This document was prepared in conjunction with work accomplished under Contract No. AT(07-2)-1 with the U. S. Department of Energy.

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DPSP 59-1564

RTR-178

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CLASSIFICATION CANCELLED BY AUTHORITY OF NAME TITLE DATE 1/20/77

[redacted]

October 21, 1959

This document consists of 16 pages, No. 11 of 31 copies, Series A.

TO: A. A. JOHNSON

COEFFICIENTS OF REACTIVITY - MARK VII-A

INTRODUCTION

Tests were performed under TA 1-620 to determine the moderator temperature coefficient of reactivity, the prompt temperature coefficient of reactivity and the power coefficient of reactivity of Mark VII-A charges. This document summarizes the results of the tests and reports values of the coefficients that are commensurate with the test data.

SUMMARY

The moderator temperature coefficient of reactivity is presented in Figure 2 as a function of the moderator temperature and fuel exposure.

The power coefficient of reactivity was measured at 46,000 MWD to be -0.000558 % Δ k/k per MW by the xenon method and -0.000654 % Δ k/k per MW by the rod oscillation method. The result of the rod oscillation method agrees better with the moderator temperature coefficient. The power coefficient as determined from the moderator temperature coefficient is plotted in Figure 3 as a function of moderator temperature and exposure. The average power coefficient of reactivity is presented in Figure 4 as a function of river water temperature, bulk moderator temperature and reactor exposure.

Qualitative analysis of the prompt coefficient test data shows that the prompt coefficient of reactivity is negative and consequently is not a major consideration in determining the degree of safety afforded by the safety circuits against fast power transients. It was felt that the data were not of sufficient accuracy and importance to warrant the necessary computer time to make a quantitative analysis. From the change in the resonance integral due to the Doppler broadening of the resonance absorption peaks, the prompt fuel coefficient was calculated to be -0.0012 % Δ k/k/ per ^{o}C of the fuel.

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CONCLUSIONS

The value of the power coefficient of reactivity measured by the rod oscillation technique is in good agreement with the theoretical extrapolation of the measured moderator temperature coefficient of reactivity. However, there still exists a discrepancy of about 15% between the value of the power coefficient as determined by the xenon method and as determined by the rod oscillation method. Work that may help resolve the discrepancy is in progress. No further experimental work is planned.

The curves of the power coefficient in Figures 3 and 4 should be used in reactor calculations.

DISCUSSION

A series of four tests were performed to determine the coefficients of reactivity of Mark VII-A charges. Tests were performed at low power (5-10 MW) to determine the moderator temperature coefficient and to make the low power measurements of the prompt coefficient test. Tests were performed at operating power levels to measure the power coefficient and to make the high power measurements of the prompt coefficient test.

Moderator Temperature Coefficient of Reactivity

The moderator temperature coefficient was measured by heating the moderator while maintaining criticality with calibrated control rods. The moderator was heated from 11.4 to 34.9°C in increments of 3 or 4°C by throttling the cooling water flow to the heat exchangers. Gang I full control rods were used to maintain criticality, and the control rods were calibrated after every increment by rod oscillation techniques.

The rod configurations, number of control rods oscillated, reactivity worth of the oscillated rods, average moderator temperature, and the reactivity worth of xenon in the reactor are presented in Table I for each increment of the test. The reactivity worth of Gang I control rod withdrawal is presented in Figure 1 as a function of the moderator temperature. Figure 1 has been corrected for the change in the reactivity worth of xenon during the test, assuming a power level of 7.5 MW. The measured values of the moderator temperature coefficient are presented in Figure 2 as a function of moderator temperature. The values were determined by measuring the slope of the curve in Figure 1 for several temperatures and subtracting -0.0012 % Δ k/k/°C from the measured value to correct for fuel heating effects (Doppler broadening). The method of extrapolation of the measured data is given in Appendix A.

Power Coefficient of Reactivity

The power coefficient was measured at a reactor exposure of 46,000 MWD by reducing power approximately 100 MW from full power using calibrated Gang I full rods. The lower power level was maintained by withdrawal of Gang I full rods until the rods were withdrawn to their original configuration. The power coefficient then was computed by two methods:

1) The calculated xenon transient during the test was divided by the power decrease to determine the coefficient by the xenon transient method.

2) The reactivity worth of the inserted Gang I control rods as determined by the rod oscillation technique was divided by the power decrease to determine the coefficient by the rod oscillation method. This value had to be adjusted to correct for the xenon transient that occurred from the instant the power reduction was started until the lower power level was stabalized and maintained by Gang I full control rods.

The initial rod configuration of Gangs I, II, and III are presented in Table II. The differential reactivity worth of the Gang I control rods, as determined by the oscillation techniques, is presented in Table III along with rod configurations.

Power was decreased from 2125 MW to 2059 MW in 4.5 minutes by the insertion of Gang I full rods from 420 veeder units to 490 veeder units. Power then was allowed to drift to 2008 MW during the next 12 minutes with no compensating motion of control rods. However, the control rods were calibrated and an axial flux measurement was obtained during this 12 minute period. Reactor power then was maintained at 2008 MW during the next 20.5 minutes, during which time the Gang I full control rods were withdrawn to their original configuration and the test was terminated.

The power coefficient was determined to be $-0.000558\ \%\ \Delta k/k/MW$* by xenon and $-0.000654\ \%\ \Delta k/k/MW$ by rod oscillation. The rod oscillation value was used in this report because the moderator coefficient test was performed also by rod oscillation techniques.

The moderator temperature coefficient and the power coefficient may be compared in the following manner. The moderator temperature was 85°C and the river water temperature was approximately 10°C, so there would be approximately a 28.3 MW change in power for a 1°C change in moderator temperature. The moderator coefficient of reactivity is equal to the power coefficient multiplied by 28.3 $MW/°C_{mod.}$ minus the fuel coefficient ($-0.0012\ \%\ \Delta k/k/°C_{fuel}$) multiplied by 3°C $fuel/°C_{mod.}$†. Thus, the value of the moderator coefficient as determined from the power coefficient at 83°C would be $-.0149\ \%\ \Delta k/k/°C$, which is in good agreement with the calculated curves (see triangular point in Figure 2).

The power coefficient of reactivity as a function of moderator temperature and fuel exposure is presented in Figure 3. The curves were generated by dividing the moderator coefficients presented in Figure 3 by 28.3 MW/°C and adding $-0.000128\ \%\ \Delta k/k$ to each value to include the effect of fuel temperature changes ($-0.0012\ \%\ \Delta k/k/°C_{fuel} \div 9.4\ MW/°C_{fuel} = -0.000128\ \%\ \Delta k/k/MW$).

* Calculated by the method discribed in DPSP 59-1260, R. L. Sanders, Parameters For Xenon Calculation, July 29, 1959, Secret.

† Calculations show that there is approximately a 3°C fuel temperature rise for each 1°C moderator temperature rise.

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The average power coefficient is presented in Figure 4 as a function of fuel exposure, river water temperature, and bulk moderator temperature. The curves were not presented with power level as a parameter, since this would increase the number of curves that would be necessary. The correct bulk moderator temperature with which to enter the curves may be determined by dividing the power level by 28.3 MW/°C and adding to this the river water temperature. The curves were generated by averaging the power coefficients presented in Figure 3 over the full range of indicated temperature.

Prompt Temperature Coefficient of Reactivity

The prompt coefficient test was performed in two parts; the low power test (5-10MW) and the full power test (1800 MW). The tests consisted of a series of truncated triangular oscillationsof Gang I full control rods. The rods were oscillated using all permutations of 1, 2, 3 and 4 second drive times and 0.5, 1.0, 1.5, 2.0, 3.1, 4.3, 5.5, and 7.1 second wait times. The purpose of the low power phase of the test was to obtain flux response data that was not affected by temperature changes. Then any differences between the high power test data and the low power test data could be attributed to reactivity changes resultant from temperature changes in the moderator, fuel, or components of the reactor.

The results of the prompt coefficient test are presented in Figure 3. Normally, the prompt coefficient and the kinetic moderator coefficient are determined quantitatively with an IBM-650 code. The code is designed to simulate pile kinetics and has a provision for the inclusion of three temperature coefficients with their associated thermal response times. The coefficients and their response times then are adjusted until the best fit is obtained to the experimental data.

It was decided, however, not to do the numerical evaluation of the prompt coefficient. The oscillations in reactor power were rather small, thus limiting the accuracy with which the flux changes could be determined. The analysis was not done because of 1) the limited accuracy, 2) the fact that the qualitative conclusions given below were sufficient to demonstrate the safety of the Mark VII-A fuel and 3) the large amount of computer time that the analysis takes.

Qualitative conclusions that may be drawn from the results of the test are as follows:

1) The prompt coefficient is negative
2) The kinetic moderator coefficient is relatively weak and slow acting (as compared to the results of the Mark VI and Mark VI J coefficients)
3) Flux transients resultant from a given reactivity change will be less severe at full power than at lower or zero power levels.

Based on measurements of the Doppler broadening of the resonance integral the fuel coefficient of reactivity is calculated for a Mark VII-A charge to be -0.0012 % Δ k/k/°C fuel.

There is no further experimentation planned for the determination of Mark VII-A reactivity coefficients.

L. W. FOX

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TABLE I

MODERATOR COEFFICIENT TEST - 3-29-58

	Sequence						
	Initial	1	2	3	4	5	6
Rod Configuration (v.u.) Gang I full rods	1753	1666	1567	1313	1230	1332	1407
Gang II full rods	1317	1317	1317	1317	1317	1317	1317
Gang III full rods	1317	1317	1317	1317	1317	1317	1317
Gang I half rods	0853	0853	0853	0853	0853	0853	0853
Gang II half rods	0853	0853	0853	0853	0853	0853	0853
Gang III half rods	0763	0763	0763	0763	0763	0763	0763
Number of Gang I rods oscillated	19	19	19	18*	18*	18*	18*
Worth of Gang I rods (% Δ k/k/v.u.)	.000332	.000388	.000439	.000825	.000855	.000815	.000456
Average moderator temperature (°C)	11.4	15.0	18.8	30.1	34.9	27.1	23.9
Worth of Xenon (% Δ k/k)	.0392	.0445	.0522	.0604	.0631	.0673	.0695
Time	0228	0332	0510	0655	0732	0832	0903

* One rod faulted midway through the test.

TABLE II

INITIAL ROD CONFIGURATION - POWER COEFFICIENT TEST - 12-11-58

Gang	I	II	III
Full rods (v.u.)	0420	0270	0216
Half rods (v.u.)	0882	0802	0707
Number of half rods	19	24	30
Alloy concentration of half rods (w/o)	1.5	1.5	1.5 (6 slug)

TABLE III

ROD WORTH - POWER COEFFICIENT TEST

Gang I configuration (v.u.)	0420	0456	.0490
Reactivity worth (%Δ k/k/v.u.)	.000673	.000645	.000605

APPENDIX A

CALCULATION OF THE MODERATOR COEFFICIENT

Measurements of the moderator temperature coefficient were made at zero exposure and covered only the range of 10 to 35°C. To extend the measurements to higher temperatures and exposures calculations based on the following expression were used.

$$\frac{1}{k_e}\frac{dk_e}{dT} = \frac{1}{\eta f}\frac{d(\eta f)}{dT} + A\frac{d\rho}{dT}$$

in which k_e is the effective multiplication constant, η is the thermal reproduction factor, f is the thermal utilization, ρ is the moderator density, T is the moderator temperature and A is taken to be a constant. The value of A really depends on the moderator density, but, since the moderator density varies by only three percent over the range of temperature of interest, it can be considered constant.

The two terms are discussed below:

I. $\frac{1}{\eta f}\frac{d(\eta f)}{dT}$

This term is independent of changes in moderator density but is dependent on fuel exposure. The contribution of this term to the coefficient was calculated as follows:

The basic equations are:

(1) $$\eta f = \frac{\nu_{49}\Sigma_{f_{49}} + \nu_{25}\Sigma_{f_{25}}}{\Sigma_a} = \frac{k_\infty}{\epsilon p}$$

and

(2) $$\Sigma_a = \Sigma_{a_{49}} + \Sigma_{a_{25}} + \Sigma_{a_{28}} + \Sigma_{a_{Al}} + \Sigma_{a_{Li}} + \Sigma_{a_{Xe}} + \Sigma_{a_{Mod}}$$

The cross sections involved were handled by converting the thermal cross sections listed in BNL-325 to Westcott's "sigma in-vee" cross sections ($\hat{\sigma}$). The value of $\hat{\sigma}$ is the 2200 m/sec cross section for an equivalent 1/v-law substance which would give the same reaction rate as does the actual substance. Or

$$\hat{\sigma}(kT) = \sigma(kT)\, f(kT)\sqrt{T/T_0}$$

in which $\sigma(kT)$ is the cross section at the temperature T, $f(kT)$ is a factor to describe the non-1/v nature of the cross section and $\sqrt{T/T_0}$ is a factor relating $\sigma(kT)$ of a 1/v absorber to σ_{2200}, the cross section at a neutron velocity of 2200 m/sec. That is,

$$\sigma(kT)\sqrt{T/T_0} = \sigma_{2200}$$

Thus $$\hat{\sigma}(kT) = \sigma_{2200}\, f(kT)$$

The temperature variation of $\hat{\sigma}$ is included only in f (kT). The $\hat{\sigma}$ cross sections were used in calculating the values of the macroscopic cross sections (Σ).

Differentiating equation (1) and dividing by ηf gives the first term of the coefficient or

$$\frac{1}{\eta f}\frac{d(\eta f)}{dT} = \frac{\Sigma_a \nu_{49}\frac{d\Sigma_{f_{49}}}{dT} - \nu_{49}\Sigma_{f_{49}}\frac{d\Sigma_a}{dT} + \Sigma_a \nu_{25}\frac{d\Sigma_{f_{25}}}{dT} - \nu_{25}\Sigma_{f_{25}}\frac{d\Sigma_a}{dT}}{\eta f\ \Sigma_a^2}$$

From equation (1)

$$\Sigma_a = \frac{\nu_{49}\Sigma_{f_{49}} + \nu_{25}\Sigma_{f_{25}}}{\eta f}$$

and from equation (2)

$$\frac{d\Sigma_a}{dT} = \frac{d\Sigma_{a_{49}}}{dT} + \frac{d\Sigma_{a_{25}}}{dT} + \frac{d\Sigma_{a_{xo}}}{dT}$$

The derivative of $\hat{\sigma}$ for the 1/v absorbers: Li, Al and D_2O is equal to zero. Then the expression for the first term of the coefficient becomes

$$(3)\quad \frac{1}{\eta f}\frac{d(\eta f)}{dT} = \frac{\left(\nu_{49}\frac{d\Sigma_{f_{49}}}{dT} + \nu_{25}\frac{d\Sigma_{f_{25}}}{dT}\right) - \eta f\left(\frac{d\Sigma_{a_{49}}}{dT} + \frac{d\Sigma_{a_{25}}}{dT} + \frac{d\Sigma_{a_{xa}}}{dT}\right)}{\nu_{49}\Sigma_{f_{49}} + \nu_{25}\Sigma_{f_{25}}}$$

Since all of the "Σ" terms now are fuel constituents, it is not necessary to flux weight or volume weight the "Σ" values to evaluate the above equation.

To evaluate equation (3) the following values were used:

$$\eta f = \frac{k_{\infty}}{\epsilon p} = \frac{1.05}{.85 \times 1.035} = 1.195$$

$$\nu_{49} = 2.91$$

$$\nu_{25} = 2.47$$

Cross sections and their rates of change with temperature are given in the table below:

CROSS SECTIONS* ($\hat{\sigma}$)

Neutron Temperature °C**	$\sigma_{f_{49}}$ barns	$\sigma_{f_{25}}$ barns	$\frac{d\sigma_{f_{49}}}{dT}$ barns/°C	$\frac{d\Sigma_{f_{25}}}{dT}$ barns/°C	$\frac{d\Sigma_{a_{49}}}{dT}$ barns/°C	$\frac{d\Sigma_{a_{25}}}{dT}$ barns	$\frac{d\sigma_{a_{Xe}}}{dT}$ †
120	938	555.8	0.865	-0.096	1.580	-0.121	-
150	966	553.1	0.961	-0.090	1.742	-0.109	-
180	996	550.5	1.054	-0.084	1.894	-0.099	-

* Values were taken from C. H. Westcott, Effective Cross Section Values for Well Moderated Thermal Reactor Spectra CRRD-680 TNCC (CAN)-7 (1957)

** These neutron temperatures correspond to moderator temperatures of 20°C, 50°C, and 80°C. The additional 100°C was added to compensate for neutron spectrum hardening in the fuel.

† The effect of xenon on the coefficient was disregarded as negligible. The value of $d\Sigma_{a_{Xe}}/dT$ at power is effectively zero, although it does become significant at lower temperatures. Power coefficient measurements made at lower power levels would have to be corrected for $d\Sigma_{a_{Xe}}/dT$ to be compatible with values presented in Figure 3.

The dependence of the atomic concentration of the materials on reactor exposure is as follows:

Atoms/cc of Fuel

Exposure (MWD)	Pu^{239}	U^{235}	Xe^{135}
0	0	3.475×10^{20}	3.44×10^{15}
30,000	1.525×10^{19}	3.245×10^{20}	
60,000	3.279×10^{19}	3.024×10^{20}	
90,000	4.856×10^{19}	2.822×10^{20}	
120,000	6.217×10^{19}	2.642×10^{20}	

The results of the calculations are tabulated below:

[redacted]

Values of $\frac{1}{\eta\xi}\frac{d(\eta\xi)}{dT}$

Exposure (MWD)	Neutron Spectrum Temperature (Moderator Temperature + 100°C)		
	120°C	150°C	180°C
0	-0.677×10^{-4}	-0.667×10^{-4}	-0.657×10^{-4}
30,000	-0.419×10^{-4}	-0.390×10^{-4}	-0.344×10^{-4}
60,000	-0.146×10^{-4}	-0.087×10^{-4}	-0.012×10^{-4}
90,000	$+0.085 \times 10^{-4}$	$+0.167 \times 10^{-4}$	$+0.264 \times 10^{-4}$
120,000	$+0.275 \times 10^{-4}$	$+0.375 \times 10^{-4}$	$+0.490 \times 10^{-4}$

II. $A\frac{d\rho}{dT}$

This term has the following form

$$\left(\frac{1}{k_{eff}}\frac{dk_{eff}}{dT}\right)_{D_2O\ \text{density}} = \frac{1}{k_{eff}}\frac{\partial k_{eff}}{\partial \rho}\frac{d\rho}{dT}$$

$$= \left[\frac{1}{f}\frac{\partial \xi}{\partial \rho} + \frac{1}{p}\frac{\partial p}{\partial \rho} - \frac{B^2}{k_\infty}\left(\frac{\partial \tau}{\partial \rho} + \frac{\partial L^2}{\partial \rho}\right)\right]\frac{d\rho}{dT}$$

The various terms in the expression can be written as

$$\frac{1}{\xi}\frac{\partial \xi}{\partial \rho} = \text{constant}$$

$$\frac{1}{p}\frac{\partial p}{\partial \rho} = \text{constant}/\rho^2$$

$$\frac{\partial \tau}{\partial \rho} = \text{constant}/\rho^3$$

$$\frac{\partial L^2}{\partial \rho} = \text{constant}/\rho^2$$

Thus $A = \text{constant} + \frac{\text{constant}}{\rho^2} + \frac{\text{constant}}{\rho^3}$

[redacted]

Over the temperature range of interest, 10°-90°C, the D_2O density, ρ varies only from 1.106 to 1.072 grams per cc. Therefore, an approximation of sufficient accuracy for this work would allow ρ to be considered constant so that A is also constant. Thus

$$\left(\frac{1}{k_{eff}} \frac{dk_{eff}}{dT}\right)_{D_2O \text{ density}} = A \frac{d\rho}{dT}$$

The constant A was evaluated by determining $\frac{d\rho}{dT}$ and matching the calculated value of the moderator coefficient at 35°C with the measured value of the coefficient. The resultant value of A is 0.194. A tabulation of the density dependent portion of the coefficient is presented below:

VALUES OF $A \frac{d\rho}{dT}$

Temperature (°C)	$d\rho/dT$	$A \frac{d\rho}{dT}$
15	-0.000039	-0.076×10^{-4}
20	-0.000114	-0.221×10^{-4}
25	-0.000191	-0.370×10^{-4}
30	-0.000264	-0.511×10^{-4}
35	-0.000329	-0.639×10^{-4}
40	-0.000384	-0.745×10^{-4}
50	-0.000474	-0.920×10^{-4}
60	-0.000551	-1.070×10^{-4}
70	-0.000617	-1.198×10^{-4}
80	-0.000676	-1.312×10^{-4}
90	-0.000731	-1.420×10^{-4}

The two terms in the moderator temperature coefficient were combined to give the family of curves of Figure 2.

[redacted]

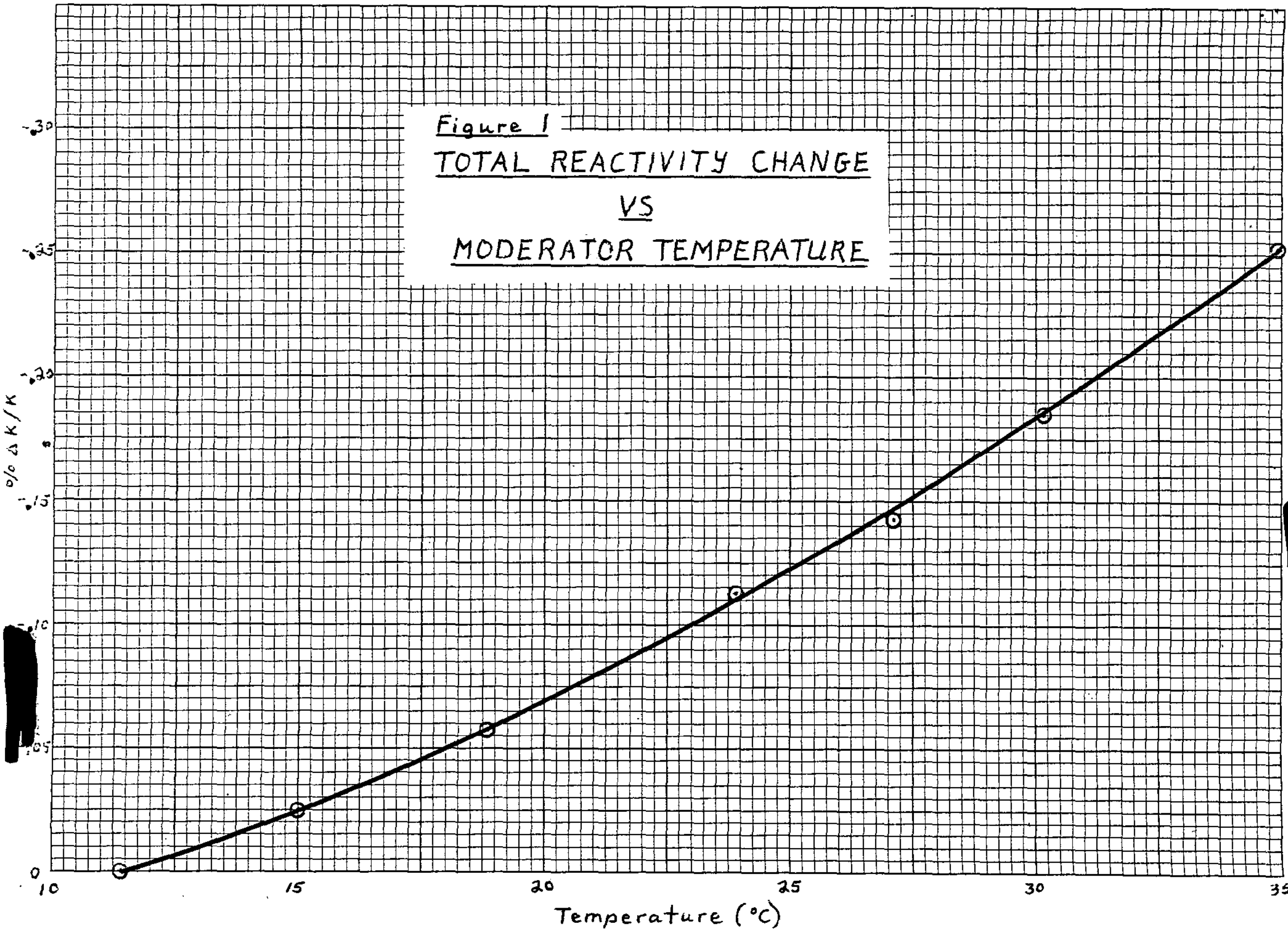

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Figure 1
TOTAL REACTIVITY CHANGE VS MODERATOR TEMPERATURE

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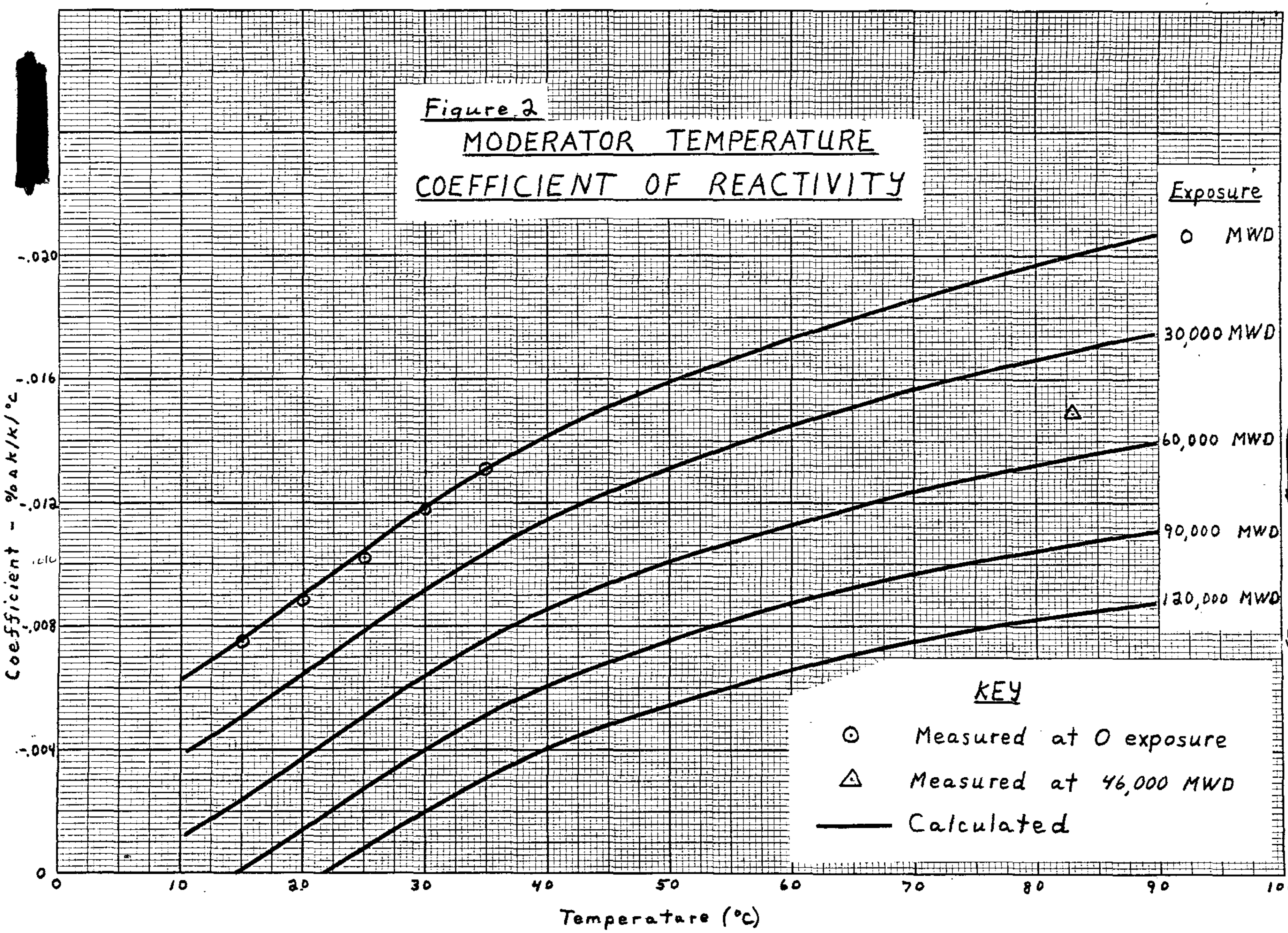

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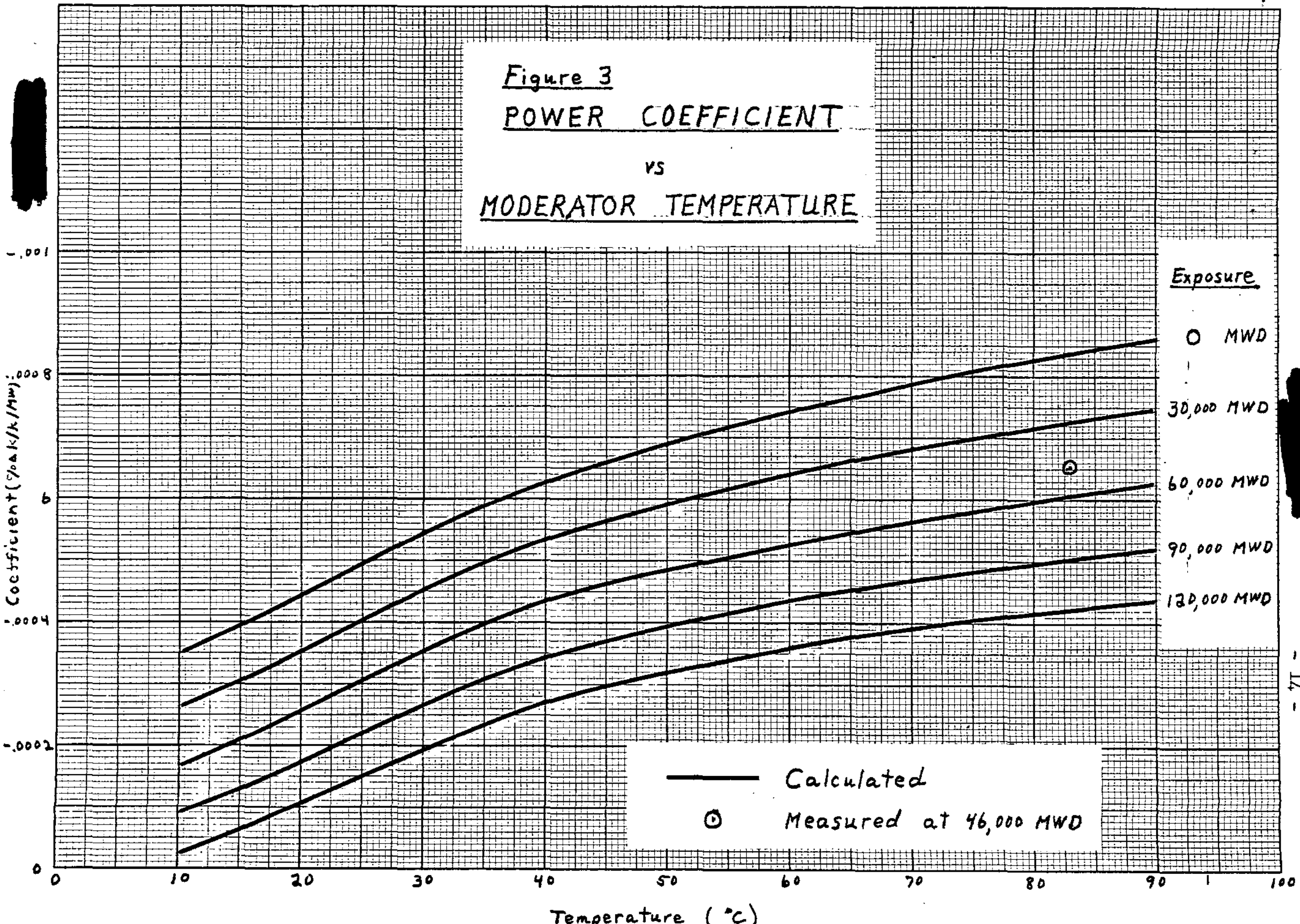

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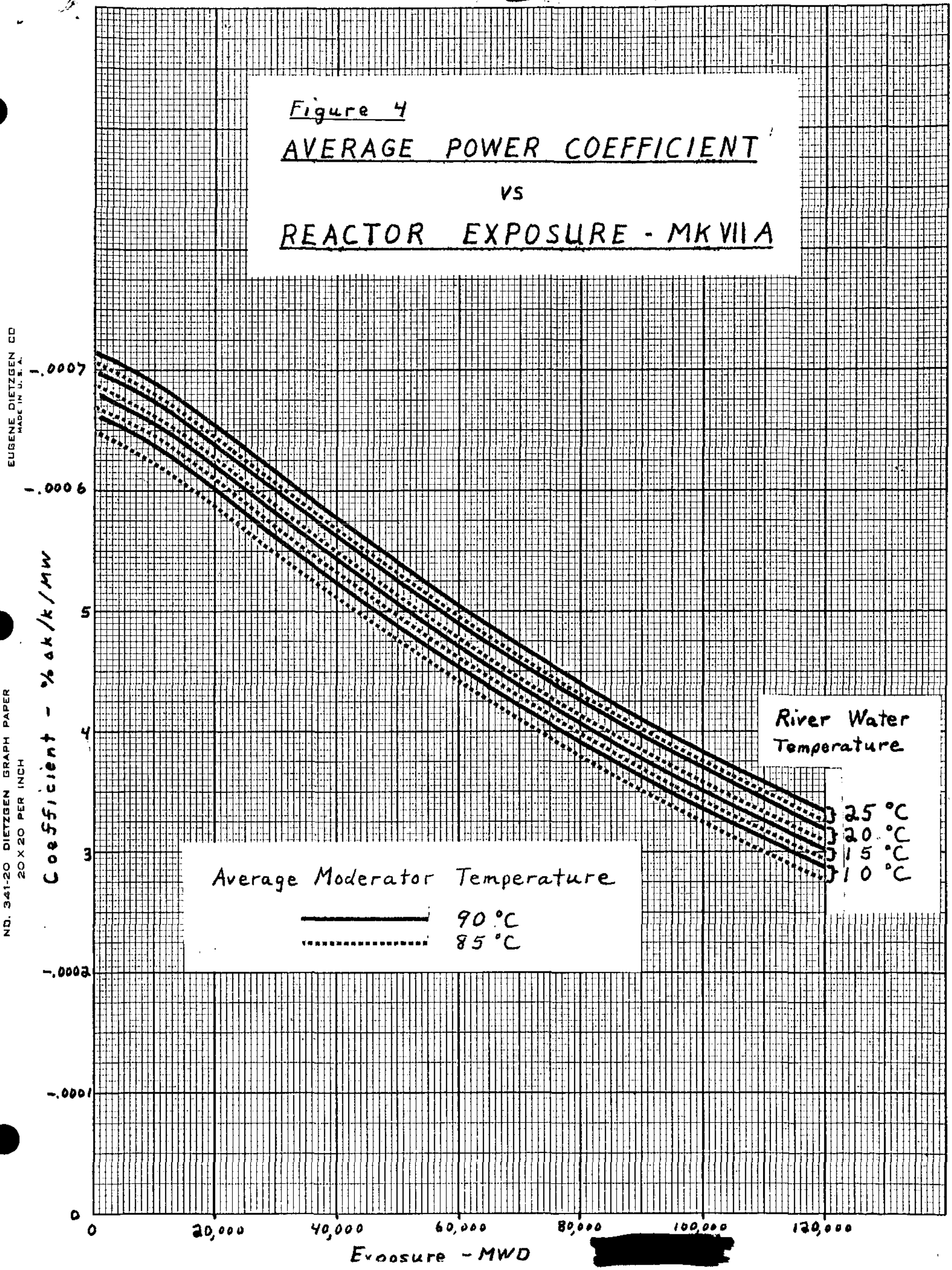

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Figure 4

AVERAGE POWER COEFFICIENT vs REACTOR EXPOSURE - MK VII A

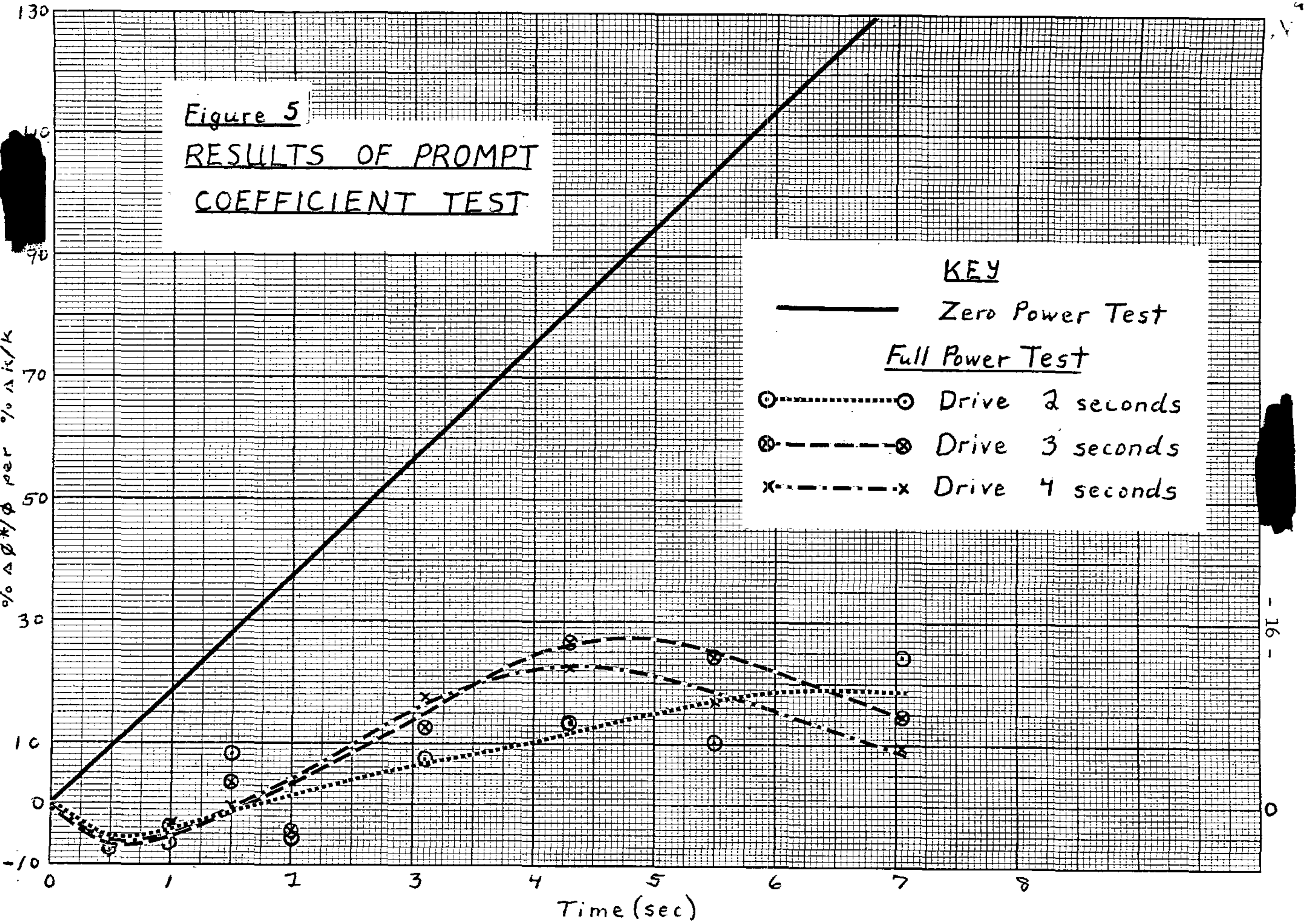
Figure 5
RESULTS OF PROMPT
COEFFICIENT TEST
KEY
Zero Power Test
Full Power Test
Drive 2 seconds
Drive 3 seconds
Drive 4 seconds
% Δφ*/φ per % Δk/k
130
[redacted]
70
50
30
10
0
-10
0
1
2
3
4
5
6
7
8
Time (sec)