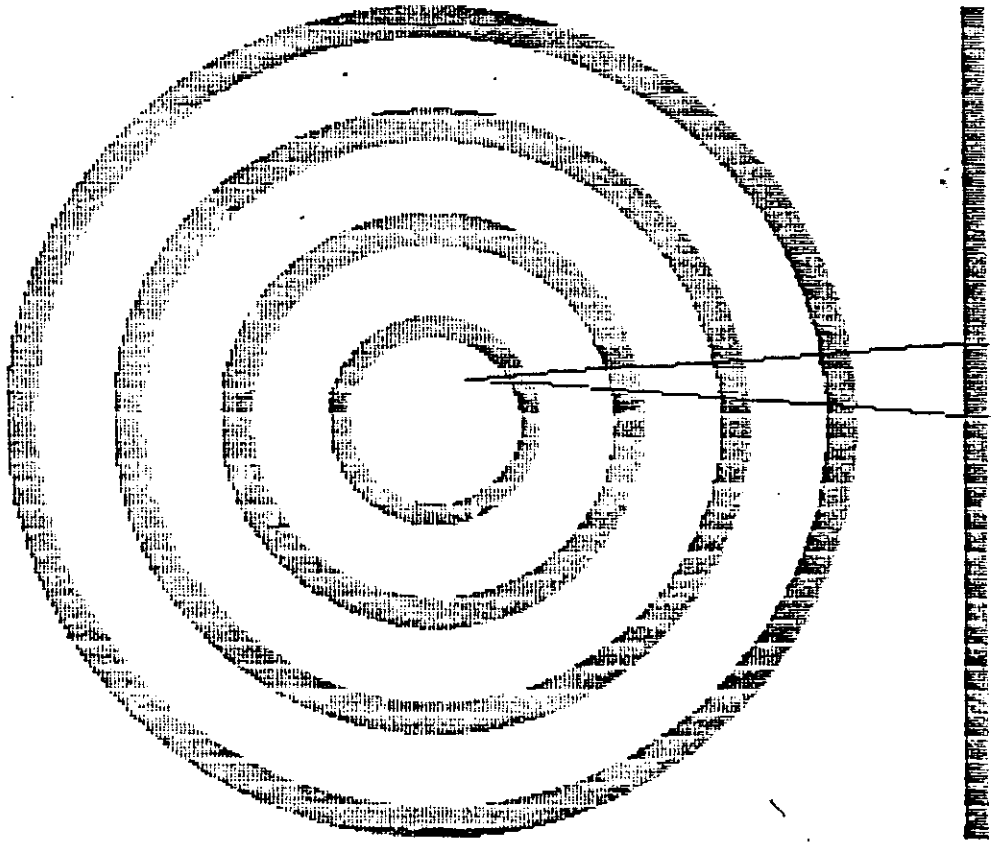


FIGURE 25

# CYLINDRICAL TEST CASE

```
REFE RENCE POINT: X0=      0.70000D+00        Y0=    0.0
DATA POINT : X=      0.30000D+00 Y=       0.40000D+00
CYLINDER       1   INNER RADIUS=       0.80000D+00   OUTER RADIUS=       0.10000D+01
CYLINDER       2   INNER RADIUS=       0.17000D+01   OUTER RADIUS=       0.20000D+01
CYLINDER       3   INNER RADIUS=       0.27000D+01   OUTER RADIUS=       0.30000D+01
CYLINDER       4   INNER RADIUS=       0.37000D+01   OUTER RADIUS=       0.40000D+01
LASER MUST BE MOVED     -0.31738D+00 IN THE HORIZONTAL DIRECTION
 LASER MUST BE MOVED     0.40632D+00 IN THE VERTICAL DIRECTION
HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM     0.42318D+01
DIRECTION OF THE VELOCITY VECTOR      0.88762D+02
```



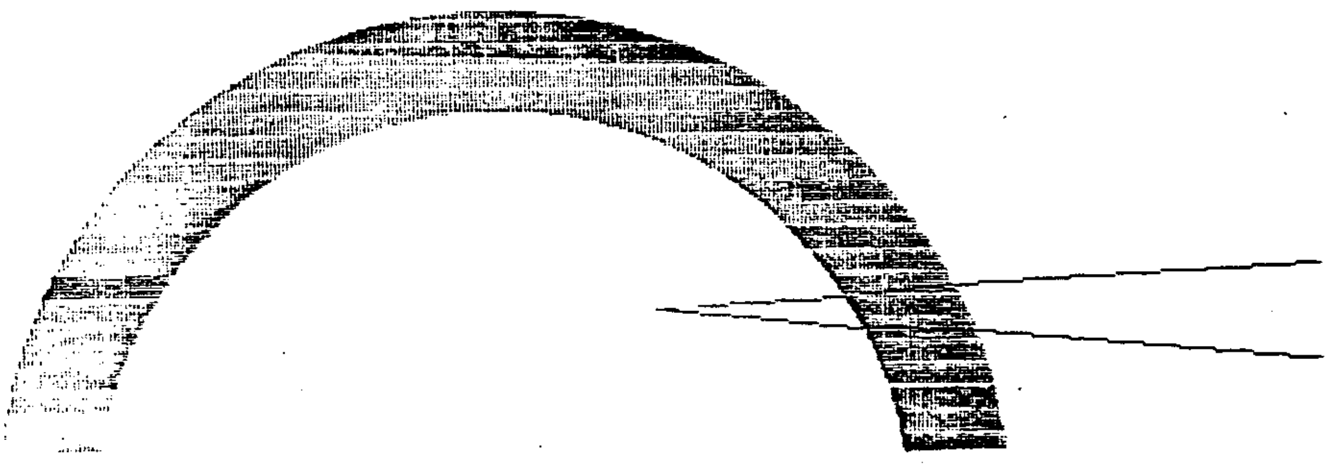


FIGURE 26

# CYLINDRICAL TEST CASE

```
REFE RENCE POINT: X0=      0.70000D+00        Y0=     0.0
DATA POINT  : X=     -0.10000D+00 Y=      0.20000D+00
CYLINDER       1  INNER RADIUS=      0.80000D+00  OUTER RADIUS=        0.10000D+01
CYLINDER       2  INNER RADIUS=      0.17000D+01  OUTER RADIUS=        0.20000D+01
CYLINDER       3  INNER RADIUS=      0.27000D+01  OUTER RADIUS=        0.30000D+01
CYLINDER       4  INNER RADIUS=      0.37000D+01  OUTER RADIUS=        0.40000D+01
LASER MUST BE MOVED     -0.61735D+00 IN THE HORIZONTAL DIRECTION
 LASER MUST BE MOVED     0.19911D+00 IN THE VERTICAL DIRECTION
HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM     0.41121D+01
DIRECTION OF THE VELOCITY VECTOR     0.89493D+02
```

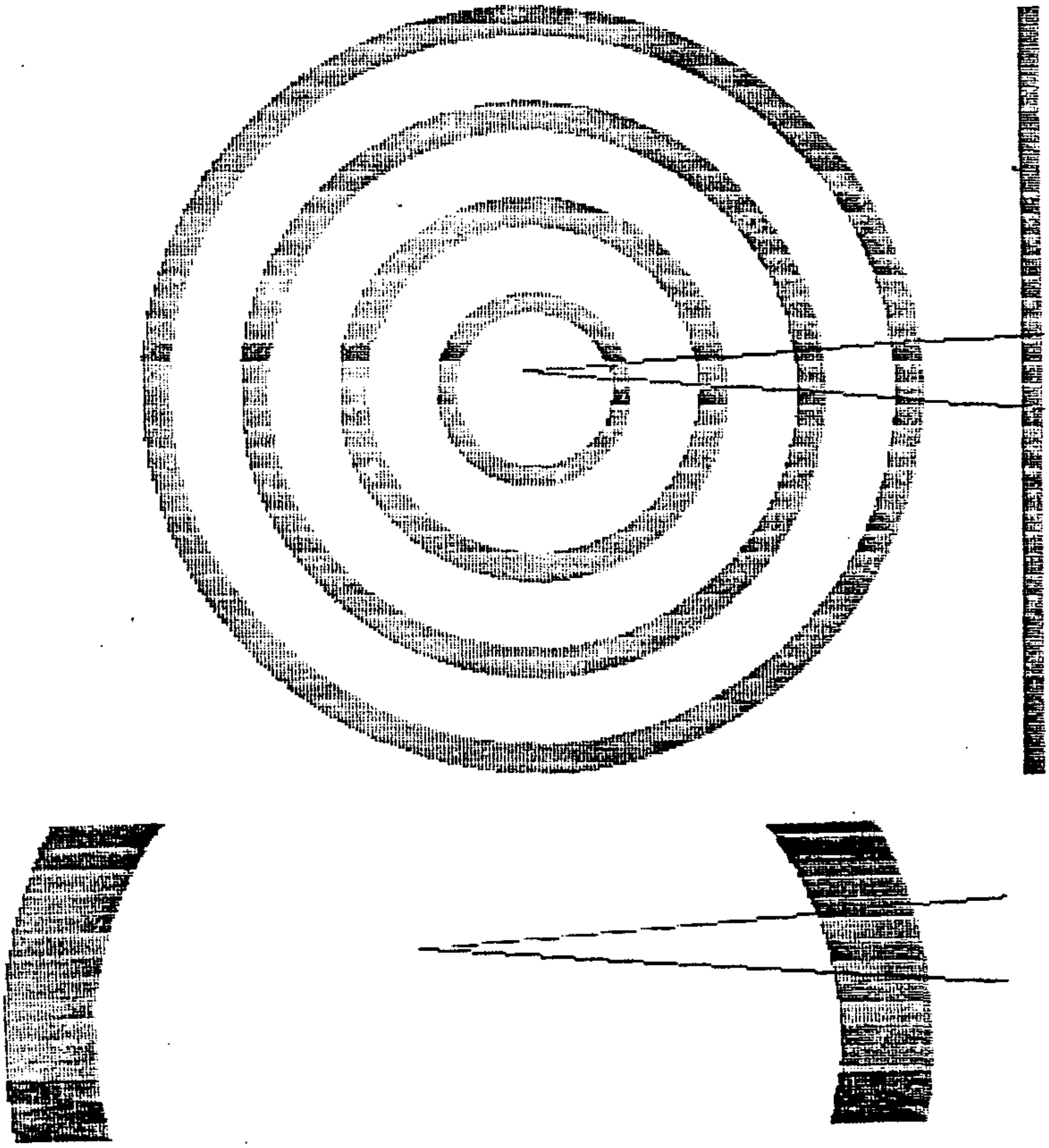
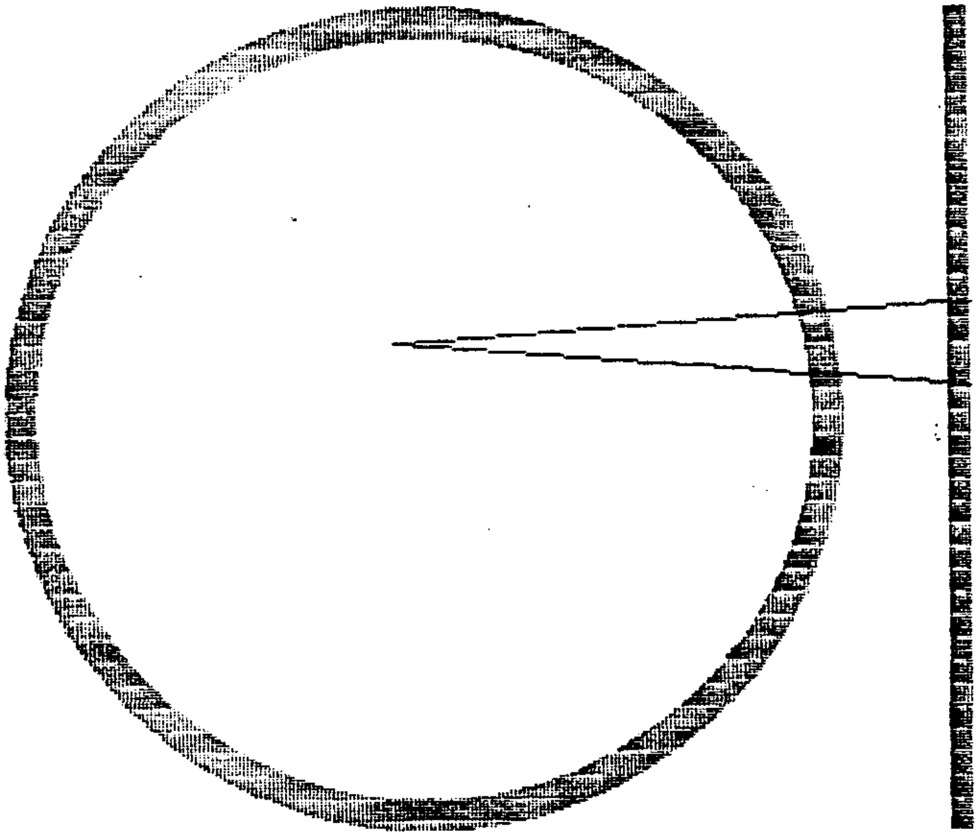


FIGURE 27

CYLINDRICAL TEST CASE

```
REFE RENCE POINT: X0=      0.70000D+00       Y0=    0.0
DATA POINT : X=     -0.30000D+00 Y=    0.72000D+00
CYLINDER       1  INNER RADIUS=      0.37000D+01  OUTER RADIUS=      0.40000D+01
CYLINDER       2  INNER RADIUS=      0.0               OUTER RADIUS=      0.0
CYLINDER       3  INNER RADIUS=      0.0               OUTER RADIUS=      0.0
CYLINDER       4  INNER RADIUS=      0.0               OUTER RADIUS=      0.0
LASER MUST BE MOVED     -0.75188D+00 IN THE HORIZONTAL DIRECTION
 LASER MUST BE MOVED      0.71951D+00 IN THE VERTICAL DIRECTION
HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM    0.41295D+01
DIRECTION OF THE VELOCITY VECTOR     0.89906D+02
```



# VII APPENDIX A

This appendix is a modified version of Algorithm I. This algorithm includes the effect that one angled surface may have on the path of a laser beam.

## ALGORITHM A-I

(1) The test section is descretized into quadralaterals.

(2) A point in a particular region is specified as the desired measurement point.

(3) Using equation (9), the direction angle in the region containing the measurement point is calculated. With the direction angle known, an equation for the beam is formulated.

(4) The point at which the beam exits from the current region is determined by the intersection of the beam equation with the region boundary.

(5) With the exit point known, a search of all regions adjacent to the current one is performed to determine the next region that the beam has entered.

(6) With the next region known, Snell's law is used to determine the direction angle in the new region (i.e., equation (5) or equation (6)). With the direction angle known, an equation for the beam in the new region is formulated.

(7) A check is made to determine if the side of the new region through which the laser beam passes is at an angle with respect to the vertical. If the beam has encountered an angled surface, the following steps are taken:

   a) A notation of the region containing the angled surface is made.

   b) The direction angle for the beam in the region containing the measurement point is recalculated using equation (10) or equations (11), (12), and (13). An equation for the beam is formulated using this new direction angle.

   c) Steps 4,5,6, & 7 are repeated until the original angled region is found.

   d) If the angled region is not found, no physical solution exists. It is not possible to make measurements at this point.

e) If the angled region is found, the direction angle for this region is calculated using equation (8) or equation (4).

f) The calculation resumes with step 8.

(8) A check is made to determine if this new region is a boundary region. If this region is a boundary region, the following steps are taken:

a) The exit point for this region is calculated.

b) The calculational procedure is terminated as the laser beam path has been determined.

(9) Steps (4) - (8) are repeated until the laser beam path has been determined.

# VIII. APPENDIX B

As previously discussed, finite regions can be used to calculate the path of a laser beam through a plexiglas model. This concept is based upon the idea of finite elements. Finite elements are currently being used in many areas of engineering. Programs to generate meshes necessary for use with finite element codes have been written. These mesh generating programs frequently require the user to first generate a very coarse mesh of many "super elements". These super elements are rather large; their only function is to define the geometry which will be discretized.

These super elements are sufficiently small for analyzing the path of a laser beam through a plexiglas model. Thus, the input data necessary for using a mesh generator code is essentially the same input data that is necessary for using AXIAL.

As with the mesh generator codes, an element connectivity matrix and node number coordinate matrix are necessary for use with AXIAL. AXIAL also requires two additional input data matrices. One matrix is an adjacent region matrix. This matrix defines for a specified region all the regions which are in physical contact with this region.

The second additional matrix is an angle specification matrix. This matrix specifies which side, if any, of a region is at an angle with respect to the vertical.

These two additional matrices are essential to the operation of AXIAL. The adjacent region matrix is used by the code AXIAL to determine the next region that the laser beam will enter as it exits a given region. The second matrix is used by AXIAL to determine if the laser beam has encountered a plexiglas-water interface at an angle with respect to the vertical. The FORTRAN listing of AXIAL starts on the next page.

```
START
COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

 1   C
 1   C                  APPENDIX B
 1   C           (FORTRAN LISTING OF PROGRAM AXIAL)
 1   C
 1   C
 1   C
 1   C***************
 1   C PROGRAM AXIAL
 1   C***************
 1   C
 7         IMPLICIT REAL*8(A-H,O-Z)
 7         COMMON TCORD(2000,2),                                                   205
 6        *XENT1(50),YENT1(50),XEXT1(50),YEXT1(50),THETA(50),
 6        *TINDX(50),XENT2(50),YENT2(50),XEXT2(50),YEXT2(50),
 6        *SCXADJ,SCYADJ,SHALF,TINW,TINP,XTEMP,YTEMP,
 6        *MATNO(1501),NPOIN,NELEM,
 6        *LNADJ(50,20),NREGIO(50),NREGI2(50),IANG(50),
 6        *INODS(1501,8),
 6        *NTITLE(10),NCOL(100),NDGRAF,
 6        *J3,J4,J,
 6        *IFLAG5,IFLAG6,IFLAG7,IFLAG8,IFLAG9
 1   C
 1   C---- THIS PROGRAM DETERMINES FOR A SPECIFIED GEOMETRY AND SPECIFIED
 1   C---- MEASURING POINT, P(X,Y), WHERE THE LASER SHOULD BE PLACED IN
 1   C---- ORDER FOR THE TWO BEAMS TO CROSS AT THIS POINT.
 1   C                                                                              203
 1   C
 1   C---- SUBROUTINE INBLOC ACCEPTS ALL INPUT DATA
 1   C                                                                              202
 7         CALL INBLOC                                                              204
 1   C
 1   C---- SUBROUTINE LASER INITIALIZES THE INPUT LASER DATA
 1   C---- AND STARTS THE PROCESS OF DETERMINING WHERE THE LASER
 1   C---- SHOULD BE PLACED.
 1   C                                                                              202
 7         CALL LASER
 1   C
 1   C---- SUBROUTINE DISSPL PLOTS THE GEOMETRY AND THE PATH OF
 1   C---- THE LASER BEAMS.
 1   C
 7         IF (IFLAG5.EQ.1)STOP
 7         IF (IFLAG6.EQ.1)STOP
 7         IF (IFLAG7.EQ.1)STOP
 7         IF (IFLAG8.EQ.1)STOP
 7         IF (IFLAG9.EQ.1)STOP
 7         IF (NDGRAF.EQ.1) CALL DISSPL
 1   C
 7         STOP
 7         END                                                                      401
 1   C     ****************
 7         SUBROUTINE INBLOC                                                        105
 1   C     ****************
 7         IMPLICIT REAL*8(A-H,O-Z)
 7         COMMON TCORD(2000,2),                                                   205
```

```
TART
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  6        *XENT1(50),YENT1(50),XEXT1(50),YEXT1(50),THETA(50),
  6        *TINDX(50),XENT2(50),YENT2(50),XEXT2(50),YEXT2(50),
  6        *SCXADJ,SCYADJ,SHALF,TINW,TINP,XTEMP,YTEMP,
  6        *MATNO(1501),NPOIN,NELEM,
  6        *LNADJ(50,20),NREGIO(50),NREGI2(50),IANG(50),
  6        *INODS(1501,8),
  6        *NTITLE(10),NCOL(100),NDGRAF,
  6        *J3,J4,J,
  6        *IFLAG5,IFLAG6,IFLAG7,IFLAG8,IFLAG9
  1   C
  1   C---- THIS SUBROUTINE ACCEPTS ALL INPUT DATA
  1   C
  1   C                     DEFINITION OF THE VARIABLES.
  1   C
  1   C
  1   C----
  1   C---- LNADJ(I,J): THIS ARRAY DEFINES ALL THE ELEMENTS WHICH ARE
  1   C---- ADJACENT TO ELEMENT I.
  1   C----
  1   C---- NREGIO(J): THIS ARRAY SPECIFIES THE ELEMENT NUMBER CORRESPONDING
  1   C---- TO THE J TH REGION THAT THE FIRST BEAM IS IN.
  1   C----
  1   C---- NREGI2(J): THIS ARRAY SPECIFIES THE ELEMENT NUMBER CORRESPONDING
  1   C---- TO THE J TH REGION THAT THE SECOND BEAM IS IN.
  1   C----
  1   C---- NTITLE: THE TITLE OF THE TEST PIECE. UP TO 48 LETTERS.
  1   C
  1   C---- NPOIN:THE NUMBER OF NODES USED TO DESCRIBE THE TEST SECTION
  1   C
  1   C---- NELEM:THE NUMBER OF INPUT BLOCKS USED TO DESCRIBE THE
  1   C---- TEST SECTION
  1   C
  1   C---- INODS(I,J): CONNECTIVITY MATRIX.THIS IS AN ARRAY SPECIFYING THE
  1   C---- NODE NUMBERS OUTLINING A GIVEN BLOCK.THE FIRST INDEX SPECIFIES THE
  1   C---- THE ELEMENT NUMBER (MAXIMUM NUMBER OF ELEMENTS IS 1500).  THE SEC-
  1   C---- OND INDEX TELLS WHICH GLOBAL NODE NUMBER CORRESPONDS TO A LOCAL
  1   C---- NODE NUMBER.  THE LOCAL NODE NUMBER ALWAYS STARTS AT THE LOWER
  1   C---- LEFT HAND CORNER OF THE ELEMENT.  THE NODE NUMBERS PROCEED AROUND
  1   C---- THE ELEMENT IN A COUNTERCLOCKWISE FASHION.
  1   C----
  1   C---- IANG(J) THIS ARRAY DEFINES ALL THE ANGLED SIDESOF ELEMENT J. A
  1   C---- VALUE OF J INDICATES THAT THERE ARE NOT ANY ANGLED SIDES.  A
  1   C---- NEGATIVE NUMBER INDICATES THE PRESENCE OF AN ANGLED SIDE.
  1   C----
  1   C---- NCOL(J) AN ARRAY USED BY THE GRAPHICS ROUTINE.  THIS ARRAY
  1   C---- INDICATES THE COLOR THAT A REGION SHOULD BE SHADED.
  1   C----
  1   C---- NDGRAF: A FLAG VARIABLE USED BY THE GRAPHICS ROUTINE. IT MUST
  1   C---- BE SET EQUAL TO 1 FOR THE GRAPHICS ROUTINE TO BE CALLED.
  1   C----
  1   C---- J3: NUMBER OF REGIONS THAT THE FIRST BEAM ENCOUNTERS.
  1   C----
  1   C---- J4: NUMBER OF REGIONS THAT THE SECOND BEAM ENCOUNTERS.
  1   C----
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C---- J: A COUNTER VARIABLE USED TO KEEP TRACK OF WHICH REGION THE
  1   C---- LASER BEAM IS IN.
  1   C----
  1   C---- IFLAG5: A FLAG VARIABLE USED TO INDICATE THAT AN ADJACENT
  1   C---- REGION HAS NOT BEEN FOUND.
  1   C----
  1   C---- IFLAG6: A FLAG VARIABLE USED TO INDICATE THAT THE LASER BEAM
  1   C---- DID NOT EXIT FROM AN ALLOWABLE SIDE OF A REGION.
  1   C----
  1   C---- IFLAG7: A FLAG VARIABLE USED TO INDICATE THAT A PHYSICAL SOLUTION
  1   C---- DOES NOT EXIST
  1   C----
  1   C---- IFLAG8: A FLAG VARIABLE USED TO INDICATE THAT A SECOND ANGLED
  1   C---- REGION HAS BEEN FOUND.
  1   C----
  1   C---- IFLAG9: A FLAG VARIABLE USED TO INDICATE THET THE LASER BEAM
  1   C---- DID NOT EXIT FROM AN ALLOWABLE SIDE OF A REGION.
  1   C----
  1   C
  1   C---- MATNO(J): AN USED AS FLAG TO INDICATE AN EXIT ELEMENT
  1   C
  1   C---- TCORD: THE TWO DIMENSIONAL ARRAY CONTIANING THE
  1   C---- COORDINATES OF THE INPUT BOUNDARY POINTS.
  1   C
  1   C----
  1   C---- SHALF: THE LASER BEAM HALF ANGLE.
  1   C----
  1   C----  SCXADJ IS A VARIABLE USED BY THE DISSPLA GRAPHICS PACKAGE TO
  1   C----  CONTROL THE SIZE OF THE PLOT IN THE X DIRECTION.
  1   C
  1   C----  SCYADJ IS A VARIABLE USED BY THE DISSPLA GRAPHICS PACKAGE TO
  1   C----  CONTROL THE SIZE OF THE PLOT IN THE Y DIRECTION.
  1   C
  1   C---- XENT1(J): AN ARRAY WHICH LISTS THE X COORDINATE OF THE POINT AT
  1   C---- WHCIH THE FIRST LASER BEAM ENTERS  REGION J.
  1   C----
  1   C---- YENT1(J): AN ARRAY WHICH LISTS THE Y COORDINATE OF THE POINT AT
  1   C---- WHCIH THE FIRST LASER BEAM ENTERS  REGION J.
  1   C----
  1   C---- XENT2(J): AN ARRAY WHICH LISTS THE X COORDINATE OF THE POINT AT
  1   C---- WHCIH THE SECOND LASER BEAM ENTERS  REGION J.
  1   C----
  1   C---- YENT2(J): AN ARRAY WHICH LISTS THE Y COORDINATE OF THE POINT AT
  1   C---- WHCIH THE SECOND LASER BEAM ENTERS  REGION J.
  1   C----
  1   C---- XEXT1(J): AN ARRAY WHICH LISTS THE X COORDINATE OF THE POINT AT
  1   C---- WHCIH THE FIRST LASER BEAM EXITS  REGION J.
  1   C----
  1   C---- YEXT1(J): AN ARRAY WHICH LISTS THE Y COORDINATE OF THE POINT AT
  1   C---- WHCIH THE FIRST LASER BEAM EXITS  REGION J.
  1   C----
  1   C---- XEXT2(J): AN ARRAY WHICH LISTS THE X COORDINATE OF THE POINT AT
  1   C---- WHCIH THE SECOND LASER BEAM EXITS  REGION J.
  1   C----
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C---- YEXT2(J): AN ARRAY WHICH LISTS THE Y COORDINATE OF THE POINT AT
  1   C---- WHCIH THE SECOND LASER BEAM EXITS  REGION J.
  1   C----
  1   C---- THETA(J): DIRECTION ANGLE OF THE LASER BEAM WHILE IN REGION
  1   C---- J.
  1   C----
  1   C---- TINDX(I):THE INDEX OF REFRACTION FOR REGION J.
  1   C----
  1   C---- TINW: THE IN DEX OF REFRACTION FOR WATER
  1   C----
  1   C---- TINP: THE INDEX OF REFRACTION FOR PLEXIGLASS.
  1   C----
  1   C---- XTEMP: X COORDINATE OF THE SPECIFIED PROBE VOLUME LOCATION
  1   C----
  1   C---- YTEMP: Y COORDINATE OF THE SPECIFIED PROBE VOLUME LOCATION.
  1   C----
  1   C---- READ AND PRINT THE TITLE OF THE MESH
  1   C
  7         READ(5,900) NTITLE                                                     203
  7         WRITE (6,901) NTITLE                                                   204
  1   C
  1   C---- READ IN THE INFOMATION DEFINING THE OUTLINE OF THE MESH
  1   C
  7         READ (5,905) NPOIN,NELEM                                               206
  7         WRITE (6,910) NPOIN,NELEM                                              207
  7         WRITE (6,920)                                                          305
  7         WRITE (6,925)                                                          307
  1   C
  1   C---- READ IN AND WRITE THE CONNECTIVITY MATRIX OF THE POINTS OUTLINING
  1   C---- THE MESH.
  1   C
  7         DO 10 IELEM=1,NELEM                                                    401
  7         READ (5,905) NUMEL,(INODS(NUMEL,INODE),INODE=1,8),                     402
  6        1MATNO(NUMEL),NCOL(NUMEL)
  4      10 WRITE(6,926) NUMEL,(INODS(NUMEL,INODE),INODE=1,8),                     402
  6        1MATNO(NUMEL)
  7         WRITE(6,999)
  7         DO 15 I=1,NELEM
 10            READ(5,1001) (LNADJ(I,J),J=1,20),TINDX(I)
 10            WRITE(6,1001) (LNADJ(I,J),J=1,20),TINDX(I)
  3     15  CONTINUE
 10            WRITE(6,1002)
  7         DO 17 I=1,NELEM
 10            READ(5,1003) IANG(I)
 10            WRITE(6,1003) I,IANG(I)
  3     17  CONTINUE
  1   C
  1   C---- READ IN AND WRITE THE COORDINATES OF THE NODE POINTS OUTLINING THE
  1   C---- BOUNDARY OF THE MESH.
  1   C
  7         WRITE (6,930)                                                          403
  7         WRITE (6,935)                                                          404
  7         DO 20 IPOIN=1,NPOIN                                                    407
  7         READ (5,940) JPOIN,(TCORD(JPOIN,IDIME),IDIME=1,2)
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  4      20 WRITE (6,941) JPOIN,(TCORD(JPOIN,IDIME),IDIME=1,2)
  1   C
  1   C---- READ IN THE INITIAL LASER DATA
  1   C
  7         J=1
  7         READ(5,960) NREGIO(J),XENT1(NREGIO(J)),YENT1(NREGIO(J)),SHALF,
  6        *TINW,TINP
  7         WRITE(6,963) TINW,TINP
  1   C
  1   C---- READ IN GRAPHICS DATA
  1   C
  7         READ(5,950) NDGRAF
  7         READ (5,955) SCXADJ,SCYADJ
  7         RETURN
  3     900 FORMAT (12A4)                                                          205
  3     901 FORMAT (///,1X,'****************     ',12A4,'*************',//)        205
  3     905 FORMAT (11I5)                                                          208
  3     910 FORMAT (/,6X,'AUTOMATIC DATA GENERATION FROM',I4,' LOCATION',          301
  6        1' POINTS AND',I3,'  DATA BLOCKS')                                      302
  3     920 FORMAT (/,6X,'DATA BLOCKS')                                            306
  3     925 FORMAT (/,' BLOCK NO.',22X,'DEFINITION POINTS',36X,'FLAG')             308
  3     926 FORMAT(1X,I8,5X,8(I5,2X),5X,I5)
  3     930 FORMAT (/,6X,'LOCATION OF POINTS')                                     405
  3     935 FORMAT (/,4X,' POINT NO',5X,'X-COORD.',10X,'Y-COORD')                  406
  3     940 FORMAT (I10,2F15.5)                                                    408
  3     941 FORMAT (1X,I10,3X,F15.5,3X,F15.5)                                      408
  3     950 FORMAT(2I5)
  3     955 FORMAT(7F10.4)
  3     960 FORMAT(I5,5F10.5)
  3     963 FORMAT(/,' INDEX OF REFRACTION FOR WATER= ',D15.5,3X,
  6        *'INDEX OF REFRACTION FOR PLEXIGLASS= ',D15.5)
  3     999 FORMAT(' ELEMENT NUMBER ,ADJACENT ELEMENTS,INDEX OF REFRACTION')
  2    1001 FORMAT(20I3,F10.3)
  2    1002 FORMAT(' ELEMNT NUMBER, ANGLED SIDE')
  2    1003 FORMAT(2(2X,I5))
  7         END                                                                    501
  1   C     ****************
  7         SUBROUTINE DISSPL
  1   C     ****************
  1   C
  7         IMPLICIT REAL*8(A-H,O-Z)
  7         COMMON TCORD(2000,2),                                                  205
  6        *XENT1(50),YENT1(50),XEXT1(50),YEXT1(50),THETA(50),
  6        *TINDX(50),XENT2(50),YENT2(50),XEXT2(50),YEXT2(50),
  6        *SCXADJ,SCYADJ,SHALF,TINW,TINP,XTEMP,YTEMP,
  6        *MATNO(1501),NPOIN,NELEM,
  6        *LNADJ(50,20),NREGIO(50),NREGI2(50),IANG(50),
  6        *INODS(1501,8),
  6        *NTITLE(10),NCOL(100),NDGRAF,
  6        *J3,J4,J,
  6        *IFLAG5,IFLAG6,IFLAG7,IFLAG8,IFLAG9
  7         REAL*4 COORD1(400),COORD2(400),WORK(6000)
  7         KGRAPH=0
  1   C
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C---- THIS ROUTINE DRAWS THE GENERATED MESH USING THE DISSPLA GRAPHICS  00040000
  1   C---- SYSTEM.
  1   C                                                                       00040000
  7         CALL DIGEST
  7         CALL NOBRDR
  1   C
  1   C---- THIS COMMAND SETS THE PHYSICAL LOCATION OF THE CORRDINATE (0,0)
  1   C     ON THE PLOT.  FOR THIS CASE THE PLOT ORIGIN IS LOCATED 0.75 INCHES
  1   C---- FROM THE LEFT EDGE AND 0.75 INCHES FROM THE BOTTOM.
  1   C
  7         XPHYS=0.75                                                        00050000
  7         YPHYS=0.75                                                        00110000
  7         CALL PHYSOR(XPHYS,YPHYS)                                          00350000
  1   C                                                                       00400000
  1   C---- THIS COMMAND SETS THE TOTAL SIZE OF PLOTTING AREA.                00410000
  1   C                                                                       00420000
  7         CALL PAGE(11.9375,8.500)                                          00430000
 73                                                                           00440000
  7         XPAGE=6.5                                                         00450000
  7         YPAGE=6.5                                                         00460000
  1   C                                                                       00600000
  1   C---- THE FOLLOWING DO LOOP DETERMINES THE STARTING POINT,              00610000
  1   C---- STEP SIZE, AND ENDPOINT FOR THE X AXIS AND Y AXISES.              00620000
  1   C                                                                       00630000
  7         YMAX=TCORD(1,2)                                                   00640000
  7         YMIN=TCORD(1,2)                                                   00650000
  7         XMAX=TCORD(1,1)                                                   00660000
  7         XMIN=TCORD(1,1)                                                   00670000
  7         DO 10 I=1,NPOIN                                                   00680000
 10            IF(XMAX.LT.TCORD(I,1))XMAX=TCORD(I,1)                          00690000
 10            IF(XMIN.GT.TCORD(I,1))XMIN=TCORD(I,1)                          00700000
 10            IF(YMAX.LT.TCORD(I,2))YMAX=TCORD(I,2)                          00710000
 10            IF(YMIN.GT.TCORD(I,2))YMIN=TCORD(I,2)                          00720000
  3     10  CONTINUE                                                          00730000
  1   C
  1   C---- THE CODE DETERIMNES WHCIH OF THE FIGURE DIMENSIONS IS THE LARGEST
  1   C
  7         SCALE1=XMAX-XMIN                                                  00740000
  7         SCALE2=YMAX-YMIN                                                  00750000
  7         SCALE3=SCALE2/SCALE1
  7         SCALE4=SCALE1/SCALE2
  1   C
  1   C---- THE CODE ADJUSTS THE OTHER DIMENSION SO THAT THE FIGURE WILL BE
  1   C---- DRAWN TO SCALE AND SO THAT ALL OF IT WILL FIT ON THE PAGE.
  1   C
  7         IF(SCALE3.GT.1.0) GO TO 15
  7         YPAGE=YPAGE*SCALE3
  7         GO TO 16
  3     15  XPAGE=XPAGE*SCALE4
  1   C
  1   C---- IF FOR SOME REASON YOU DO NOT WANT THE FIGURE DRAWN TO SCALE,USE
  1   C     THE VARIABLES SCXADJ AND SCXYADJ TO ADJUST THE SCALE OF THE X AND
  1   C     Y AXIS.  A VALUE OF 1.0 FOR SCXADJ AND SCYADJ MEANS THAT SCALE FOR
  1   C     X AND Y AXIS WILL BE THE SAME.  A VALUE OF 1.0 FOR SCXADJ
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C      AND 2.0 FOR SCYADJ MEANS THAT THE SCALE OF THE Y AXIS WILL BE
  1   C      TWICE THE SCALE OF THE X AXIS.  A VALUE OF 1.0 FOR SCYADJ
  1   C      AND 2.0 FOR SCXADJ MEANS THAT THE SCALE OF THE X AXIS WILL BE
  1   C----  TWICE THE SCALE OF THE Y AXIS.
  1   C
  1   C
  3     16   XPAGE=XPAGE*SCXADJ
  7          YPAGE=YPAGE*SCYADJ
  1   C                                                                     00470000
  1   C                                                                     00470000
  1   C----  THIS COMMAND MAKES THE FIGURE FIT WITHIN THE PLOTTING            00480000
  1   C----  AREA.                                                          00490000
  1   C                                                                     00500000
  1   C                                                                     00500000
  7          CALL AREA2D(XPAGE,YPAGE)
  1   C
  1   C
  1   C---   DISSPLA HEADING COMMAND
  1   C
  1   C
  7          CALL HEADIN(NTITLE,100,3,1)                                    00590000
  7          XSTEP=(XMAX-XMIN)/10.                                          00740000
  7          YSTEP=(YMAX-YMIN)/10.                                          00750000
  1   C                                                                     00760000
  1   C                                                                     00760000
  1   C----  THIS COMMAND SETS THE SCALE OF THE GRAPH ACCORDING TO VARIALBES IN00770000
  1   C----  THE ARGUMENT.
  1   C                                                                     00780000
  1   C                                                                     00780000
  7          CALL GRAF(XMIN,XSTEP,XMAX,YMIN,YSTEP,YMAX)                     00790000
  1   C
  1   C
  1   C----  THIS SECTION OF THE GRAPHICS ROUTINE DRAWS THE INPUT BLOCK AND
  1   C----  COLORS IT.
  1   C
  1   C
  7          DO 42 IBLOC=1,NELEM
  1   C
  1   C----  SET UP THE COORDINATE ARRAYS TO OUTLINE THE BLOCK.
  1   C
 10             DO 35 ISIDE=1,8
 16                   COORD1(ISIDE)=TCORD(INODS(IBLOC,ISIDE),1)
 16                   COORD2(ISIDE)=TCORD(INODS(IBLOC,ISIDE),2)
  3     30          CONTINUE
  3     35      CONTINUE
  1   C
  1   C
  1   C----  THE STARTING POINTS AND END POINTS FOR THE OUTLINE MUST BE THE
  1   C----  SAME. THE CODE ADJUSTS THE OUTLINE ARRAYS TO REFLECT THIS.
  1   C
  1   C
 10             COORD1(ISIDE)=TCORD(INODS(IBLOC,1),1)
 10             COORD2(ISIDE)=TCORD(INODS(IBLOC,1),2)
  3     40      CONTINUE
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C
  1   C
  1   C---- THE CODE COLORS IN THE INPUT BLOCK ONLY IF A COMMAND IS GIVEN
  1   C     TO DO THIS IN A CERTAIN COLOR.  IF NO COLOR, IS GIVEN THE CODE
  1   C     ASSUMES THE USER DOES NOT WANT THE INPUT BLOCK SHADED IN. A
  1   C     DEFALUT VALUE OF 0 FOR COLOR INSTRUCTS THE CODE TO NOT SHADE
  1   C---- IN THE INPUT BLOCK.
  1   C
  1   C
 10            IF(NCOL(IBLOC).EQ.0)GOTO 42
 10            IF(NCOL(IBLOC).EQ.8)NCOL(IBLOC)=0
 10            CALL NEWPEN(NCOL(IBLOC))
  1   C
  1   C
  1   C---- THE CODE USES THE DISSPLA SHADE COMMAND TO COLOR THE BLOCK IN.
  1   C
  1   C
 10            CALL SHADE(COORD1,COORD2,ISIDE,90.0,0.010,1,WORK,6000)
  3     42  CONTINUE
  1   C
  1   C
  1   C---- THE CODE NOW DRAWS THE MESH ON TOP OF INPUT BLOCK.  IT
  1   C     ACCOMPLISHES THIS TASK BY CONNECTING POINTS. THE CODE GOES THROUGH
  1   C     THE ELEMENT CONNECTIVITY MATRIX AND DRAWS A LINE CONMNECTING EACH
  1   C---- ONE OF THE NODE POINTS IN THE PROPER FASHION.
  1   C
  1   C
  7         DO 60 IBLOC=1,NELEM
 10            DO 50 INODE=1,9
 13               IPLOT2=INODE
 13               IF(INODE.EQ.9)IPLOT2=1
 13               NNPOIN=INODS(IBLOC,IPLOT2)
 13               COORD1(INODE)=TCORD(NNPOIN,1)
 13               COORD2(INODE)=TCORD(NNPOIN,2)
  3     50     CONTINUE
  7         CALL NEWPEN(1)
 10            DO 55 I=1,8
 13               II=I+1
 13               CALL RLVEC(COORD1(I),COORD2(I),COORD1(II),COORD2(II),0)
  3     55     CONTINUE
  3     60  CONTINUE
  7         CALL NEWPEN(6)
  1   C
  1   C---- NOW DRAW THE PATH OF THE FIRST LASER BEAM
  1   C
 10            DO 65 I3=1,J4
 13               II=2
 13               I=1
 13               COORD1(I)=XENT1(NREGIO(I3))
 13               COORD2(I)=YENT1(NREGIO(I3))
 13               COORD1(II)=XEXT1(NREGIO(I3))
 13               COORD2(II)=YEXT1(NREGIO(I3))
 13               CALL RLVEC(COORD1(I),COORD2(I),COORD1(II),COORD2(II),0)
  3     65     CONTINUE
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C
  1   C---- NOW DRAW THE PATH OF THE SECOND LASER BEAM
  1   C
 10             DO 85 I3=1,J3
 13                II=2
 13                I=1
 13                COORD1(I)=XENT2(NREGI2(I3))
 13                COORD2(I)=YENT2(NREGI2(I3))
 13                COORD1(II)=XEXT2(NREGI2(I3))
 13                COORD2(II)=YEXT2(NREGI2(I3))
 13                CALL RLVEC(COORD1(I),COORD2(I),COORD1(II),COORD2(II),0)
  3     85      CONTINUE
  7         CALL ENDPL(0)
  7         CALL DONEPL(0)                                                  00010000
  7         RETURN
  7         END
  1   C     ****************
  7         SUBROUTINE LASER                                                     203
  1   C     ****************
  1   C
  7         IMPLICIT REAL*8(A-H,O-Z)
  7         COMMON TCORD(2000,2),                                                205
  6        *XENT1(50),YENT1(50),XEXT1(50),YEXT1(50),THETA(50),
  6        *TINDX(50),XENT2(50),YENT2(50),XEXT2(50),YEXT2(50),
  6        *SCXADJ,SCYADJ,SHALF,TINW,TINP,XTEMP,YTEMP,
  6        *MATNO(1501),NPOIN,NELEM,
  6        *LNADJ(50,20),NREGIO(50),NREGI2(50),IANG(50),
  6        *INODS(1501,8),
  6        *NTITLE(10),NCOL(100),NDGRAF,
  6        *J3,J4,J,
  6        *IFLAG5,IFLAG6,IFLAG7,IFLAG8,IFLAG9
  7         COMMON /REG/SLOPE2,B2,THPLEX,THWAT,IFLAG,IFLAG2,ITEMP
  1   C
  1   C---- THIS SUBROUTINE INITIALIZES THE LASER DATA. IT CALLS THE OTHER
  1   C---- TWO OTHER LASER ROUTINES WHICH TRACE THE PATH OF THE BEAM.
  1   C
  1   C----
  1   C---- LOCAL SUBROUTINE VARIABLES
  1   C----
  1   C---- SLOPE2: THE SLOPE OF A SIDE OF AN ELEMENT. IT IS USED IN
  1   C---- CALCULATING THE EQUATION FOR A SIDE OF AN ELEMENT.
  1   C----
  1   C---- B2: THE Y INTERCEPT OF THE EQUATION OF FOR AN
  1   C---- ELEMENT.
  1   C----
  1   C---- THPLEX: THE DIRECTION ANLGE OF THE LASER BEAM IN THE PLEXIGLASS.
  1   C---- ( ASSUMES NO ANGLED SURFACE).
  1   C----
  1   C---- THWAT: THE DIRECTION ANLGE OF THE LASER BEAM IN THE WATER.
  1   C---- ( ASSUMES NO ANGLED SURFACE).
  1   C----
  1   C---- IFLAG: A FLAG VARIABLE USED TO INDICATE IF THE LASER BEAM HAS
  1   C---- ENTERED A BOUNDARY REGION.
  1   C----
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C---- IFLAG2: A FLAG VARIABLE USED TO INDICATE IF THE LASER BEAM HAS
  1   C---- ENTERED AN ANGLED REGION.
  1   C----
  1   C---- ANG1(J): THE DIRECTION ANGLE OF THE FIRST LASER BEAM IN REGION J.
  1   C----
  1   C---- ANG2(J): THE DIRECTION ANGLE OF THE SECOND LASER BEAM IN REGION J.
  1   C----
  1   C---- X11: X COORDINATE OF THE REFERENCE POINT.
  1   C----
  1   C---- Y11: Y COORDINATE OF THE REFERENCE POINT.
  1   C----
  1   C---- Y1: Y COORDINATE OF THE POINT AT WHICH THE FIRST REFERENCE BEAM
  1   C---- THE TEST SECTION.
  1   C----
  1   C---- Y2: Y COORDINATE OF THE POINT AT WHICH THE SECOND REFERENCE BEAM
  1   C---- THE TEST SECTION.
  1   C----
  1   C---- IBLOC: INDICATES WHICH ELEMENT THE FOLLOWING NTPOIN POINTS ARE IN.
  1   C----
  1   C---- NTPOIN: THE NUMBER OF POINTS IN ELEMENT IBLOC AT WHICH VELCOITY
  1   C---- MEAUREMENTS WILL BE MADE.
  1   C----
  1   C---- YREF: Y COORDINATE OF THE LASER CENTER POSITION.
  1   C----
  1   C---- SEP1: SEPARATION DISTANCE FOR THE TWO REFERENCE LASER BEAMS AT THE
  1   C---- BOUNDARY ELEMENT.
  1   C----
  1   C---- XCENT: DISTANCE THE LASER SHOULD BE MOVED IN THE X DIRECTION FROM
  1   C---- THE REFERENCE POINT.
  1   C----
  1   C---- YCENT: DISTANCE THE LASER SHOULD BE MOVED IN THE Y DIRECTION FROM
  1   C---- THE REFERENCE POINT.
  1   C----
  7         SHALF=SHALF*3.141592654D0/180.0D0
  1   C
  1   C---- COMPUTE HALF ANGLE OF THE LASER IN WATER AND PLEXIGLASS ASSUMING
  1   C---- NO ANGLED SURFACES
  1   C
  7         THWAT=DARSIN(1./TINW*DSIN(SHALF))
  7         THETA(NREGIO(J))=THWAT
  7         THPLEX=DARSIN(1.000/TINP*DSIN(SHALF))
  1   C
  1   C---- CALCULATE THE REFERENCE POSITION OF THE LASER.
  1   C
  7         THW2=THWAT*180.0D0/3.141592654D0
  7         WRITE(6,969) XENT1(NREGIO(1)),YENT1(NREGIO(1)),THW2
  7         Y11=YENT1(NREGIO(J))
  7         X11=XENT1(NREGIO(J))
  7         CALL YPOINT
  7         IF(IFLAG5.EQ.1) GOTO 681
  7         IF(IFLAG6.EQ.1) GOTO 685
  7         IF(IFLAG7.EQ.1) GOTO 690
  7         IF(IFLAG8.EQ.1) GOTO 693
  7         IF(IFLAG9.EQ.1) GOTO 696
```

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  7          IFLAG=0
  7          Y1=YEXT1(NREGIO(J))
  7          DO 40 I=2,J
 10             NREGIO(J)=0
  1    40    CONTINUE
  7          J=1
  7          SHALF=-SHALF
  1    C
  1    C---- COMPUTE HALF ANGLE OF THE LASER IN WATER AND PLEXIGLASS ASSUMING
  1    C---- NO ANGLED SURFACES
  1    C
  7          THWAT=DARSIN(1./TINW*DSIN(SHALF))
  7          THETA(NREGIO(J))=THWAT
  7          THPLEX=DARSIN(1.000/TINP*DSIN(SHALF))
  7          CALL YPOINT
  7          IF(IFLAG5.EQ.1) GOTO 681
  7          IF(IFLAG6.EQ.1) GOTO 685
  7          IF(IFLAG7.EQ.1) GOTO 690
  7          IF(IFLAG8.EQ.1) GOTO 693
  7          IF(IFLAG9.EQ.1) GOTO 696
  7          Y2=YEXT1(NREGIO(J))
  7          DO 42 I=2,J
 10             NREGIO(J)=0
  1    42    CONTINUE
  1    C
  1    C---- CALCULATE EXIT POINT FOR THE FIRST BEAM ASSUMING NO ANGLED
  1    C---- SURFACES
  1    C
  2     45   CONTINUE
  7          READ(5,995)IBLOC,NTPOIN
  7          IF(IBLOC.EQ.0)GO TO 800
  7          DO 500 I5=1,NTPOIN
  7          IFLAG=0
  7          J=1
  7          NREGIO(J)=IBLOC
  7          READ(5,1200) XENT1(NREGIO(J)),YENT1(NREGIO(J))
  7          SHALF=DABS(SHALF)
  1    C
  1    C---- COMPUTE HALF ANGLE OF THE LASER IN WATER AND PLEXIGLASS ASSUMING
  1    C---- NO ANGLED SURFACES
  1    C
  7          THPLEX=DARSIN(1.000/TINP*DSIN(SHALF))
  7          THWAT=DARSIN(1./TINW*DSIN(SHALF))
  7          THETA(NREGIO(J))=THWAT
  7          IBEAM=1
  7          CALL YPOINT
  7          IF(IFLAG5.EQ.1) GOTO 681
  7          IF(IFLAG6.EQ.1) GOTO 685
  7          IF(IFLAG7.EQ.1) GOTO 690
  7          IF(IFLAG8.EQ.1) GOTO 693
  7          IF(IFLAG9.EQ.1) GOTO 696
  1    C
  1    C---- STORE THE POINTS THROUGH WHICH THE FIRST BEAM ENTERS AND EXITS
  1    C---- EACH REGION.  THESE WILL BE SUBSEQUENTLY USED IN THE GRAPHICS
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C---- ROUTINE
  1   C
  7         ANGL1=THETA(NREGIO(1))
  7         IFLAG=0
  7         J3=J
  7         DO 60 I=1,J3
  7         NREGI2(I)=NREGIO(I)
  7         YEXT2(NREGI2(I))=YEXT1(NREGIO(I))
  7         YENT2(NREGI2(I))=YENT1(NREGIO(I))
  7         XEXT2(NREGI2(I))=XEXT1(NREGIO(I))
  7         XENT2(NREGI2(I))=XENT1(NREGIO(I))
  7         NREGIO(I)=0
  7         NREGIO(1)=NREGI2(1)
  1   60    CONTINUE
  1   C
  1   C---- NOW COMPUTE THE PATH FOR THE SECOND BEAM
  1   C
  7         J=1
  7         SHALF=-SHALF
  1   C
  1   C---- COMPUTE HALF ANGLE OF THE LASER IN WATER AND PLEXIGLASS ASSUMING
  1   C---- NO ANGLED SURFACES
  1   C
  7         THWAT=DARSIN(1./TINW*DSIN(SHALF))
  7         THPLEX=DARSIN(1.000/TINP*DSIN(SHALF))
  7         THETA(NREGIO(J))=THWAT
  7         CALL YPOINT
  7         IF(IFLAG5.EQ.1) GOTO 681
  7         IF(IFLAG6.EQ.1) GOTO 685
  7         IF(IFLAG7.EQ.1) GOTO 690
  7         IF(IFLAG8.EQ.1) GOTO 693
  7         IF(IFLAG9.EQ.1) GOTO 696
  7         ANGL2=THETA(NREGIO(1))
  7         J4=J
  1   C
  1   C---- NOW COMPUTE THE ACTUAL HALF ANGLE OF THE LASER BEAMS IN WATER
  1   C
  7         THALF=(ANGL1-ANGL2)/2.0D0
  1   C
  1   C---- NOW COMPUTE THE DIRECTION OF THE VELOCITY VECTOR
  1   C
  7         TDIR=90.0D0*3.141592654D0/180.0D0+((ANGL1+ANGL2)/2.0D0)
  7         THALF=THALF*180.0D0/3.141592654D0
  7         TDIR=TDIR*180.0D0/3.141592654D0
  7         SHALF=-SHALF
  1   C
  1   C---- NOW CALCULATE HOW FAR THE LASER SHOULD BE MOVED FROM THE
  1   C---- POSITION SO THAT WE CAN MEASURE AT THE DESIRED POINT.
  1   C
  7         YREF=(Y1+Y2)/2.0D0
  7         SEP1=(Y1-Y2)/2.0D0
  7         YCENT=(YEXT2(NREGI2(J3))+YEXT1(NREGIO(J4)))/2.0D0
  7         YCENT=YCENT-YREF
  7         XCEN1=(YEXT2(NREGI2(J3))-YEXT1(NREGIO(J4)))/2.0D0
```

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  7          XCENT=(SEP1-XCEN1)/DTAN(SHALF)
  1    C
  1    C---- PRINT OUT THE RESULTS
  1    C
  7          SHAL2=SHALF*180.0D0/3.141592654D0
  7          WRITE(6,950)
  7          WRITE(6,951) THALF,TDIR,SHAL2
  7          WRITE(6,968) XENT1(NREGIO(1)),YENT1(NREGIO(1))
  7          WRITE(6,967) YCENT,XCENT
  7          WRITE(15,970) XENT1(NREGIO(1)),YENT1(NREGIO(1)),XCENT,YCENT,
  6         *X11,Y11,THALF,TDIR
  1    500   CONTINUE
  7          GO TO 45
  1    681   CONTINUE
  7          WRITE(6,950)
  7          WRITE(6,968) XENT1(NREGIO(1)),YENT1(NREGIO(1))
  7          WRITE(6,966)
  7          XENT1(NREGIO(1))=0.0
  7          YENT1(NREGIO(1))=0.0
  7          XCENT=0.0
  7          YCENT=0.0
  7          THALF=0.0
  7          TDIR=0.0
  7          WRITE(15,970) XENT1(NREGIO(1)),YENT1(NREGIO(1)),XCENT,YCENT,
  6         *X11,Y11,THALF,TDIR
  7          GO TO 45
  1    685   CONTINUE
  7          WRITE(6,950)
  7          WRITE(6,968) XENT1(NREGIO(1)),YENT1(NREGIO(1))
  7          WRITE(6,1250)
  7          XENT1(NREGIO(1))=0.0
  7          YENT1(NREGIO(1))=0.0
  7          XCENT=0.0
  7          YCENT=0.0
  7          THALF=0.0
  7          TDIR=0.0
  7          WRITE(15,970) XENT1(NREGIO(1)),YENT1(NREGIO(1)),XCENT,YCENT,
  6         *X11,Y11,THALF,TDIR
  7          GO TO 45
  1    690   CONTINUE
  7          WRITE(6,950)
  7          WRITE(6,968) XENT1(NREGIO(1)),YENT1(NREGIO(1))
  7          WRITE(6,1150)
  7          XENT1(NREGIO(1))=0.0
  7          YENT1(NREGIO(1))=0.0
  7          XCENT=0.0
  7          YCENT=0.0
  7          THALF=0.0
  7          TDIR=0.0
  7          WRITE(15,970) XENT1(NREGIO(1)),YENT1(NREGIO(1)),XCENT,YCENT,
  6         *X11,Y11,THALF,TDIR
  7          GO TO 45
  1    693   CONTINUE
  7          WRITE(6,950)
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  7         WRITE(6,968) XENT1(NREGIO(1)),YENT1(NREGIO(1))
  7         WRITE(6,1040)
  7         XENT1(NREGIO(1))=0.0
  7         YENT1(NREGIO(1))=0.0
  7         XCENT=0.0
  7         YCENT=0.0
  7         THALF=0.0
  7         TDIR=0.0
  7         WRITE(15,970) XENT1(NREGIO(1)),YENT1(NREGIO(1)),XCENT,YCENT,
  6        *X11,Y11,THALF,TDIR
  7         GO TO 45
  1   696   CONTINUE
  7         WRITE(6,950)
  7         WRITE(6,968) XENT1(NREGIO(1)),YENT1(NREGIO(1))
  7         WRITE(6,1041)
  7         XENT1(NREGIO(1))=0.0
  7         YENT1(NREGIO(1))=0.0
  7         XCENT=0.0
  7         YCENT=0.0
  7         THALF=0.0
  7         TDIR=0.0
  7         WRITE(15,970) XENT1(NREGIO(1)),YENT1(NREGIO(1)),XCENT,YCENT,
  6        *X11,Y11,THALF,TDIR
  7         GO TO 45
  1   800   CONTINUE
  7         RETURN
  1   950   FORMAT(//'  ')
  1   951   FORMAT(' ALTERED HALF ANGLE =',2X,D15.5,' ANGLE OF VELOCITY VECTOR
  6        *(MEASURED FROM THE HORIZONTAL)=',2X,D15.5,/,
  6        *' HALF ANGLE OF THE LASER IN AIR',2X,D15.5)
  1   966   FORMAT('  ERROR: NO ADJACENT NREGIO FOUND',2(2X,I5))
  1   967   FORMAT( ' DELTA Y =',F15.5,' DELTA X = ',F15.5)
  1   968   FORMAT('  MEASUREMENT POINT IS  X = ',D15.5,' Y =',D15.5)
  1   969   FORMAT(' REFERENCE POINT  IS X=',D15.5,2X,' Y=',D15.5,/
  6        *,' HALF ANGLE OF THE BEAM IN WATER WITH NO ANGLED SURFACES =',
  6        *D15.5)
  1   970   FORMAT(6(2X,D10.3),/,2(2X,D15.8))
  1   995   FORMAT(2I10)
  1   1040  FORMAT(' HAVE ENTERED A DIFFERENT ANGLED REGION, OR HAVE ENTERED
  6        *A SECOND ANGLED REGION ---NO SOLUITON EXISTS')
  1   1041  FORMAT(' ERROR NO EXIT POINT HAS BEEN FOUND')
  1   1150  FORMAT(' DID NOT ENTER ORIGINAL ANGLED REGION')
  1   1200  FORMAT( D15.5,D15.5)
  1   1250  FORMAT(' HAVE EXITED FROM THE WRONG SIDE OF A REGION')
  7         END
  1   C     ****************
  7         SUBROUTINE YPOINT                                                   203
  1   C     ****************
  7         IMPLICIT REAL*8(A-H,O-Z)
  7         COMMON TCORD(2000,2),                                               205
  6        *XENT1(50),YENT1(50),XEXT1(50),YEXT1(50),THETA(50),
  6        *TINDX(50),XENT2(50),YENT2(50),XEXT2(50),YEXT2(50),
  6        *SCXADJ,SCYADJ,SHALF,TINW,TINP,XTEMP,YTEMP,
  6        *MATNO(1501),NPOIN,NELEM,
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  6        *LNADJ(50,20),NREGIO(50),NREGI2(50),IANG(50),
  6        *INODS(1501,8),
  6        *NTITLE(10),NCOL(100),NDGRAF,
  6        *J3,J4,J,
  6        *IFLAG5,IFLAG6,IFLAG7,IFLAG8,IFLAG9
  7         COMMON /REG/SLOPE2,B2,THPLEX,THWAT,IFLAG,IFLAG2,ITEMP
  1   C
  1   C---- THIS SUBROUTINE DETERMINES THE EXIT POINT OF THE LASER BEAM AS
  1   C---- LEAVES ONE REGION AND ENTERS ANOTHER REGION.
  1   C
  1   C---- THIS SUBROUTINE DETERMINES THE EXIT POINT OF THE LASER BEAM AS
  1   C---- LEAVES ONE REGION AND ENTERS ANOTHER REGION.
  1   C
  1   C---- SET UP SOME INTITIAL FLAG VARIABLES
  1   C
  1   C----   IFLAG2: INDICATES IF AN ANGLED REGION HAS BEEN FOUND.
  1   C
  1   C----   IFLAG3: KEEPS TRACK OF THE FIRST ANGLED REGION ENCOUNTERED.
  1   C
  1   C----   JCHEC: A FLAG VARIABLE USED TO KEEP TRACK OF THE NUMBER OF
  1   C----   ANGLED REGIONS FOUND.
  1   C
  1   C---- ITEMP: A TEMPORARY VARIABLE USED BY SUBROUTINE YPOINT.  THIS
  1   C---- VARIABLE KEEPS TRACK OF WHICH ELEMENT THE BEAM IS IN AS THE
  1   C---- CODE WORKS ITS WAY THROUGH THE VARIOUS REGIONS.
  1   C----
  1   C---- SLOPE1: THE CODE DETERMINES A LINEAR EQUATION WHICH DESCRIBES THE
  1   C---- PATH OF THE LASER BEAM THROUGH EACH REGION.  SLOPE1 IS THE SLOPE
  1   C---- OF THIS EQUATION.
  1   C----
  1   C---- B1: THE CODE DETERMINES A LINEAR EQUATION WHICH DESCRIBES THE
  1   C---- PATH OF THE LASER BEAM THROUGH EACH REGION.  B1 IS THE Y INTERCEPT
  1   C---- OF THIS EQUATION.
  1   C----
  1   C---- XINT: THE X COORDINATE OF THE CALCULATED POINT AT WHICH THE LASER
  1   C---- BEAM EXITS THE REGION.
  1   C----
  1   C---- YINT:  THE Y COORDINATE OF THE CALULATED POINT AT WHICH THE LASER
  1   C---- BEAM EXITS THE REGION
  1   C----
  1   C---- IFLAG3: A VARIABLE WHICH KEEPS TRACK OF THE FIRST ANGLED REGION
  1   C---- ENCOUNTERED.
  1   C----
  1   C---- ALPH2: THE ANGLE BETWEEN A SLANTED SIDE OF AN ELEMENT AND THE
  1   C---- HORIZONTAL.
  1   C----
  7         JCHEC=0
  7         IFLAG3=0
  7         IFLAG7=0
  7         IFLAG8=0
  7         IFLAG9=0
  1   10    CONTINUE
  7         IF(J.LT.2)GO TO 20
  1   C
```

```
START
COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

 1   C---- DETERMINE THE HALF ANGLE OF THE LASER BEAM FOR THE CURRENT REGION
 1   C
 7         THETA(NREGIO(J))=DARSIN(TINDX(NREGIO(J-1))/TINDX(NREGIO(J))*
 6        *DSIN(THETA(NREGIO(J-1))))
 1   15    CONTINUE
 1   20    CONTINUE
 1   C
 1   C---- DETERMINE THE EQUATION WHICH DESCRIBES THE PATH OF THE RAY
 1   C---- IN THE GIVEN REGION
 7         SLOPE1=DTAN(THETA(NREGIO(J)))
 7         B1=YENT1(NREGIO(J))-SLOPE1*XENT1(NREGIO(J))
 1   C
 1   21    CONTINUE
 7         I2=1
 7         I3=3
 7         DO 110 I=1,3
 1   C
 1   C---- FOR A GIVEN SIDE OF THE CURRENT REGION, DETERMINE YMAX,YMIN,XMAX,
 1   C---- AND XMIN FOR THAT SIDE.
 1   C
 7         YMAX=DMAX1(TCORD(INODS(NREGIO(J),I2),2),TCORD(INODS(NREGIO(J),I3),
 6        *2))
 7         YMIN=DMIN1(TCORD(INODS(NREGIO(J),I2),2),TCORD(INODS(NREGIO(J),I3),
 6        *2))
 7         XMAX=DMAX1(TCORD(INODS(NREGIO(J),I2),1),TCORD(INODS(NREGIO(J),I3),
 6        *1))
 7         XMIN=DMIN1(TCORD(INODS(NREGIO(J),I2),1),TCORD(INODS(NREGIO(J),I3),
 6        *1))
 1   C
 1   C---- DETERMINE IF THIS SIDE IS A SIDE OF CONSTANT X, CONSTANT Y, OR IF
 1   C---- THE SIDE IS SLOPED.
 1   C
10              IF(TCORD(INODS(NREGIO(J),I2),1).EQ.TCORD(INODS(NREGIO(J),I3),
 6        *     1))GO TO 30
10              IF(TCORD(INODS(NREGIO(J),I2),2).EQ.TCORD(INODS(NREGIO(J),I3),
 6        *     2))GO TO 40
10              SLOPE2=(TCORD(INODS(NREGIO(J),I2),2)-TCORD(INODS(NREGIO(J),I3),
 6        *     2))/(TCORD(INODS(NREGIO(J),I2),1)-TCORD(INODS(NREGIO(J),I3),1)
 6        *     )
10              B2=TCORD(INODS(NREGIO(J),I2),2)-SLOPE2*TCORD(INODS(NREGIO(J),I2
 6        *     ),1)
 1   C
 1   C---- DETERMINE THE EQUATION OF THE SLOPED SIDE. USING THIS EQUATION AND
 1   C---- THE EQUATION FOR THE RAY, DETERMINE THE INTERSECTION POINT. THEN
 1   C---- CHECK TO SEE IF THIS INTERSECTION POINT FALLLS WITHIN THE BOUNDARY
 1   C---- OF THE REGION (LABEL 100).
 1   C
10              DCRAM=SLOPE1-SLOPE2
10              YNUM=B1*SLOPE2*(-1.0)+B2*SLOPE1
10              XNUM=B2-B1
10              YINT=YNUM/DCRAM
10              XINT=XNUM/DCRAM
10              GO TO 100
 1   C
```

```
START
COL    ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1    C---- SINCE X IS CONSTANT ALONG A SIDE, PLUG THIS VALUE OF X INTO THE
  1    C---- EQUATION FOR THE RAY AND DETERMINE THE VALUE OF Y.  THEN CHECK TO
  1    C---- SEE IF THIS INTERSECTION POINT FALLS WITHIN THE BOUNDARY OF THE
  1    C---- REGION (LABEL 100).
  1    C
  1    30          CONTINUE
 10                XINT=TCORD(INODS(NREGIO(J),I2),1)
 10                YINT=B1+SLOPE1*XINT
 10                SLOPE2=-1.0D70
 10                B2=-1.0D70
 10                GO TO 100
  1    C
  1    C---- SINCE Y IS CONSTANT ALONG A SIDE, PLUG THIS VALUE OF Y INTO THE
  1    C---- EQUATION FOR THE RAY AND DETERMINE THE VALUE OF X.  THEN CHECK TO
  1    C---- SEE IF THIS INTERSECTION POINT FALLS WITHIN THE BOUNDARY OF THE
  1    C---- REGION (LABEL 100).
  1    C
  1    40          CONTINUE
 10                YINT=TCORD(INODS(NREGIO(J),I2),2)
 10                XINT=(YINT-B1)/SLOPE1
 10                SLOPE2=1.0D70
 10                B2=1.0D70
  1    100         CONTINUE
 10                DDIF=YENT1(NREGIO(J))
 10                DDI2=XENT1(NREGIO(J))
 10                IF(DABS(DDI2).LT.0.001D0)DDI2=1.0D0
 10                IF(DABS(DDIF).LT.0.001D0)DDIF=1.0D0
 10                DDIF=(YINT-YENT1(NREGIO(J)))/DDIF
 10                DDI2=(XINT-XENT1(NREGIO(J)))/DDI2
 10                IF(SLOPE1.GT.0.0D0.AND.YINT.LT.YENT1(NREGIO(J)))GO TO 109
 10                IF(SLOPE1.LT.0.0D0.AND.YINT.GT.YENT1(NREGIO(J)))GO TO 109
 10                IF(DABS(DDIF).LT.0.0001D0.AND.DABS(DDI2).LT.0.0001D0)
  6         *      GO TO 109
 10                IF(YINT.LE.YMAX.AND.YINT.GE.YMIN.AND.XINT.LE.XMAX.AND.
  6         *      XINT.GE.XMIN)GO TO 120
  1    109         CONTINUE
 10                IF(I.EQ.3)GO TO 130
 10                I2=I2+2
 10                I3=I3+2
  1    110   CONTINUE
  1    120   CONTINUE
  7          XEXT1(NREGIO(J))=XINT
  7          YEXT1(NREGIO(J))=YINT
  7          GO TO 140
  1    C
  1    C---- ONCE THE EXIT POINT FOR REGION J IS FOUND, DETERMINE THE NEXT
  1    C---- REGION INTO WHICH THE RAY IS ENTERING.
  1    C
  1    130   CONTINUE
  7          IFLAG9=1
  7          RETURN
  1    140   CONTINUE
  1    C
  1    C---- CHECK TO MAKE SURE THAT WE ARE NOT ALREADY IN THE LAST REGION.
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C----
  7         IF(IFLAG.EQ.1)GO TO 500
  7         CALL REGI
  7         IF(IFLAG2.GT.0)GO TO 900
  7         GO TO 10
  1   500   CONTINUE
  1   C
  1   C---- IF THE BEAM ENTERED AN ANGLED REGION BEFORE THE CODE ADJUSTED
  1   C---- THE HALF ANGLE, CHECK TO MAKE SURE THAT THE BEAM ENTERED THIS
  1   C---- SAME ANGLED REGION AFTER THE HALF ANGLE WAS ADJUSTED.
  1   C
  7         IF(IFLAG3.GT.0)GO TO 1100
  7         RETURN
  1   900   CONTINUE
  1   C
  1   C---- BECAUSE THIS IS AN ANGLED REGION, THE CODE NOW CHECKS FOR THE
  1   C---- FOLLOWING:
  1   C----                (1) IS THIS THE FIRST TIME AN ANGLED REGION HAS
  1   C----                    BEEN ENCOUNTERED?
  1   C----                (2) HAVE TWO OR MORE ANGLED REGIONS BEEN ENCOUNTERED?
  1   C
  7         IF(IFLAG3.NE.IFLAG2.AND.JCHEC.NE.0)GO TO 1000
  7         IF(IFLAG3.EQ.IFLAG2.AND.JCHEC.EQ.1)GO TO 1050
  1   C
  1   C---- IF THE CODE MAKES TO HERE THIS IS THE FIRST TIME AN ANGLED REGION
  1   C---- HAS BEEN ENCOUNTERED.
  1   C
  7         ALPH2=DATAN(SLOPE2)
  7         IF(SLOPE2.LT.0.0D0)ALPH2=3.141592654D0+ALPH2
  7         JCHEC=1
  7         IFLAG3=IFLAG2
  1   C
  1   C---- THE CODE NOW ADJUSTS THE HALF ANGLE OF THE LASER BEAM IN WATER.
  1   C---- TO DO THIS, THE CODE DETERMINES THE ORDER IN WHCIH THE BEAM
  1   C---- ENTERS AN ANGLED SURFACE,I.E., DOES THE BEAM ENTER A REGION
  1   C---- WHERE THE ANGLED INTERFACE HAS WATER ON THE LEFT AND PLEXIGLASS
  1   C---- ON THE RIGHT?  OR DOES THE BEAM ENTER A REGION WITH WATER ON THE
  1   C---- RIGHT AND PLEXIGLASS ON THE LEFT?
  1   C
  7         D1=TINDX(NREGIO(J))-TINP
  7         D2=TINDX(ITEMP)-TINW
  7         IF(DABS(D1).LT.1.0D-05.AND.DABS(D2).LT.1.0D-05)
  6        *GO TO 910
  7         THETA(NREGIO(1))=DARSIN(TINP/TINW*DSIN(THPLEX+1.5707963D0-
  6        *ALPH2))+ALPH2-1.5707963D0
  7         GO TO 920
  1   910   CONTINUE
  7         TPLEX=DARSIN(TINW/TINP*DSIN(THWAT+1.5707963D0-ALPH2))+ALPH2-
  6        *1.5707963D0
  7         THETA(NREGIO(1))=DARSIN(TINP/TINW*DSIN(TPLEX))
  1   920   CONTINUE
  1   C
  1   C---- REDO ALL CALCULATIONS WITH THE CORRECTED ANGLE.
  1   C
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

 7          J=1
 7          GO TO 10
 1    1000  CONTINUE
 7          IFLAG8=1
 7          RETURN
 1    1050  CONTINUE
 1    C
 1    C---- NOW FOR THE ANGLED REGION WHICH HAS A VERTICAL SURFACE ON THE
 1    C---- RIGHT AND AN ANGLED SURFACE ON THE LEFT, SET THE HALF ANGLE
 1    C---- OF THE LASER TO ITS KNOWN VALUE.
 1    C
 7          JCHEC=JCHEC+1
 7          J2=J
 7          J=J+1
 7          NREGIO(J)=ITEMP
 7          YENT1(ITEMP)=YEXT1(NREGIO(J2))
 7          XENT1(ITEMP)=XEXT1(NREGIO(J2))
 7          D11=TINDX(ITEMP)-TINP
 7          D12=TINDX(ITEMP)-TINW
 7          IFLAG3=0
 7          IF(D11.LT.1.0D-05)THETA(ITEMP)=THPLEX
 7          IF(D12.LT.1.0D-05)THETA(ITEMP)=THWAT
 7          GO TO 15
 1    1100  CONTINUE
 7          IFLAG7=1
 7          RETURN
 7          END
 1    C     ****************
 7          SUBROUTINE REGI                                                   203
 1    C     ****************
 1    C
 1    C---- FOR A RAY WITH A KNOWN EXIT POINT IN A REGION, THIS SUBROUTINE
 1    C---- DETERMINES THE NEXT REGION WHICH THE BEAM WILL ENTER.
 1    C
 7          IMPLICIT REAL*8(A-H,O-Z)
 7          COMMON TCORD(2000,2),                                             205
 6         *XENT1(50),YENT1(50),XEXT1(50),YEXT1(50),THETA(50),
 6         *TINDX(50),XENT2(50),YENT2(50),XEXT2(50),YEXT2(50),
 6         *SCXADJ,SCYADJ,SHALF,TINW,TINP,XTEMP,YTEMP,
 6         *MATNO(1501),NPOIN,NELEM,
 6         *LNADJ(50,20),NREGIO(50),NREGI2(50),IANG(50),
 6         *INODS(1501,8),
 6         *NTITLE(10),NCOL(100),NDGRAF,
 6         *J3,J4,J,
 6         *IFLAG5,IFLAG6,IFLAG7,IFLAG8,IFLAG9
 7          COMMON /REG/SLOPE2,B2,THPLEX,THWAT,IFLAG,IFLAG2,ITEMP
 1    C----
 1    C---- LOCAL SUBROUTINE VARIABLES
 1    C----
 1    C---- ITEMP: SEE SUBROUTINE YPOINT
 1    C----
 1    C---- SLOP2: SLOPE FOR SIDE I5 OF REGION ITEMP
 1    C----
 1    C---- B8: Y INTERCEPT FOR SIDE I5 OF REGION ITEMP
```

```
START
COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C----
  7         IFLAG2=0
  7         IFLAG5=0
  7         IFLAG6=0
  7         DO 600 I=1,20
 10            ITEMP=LNADJ(NREGIO(J),I)
 10            IF(ITEMP.EQ.0)GO TO 601
 10            IF(ITEMP.EQ.NREGIO(J))GO TO 600
  1   C
  1   C---- DETERMINE THE MAXIMUM AND MINIMUM BOUNDARIES OF THE
  1   C---- ADJACENT REGIONS.
  1   C
  7         YMAX=DMAX1(TCORD(INODS(ITEMP,2),5),TCORD(INODS(ITEMP,7),
  6        *2))
  7         YMIN=DMIN1(TCORD(INODS(ITEMP,1),2),TCORD(INODS(ITEMP,3),
  6        *2))
  7         XMAX=DMAX1(TCORD(INODS(ITEMP,3),1),TCORD(INODS(ITEMP,5),
  6        *1))
  7         XMIN=DMIN1(TCORD(INODS(ITEMP,1),1),TCORD(INODS(ITEMP,7),
  6        *1))
  7         DO 598 I5=1,3
  1   C
  1   C---- CHECK EACH SIDE OF THE ELEMENT AND CHECK TO SEE IF THE SIDE IS
  1   C---- HORIZONTAL, VERTICAL OR SLANTED.
  1   C
 10            IF(I5.EQ.1)I2=1
 10            IF(I5.EQ.1)I3=3
 10            IF(I5.EQ.2)I2=5
 10            IF(I5.EQ.2)I3=7
 10            IF(I5.EQ.3)I2=7
 10            IF(I5.EQ.3)I3=1
 10            IF(TCORD(INODS(ITEMP,I2),1).EQ.TCORD(INODS(ITEMP,I3),
  6        *   1))GO TO 30
 10            IF(TCORD(INODS(ITEMP,I2),2).EQ.TCORD(INODS(ITEMP,I3),
  6        *   2))GO TO 40
  1   C
  1   C---- FOR THE GIVEN REGION AND SIDE, COMPUTE THE EQUATION FOR THIS SIDE.
  1   C
 10            SLOP2=(TCORD(INODS(ITEMP,I2),2)-TCORD(INODS(ITEMP,I3),
  6        *   2))/(TCORD(INODS(ITEMP,I2),1)-TCORD(INODS(ITEMP,I3),1)
  6        *   )
 10            B8=TCORD(INODS(ITEMP,I2),2)-SLOP2*TCORD(INODS(ITEMP,I2
  6        *   ),1)
 10            GO TO 100
  1   C
  1   C     FOR X CONSTANT ALONG A SIDE
  1   C
  1   30       CONTINUE
 10            SLOP2=-1.0D70
 10            B8=-1.0D70
 10            GO TO 100
  1   C
  1   C     FOR Y CONSTANT ALONG AN EDGE
  1   C
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   40        CONTINUE
 10             SLOP2=1.0D70
 10             B8=1.0D70
  1   100       CONTINUE
  1   C
  1   C---- CHECK TO SEE IF THE EQUATION FOR THIS PARTICULAR SIDE OF THIS
  1   C---- PARTICULAR ELEMENT MATCHES THE EQUATION OF THE KNOWN SIDE.
  1   C
 10             DIFF1=(SLOPE2-SLOP2)/SLOPE2
 10             DIFF2=(B2-B8)/B2
 10             IF(DABS(DIFF1).LT.1.0D-02.AND.DABS(DIFF2).LT.1.0D-02)GO TO 590
 10             GO TO 595
  1   590       CONTINUE
  1   C
  1   C---- THE EQUATIONS FOR THE TWO SIDES MATCHED, SO NOW CHECK TO
  1   C---- MAKE SURE THAT THE EXIT POINTS FALL WITHIN THE SUSPECTED REGION.
  1   C
 10             IF(YEXT1(NREGIO(J)).LE.YMAX.AND.YEXT1(NREGIO(J)).GE.YMIN.AND.
  6          *  XEXT1(NREGIO(J)).LE.XMAX.AND.XEXT1(NREGIO(J)).GE.XMIN)GO TO 700
  1   595       CONTINUE
  1   598   CONTINUE
  1   600   CONTINUE
  1   601   CONTINUE
  7         RETURN
  1   700   CONTINUE
  1   C
  1   C---- CHECK TO SEE IF THIS IS AN ANGLED REGION.
  1   C
  7         IF(I5.EQ.1)NSIDE=-1
  7         IF(I5.EQ.2)NSIDE=-3
  7         IF(I5.EQ.3)NSIDE=-4
  7         IF(IANG(ITEMP).EQ.NSIDE)GO TO 800
  7         J2=J
  7         J=J+1
  7         NREGIO(J)=ITEMP
  7         YENT1(ITEMP)=YEXT1(NREGIO(J2))
  7         XENT1(ITEMP)=XEXT1(NREGIO(J2))
  7         IF(MATNO(ITEMP).LT.0)IFLAG=1
  7         RETURN
  1   800   CONTINUE
  1   C
  1   C---- CHECK THE FOLLOWING:
  1   C----                     (1) MAKE SURE THAT THE RAY ENTERS ON SIDE
  1   C----                         4, OR THAT THE INDICES OF REFRACTION
  1   C----                         ARE THE SAME.
  1   C
  7         IF(IANG(ITEMP).EQ.-4.AND.
  6        *TINDX(ITEMP).NE.TINDX(NREGIO(J)))GO TO 820
  7         IF(TINDX(ITEMP).NE.TINDX(NREGIO(J)))GO TO 830
  7         J2=J
  7         J=J+1
  7         NREGIO(J)=ITEMP
  7         YENT1(ITEMP)=YEXT1(NREGIO(J2))
  7         XENT1(ITEMP)=XEXT1(NREGIO(J2))
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  7         IF(MATNO(ITEMP).LT.0)IFLAG=1
  7         RETURN
  1   820   CONTINUE
  7         IFLAG2=NREGIO(J)
  7         RETURN
  1   830   CONTINUE
  7         RETURN
  7         END
```

# IX. APPENDIX C

An analysis determining the path of a laser beam through a plexiglas cylinder sitting inside a plexiglas box has already been performed [C1]. This analysis is extended here to include multiple cylinders. Figures C-1 through C-6 outline the equations for two cylinders. Although it is not shown in this appendix, these equations can easily be extended to multiple cylinders using indicial notation.

Figure C-1 is a cross-sectional view of two vertical cylinders sitting inside a plexiglas box. The point P(x,y) has been chosen as the point at which velocity measurements will be made. The remaining angles and distances specified in Figure C-1 have been defined in Table C-I.

Figure C-2 is a close up of the inner cylinder which contains the specified measurement point. By drawing a radius from the origin through the point at which the laser beam intersects the inner radius of the inner cylinder, it can be observed that two right triangles with a common side are formed. As shown in Figure C-2 a relationship for $\alpha_{1,1,1}$ interms of $R_p$, $R_{1,1}$, B, and ALPH4G can be obtained from these two triangles. At the inner radius of the inner cylinder, Snell's law can be applied to determine $\alpha_{1,1,2}$. The resulting relationship for $\alpha_{1,1,2}$ is also shown in Figure C-2.

Figure C-3 shows the path of the laser beam through the first cylinder. By examining the figure, it can be observed that two right triangles with a common side are formed. From these two triangles a relationship can be obtained for $\alpha_{1,2,1}$. By applying Snell's law at the outer radius of the first cylinder a relationship for $\alpha_{1,2,2}$ can also be obtained.

Figure C-4 shows the two plexiglas cylinders. Again by examining the figure, it can be seen that two right triangles with a common side are formed. From these two triangles, a relationship for $\alpha_{2,1,1}$ can be obtained. By applying Snell's law at the inner radius of the outer cylinder a relationship for $\alpha_{2,1,2}$ can be obtained.

Figure C-5 shows the two plexiglas cylinders and the plexiglas box. Again by examining the figure, it will be noticed that two right triangles with a common side are formed. From these two triangles a relationship for $\alpha_{2,2,1}$ can be obtained. By applying Snell's law at the outer radius of the outer cylinder, a relationship for $\alpha_{2,2,2}$ can be obtained.

Many triangles are formed by the radii passing through the points at which the laser beam enters and exits each cylinder. By examining each triangle a relationship for ALPHW can be obtained. By examining these same triangles, a relationship for ANGLE can also be obtained. Relationships for ANGLE and ALPHW are shown in Figure C-5.

As shown in Figure C-5, Snell's law can be applied at the interface between the water and the plexiglas box to determine ALPHP. Snell's law can also be applied at the interface between the plexiglas box and the air to determine ALPHGH. Relationships for both ALPHP and ALPHGH are shown in Figure C-5.

At this point an equation set which completely defines the laser beam path has been determined and is listed in Table C-II. However, a direct solution is not possible.

The computer program cylinder uses an iterative technique to solve the equation set. The basic approach is to assume a value for ALPH4G, and then to use equations (C-1) through (C-14) to calculate ALPHGH. Once a value of ALPH4G is obtained which gives a value of ALPHGH sufficiently close to the true value the iteration is stopped.

Figure C-6 is a drawing of the outer cylinder and the plexiglas box. This figure shows how the relationship for Y1 is obtained.

---

C1. Hanle, D. D., "In Vitro Fluid Dynamics of Prosthetic Aortic Heart Valves in Steady and Pulsatile Flow", Ph.D. Thesis, California Institute of Technology, Pasadena, California, 1984.

## TABLE C-1

### DEFINITION OF NOMENCLATURE

| | | |
|---|---|---|
| x | = | x coordinate of laser beam crossing point |
| y | = | y coordinate of laser beam crossing point |
| $R_p$ | = | radius of laser beam crossing point |
| $R_{i,j}$ | = | radius of a particular cylinder. i indicates the cylinder (from center outwards). j indicates which radius (1 means inner radius, 2 means outer radius) |
| $\alpha_{i,j,k}$ | = | angle that the laser beam makes with respect to the normal at the cylinder-water interface. i indicates the cylinder (from center outwards). j indicates inner or outer radius (1 means inner radius, 2 means outer radius). k indicates the angle (1 means inner angle, 2 means outer angle). |
| ALPHW | = | direction angle for the laser beam in the water between the outer cylinder and the plexiglas box |
| ALPHP | = | direction angle for the laser beam in the plexiglas box |
| ALPH4G | = | direction angle of the laser beam in the fluid at the beam crossing. |
| ALPHGH | = | Half angle of the LDA in air. |
| Y1 | = | y coordinate of the point at which the laser beam exits the plexiglas box. |
| D | = | distance between the origin and the inside of the plexiglas wall. |
| t | = | thickness of the plexiglas box |
| ANGLE | = | angle between the line from the origin through the point where the laser beam exits the outermost cylinder, and the horizontal. |
| i | = | number of cylidners. |

TABLE C-II
EQUATIONS WHICH DESCRIBE THE PATH OF A
LASER BEAM THROUGH CYLINDRICALLY LAYERED MEDIA

(C-1) $\beta = \mathrm{INVTAN}\ (y/x)$

(C-2) $R_p = x/\ \cos\ (\beta)$

(C-3) $\alpha_{1,1,1} = \mathrm{INVSIN}\ [\frac{R_p}{R_{1,1}} \sin\ [\beta - \mathrm{ALPH4G}]]$

(C-4) $\alpha_{1,1,2} = \mathrm{INVSIN}\ [\frac{n_{water}}{n_{plex}} \sin\ (\alpha_{1,1,1})]$

(C-5) $\alpha_{1,2,1} = \mathrm{INVSIN}\ [\frac{R_{1,1}}{R_{1,2}} \sin\ (\alpha_{1,1,2})]$

(C-6) $\alpha_{1,2,2} = \mathrm{INVSIN}\ [\frac{n_{plex}}{n_{water}} \sin\ (\alpha_{1,2,1})]$

(C-7) $\alpha_{2,1,1} = \mathrm{INVSIN}\ [\frac{R_{1,2}}{R_{2,1}} \sin\ (\alpha_{1,2,2})]$

(C-8) $\alpha_{2,1,2} = \mathrm{INVSIN}\ [\frac{n_{water}}{n_{plex}} \sin\ (\alpha_{2,1,1})]$

(C-9) $\alpha_{2,2,1} = \mathrm{INVSIN}\ [\frac{R_{2,1}}{R_{2,2}} \sin\ (\alpha_{2,1,2})]$

(C-10) $\alpha_{2,2,2} = \mathrm{INVSIN}\ [\frac{n_{plex}}{n_{water}} \sin\ (\alpha_{2,2,2})]$

$$\alpha_{i,1,1} = \mathrm{INVSIN}\ [\frac{R(i-1),2}{Ri,1} \sin(\alpha_{(i-1)},2,2)]$$

$$\alpha_{i,1,2} = \mathrm{INVSIN}\ [\frac{n_{water}}{n_{plex}} \sin(\alpha_{i,1,1})]$$

$$\alpha_{i,2,1} = \mathrm{INVSIN}\ [\frac{R_{i,1}}{R_{i,2}} \sin\ (\alpha i,1,2)]$$

$$\alpha_{i,2,2} = \mathrm{INVSIN}\ [\frac{n_{plex}}{n_{water}} \sin\ (\alpha i,2,1)]$$

TABLE C-II (cont'd)

EQUATIONS WHICH DESCRIBE THE PATH OF A LASER BEAM THROUGH CYLINDRICALLY LAYERED MEDIA

(C-11) $$\mathrm{ALPHW} = \mathrm{ALPH4G} + \alpha_{1,1,1} - \alpha_{1,1,2} + \alpha_{1,2,1} - \alpha_{1,2,2} + \alpha_{2,1,1} + \alpha_{2,1,2} + \alpha_{2,2,1} - \alpha_{2,2,2} \;\ldots\ldots\; + \alpha_{i,1,1} - \alpha_{i,1,2} + \alpha_{i,2,1} - \alpha_{i,2,2}$$

(C-12) $$\mathrm{ALPHP} = \mathrm{INVSIN}\left[\frac{n_{water}}{n_{plex}} \sin(\mathrm{ALPHW})\right]$$

(C-13) $$\mathrm{ALPHGH} = \mathrm{INVSIN}\left[\frac{n_{plex}}{n_{air}} \sin(\mathrm{ALPHP})\right]$$

(C-14) $$\mathrm{ANGLE} = \mathrm{ALPHW} + \alpha_{i,2,2}$$

(C-15) $$\mathrm{Y1} = R_{i,2} * \sin(\mathrm{ANGLE}) + [D - R_{i,2} \cos(\mathrm{ANGLE})] * [\tan(\mathrm{ALPHW})] + t * \tan(\mathrm{ALPHP})$$

CYLINDRICALLY LAYERED INTERFACES

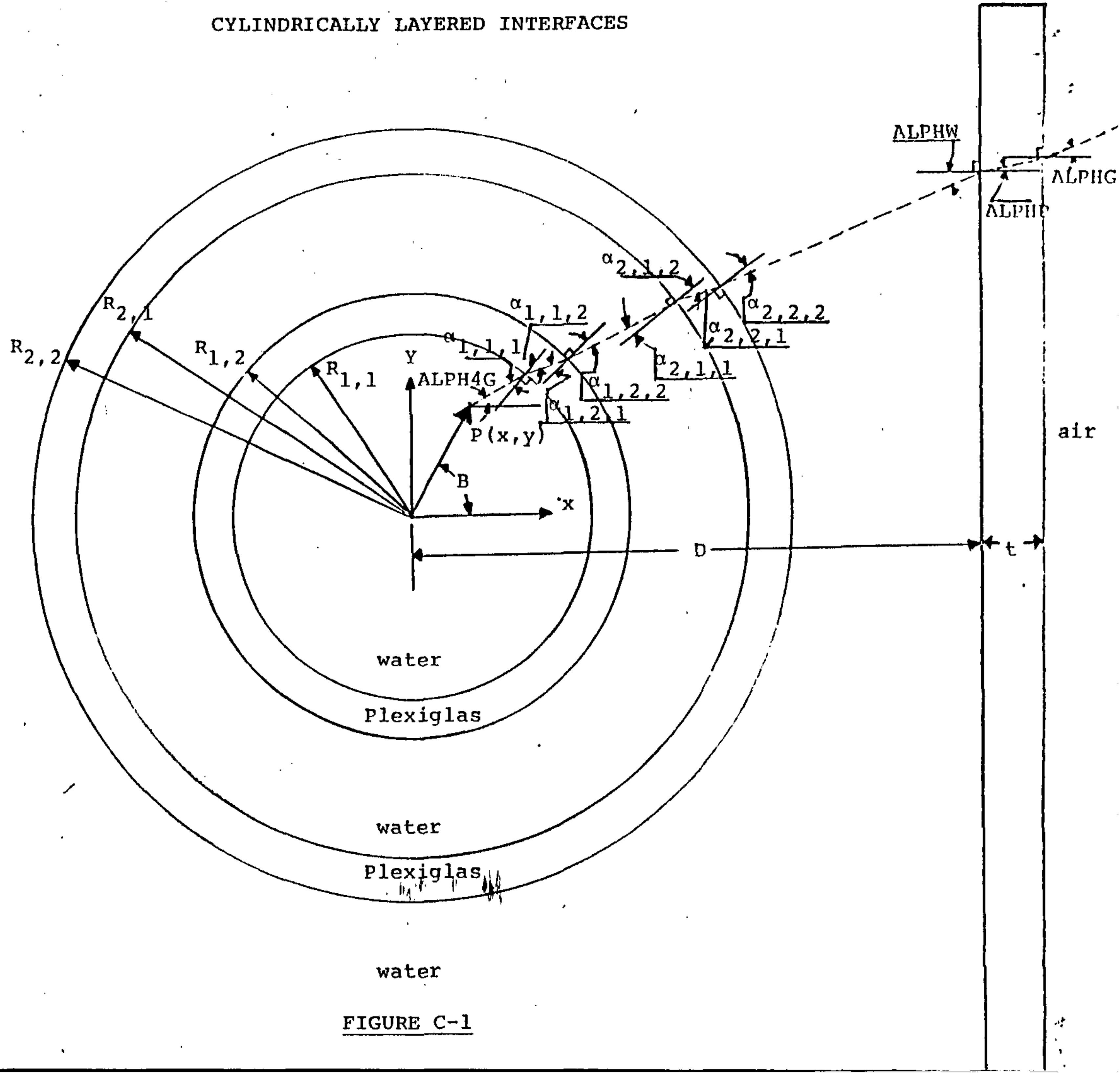


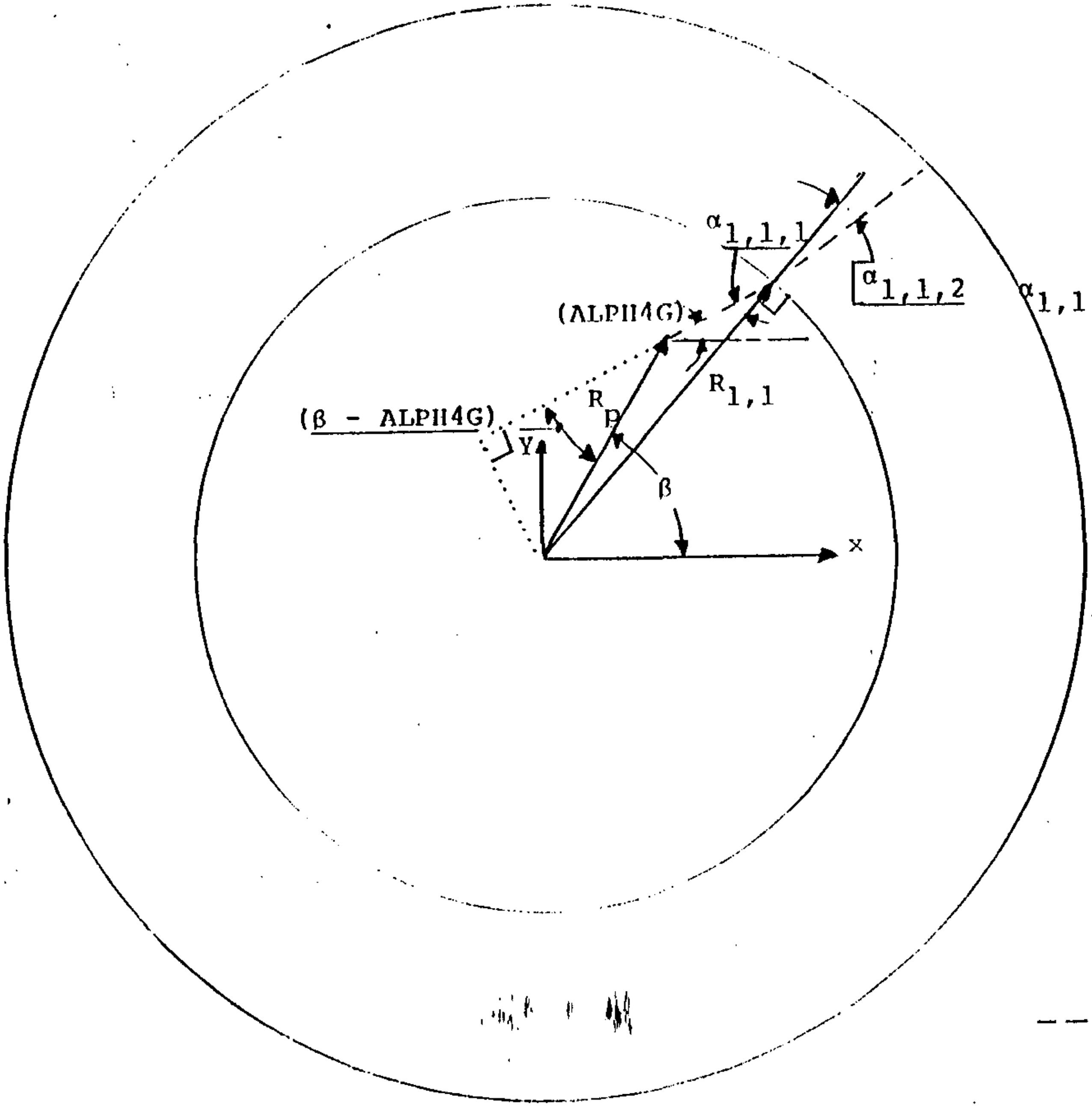

FIGURE C-1

DETERMINATION OF $\beta$, $R_p$, $\alpha_{1,1,1}$ AND $\alpha_{1,1,2}$

(ONLY INNER CYLINDER IS SHOWN)

$\beta$ INVTAN $\left(\frac{y}{x}\right)$

$$R_p = \frac{x}{\cos(\beta)}$$




From Geometry

$$\alpha_{1,1,1} = \text{INVSIN}\left[\frac{R_p}{R_{1,1}} \sin (\beta - \text{ALPH4G})\right]$$

From Snell's law

$$\alpha_{1,1,2} = \text{INVSIN}\left[\frac{n_{wate}}{n_{plex}} \sin\alpha_{1,1,1}\right]$$

---- laser beam

FIGURE C-2

DETERMINATION OF $\alpha_{1,2,1}$ and $\alpha_{1,2,2}$

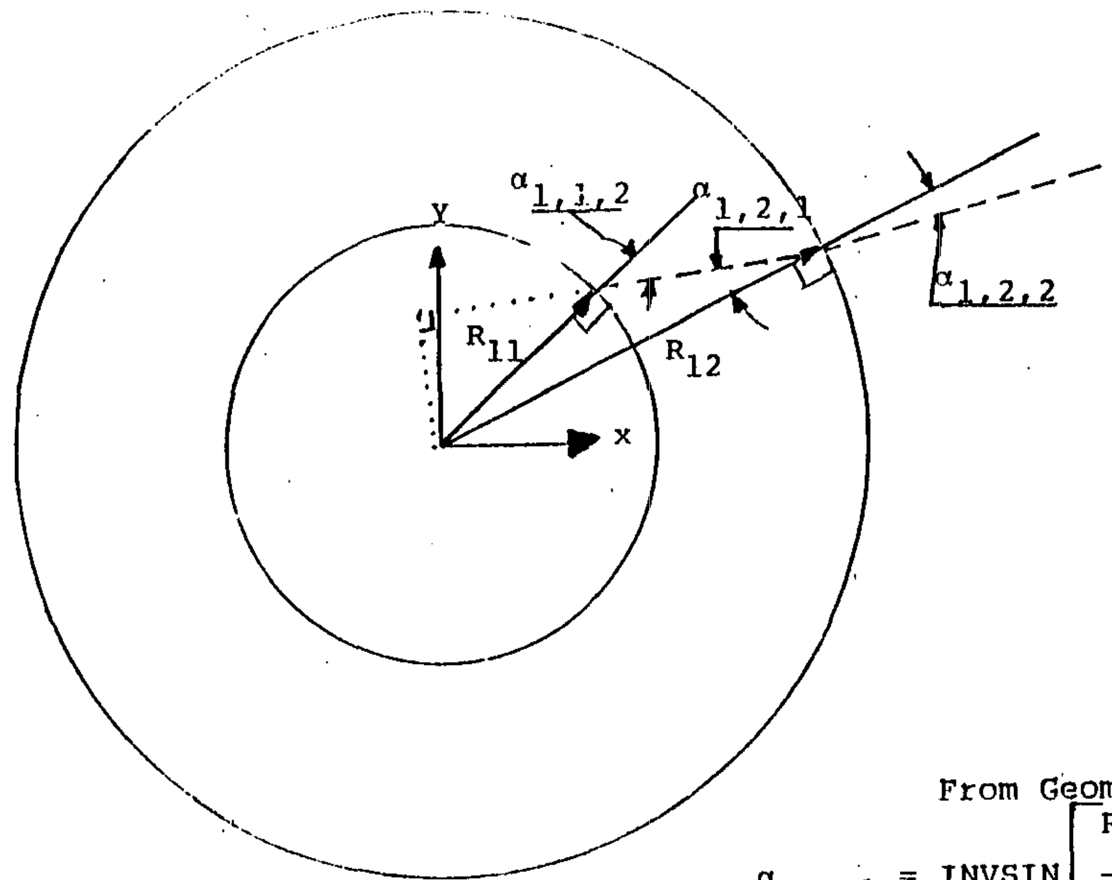



FIGURE C-3

From Geometry

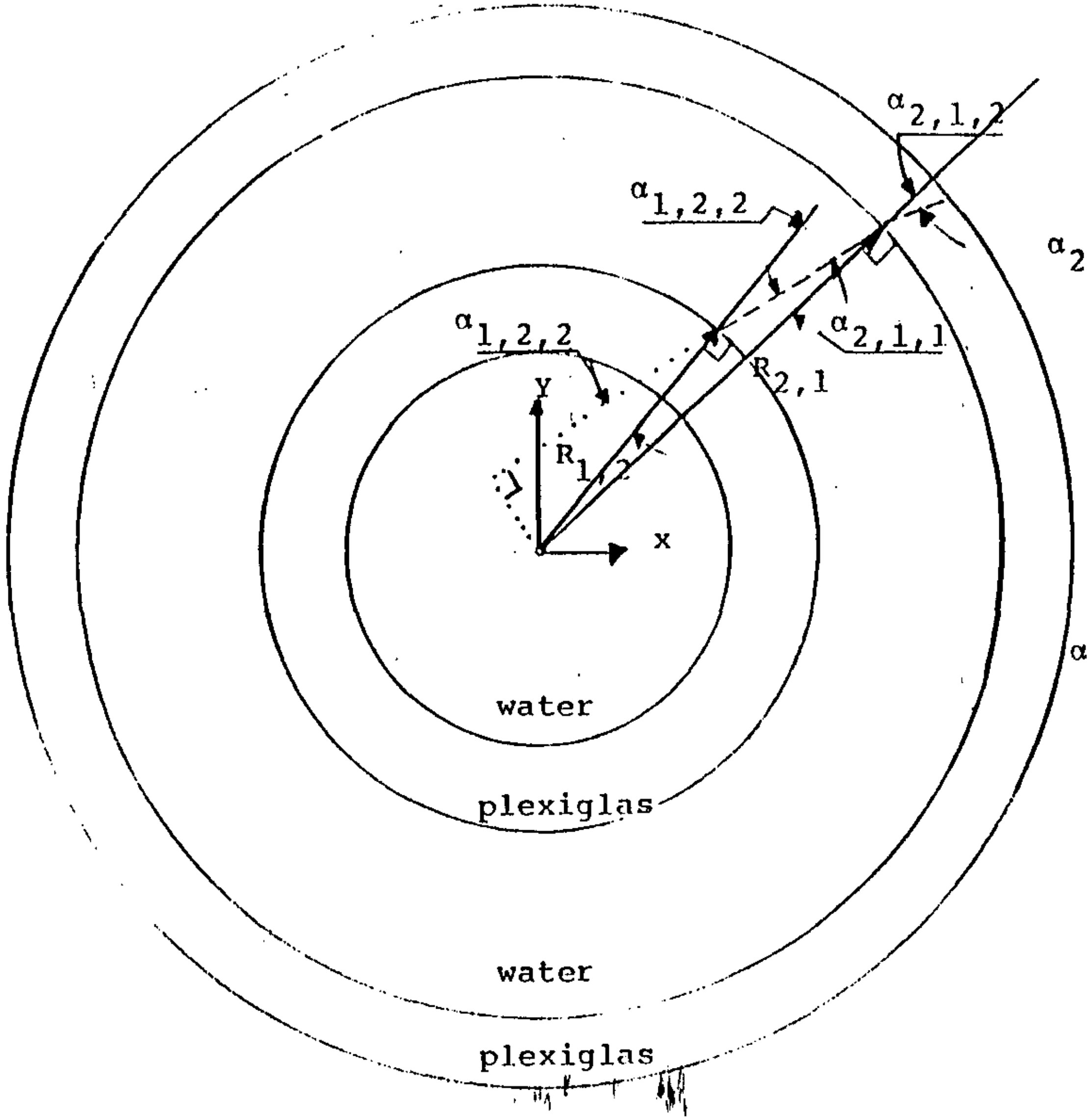
$$\alpha_{1,2,1} = \text{INVSIN}\left[\frac{R_{1,1}}{R_{1,2}} \sin \alpha_{1,1,2}\right]$$

From Snell's Law

$$\alpha_{1,2,2} = \text{INVSIN}\left[\frac{n_{plexiglas}}{n_{water}} \sin \alpha_{1,2,1}\right]$$

DETERMINATION OF $\alpha_{2,2,1}$ and $\alpha_{2,1,2}$

(OUTER BOX IS NOT SHOWN)




From Geometry

$$\alpha_{2,1,1} = \text{INVSIN}\left[\frac{R_{1,2}}{R_{2,1}} \sin \alpha_{1,2,2}\right]$$

From Snell's law

$$\alpha_{2,1,2} = \text{INVSIN}\left[\frac{n_{water}}{n_{plexiglas}} \sin \alpha_{2,1,1}\right]$$

– – – laser beam

FIGURE C-4

DETERMINATION OF $\alpha_{2,2,1}$, $\alpha_{2,2,2}$, ALPHW, ALPHP, ALPHGH

rom Geometry

$${}_{2,2,1} = \text{INVSIN} \; \frac{R_{2,1}}{R_{2,2}} \sin\alpha_{2,1,2}$$

rom Snell's law

$${}_{2,2,2} = \text{INVSIN} \left[ \frac{n_{plex}}{n_{water}} \sin\alpha_{2,2,1} \right]$$

$$\text{LPHP} = \text{INVSIN} \left[ \frac{n_{water}}{n_{plex}} \sin (\text{ALPH}_W) \right]$$

$$\text{LPHGH} = \text{INVSIN} \left[ \frac{n_{plex}}{n_{air}} \sin(\text{ALPHP}) \right]$$

rom simple addition of angles

LPHW = ALPH4G

$$\alpha_{1,1,1} - \alpha_{1,1,2} + \alpha_{1,2,1} - \alpha_{1,2,2}$$

$$\alpha_{2,1,1} - \alpha_{2,1,2} + \alpha_{2,2,1} - \alpha_{2,2,2}$$

$$\text{NGLE} = \text{ALPHW} + \alpha_{2,2,2}$$

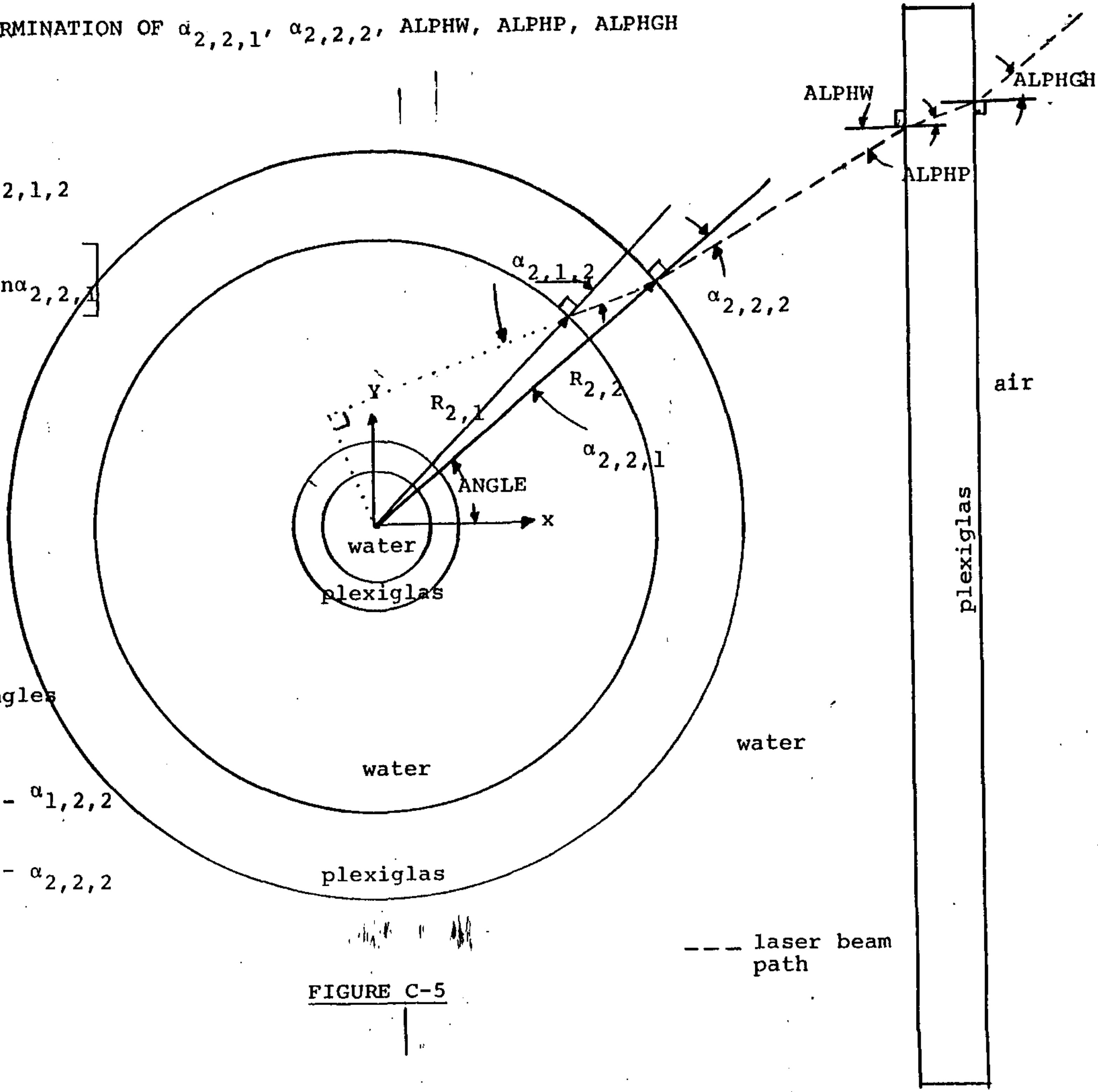



FIGURE C-5

DETERMINATION OF THE Y COORDINATE AT WHICH THE LASER BEAM EXITS THE PLEXIGLAS BOX

$$Y11 = R \cdot SIN\ (ANGLE)$$

$$Y12 = \left[D - R22\ \cos(ANGLE)\right] \cdot TAN\ (ALPHW)$$

$$Y13 = T \cdot TAN\ (ALPHP)$$

$$Y1 = Y11 + Y12 + Y13$$

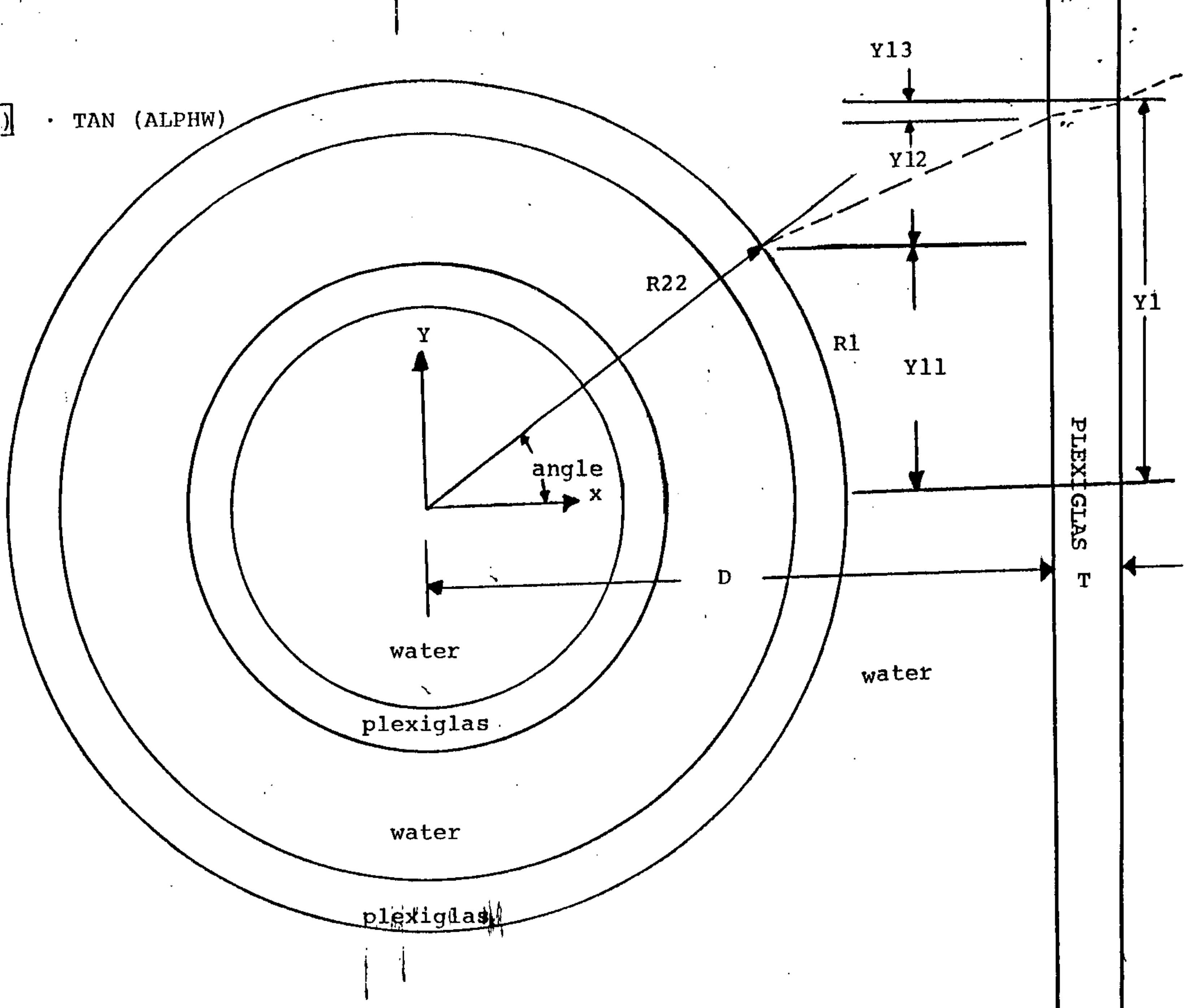



FIGURE C-6

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C
  1   C                    APPENDIX D
  1   C          (FORTRAN LISITING OF PROGRAM CYLINDER)
  1   C
  1   C
  1   C
  1   C********************
  1   C    PROGRAM CYLINDER
  1   C********************
  1   C
  9           IMPLICIT REAL*8(A-H,O-Z)
  9           COMMON R(4,2),ALPH(4,2,2),TINW,TINP,ERR,ALPHH,ALPHP,RP,
  6        *  ALPHW,T,D,TINA,ANG1(11),ANG2(11),BETA,NTITLE(10),IS
  1   C
  9           READ(5,10) NTITLE
  1   10     FORMAT(12A4)
  9           ERR=0.00001D0
  1   C----
  1   C----   READ IN DATA
  1   C----
  1   C----   DEFINITION OF VARIABLES
  1   C----
  1   C---- XO= X COORDINATE OF REFERENCE POINT
  1   C----
  1   C---- YO= Y COORDINATE OF REFERENCE POINT
  1   C----
  1   C---- X= X COORDINATE OF DATA POINT
  1   C----
  1   C---- Y= Y COORDINATE OF DATA POINT
  1   C----
  1   C---- T = THICKNESS OF PLEXIGLASS WALL
  1   C----
  1   C---- D = DISTANCE FROM CENTER OF TUBE TO PLEXIGLASS WALL
  1   C----
  1   C---- TINW = INDEX OF REFRACTION FOR WATER
  1   C----
  1   C---- TINA = INDEX OF REFRACTION FOR AIR
  1   C----
  1   C---- TINP = INDEX OF REFRACTION FOR PLEXIGLASS
  1   C----
  1   C---- ALPHH= LASER DOPPLER ANEMONETER HALF ANGLE IN AIR
  1   C----
  9           READ(5,20) XO,YO,X,Y
  9           WRITE(6,19) XO,YO,X,Y
  1   19     FORMAT(' REFEDRENCE POINT: XO= ',D15.5,5X,' YO=',D15.5,/,
  6        * ' DATA POINT : X= ',D15.5,' Y= ',D15.5)
  9           READ(5,20) T,D
  1   20      FORMAT(4D15.8)
  9           READ(5,21) TINW,TINA,TINP,ALPHH
  1   21      FORMAT(4D15.8)
  9           DO 35 I=1,4
 12              READ(5,30) (R(I,J),J=1,2)
 12              WRITE(6,32) I,(R(I,J),J=1,2)
  1   30         FORMAT(2D15.5)
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   32        FORMAT(' CYLINDER',2X,I5,2X,'INNER RADIUS= ',D15.5,2X,
  6        *    'OUTER RADIUS= ',D15.5)
  1   35     CONTINUE
  1   C----
  1   C---- DETERMINE HOW MANY CYLINDERS ARE PRESENT
  1   C----
  7         IS=4
  7         IF(R(4,2).LT.0.00001D0)IS=3
  7         IF(R(3,2).LT.0.00001D0)IS=2
  7         IF(R(2,2).LT.0.00001D0)IS=1
  9           ALPHH=ALPHH/180.0*3.141592654D0
  9           ALPHHT  = ALPHH
  1   C----
  1   C----  COMPUTE PATH OF LASER BEAMS FOR REFERENCE POINT
  1   C----
  9           CALL ANGLE(XO,YO,ALPHHT,ALPH4G)
  9           H0      = HCALC(ALPHHT,ALPH4G)
  9           ALPHHT =-ALPHH
  9           CALL ANGLE(XO,YO,ALPHHT,ALPH4G)
  9           H01     = HCALC(ALPHHT,ALPH4G)
  9           ALPHHT  = ALPHH
  1   C----
  1   C----  COMPUTE PATH OF LASER BEAMS FOR ACTUAL DATA POINT
  1   C----
  9           CALL ANGLE(X,Y,ALPHHT,ALPH4G)
  9           H1      = HCALC(ALPHHT,ALPH4G)
  9           ANG1(1)=BETA
  9           ANG1(2)=ALPH4G+ALPH(1,1,1)
  9           ANG1(3)=ANG1(2)-ALPH(1,1,2)+ALPH(1,2,1)
  9           ANG1(4)=ANG1(3)-ALPH(1,2,2)+ALPH(2,1,1)
  9           ANG1(5)=ANG1(4)-ALPH(2,1,2)+ALPH(2,2,1)
  9           ANG1(6)=ANG1(5)-ALPH(2,2,2)+ALPH(3,1,1)
  9           ANG1(7)=ANG1(6)-ALPH(3,1,2)+ALPH(3,2,1)
  9           ANG1(8)=ANG1(7)-ALPH(3,2,2)+ALPH(4,1,1)
  9           ANG1(9)=ALPHW+ALPH(IS,2,2)
  9           ANG1(10)=ALPHW
  9           ANG1(11)=ALPHP
  9           A1=ALPH4G
  9           ALPHHT  = -ALPHH
  9           CALL ANGLE(X,Y,ALPHHT,ALPH4G)
  9           ANG2(1)=BETA
  9           ANG2(2)=ALPH4G+ALPH(1,1,1)
  9           ANG2(3)=ANG2(2)-ALPH(1,1,2)+ALPH(1,2,1)
  9           ANG2(4)=ANG2(3)-ALPH(1,2,2)+ALPH(2,1,1)
  9           ANG2(5)=ANG2(4)-ALPH(2,1,2)+ALPH(2,2,1)
  9           ANG2(6)=ANG2(5)-ALPH(2,2,2)+ALPH(3,1,1)
  9           ANG2(7)=ANG2(6)-ALPH(3,1,2)+ALPH(3,2,1)
  9           ANG2(8)=ANG2(7)-ALPH(3,2,2)+ALPH(4,1,1)
  9           ANG2(9)=ALPHW+ALPH(IS,2,2)
  9           ANG2(10)=ALPHW
  9           ANG2(11)=ALPHP
  9           A2=ALPH4G
  9           H2      = HCALC(ALPHHT,ALPH4G)
  1   C----
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1   C---- COMPUTE HOW FAR THE LASER MUST BE MOVED FROM THE REFERNCE POINT
  1   C----
  9           HRZ     = (((H1-H2)/2.0-(H0-H01)/2.0)/DTAN(ALPHH))
  9           HRZ     = -HRZ
  9           VRT     = ((H1+H2)/2.0)-((H0+H01)/2.0)
  1   C----
  1   C----  COMPUTE THE NEW LDA HALF ANGLE IN THE FLUID MEDIUM
  1   C----
  9           TDIR=(A1-A2)/2.0
  9           TDIR=TDIR*180.0/3.141592654
  1   C----
  1   C----  COMPUTE THE DIRECTION OF THE VELOCITY VECTOR IN THE FLUID
  1   C----  MEDIUM
  1   C----
  9           TDIR2=3.141592654/2.0+(A1+A2)/2.0
  9           TDIR2=TDIR2*180.0/3.141592654
  1   C----
  1   C---- PRINT OUT THE RESULTS
  1   C----
  9           WRITE(6,100)HRZ,VRT
  9           WRITE(6,115)TDIR,TDIR2
  1   100     FORMAT(' LASER MUST BE MOVED ',D15.5,' IN THE HORIZONTAL ',
  6        *  'DIRECTION',/,'  LASER MUST BE MOVED ',D15.5,' IN THE VERTICAL'
  6        *  ,' DIRECTION')
  1   115     FORMAT(' HALF ANGLE OF THE TWO BEAMS IN THE FLUID MEDIUM',
  6        *  ,D15.5,/,' DIRECTION OF THE VELOCITY VECTOR ',D15.5)
  1   C----
  1   C---- PLOT THE RESULTS
  1   C----
  9           CALL DISSPL
  9           STOP
  9           END
  1   C
  9           SUBROUTINE ANGLE(X,Y,ALPHHT,ALPH4G)
  9           IMPLICIT REAL*8(A-H,O-Z)
  9           COMMON R(4,2),ALPH(4,2,2),TINW,TINP,ERR,ALPHH,ALPHP,RP,
  6        *  ALPHW,T,D,TINA,ANG1(11),ANG2(11),BETA,NTITLE(10),IS
  1   C----
  1   C---- THIS ROUTINE CALCULATES THE ANGLE ASSOCIATED WITH THE BEAM
  1   C----CROSSSING. A REGULA FALSI METHOD IS USED TO SOLVE THE EQUATION SET.
  1   C----
  9           IFLAG   = 0
  9           ALPH4G  = 0.0
  1   25      CALL GEOM(X,Y,ALPH4G,ALPHGH)
  9           IF (ALPHGH .GE. ALPHHT) GO TO 15
  9           ALPHB   = ALPHGH
  9           ALPH42  = ALPH4G
  9           ALPH4G  = ALPH42+ALPHH/2.0
  9           IFLAG   = 1
  9           GO TO 25
  1   15      ALPHA   = ALPHGH
  9           ALPH41  = ALPH4G
  9           IF (IFLAG .EQ. 1) GO TO 35
  9           ALPH4G  = ALPH41-ALPHH/2.0
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  9            GO TO 25
  1    35      ALPH4G  = ALPH42+(ALPH41-ALPH42)/(ALPHA-ALPHB)*(ALPHHT-ALPHB)
  9            CALL GEOM(X,Y,ALPH4G,ALPHGH)
  9            IF (DABS(ALPHGH-ALPHHT) .LT. ERR) RETURN
  9            IF (ALPHHT .GE. ALPHGH) GO TO 45
  9            ALPHA   = ALPHGH
  9            ALPH41  = ALPH4G
  9            GO TO 35
  1    45      ALPHB   = ALPHGH
  9            ALPH42  = ALPH4G
  9            GO TO 35
  9            END

  9            SUBROUTINE GEOM(X,Y,ALPH4G,ALPHGH)
  1    C
  1    C---- THIS ROUTINE CALCULATES THE VARIOUS ANGLES PERTINENT TO THE
  1    C---- FLOW SECTION GEOMETRY GIVEN THE LATEST GUESS OF  THE  ANGLE
  1    C---- ASSOCIATED WITH THE BEAM CROSSING.
  1    C
  9            IMPLICIT REAL*8(A-H,O-Z)
  9            COMMON R(4,2),ALPH(4,2,2),TINW,TINP,ERR,ALPHH,ALPHP,RP,
  6          * ALPHW,T,D,TINA,ANG1(11),ANG2(11),BETA,NTITLE(10),IS
  1    C----
  9            ALPH(1,1,1)=0.0
  9            ALPH(1,1,2)=0.0
  9            ALPH(1,2,1)=0.0
  9            ALPH(1,2,2)=0.0
  9            ALPH(2,1,1)=0.0
  9            ALPH(2,1,2)=0.0
  9            ALPH(2,2,1)=0.0
  9            ALPH(2,2,2)=0.0
  9            ALPH(3,1,1)=0.0
  9            ALPH(3,1,2)=0.0
  9            ALPH(3,2,1)=0.0
  9            ALPH(3,2,2)=0.0
  9            ALPH(4,1,1)=0.0
  9            ALPH(4,1,2)=0.0
  9            ALPH(4,2,1)=0.0
  9            ALPH(4,2,2)=0.0
  1    C----
  9            IF (X .EQ. 0.0) GO TO 15
  9            BETA    = DATAN(Y/X)
  9            RP      = X/DCOS(BETA)
  9            GO TO 25
  1    15      BETA    = 3.141592654/2.0
  9            RP      = Y
  1    C----
  1    25      ALPH(1,1,1)=DARSIN(RP/R(1,1)*DSIN(BETA-ALPH4G))
  9            ALPH(1,1,2)=DARSIN(TINW/TINP*DSIN(ALPH(1,1,1)))
  9            ALPH(1,2,1)=DARSIN(R(1,1)/R(1,2)*DSIN(ALPH(1,1,2)))
  9            ALPH(1,2,2)=DARSIN(TINP/TINW*DSIN(ALPH(1,2,1)))
  9            IF(R(2,2).LT.0.00001D0)GO TO 50
  9            ALPH(2,1,1)=DARSIN(R(1,2)/R(2,1)*DSIN(ALPH(1,2,2)))
  9            ALPH(2,1,2)=DARSIN(TINW/TINP*DSIN(ALPH(2,1,1)))
```

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  9            ALPH(2,2,1)=DARSIN(R(2,1)/R(2,2)*DSIN(ALPH(2,1,2)))
  9            ALPH(2,2,2)=DARSIN(TINP/TINW*DSIN(ALPH(2,2,1)))
  9            IF(R(3,2).LT.0.00001)GO TO 50
  9            ALPH(3,1,1)=DARSIN(R(2,2)/R(3,1)*DSIN(ALPH(2,2,2)))
  9            ALPH(3,1,2)=DARSIN(TINW/TINP*DSIN(ALPH(3,1,1)))
  9            ALPH(3,2,1)=DARSIN(R(3,1)/R(3,2)*DSIN(ALPH(3,1,2)))
  9            ALPH(3,2,2)=DARSIN(TINP/TINW*DSIN(ALPH(3,2,1)))
  9            IF(R(4,2).LT.0.00001D0)GO TO 50
  9            ALPH(4,1,1)=DARSIN(R(3,2)/R(4,1)*DSIN(ALPH(3,2,2)))
  9            ALPH(4,1,2)=DARSIN(TINW/TINP*DSIN(ALPH(4,1,1)))
  9            ALPH(4,2,1)=DARSIN(R(4,1)/R(4,2)*DSIN(ALPH(4,1,2)))
  9            ALPH(4,2,2)=DARSIN(TINP/TINW*DSIN(ALPH(4,2,1)))
  1    50      CONTINUE
  9            ALPHW=ALPH4G+ALPH(1,1,1)-ALPH(1,1,2)+ALPH(1,2,1)-ALPH(1,2,2)
  6         *  +ALPH(2,1,1)-ALPH(2,1,2)+ALPH(2,2,1)-ALPH(2,2,2)
  6         *  +ALPH(3,1,1)-ALPH(3,1,2)+ALPH(3,2,1)-ALPH(3,2,2)
  6         *  +ALPH(4,1,1)-ALPH(4,1,2)+ALPH(4,2,1)-ALPH(4,2,2)
  9            ALPHP=DARSIN(TINW/TINP*DSIN(ALPHW))
  9            ALPHGH=DARSIN(TINP/TINA*DSIN(ALPHP))
  1    C----
  9            RETURN
  9            END

  1    C----
  9            FUNCTION HCALC(ALPHHT,ALPH4G)
  9            IMPLICIT REAL*8(A-H,O-Z)
  9            COMMON R(4,2),ALPH(4,2,2),TINW,TINP,ERR,ALPHH,ALPHP,RP,
  6         *  ALPHW,T,D,TINA,ANG1(11),ANG2(11),BETA,NTITLE(10),IS
  1    C----
  1    C---- THIS SUBROUTINE CALCULATES THE Y COORDINATE OF THE LASER BEAM AT
  1    C---- THE PLEXIGLASS WALL
  1    C----
  9            ANGLE   = ALPHW+ALPH(IS,2,2)
  9            HCALC   = R(IS,2)*DSIN(ANGLE)+(D-R(IS,2)*DCOS(ANGLE))*DTAN(ALPHW
  6         *  ) + T*DTAN(ALPHP)
  9            RETURN
  1    C
  9            END
  1    C
  7          SUBROUTINE DISSPL
  1    C
  7          IMPLICIT REAL*8(A-H,O-Z)
  9            COMMON R(4,2),ALPH(4,2,2),TINW,TINP,ERR,ALPHH,ALPHP,RP,
  6         *  ALPHW,T,D,TINA,ANG1(11),ANG2(11),BETA,NTITLE(10),IS
  7          REAL*4 COORD1(1450),COORD2(1450),WORK(9000)
  7          KGRAPH=0
  1    C
  1    C---- THIS ROUTINE DRAWS THE LASER BEAMS AND TEST SECTION USING THE       00040000
  1    C---- DISSPLA GRAPHICS SYSTEM.
  1    C                                                                         00040000
  7          CALL DIGEST
  7          CALL NOBRDR
  1    C
  1    C---- THIS COMMAND SETS THE PHYSICAL LOCATION OF THE CORRDINATE (0,0)
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  1  C      ON THE PLOT.  FOR THIS CASE THE PLOT ORIGIN IS LOCATED 0.75 INCHES
  1  C----- FROM THE LEFT EDGE AND 0.75 INCHES FROM THE BOTTOM.
  1  C
  7        XPHYS=0.75                                                   00050000
  7        YPHYS=0.75                                                   00110000
  7        CALL PHYSOR(XPHYS,YPHYS)                                     00350000
  1  C                                                                  00400000
  1  C---- THIS COMMAND SETS THE TOTAL SIZE OF PLOTTING AREA.            00410000
  1  C                                                                  00420000
  7        CALL PAGE(11.9375,8.500)                                     00430000
 73                                                                     00440000
  7        XPAGE=6.5                                                    00450000
  7        YPAGE=6.5                                                    00460000
  7        YMAX1=R(IS,2)*DSIN(ANG1(9))+(D-R(IS,2)*DCOS(ANG1(9)))*DTAN(ANG1(10
  6       *))+T*DTAN(ANG1(11))
  7        YMIN1=R(IS,2)*DSIN(ANG2(9))+(D-R(IS,2)*DCOS(ANG2(9)))*DTAN(ANG2(10
  6       *))+T*DTAN(ANG2(11))
  7        YMAX=D+T
  7        YMIN=-(D+T)
  7        XMIN=-(D+T)
  7        XMAX=D+T
  1  C                                                                  00470000
  1  C---- THIS COMMAND MAKES THE FIGURE FIT WITHIN THE PLOTTING          00480000
  1  C---- AREA.                                                        00490000
  1  C                                                                  00500000
  7        CALL AREA2D(XPAGE,YPAGE)
  1  C
  1  C---  DISSPLA HEADING COMMAND
  1  C
  7        CALL HEADIN(NTITLE,100,3,1)                                  00590000
  7        XSTEP=1.0
  7        YSTEP=1.0
  1  C                                                                  00760000
  1  C                                                                  00760000
  1  C---- THIS COMMAND SETS THE SCALE OF THE GRAPH ACCORDING TO VARIALBES IN00770000
  1  C---- THE ARGUMENT.
  1  C                                                                  00780000
  1  C                                                                  00780000
  7        CALL GRAF(XMIN,XSTEP,XMAX,YMIN,YSTEP,YMAX)                   00790000
  1  C
  1  C
  1  C---- THIS SECTION OF THE GRAPHICS ROUTINE DRAWS THE INPUT BLOCK AND
  1  C---- COLORS IT.
  1  C
  1  C
  7        CALL NEWPEN(2)
  1  C----
  1  C---- THIS SECTION DRAWS THE CYLINDER
  1  C----
  7        DO 200 J1=1,4
  9          II=0
 10           DO 150 J2=1,2
 13              DTHET=360.0/180.0*3.141592654D0/720.0
 13              TH=0.0
```

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

 13                DO 100 I=1,720
 16                   II=1+II
 16                   COORD1(II)=R(J1,J2)*DCOS(TH)
 16                   COORD2(II)=R(J1,J2)*DSIN(TH)
 16                   TH=TH+DTHET
  1    100         CONTINUE
 13                TH=TH +DTHET
 16                   II=1+II
 13                COORD1(II)=R(J1,J2)*DCOS(TH)
 13                COORD2(II)=R(J1,J2)*DSIN(TH)
  1    150      CONTINUE
 10             CALL SHADE(COORD1,COORD2,II,90.0,0.010,1,WORK,9000)
  1    200   CONTINUE
  1    C----
  1    C---- THIS SECTION DRAWS THE PLEXIGLASS WALL
  1    C----
  7          COORD1(1)=D
  7          COORD2(1)=-R(IS,2)
  7          COORD1(2)=D+T
  7          COORD2(2)=-R(IS,2)
  7          COORD1(3)=D+T
  7          COORD2(3)=R(IS,2)
  7          COORD1(4)=D
  7          COORD2(4)=R(IS,2)
  7          COORD1(5)=D
  7          COORD2(5)=-R(IS,2)
 10             II=5
 10             CALL SHADE(COORD1,COORD2,II,90.0,0.010,1,WORK,9000)
  1    C----
  1    C---- THIS SECTION DRAWS THE LASER BEAMS
  1    C----
  7          CALL NEWPEN(4)
  7          COORD1(1)=RP*DCOS(ANG1(1))
  7          COORD2(1)=RP*DSIN(ANG1(1))
  7          COORD1(2)=R(1,1)*DCOS(ANG1(2))
  7          COORD2(2)=R(1,1)*DSIN(ANG1(2))
 13                DO 92 I=1,11
  1    92          CONTINUE
 13                CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
  7          COORD1(1)=R(1,1)*DCOS(ANG1(2))
  7          COORD2(1)=R(1,1)*DSIN(ANG1(2))
  7          COORD1(2)=R(1,2)*DCOS(ANG1(3))
  7          COORD2(2)=R(1,2)*DSIN(ANG1(3))
 13                CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 13                IF(R(2,2).LT.0.00001D0)GO TO 50
  7          COORD1(1)=R(1,2)*DCOS(ANG1(3))
  7          COORD2(1)=R(1,2)*DSIN(ANG1(3))
  7          COORD1(2)=R(2,1)*DCOS(ANG1(4))
  7          COORD2(2)=R(2,1)*DSIN(ANG1(4))
 13                CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
  7          COORD1(1)=R(2,1)*DCOS(ANG1(4))
  7          COORD2(1)=R(2,1)*DSIN(ANG1(4))
  7          COORD1(2)=R(2,2)*DCOS(ANG1(5))
  7          COORD2(2)=R(2,2)*DSIN(ANG1(5))
```

```
START
COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
13             IF(R(3,2).LT.0.00001D0)GO TO 50
 7       COORD1(1)=R(2,2)*DCOS(ANG1(5))
 7       COORD2(1)=R(2,2)*DSIN(ANG1(5))
 7       COORD1(2)=R(3,1)*DCOS(ANG1(6))
 7       COORD2(2)=R(3,1)*DSIN(ANG1(6))
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 7       COORD1(1)=R(3,1)*DCOS(ANG1(6))
 7       COORD2(1)=R(3,1)*DSIN(ANG1(6))
 7       COORD1(2)=R(3,2)*DCOS(ANG1(7))
 7       COORD2(2)=R(3,2)*DSIN(ANG1(7))
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
13             IF(R(4,2).LT.0.00001D0)GO TO 50
 7       COORD1(1)=R(3,2)*DCOS(ANG1(7))
 7       COORD2(1)=R(3,2)*DSIN(ANG1(7))
 7       COORD1(2)=R(4,1)*DCOS(ANG1(8))
 7       COORD2(2)=R(4,1)*DSIN(ANG1(8))
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 7       COORD1(1)=R(4,1)*DCOS(ANG1(8))
 7       COORD2(1)=R(4,1)*DSIN(ANG1(8))
 7       COORD1(2)=R(4,2)*DCOS(ANG1(9))
 7       COORD2(2)=R(4,2)*DSIN(ANG1(9))
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 1  50   CONTINUE
 7       COORD1(1)=R(IS,2)*DCOS(ANG1(9))
 7       COORD2(1)=R(IS,2)*DSIN(ANG1(9))
 7       COORD1(2)=D
 7       COORD2(2)=YMAX1-T*DTAN(ANG1(11))
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 7       COORD1(1)=D
 7       COORD2(1)=YMAX1-T*DTAN(ANG1(11))
 7       COORD1(2)=D+T
 7       COORD2(2)=YMAX1
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 7       COORD1(1)=RP*DCOS(ANG2(1))
 7       COORD2(1)=RP*DSIN(ANG2(1))
 7       COORD1(2)=R(1,1)*DCOS(ANG2(2))
 7       COORD2(2)=R(1,1)*DSIN(ANG2(2))
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 7       COORD1(1)=R(1,1)*DCOS(ANG2(2))
 7       COORD2(1)=R(1,1)*DSIN(ANG2(2))
 7       COORD1(2)=R(1,2)*DCOS(ANG2(3))
 7       COORD2(2)=R(1,2)*DSIN(ANG2(3))
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
13             IF(R(2,2).LT.0.00001D0)GO TO 70
 7       COORD1(1)=R(1,2)*DCOS(ANG2(3))
 7       COORD2(1)=R(1,2)*DSIN(ANG2(3))
 7       COORD1(2)=R(2,1)*DCOS(ANG2(4))
 7       COORD2(2)=R(2,1)*DSIN(ANG2(4))
13             CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 7       COORD1(1)=R(2,1)*DCOS(ANG2(4))
 7       COORD2(1)=R(2,1)*DSIN(ANG2(4))
 7       COORD1(2)=R(2,2)*DCOS(ANG2(5))
 7       COORD2(2)=R(2,2)*DSIN(ANG2(5))
```

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

 13              CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 13              IF(R(3,2).LT.0.00001D0)GO TO 70
  7        COORD1(1)=R(2,2)*DCOS(ANG2(5))
  7        COORD2(1)=R(2,2)*DSIN(ANG2(5))
  7        COORD1(2)=R(3,1)*DCOS(ANG2(6))
  7        COORD2(2)=R(3,1)*DSIN(ANG2(6))
 13              CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
  7        COORD1(1)=R(3,1)*DCOS(ANG2(6))
  7        COORD2(1)=R(3,1)*DSIN(ANG2(6))
  7        COORD1(2)=R(3,2)*DCOS(ANG2(7))
  7        COORD2(2)=R(3,2)*DSIN(ANG2(7))
 13              CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
 13              IF(R(4,2).LT.0.00001D0)GO TO 70
  7        COORD1(1)=R(3,2)*DCOS(ANG2(7))
  7        COORD2(1)=R(3,2)*DSIN(ANG2(7))
  7        COORD1(2)=R(4,1)*DCOS(ANG2(8))
  7        COORD2(2)=R(4,1)*DSIN(ANG2(8))
 13              CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
  7        COORD1(1)=R(4,1)*DCOS(ANG2(8))
  7        COORD2(1)=R(4,1)*DSIN(ANG2(8))
  7        COORD1(2)=R(4,2)*DCOS(ANG2(9))
  7        COORD2(2)=R(4,2)*DSIN(ANG2(9))
 13              CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
  1    70  CONTINUE
  7        COORD1(1)=R(IS,2)*DCOS(ANG2(9))
  7        COORD2(1)=R(IS,2)*DSIN(ANG2(9))
  7        COORD1(2)=D
  7        COORD2(2)=YMIN1-T*DTAN(ANG2(11))
 13              CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
  7        COORD1(1)=D
  7        COORD2(1)=YMIN1-T*DTAN(ANG2(11))
  7        COORD1(2)=D+T
  7        COORD2(2)=YMIN1
 13              CALL RLVEC(COORD1(1),COORD2(1),COORD1(2),COORD2(2),0)
  7        CALL ENDPL(0)
  7        CALL DONEPL(0)                                                        00010000
  7        RETURN
  7        END
```

```
START
 COL  ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

 21                APPENDIX E
 11       (INPUT DATA FILE FOR FIGURES 16-18)

  1   MARK 22 BOTTOM FITTING INSERT$

  3     225    44     8     2
  5       1     5     6     7    19    32    31    30    18     1     2
  5       2     3     4     5     9    15    14    11     8     1     2
  5       3    11    12    13    17    23    22    21    16     1     2
  5       4    21    23    24    26    30    29    28    25     1     2
  5       5     1     2     3    20    28   108    27    10     1     0
  5       6    30    31    32    38    52    50    48    47     1     2
  5       7    30    36    37    46    45    44    43    35     1     2
  5       8    28    29    30    35    43    42    41    34     1     2
  5       9    27   108    28    34    41    40    39    33     1     0
  4      10    39    40    41    54   112    58    55    53     1     0
  4      11    41    43    45   215   218   217   216   214     1     2
  4      12    45    48    49   114   105   104   103   113     1     0
  4      13    49    51    52   115   107   106   105   114     1     2
  4      14    55    56    57    63    67    66    65    62     1
  4      15    65    66    67    71    75    74    73    70     1     0
  4      16    73    74    75    81    85    84    83    80     1     0
  4      17    57    58    59    64    69    68    67    63     1     2
  4      18    67    68    69    72    77    76    75    71     1     2
  4      19    75    76    77    82    87    86    85    81     1     2
  4      20    59    60    61    79    89    88    87    78     1     0
  4      21   110   111   112    92   101   100    99    91     1     0
  4      22    61   109   110    91    99    98    97    90     1     2
  4      23    83    86    89    94    97    96    95    93     1     0
  4      24   208   209   116   137   213   212   211   210     1     0
  4      25   140   141   142   146   150   149   213   145     1     0
  4      26   148   149   150   154   167   161   156   153     1     0
  4      27   118   119   120   139   144   143   142   138     1     2
  4      28   120   121   122   173   205   204   203   172     1     0
  4      29   122   123   124   174   207   206   205   173     1     2
  4      30   126   127   128   131   133   136   207   125     1     0
  4      31   128   129   130   132   135   134   133   131    -1     2
  4      32   142   143   144   147   152   151   150   146     1     2
  4      33   150   151   152   155   169   168   167   154     1     2
  4      34   165   166   167   182   201   200   199   181     1     0
  4      35   164   163   165   181   199   198   197   180     1     2
  4      36   160   162   164   180   197   196   195   179     1     0
  4      37   158   159   160   179   195   194   193   178     1     2
  4      38   156   157   158   171   177   176   175   170     1     0
  4      39   175   176   177   185   188   187   186   184     1     0
  4      40   186   187   188   190   193   192   191   189     1     0
  4      41   116   117   118   138   142   141   140   137     1     0
  4      42   167   168   169   183   203   202   201   182     1     2
  4      43   216   217   218   220   223   222   221   219     1     0
  4      44   221   222   223   225   120   119   118   224     1     2
  3     1  2  4  6 30                                                  1.490
  3     2  1  3  5                                                     1.490
```

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

 3     3  4  2  5                                                   1.490
 3     4  3  5  8  1                                                1.490
 3     5  2  3  4  9                                                1.330
 3     6  7 12 13  1  4 30                                          1.490
 3     7 11  8  6                                                   1.490
 3     8  9  4  7 11                                                1.490
 3     9  5  8 10                                                   1.330
 2    10  9  8 11 21 22 20 17 14 43 44                              1.330
 2    11 10  8 12  7 21 43                                          1.490
 2    12 11  6 13 43 44 28                                          1.330
 2    13 12  6 30 29                                                1.490
 2    14 10 17 15                                                   1.330
 2    15 14 18 16                                                   1.330
 2    16 15 19 23                                                   1.33
 2    17 14 18 10 20                                                1.490
 2    18 15 17 20 19                                                1.490
 2    19 16 23 20 18                                                1.490
 2    20 17 18 19 23 22 10                                          1.330
 2    21 22 10 44 41                                                1.330
 2    22 20 23 21 10 41                                             1.490
 2    23 16 19 20 22 41                                             1.330
 2    24 23 25 41                                                   1.330
 2    25 24 41 26 32                                                1.330
 2    26 25 38 37 36 35 34 33                                       1.330
 2    27 41 32 33 28 44                                             1.490
 2    28 27 33 42 29 12 32                                          1.330
 2    29 28 30 13                                                   1.49
 2    30 29 13  6  1 31                                             1.330
 2    31 30                                                         1.490
 2    32 25 33 28 27                                                1.490
 2    33 26 42 28 32                                                1.490
 2    34 35 42 26                                                   1.33
 2    35 36 34 26                                                   1.49
 2    36 37 35 26                                                   1.33
 2    37 38 39 40 26 36                                             1.490
 2    38 39 37 26                                                   1.33
 2    39 40 37 38                                                   1.33
 2    40 39 37                                                      1.33
 2    41 24 25 27 23 22 21                                          1.330
 2    42 34 28 33                                                   1.49
 2    43 10 44 12 11                                                1.33
 2    44 21 27 12 43 10                                             1.49
 7         0
 6        -4
 6        -4
 6        -4
 6        -2
 6        -4
 6        -1
 6        -4
 6        -2
 7         0
 7         0
 7         0
```

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  7        0
  6       -2
  7        0
  6       -2
  6       -4
  7        0
  6       -4
  7        0
  7        0
  7        0
  7        0
  6       -3
  6       -2
  7        0
  7        0
  7        0
  7        0
  7        0
  6       -4
  7        0
  7        0
  7        0
  7        0
  7        0
  7        0
  6       -4
  7        0
  7        0
  7        0
  7        0
  7        0
 10          1        0.00        0.00
 10          2     0.79260        0.00
 10          3      1.5852        0.00
 10          4      1.6501        0.00
 10          5      1.7150        0.00
 10          6      2.0140        0.00
 10          7      2.3120        0.00
 10          8      1.5297     0.63375
 10          9      1.7150     0.63375
  9         10        0.00     1.20125
  9         11      1.4743     1.26750
  9         12    1.537154     1.26750
  9         13      1.6000     1.26750
  9         14    1.594654     1.26750
  9         15      1.7150     1.26750
  9         16     1.46293     1.20125
  9         17     1.60000     1.39750
  9         18     1.71550     1.20125
  9         19     2.31200     1.20125
  9         20    1.480104     1.20125
  9         21    1.451560      1.5275
  9         22     1.56250      1.5275
```

START COL ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

| START COL | | | |
|---|---|---|---|
| 9 | 23 | 1.6000 | 1.5275 |
| 9 | 24 | 1.7150 | 1.5275 |
| 9 | 25 | 1.41328 | 1.9650 |
| 9 | 26 | 1.7150 | 1.9650 |
| 9 | 27 | 0.00 | 2.4025 |
| 9 | 28 | 1.375 | 2.4025 |
| 9 | 29 | 1.545 | 2.4025 |
| 9 | 30 | 1.715 | 2.4025 |
| 9 | 31 | 2.0135 | 2.4025 |
| 9 | 32 | 2.3120 | 2.4025 |
| 9 | 33 | 0.00 | 2.49625 |
| 9 | 34 | 1.46875 | 2.49625 |
| 9 | 35 | 1.71500 | 2.49625 |
| 9 | 36 | 1.7775 | 2.465 |
| 9 | 37 | 1.8400 | 2.5275 |
| 9 | 38 | 2.3120 | 2.5275 |
| 9 | 39 | 0.00 | 2.59 |
| 9 | 40 | 0.78125 | 2.59 |
| 9 | 41 | 1.56250 | 2.59 |
| 9 | 42 | 1.63875 | 2.59 |
| 9 | 43 | 1.71500 | 2.59 |
| 9 | 44 | 1.77750 | 2.59 |
| 9 | 45 | 1.84000 | 2.59 |
| 9 | 46 | 1.84000 | 2.558750 |
| 9 | 47 | 1.80875 | 2.49625 |
| 9 | 48 | 1.90250 | 2.59 |
| 9 | 49 | 2.06200 | 2.59 |
| 9 | 50 | 2.10725 | 2.59 |
| 9 | 51 | 2.187 | 2.59 |
| 9 | 52 | 2.312 | 2.59 |
| 9 | 53 | 0.00 | 3.16 |
| 9 | 54 | 1.5625 | 3.16 |
| 9 | 55 | 0.00 | 3.73 |
| 9 | 56 | 0.3250 | 3.73 |
| 9 | 57 | 0.65 | 3.73 |
| 9 | 58 | 0.70 | 3.73 |
| 9 | 59 | 0.75 | 3.73 |
| 9 | 60 | 0.9885 | 3.73 |
| 9 | 61 | 1.2270 | 3.73 |
| 9 | 62 | 0.00 | 3.85125 |
| 9 | 63 | 0.580 | 3.85125 |
| 9 | 64 | 0.75 | 3.85125 |
| 9 | 65 | 0.00 | 3.9725 |
| 9 | 66 | 0.255 | 3.97250 |
| 9 | 67 | 0.51 | 3.9725 |
| 9 | 68 | 0.630 | 3.9725 |
| 9 | 69 | 0.750 | 3.9725 |
| 9 | 70 | 0.00 | 4.035 |
| 9 | 71 | 0.510 | 4.035 |
| 9 | 72 | 0.750 | 4.035 |
| 9 | 73 | 0.00 | 4.0975 |
| 9 | 74 | 0.255 | 4.0975 |
| 9 | 75 | 0.510 | 4.0975 |
| 9 | 76 | 0.630 | 4.0975 |

START COL ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

| START COL | | | |
|---|---|---|---|
| 9 | 77 | 0.750 | 4.0975 |
| 9 | 78 | 0.750 | 3.9450 |
| 9 | 79 | 1.227 | 3.9450 |
| 9 | 80 | 0.000 | 4.12875 |
| 9 | 81 | 0.54125 | 4.12875 |
| 9 | 82 | 0.75000 | 4.12875 |
| 9 | 83 | 0.000 | 4.16000 |
| 9 | 84 | 0.28625 | 4.16000 |
| 9 | 85 | 0.57250 | 4.16000 |
| 9 | 86 | 0.66125 | 4.16000 |
| 9 | 87 | 0.75000 | 4.16000 |
| 9 | 88 | 0.98850 | 4.16000 |
| 9 | 89 | 1.22700 | 4.16000 |
| 9 | 90 | 1.22700 | 4.25500 |
| 9 | 91 | 1.35750 | 4.25500 |
| 9 | 92 | 1.56250 | 4.25500 |
| 9 | 93 | 0.00000 | 4.47000 |
| 9 | 94 | 1.22700 | 4.47000 |
| 9 | 95 | 0.0 | 4.78000 |
| 9 | 96 | 0.61350 | 4.78000 |
| 9 | 97 | 1.22700 | 4.78000 |
| 9 | 98 | 1.29225 | 4.78000 |
| 9 | 99 | 1.35750 | 4.78000 |
| 8 | 100 | 1.46000 | 4.78000 |
| 8 | 101 | 1.56250 | 4.78000 |
| 8 | 102 | 1.70125 | 4.78000 |
| 8 | 103 | 1.84000 | 4.78000 |
| 8 | 104 | 1.95100 | 4.78000 |
| 8 | 105 | 2.06200 | 4.78000 |
| 8 | 106 | 2.18700 | 4.78000 |
| 8 | 107 | 2.31200 | 4.78000 |
| 8 | 108 | 0.68750 | 2.40250 |
| 8 | 109 | 1.29225 | 3.730 |
| 8 | 110 | 1.35750 | 3.730 |
| 8 | 111 | 1.46000 | 3.730 |
| 8 | 112 | 1.56250 | 3.730 |
| 8 | 113 | 1.840 | 3.730 |
| 8 | 114 | 2.062 | 3.730 |
| 8 | 115 | 2.312 | 3.730 |
| 8 | 116 | 0.451 | 4.780 |
| 8 | 117 | 1.00675 | 4.780 |
| 8 | 118 | 1.56250 | 4.780 |
| 8 | 119 | 1.70125 | 4.780 |
| 8 | 120 | 1.84000 | 4.780 |
| 8 | 121 | 1.95100 | 4.780 |
| 8 | 122 | 2.06200 | 4.780 |
| 8 | 123 | 2.1870 | 4.780 |
| 8 | 124 | 2.3120 | 4.780 |
| 8 | 125 | 2.312 | 4.125 |
| 8 | 126 | 2.312 | 0.000 |
| 8 | 127 | 4.156 | 0.000 |
| 8 | 128 | 6.000 | 0.000 |
| 8 | 129 | 6.125 | 0.000 |
| 8 | 130 | 6.250 | 0.000 |

START COL ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

| | | | |
|---|---|---|---|
| 8 | 131 | 6.000 | 4.215 |
| 8 | 132 | 6.250 | 4.125 |
| 8 | 133 | 6.000 | 8.430 |
| 8 | 134 | 6.125 | 8.430 |
| 8 | 135 | 6.250 | 8.430 |
| 8 | 136 | 4.156 | 8.430 |
| 8 | 137 | 0.451 | 5.368750 |
| 8 | 138 | 1.56250 | 5.368750 |
| 8 | 139 | 1.84000 | 5.368750 |
| 8 | 140 | 0.451 | 5.95750 |
| 8 | 141 | 1.006750 | 5.95750 |
| 8 | 142 | 1.56250 | 5.95750 |
| 8 | 143 | 1.70125 | 5.95750 |
| 8 | 144 | 1.8400 | 5.9575 |
| 8 | 145 | 0.45100 | 6.05875 |
| 8 | 146 | 1.66625 | 6.05875 |
| 8 | 147 | 1.84000 | 6.05875 |
| 8 | 148 | 0.65350 | 6.16000 |
| 8 | 149 | 1.32925 | 6.16000 |
| 8 | 150 | 1.7700 | 6.160 |
| 8 | 151 | 1.805 | 6.160 |
| 8 | 152 | 1.840 | 6.160 |
| 8 | 153 | 0.6535 | 6.345 |
| 8 | 154 | 1.77000 | 6.345 |
| 8 | 155 | 1.84000 | 6.345 |
| 8 | 156 | 0.6535 | 6.530 |
| 8 | 157 | 0.8255 | 6.530 |
| 8 | 158 | 0.9975 | 6.530 |
| 8 | 159 | 1.087 | 6.530 |
| 8 | 160 | 1.17650 | 6.530 |
| 8 | 161 | 1.21175 | 6.530 |
| 8 | 162 | 1.31125 | 6.530 |
| 8 | 163 | 1.5230 | 6.530 |
| 8 | 164 | 1.446 | 6.530 |
| 8 | 165 | 1.60 | 6.530 |
| 8 | 166 | 1.685 | 6.530 |
| 8 | 167 | 1.770 | 6.530 |
| 8 | 168 | 1.805 | 6.530 |
| 8 | 169 | 1.840 | 6.530 |
| 8 | 170 | 0.65350 | 6.980 |
| 8 | 171 | 0.9975 | 6.980 |
| 8 | 172 | 1.840 | 6.6050 |
| 8 | 173 | 2.062 | 6.6050 |
| 8 | 174 | 2.312 | 6.6050 |
| 8 | 175 | 0.6535 | 7.430 |
| 8 | 176 | 0.8255 | 7.430 |
| 8 | 177 | 0.9975 | 7.430 |
| 8 | 178 | 0.9975 | 7.480 |
| 8 | 179 | 1.17650 | 7.480 |
| 8 | 180 | 1.44600 | 7.480 |
| 8 | 181 | 1.60000 | 7.480 |
| 8 | 182 | 1.77000 | 7.480 |
| 8 | 183 | 1.84000 | 7.480 |
| 8 | 184 | 0.72425 | 7.730 |

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

  8          185        0.99750         7.730
  8          186         0.7950         8.030
  8          187        0.89625         8.030
  8          188         0.9975         8.030
  8          189          0.795         8.230
  8          190         0.9975         8.230
  8          191          0.795         8.430
  8          192        0.89625         8.430
  8          193         0.9975         8.430
  8          194          1.807         8.430
  8          195        1.17650         8.430
  8          196        1.31125         8.430
  8          197        1.44600         8.430
  8          198          1.523         8.430
  8          199          1.60          8.430
  8          200          1.685         8.430
  8          201          1.770         8.430
  8          202          1.805         8.430
  8          203          1.840         8.430
  8          204          1.951         8.430
  8          205          2.062         8.430
  8          206          2.187         8.430
  8          207          2.312         8.430
  8          208          0.000         4.780
  8          209        0.22550         4.780
  8          210        0.0           5.00550
  8          211        0.0           5.70900
  8          212        0.22550       5.93450
  8          213        0.45100       6.16000
  8          214        1.56250       2.76500
  8          215        1.84000       2.76500
  8          216        1.56250       2.94000
  8          217        1.70125       2.94000
  8          218        1.84000       2.94000
  8          219        1.56250       3.25000
  8          220        1.84000       3.25000
  8          221        1.56250       3.56000
  8          222        1.70125       3.56000
  8          223        1.84000       3.56000
  8          224        1.56250       4.17000
  8          225        1.84000       4.17000
  4     10      0.00       3.16       5.5       1.33      1.490
  5     1
  8          1.0        1.0
  9          20          1
  9          0.98550         3.945
 10          0
```

```
START
 COL   ----+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8

 24                      APPENDIX F
 14               (INPUT DATA FILE FOR FIGURES 19-26)
  1    CYLINDRICAL TEST CASES$
 13              0.7            0.0             -0.3            0.72
 12             0.25           5.00
 12             1.33            1.0             1.49             5.5
 12             3.70           4.00
 12             0.80         1.0000
 12             1.70           2.00
 12             2.70           3.00
```