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662-E Solid Waste Silo - Plug Lifting Analysis (U)

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March 1993

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SRTC

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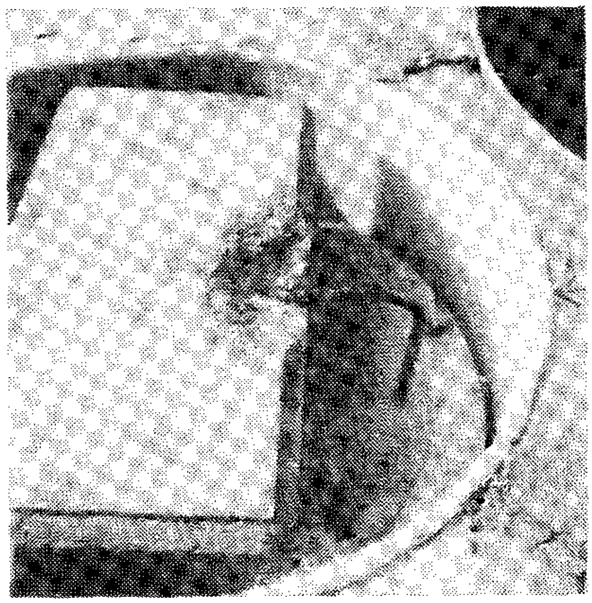
Executive Summary

The Intermediate Level Tritium Vault #1, 662-E, Cell #1 contains 140 waste silos. Each silo is approximately 25' deep, 30" in diameter at the top and covered by a reinforced concrete plug. Two #4 reinforcing bars project from the top of each plug for lifting. During lifting operations, the 1.5" concrete cover over the lifting bars spalled off 16% of the silo plugs. The #4 reinforcing bars were also distorted on many of the silo plugs. Thirteen of the plugs have been repaired to date.



Silo Plug with Spalled Concrete

The existing silo plug lifting bars have a safe working load of 480 pounds per plug, which is less than 1/3 of the dead weight of the silo plug. The safe working load was calculated using the minimum design factor of 3 based on the yield strength or 5 based on the ultimate strength of the material, as per the Savannah River Site Hoisting and Rigging Manual.

The existing design calculations were reviewed, and the following items are noted:

(1) Adequate concrete cover was not provided over the horizontal portion of the lifting bars.
(2) The lifting bars were allowed to yield in bending, which violates the requirements of the Savannah River Site Hoisting and Rigging Manual.
(3) The ultimate strain of the lifting bars would be exceeded before the calculated ultimate strength was achieved.

Alternative lifting devices are also identified.

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1. Introduction

The Intermediate Level Tritium Vault #1, 662-E, Cell #1 contains 140 waste silos. Each silo is approximately 25' deep and 30" in diameter at the top. The silos are closed by a conical plug that is 42" tall and 29" in diameter at the top. The silo plug tapers from 29" in diameter to 25" in diameter over an 18" length. Two #4 reinforcing bars projecting from the top of each silo plug are provided for lifting, as shown in Figure 1.

ASTM A615 Grade 60 #4 reinforcing bars have a nominal bar diameter, d_b, of 0.5", a yield stress of 60 ksi, at not more than 0.35% strain and an ultimate stress of 90 ksi with 9% minimum elongation in 8 inches [16, 5]. Additionally, ASTM A615 specifies that the reinforcing bar not crack when bent 180° around a pin with a diameter of $3.5d_b$. The concrete has a minimum compressive strength of 4,000 psi and a unit weight of 147 pcf [16].

The silo plugs were lifted using a two hook sling, shown in Figure 2. During lifting operations, the 1.5" concrete cover over the lifting bars spalled off of several silo plugs. Thirteen of the plugs have been repaired.

The weight of the silo plugs is less than 2000 lbs. There is anecdotal evidence that some of the silo plugs were stuck in the silos causing additional loading when they were removed. Irregularities in the silo wall and fusion of the HDPE form plug with the HDPE silo liner have been suggested as possible causes of sticking.

The Savannah River Site Hoisting and Rigging Manual [1] requires that "lift devices shall be designed with a minimum design factor of 3 based on the yield strength or 5 based on the ultimate strength of the material, whichever is more conservative." The purpose of this report is to review the design of the silo plug lifting bar against the requirements of the site hoisting and rigging manual and to provide options if the design is not adequate.

2. Walkdown

On March 2, 1993, the author performed a walkdown of the waste silo plugs in Building 662-E Cell #1. Various amounts of spalled concrete was observed on 15 silo plugs in addition to the 13 silo plugs that had already been repaired, as shown in Table 1. The failure rate for the silo plugs, ignoring the plugs with slight damage, is $(13+9)/140 \approx 16\%$.

The spalled regions were approximately conical in shape, starting at the top of the reinforcing bar and spreading to the top of the lifting plug, as shown in Figure 3. The clear cover over all of the lifting bars was observed to be[1] 1.5" ± 0.125", which is consistent with the 1.5" of cover specified on the drawings [2], and within the ACI-318 allowable tolerance on cover [11].

In addition to spalled concrete, cracks were observed in the concrete above the horizontal lifting bars in silo plugs #7 and #92. These cracks are a precursor to future spalling.

Distorted lifting bars were observed on many of the silo plugs. Two patterns of distortion were noted: (1) distortion due to vertical loading, and (2) distortion due to vertical and lateral loading. The magnitude of most, if not all, lateral distortions is less than 1.25", as measured on silo plug #85. The magnitude of vertical distortion was not quantified but is consistent with plastic deformation of the lifting bar due to vertical overload.

Measurement of the lifting bar diameter confirms that #4 bars were used, as specified on Drawing W2020320 [2]. The inside bend diameter of two typical silo plug's reinforcing bars was measured and varied from 2" to 2.5" ($4d_b$ to $5\ d_b$) on silo plugs #104 and #112.

The inside of the silos are lined with 0.25" thick HDPE. Several locations were observed where the HDPE liner had pulled away from the silo wall, creating a ≈0.25" gap between the liner and silo wall (silo #97). A similar gap is shown in Figure 4. The concrete surface between silos is rough, showing signs of rework. Chips of concrete are present on this surface and one concrete chip was observed wedged in a gap between the silo wall and HDPE liner. As this facility is used, additional chipping of the concrete surface should be expected, and the potential for chips falling in the crack between liner and silo wall or between the silo plug and liner is high. Concrete chips in these locations may jam against the silo plug, causing future difficulty when extracting silo plugs.

The lifting bars have surface rust, which easily brushes off with a wire brush. The current amount of corrosion is not structurally significant.

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1 The 1/8" dimension reflects the accuracy of field measurements and does not imply that cover dimensions ranging from 1.375" to 1.625" were observed.

A lifting sling that may have been used to remove the silo plugs was observed in the corner of cell #1. This sling is approximately 26" from its center to each of the two lifting hooks.

Table 1 March 2, 1993 Walkdown Observations

Plug Number	Plug Repaired	Moderate Damage	Slight Damage	Concrete Spall Dimensions[2] Cover (in)	Depth (in)	Width (in)
1	X					
3	X					
5			X			
6		X		1.5	1	4
7			X[3]			
8			X			
10	X					
14	X					
15		X		1.5	2.5	6
18	X					
26		X		1.5	2	5
36			X			
39	X					
50		X		1.5	3.5	8
51	X					
68		X		1.5	2	5
81		X		1.5	3.5	8
82	X					
84		X		1.5	2.5	4
92			X[3]			
95			X			
106	X					
115		X		1.5	2	4
117		X		1.5	3	8
128	X					
129	X					
132	X					
142	X					
Totals	13	9	6			

2 Cover dimension is ± 1/8", the depth and width of the spall are shown in Figure 3 and are approximate dimensions.

3 Concrete is cracked above the horizontal reinforcing bar but has not spalled yet.

3. Structural Response to Lifting

3.1. Material Limitations

This section addresses: (1) the shear capacity of the horizontal lifting bar, which is reduced due to spalling of the concrete cover as shown in Figure 3; and (2) reinforcing bar strain limits. The tensile and bending capacity of the reinforcing steel is addressed later in this document. Bond strength and development lengths are addressed in Reference 3.

3.1.1. Concrete Cover over Horizontal Reinforcing Steel

The spacing between a reinforcing bar and a free edge, in the direction of applied shear, must be sufficiently large enough to preclude a tensile failure of the concrete above the reinforcing bar, as shown in Figure 5. This detail is not common in conventional concrete construction, and ACI-318 does not include the appropriate design provisions.

A similar detail is commonly encountered in precast concrete construction with headed anchors and weld plates. The 'shear cone' failure mechanism for a long headed anchor and a reinforcing bar are similar; and design equations, developed for headed anchors [4], are used to determine the shear capacity of the reinforcing bar in Figure 5.

$$V = S_{uc}\left(\frac{Des\text{-}1}{8d_b} C \sqrt{\frac{f'c}{5000}} \right)$$

Where S_{uc} is the ultimate shear capacity of the bar,
Des is the distance from the free edge to the center of the bar,
d_b is the bar diameter,
C equals one for normal weight concrete, and
f'c is the ultimate compressive strength of the concrete.

For the 1.5" clear cover and #4 reinforcing bars, $V = 0.112\ S_{uc}$, or 11% of the ultimate shear capacity with adequate concrete cover. To develop the full shear capacity of the reinforcing bar would require 5.5" of concrete cover over the #4 horizontal reinforcing bar.

The ultimate shear capacity of a bar with adequate concrete cover, S_{uc}, is given by [4]

$$S_{uc} = \phi\ 0.00666\ As\ f'c^{0.3}\ Ec^{0.44} \leq 0.9\ As\ Fy = 10.6 \text{ kips}$$

Where $\phi = 0.85$,
As is the bar area,
$Ec = W^{1.5}\ 33\ \sqrt{f'c}$,
W is the unit weight of the concrete in pcf, W=147, and

Fy is the steel yield strength.

Thus, the ultimate shear capacity with 1.5" of concrete cover is V= 1.2 kips. Shear loading larger than 1.2 kips may cause the concrete above the reinforcing steel to spall, reducing the load carrying capacity.

Note that the shear in the reinforcing bar is only 1.2/0.2=6 ksi, which is well below the limit load shear capacity of the bar, 0.55 Fy=33 ksi [12].

3.1.2. Reinforcing Steel Strain Limits

The specified minimum elongation of the reinforcing steel is 9% over an 8" gauge length. Additionally, the reinforcing steel is specified not to crack when bent 180° around a pin with a diameter of $3.5d_b$, where d_b is the bar diameter [5].

The extreme fiber bending strain in a reinforcing bar, bent to a center line radius, r is [6]

$$\varepsilon = \frac{d_b}{2r}$$

The extreme fiber bending strains are calculated for several different bend radii in Table 2. ACI limits on the minimum bend diameter [11] yield a maximum strain of 14% in primary reinforcing bars and 20% in stirrups and ties. The ASTM A615 bend test assures that the reinforcing bar has a minimum ultimate strain of 22%.

Plastically deforming a reinforcing bar by pulling on it with a lifting hook will eventually wrap the bar around the lifting hook causing high bending strains and possibly leading to rupture. Estimating the inside bend diameter to be 1 to 2 bar diameters gives extreme fiber bending strains between 33% and 50%. These bending strains alone exceed the ductility of the reinforcing steel. Tensile loads in the bar, due to lifting, would increase the strain and cause rupture at reduced load levels.

For primary load carrying members, such as the lifting bars, the minimum inside bend diameter should be limited to $6d_b$. Smaller bend diameters will reduce the bar's capacity. Bending a reinforcing bar around a crane hook will eventually crack the reinforcing bar, leading to failure. Bar diameters as small as $4d_b$ were observed during a walkdown, as discussed in Section 2.

The load capacity of wire rope is similarly reduced when bent to a tight radius. A thimble is inserted into the loop at the end of a wire rope to maintain a safe minimum radius.

Table 2 Reinforcing Bar Strain Due To Bending

Inside Bend Diameter	Centerline radius	Extreme Fiber Bending Strain	Comment
6 d_b = 3"	1.75"	14%	ACI-318 Limit for #4 bar
4 d_b = 2"	1.25"	20%	ACI-318 Limit for #4 stirrups & ties
3.5 d_b = 1.75"	1.125	22%	ASTM A615 bend test requirement for #4 bar
2 d_b = 1"	0.75"	33%	Upper range of bend diameter over a crane hook
1 d_b = 0.5"	0.5"	50%	Lower range of bend diameter over a crane hook

3.2. Stress Analysis

The load distribution in the lifting bars is highly dependent on the location of the lifting hooks. Figure 2, Cases A, B and C demonstrate several examples of lifting hook placement. Elastic and ultimate strength analyses are performed to determine the safe lifting loads.

3.2.1. Elastic Analysis

In Case A, the lifting hooks are located as far outboard as possible. An elastic stress analysis of this loading case was performed using the ABAQUS finite element program [7]. The reinforcing bars were modeled with 15 second order beam elements, B32. Lifting bars with inside bend diameters of 6d_b [8] and 4d_b [9] were both analyzed; other dimensions are shown in Figure 6. The bars were assumed to be fixed at the face of the concrete, and the lifting load was applied as a concentrated nodal load.

The response of the reinforcing bar is dominated by bending, as shown by the bending moment diagram in Figure 6. These moment diagrams were obtained by scaling the elastic ABAQUS results in References 8 and 9 up to the yield stress. The yield moment of a #4 Grade 60 reinforcing bar, without axial forces, is 0.736 in-kip. When combined with an axial loading, the yield moment is reduced. The minimum yield load for these two cases is 1.14 kips per bar.

In Case B, the lifting hooks are located as far inboard as possible. Most of the applied loading is transferred, in direct shear, to the adjacent concrete. As shown in Section 3.1.1, the applied load is limited by failure of the concrete cover over the horizontal reinforcing steel to 1.2 kips per bar.

Loading Case C, with a concentrated load located 2" from the face of the concrete, was found to give the lowest yield load of 0.87 kips per bar [10], with the bar geometry in Figure 6A.

3.2.2. Nonlinear Analysis - Ultimate Capacity

Plastic hinges will form in the bars, as the load is increased beyond the initial yield load, allowing large rotations and gross changes in the geometry, as shown in Figure 7. Assuming that the ultimate strain in the bar is not exceeded, eventually, the bars will rotate into the two bar truss geometry as shown in Figure 7(D).

The ultimate load is calculated for the two bar truss geometry in Attachment A, neglecting the strain limitation. Depending on the location of the applied loading, the ultimate load varies between 1.2 and 18 kips per bar, as shown in Figure 8. If the load is applied at a distance less than 3.5" from the face of the concrete (x=3.5" in Figure 8), then spalling of the concrete controls the ultimate strength. Loads greater than 3.5" from the face of the concrete are controlled by rupture of the reinforcing steel.

Additional loads may be carried by the steel reinforcing once the concrete spalls and the geometry changes. A second calculation in Attachment A, assumes that 4" of concrete has spalled off, and shows an increased load carrying capacity for some loading cases, with the maximum load capacity approximately 18 kips.

A nonlinear finite element analysis of the loading configuration shown in Figure 6A was performed to determine the strain when loaded beyond the elastic limit [14]. The same finite element model used for the elastic analysis was also used for the nonlinear analysis. An elastic-plastic material model having a yield point of 60 ksi, Young's modulus of 29,000 ksi, and a Poisson's ratio of 0.3 was used in the nonlinear analysis. The elastic-plastic material model is accurate up to 1.5% strain [13]; beyond that point, the elastic-plastic material model is conservative.

Equivalent plastic strain is given in Table 3 at different load levels for element #17 and element #29. Element #17 is representative of the reinforcing bar strain under the lifting hook. Figure 9 shows the deformed shape of the reinforcing bar at a load of 3 kips, along with the locations of elements #17 and #29. The deformed shape of the reinforcing bars in Figure 9 is similar to the bar's shape observed during the walkdown.

The maximum strain is underestimated because beam elements were used to model the reinforcing bar under the crane hook. Continuum elements with contact against a finite width crane hook would be needed to accurately determine the strain at this location. However, these results demonstrate that the strains in the reinforcing bar approach the ultimate strain at low loads.

A second nonlinear analysis with loading Case C was performed, and the equivalent plastic strains are given in Table 4 for elements #23 and #29. The strain in element #23 represents the strain in the reinforcing bar under the lifting hook, load Case C.

Recall that the maximum strain in the bend test is 22%, which corresponds to an ultimate load of 2.5 kips for the analysis represented in Tables 3 and 4. The use of 20% plastic strain as a design criteria would be very difficult to technically justify. Primary strains of limits of 1 to 2% are much more common. Using a maximum strain limit of 2% yields an ultimate capacity of 2.2 kips. As shown in Section 3.1.1, load Case B (Figure 2) has the minimum ultimate capacity of 1.2 kips, which is due to failure of the concrete cover, as discussed above.

Table 3 Plastic Strain vs. Loading, Case A [14]

Load (kips)	Equivalent Plastic Strain Element 17	Equivalent Plastic Strain Element 29
1.14	0	0
2.00	0.3%	0
2.37	1.7%	0.3%
2.53	3%	0.7%
2.61	4.4%	1.1%
2.76	10%	3%
2.81	12.3%	3.8%
3.00	20%	6.4%

Table 4 Plastic Strain vs. Load, Case C [15]

Load (kips)	Equivalent Plastic Strain Element 23	Equivalent Plastic Strain Element 29
0.87	0	0
2.00	0.3%	1.5%
2.18	0.3%	1.7%
2.30	0.3%	3.25%
2.39	0.3%	5.3%
2.46	0.4%	10%
2.56	0.5%	20%

3.3. Loading and Comparison to Structural Capacity

The weight of the silo plug has been calculated at 1.85 kips [3]; a 25% increase for impact [12] gives a working load of 2.32 kips. Additional loading is possible if the plug becomes stuck in the silo. Since the annular gap between the concrete silo plug and concrete silo is between 0.5" to 1", and the annular gap contains a 0.25" thick HDPE liner and possibly concrete chips, the sticking force cannot be calculated with any degree of certainty. For the purposes of discussion, the sticking force is estimated to be equal to the weight of the silo plug. If concrete chips fall in the annular gap and wedge between the silo plug, liner and silo wall, then this load estimate may be unconservative.

Thus, the working load due to the silo plug, impact, and an estimated sticking load becomes 2.32 + 2.32 ≈ 5 kips. The loading corresponding to initial yield and ultimate strength are summarized in Table 51, along with the design factors and the safe working load. The safe working load is taken as the minimum initial yield or ultimate strength, divided by the appropriate design factor. The minimum safe working load is 480 pounds, as shown in Table 5, which is less than the weight of the silo cap.

Table 5 Summary of Load Capacities and Safe Working Load

	Initial Yield	Ultimate Strength	Two Bar Tensile Capacity[4]
Load per bar (kips)	.87 to 1.35	1.2 to 2.5	1.2 to 18
Total Load (kips)	1.74 to 2.7	2.4 to 5	2.4 to 36
Design Factor	3	5	5
Total Allowable Load (kips)	0.58 to 0.9	0.48 to 1	0.48 to 7.2

Safe Working Load - Without Restrictions = 480 lbs

[4] Neglecting strain limits.

4. Design Review

The design of the lifting bars is documented in Calc. Note C-CLC-E-00007. This calculation checks the bar capacity in direct shear, the bar capacity in direct tension, and the bar embedment lengths.

Sheet 5 [3] specifies 1.5" of cover over the lifting bar, and the design assumes that the bar can develop its full shear strength. In section 3.1.1, it was shown that 5.5" of cover over the lifting bar is required to develop the bar's full shear strength.

Sheets 5c through 5e [3] calculate the deformed bar geometry assuming that the bending stress in the bar has exceeded the yield stress. Section 6.4.3 of the site hoisting and rigging manual [1] requires a minimum design factor of 3 on the yield stress. The design calculations clearly do not meet the yield stress criteria of Reference 1.

The ultimate strength calculation in sheets 5c and 5d assumes that each of the bars has the same load, T=1.759 kips. Checking horizontal equilibrium

$$T_{x1} = T\,\mathrm{Cos}(\theta) = 0.337 \text{ kip} \neq T_{x2} = T\,\mathrm{Cos}(\alpha) = 1.736 \text{ kip}$$

Where $\theta = 1.357$ rad, and
$\alpha = 0.16$ rad, and
T, θ, and α are defined in Reference 3,

shows that this solution is invalid. Bar 2 has to carry a smaller load to be in equilibrium, and the total applied load

$$\text{Load} = T_{y1} + T_{y2} = T_1\,\mathrm{Sin}(\theta) + T_2\,\mathrm{Sin}(\alpha)$$

is overestimated.

The ultimate strength calculation does not check the strain in the reinforcing bar. As shown previously, strain controls the ultimate capacity of the bars.

5. Lifting Options

The current design of attaching lifting hooks to #4 reinforcing bars projecting from the silo plugs has been shown to be deficient. Several options for lifting the silo plugs, in conceptual form, are presented below.

Option 1 Use the same plate and concrete anchor detail that was used to repair the 13 existing plugs.
Advantages: Simple; some are already in use.
Disadvantages: Requires extensive labor to fabricate material, drill holes, and attach plates.

Option 2 Use a modified barrier lifter to pull the plugs by attaching to the stem on the concrete plugs, as shown in Figure 10.
Advantages: Simple; no field work required; proven technology. Barrier lifters are available 'off the shelf' in capacities up to 14 kip.
Disadvantages: The lifter will have to be redesigned and fabricated to reach inside the 30" silos, around the existing lifting bars, and grab the concrete stem.

Option 3 Lift the silo plugs from the vertical bar only, using the eccentric cam tool shown in Figure 11.
Advantages: No field work required; uses existing lifting bar.
Disadvantages: The tool would have to be designed, built, and tested.

Option 4 Lift the silo plugs from the vertical bar only, using a prestressing wedge cone, as shown in Figure 12.
Advantages: Simple; prestressing wedge cones are a proven technology.
Disadvantages: May require cutting off a portion of the existing reinforcing steel. Lifting gear would have to be designed and built. Prestressing wedge cones would have to remain on the bar in the field.

6. References

1 WSRC-TR-90-7, Rev. 1, Savannah River Site Hoisting and Rigging Manual.

2 Drwg W2020320 Rev 2, *Burial Ground Expansion, ILT Vault Crucible Silos, Plan and Sections, Concrete.*

3 Calc. Note C-CLC-E-00007, *Burial Ground Expansion - 2889, Miscellaneous Title III Calculations*, 4/24/92.

4 Embedment Properties of Headed Studs, TRW Nelson Division, 1977.

5 ASTM A615, *Standard Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement*, 1990.

6 *Strength of Materials, Third Edition*, S.J. Timoshenko, 1956.

7 ABAQUS Version 4.9.1N, Hibbitt, Karlsson & Sorensen, Inc. Pawtucket, RI.

8 Computer Output bar1a.*, Stored in CFS Directory /a9517/waste/silo_plug/, 10:08, 3/7/93.

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10 Computer Output bar1e.*, Stored in CFS Directory /a9517/waste/silo_plug/, 13:54, 3/7/93.

11 ACI-318, *Building Code Requirements for Reinforced Concrete*, American Concrete Institute, 1989.

12 AISC, *Manual of Steel Construction*, American Institute of Steel Construction, 1980.

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15 Computer Output barx5.*, Stored in CFS Directory /a9517/waste/silo_plug/, 11:11 3/9/93.

16 Drwg SE5-6-2003318 SC1 Rev. 2, *Burial Ground Expansion, ILNT Vault, General Notes and Legend Concrete.*

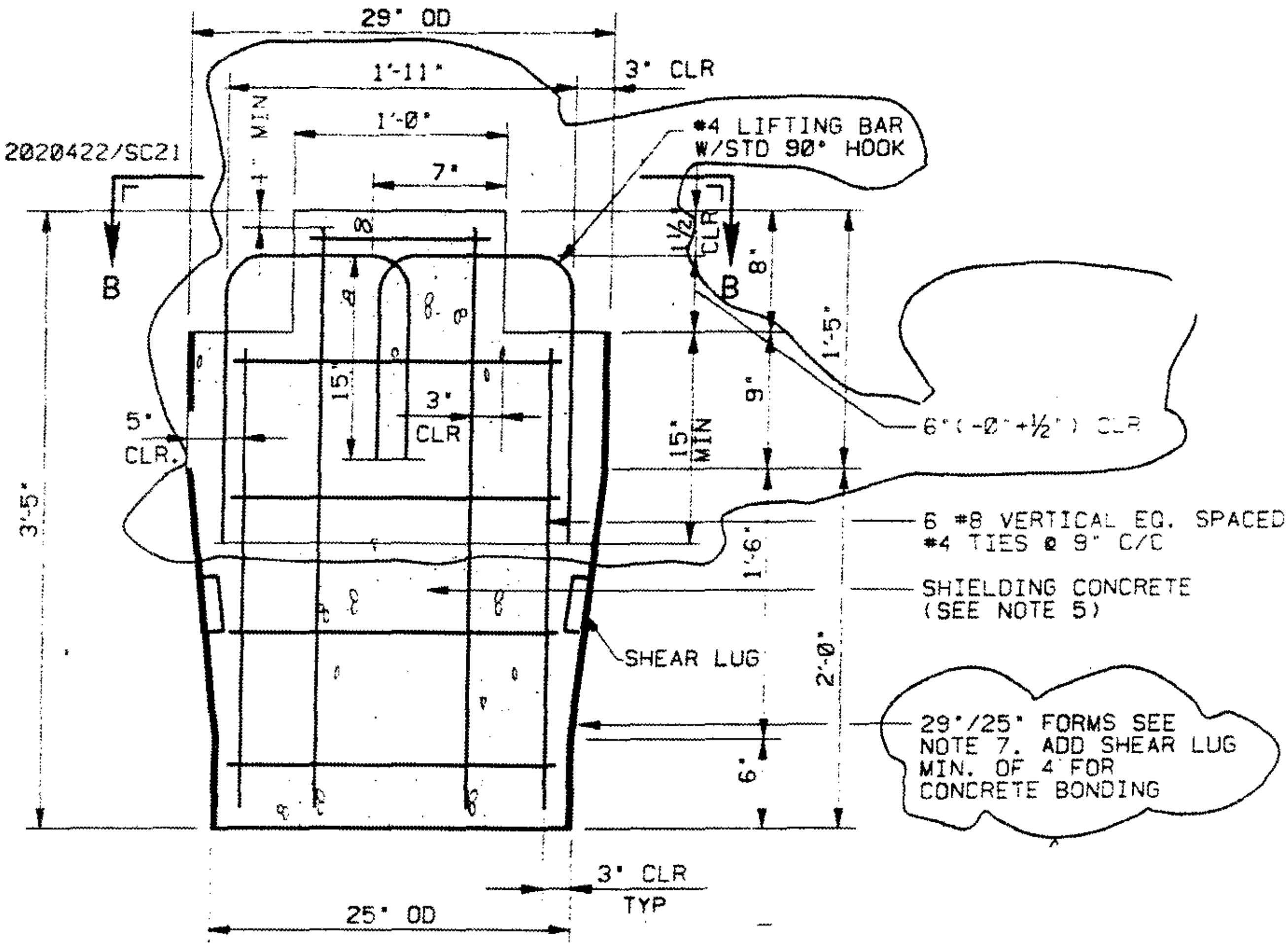


Figure 1 Silo Plug Lifting Bars [2]

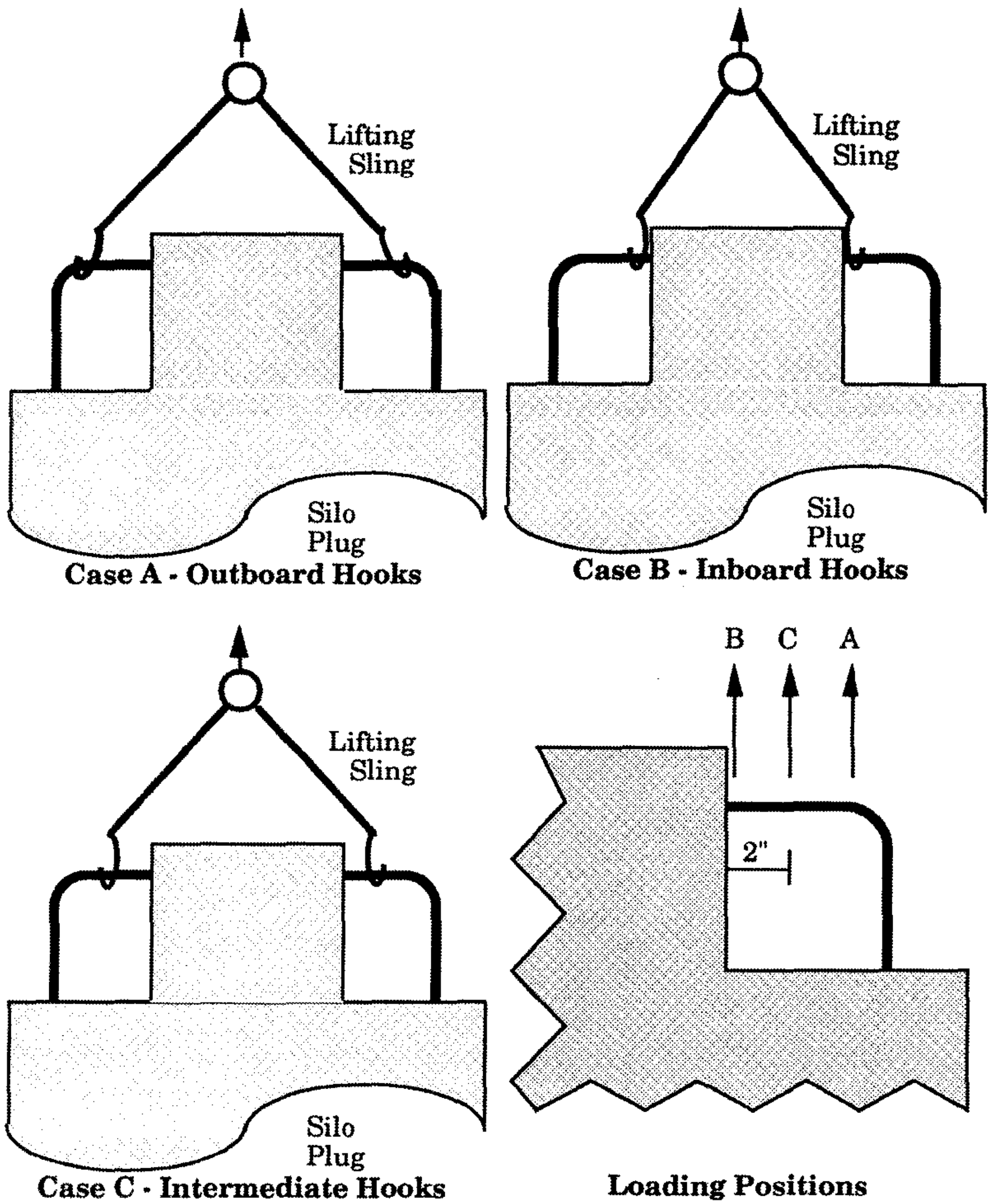


Figure 2 Silo Plug Lifting Configurations

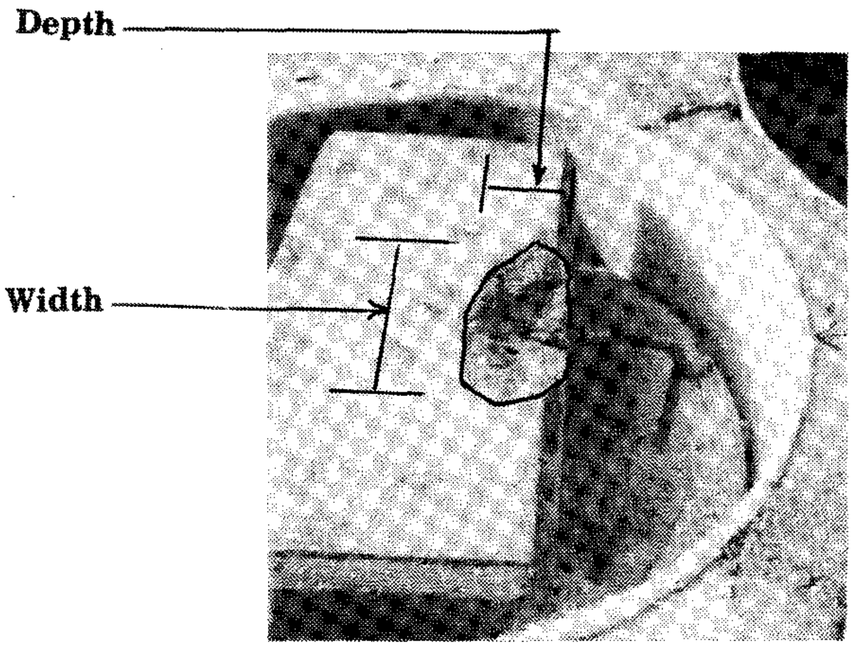


Figure 3 Spalled Concrete on Silo Plug #81

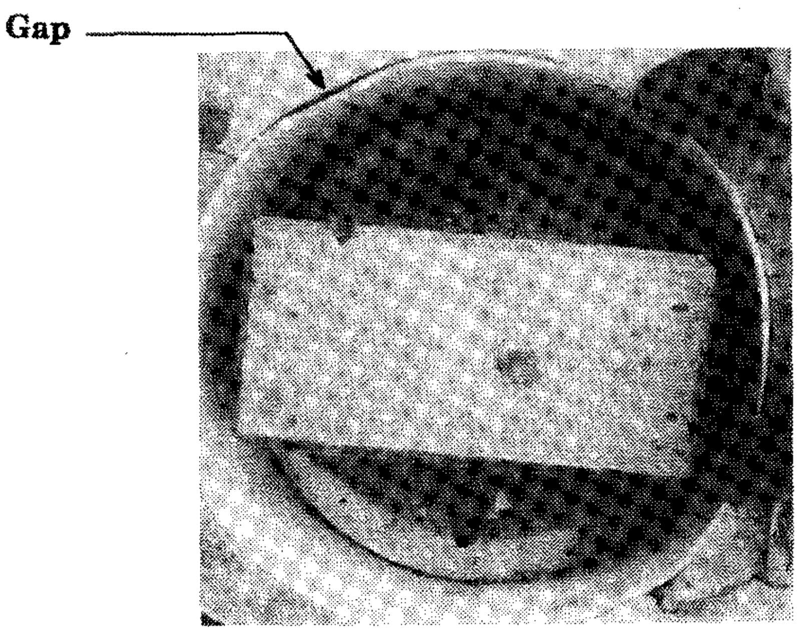


Figure 4 Top View of Concrete on Silo Plug

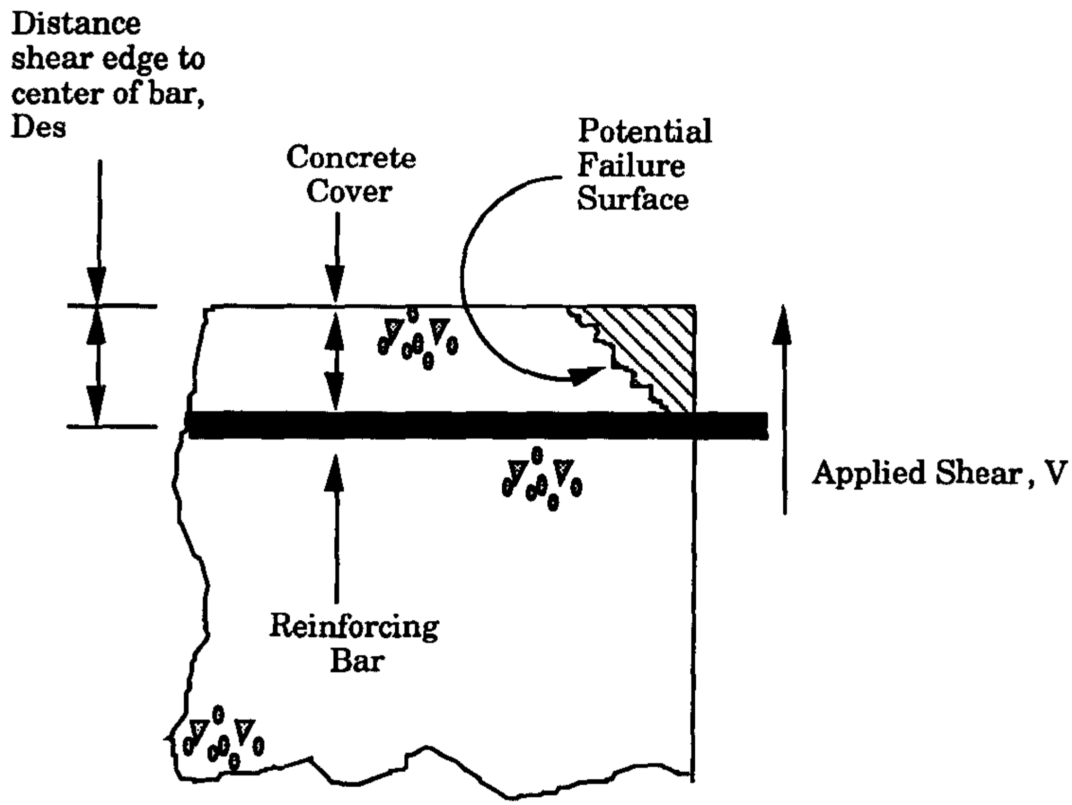


Figure 5 Failure Through Inadequate Concrete Cover

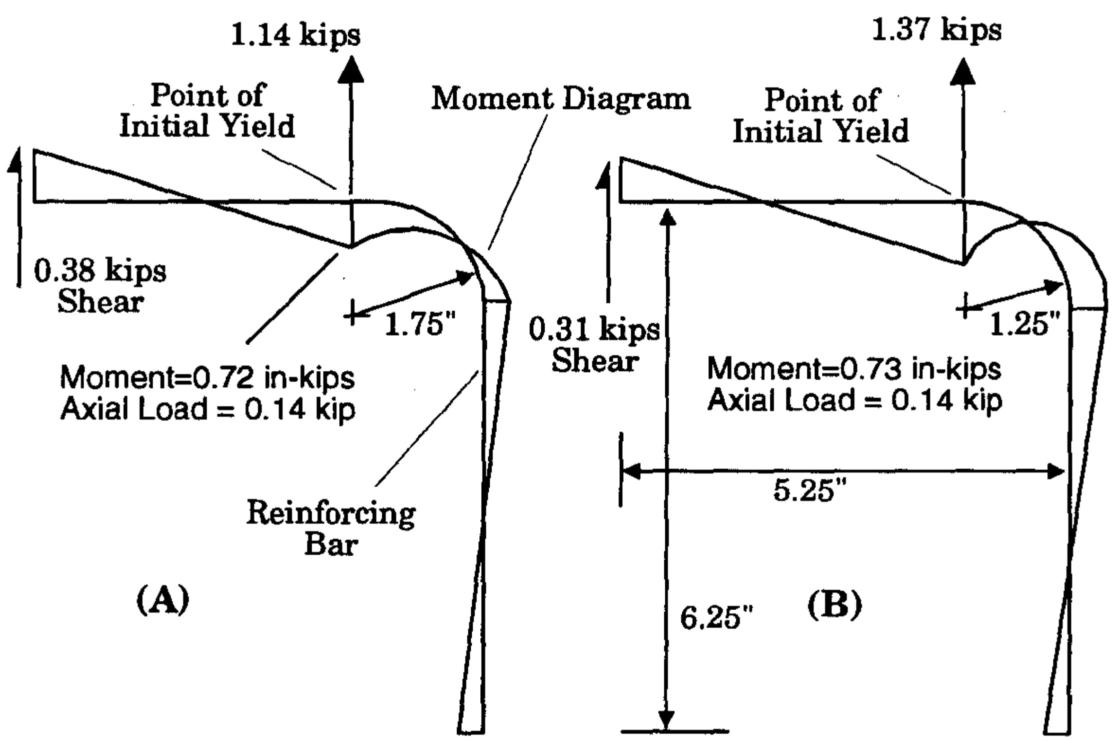


Figure 6 Moment Diagrams at the Initial Yield Load Loading Case 'A'

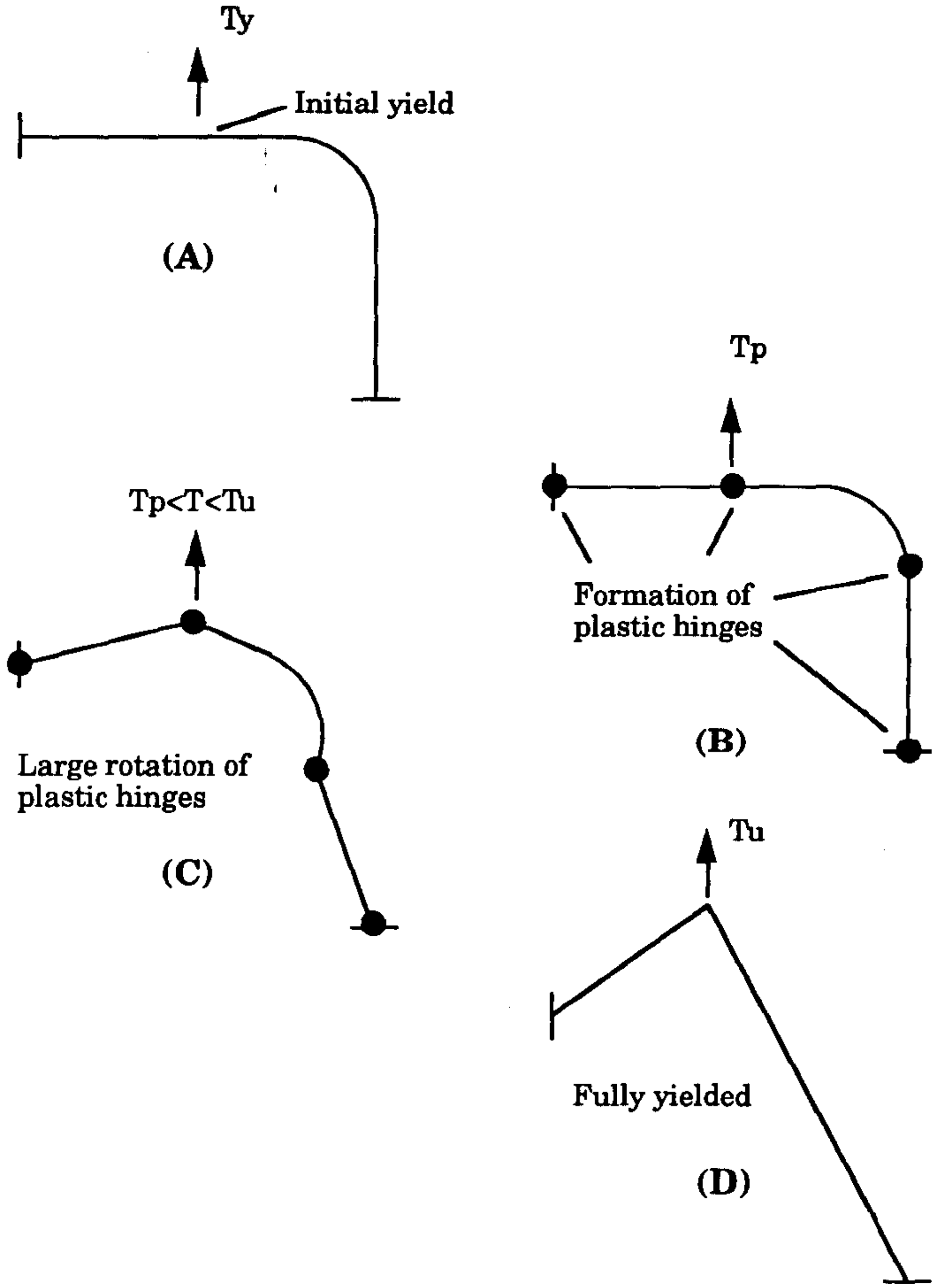


Figure 7 Transition from First Yield to the Ultimate Capacity

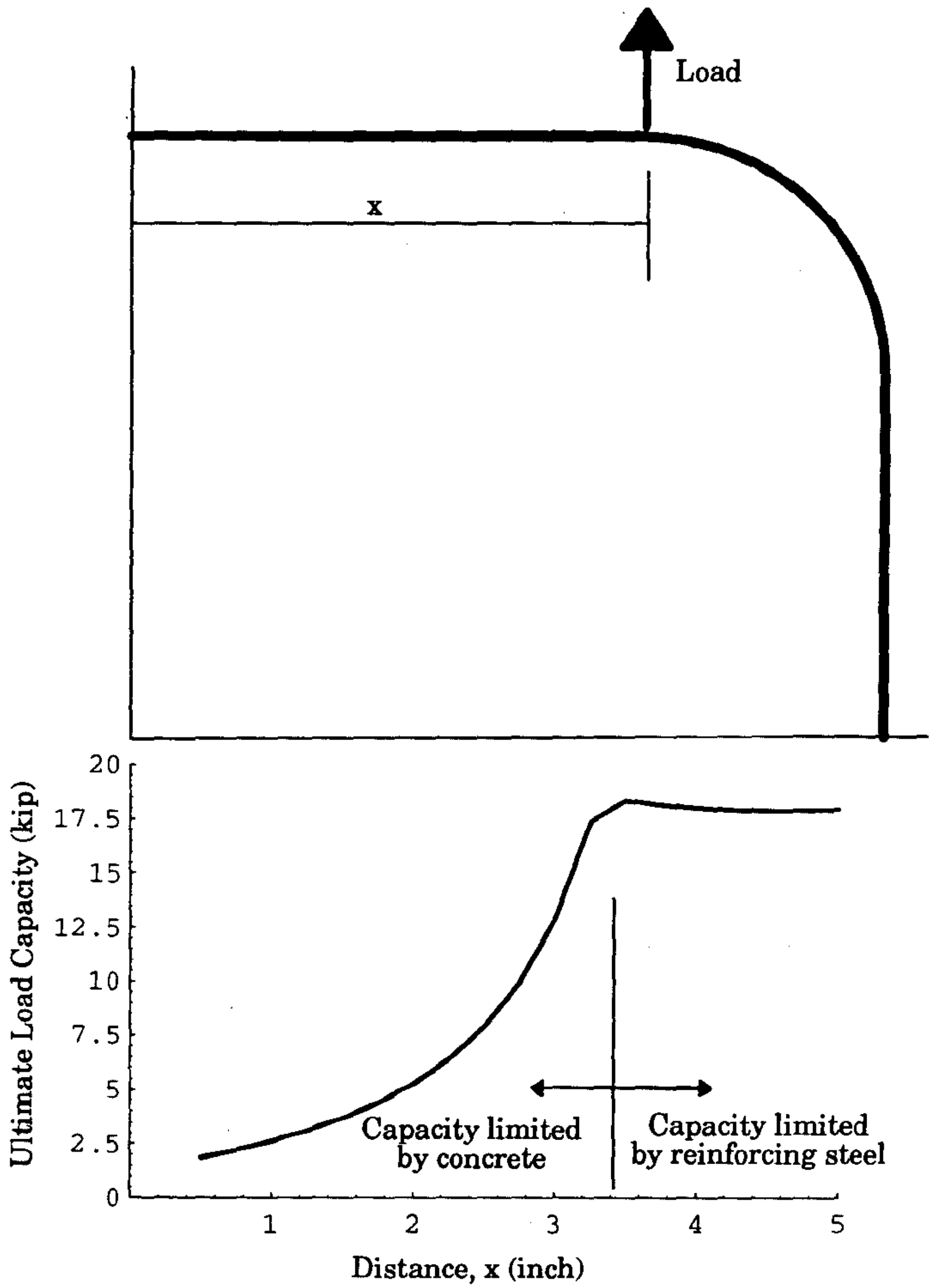


Figure 8 Ultimate Load Capacity per Bar

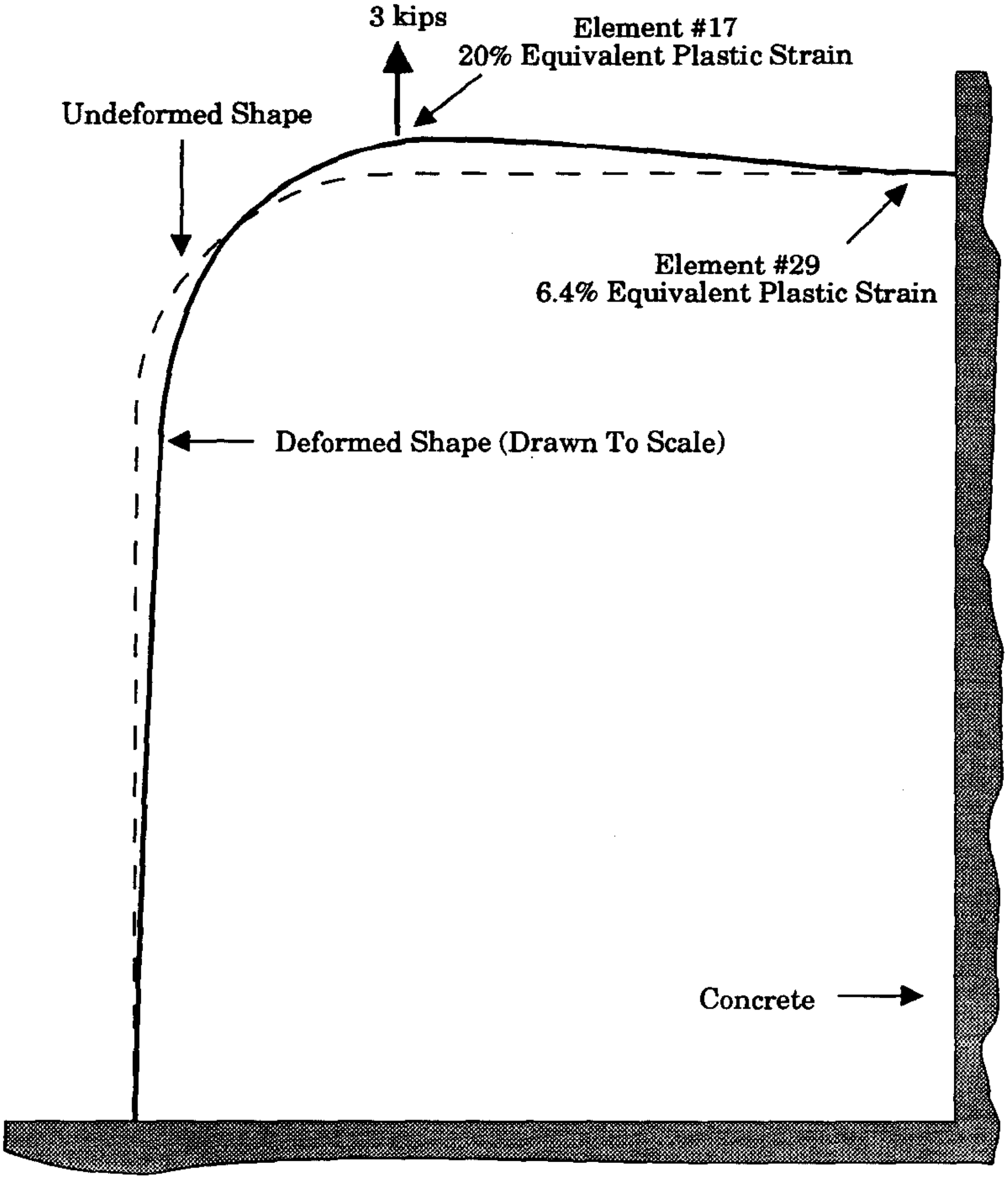
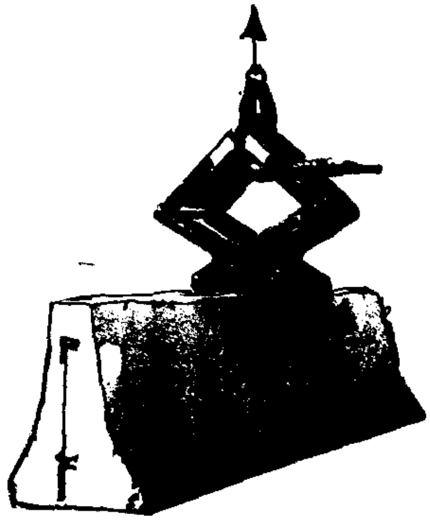


Figure 9 Displaced Shape, Loading at Position A [14]



Concrete Barrier Lifter

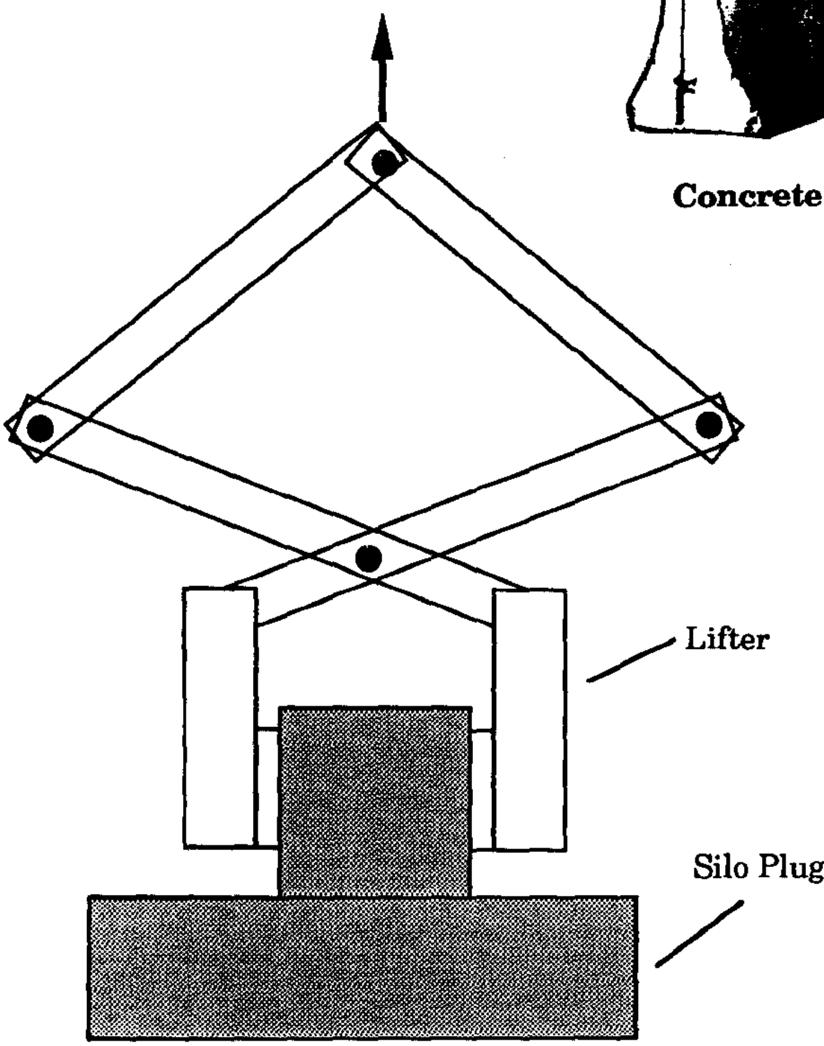


Figure 10 Modified Barrier Lifter Used to Lift Plug

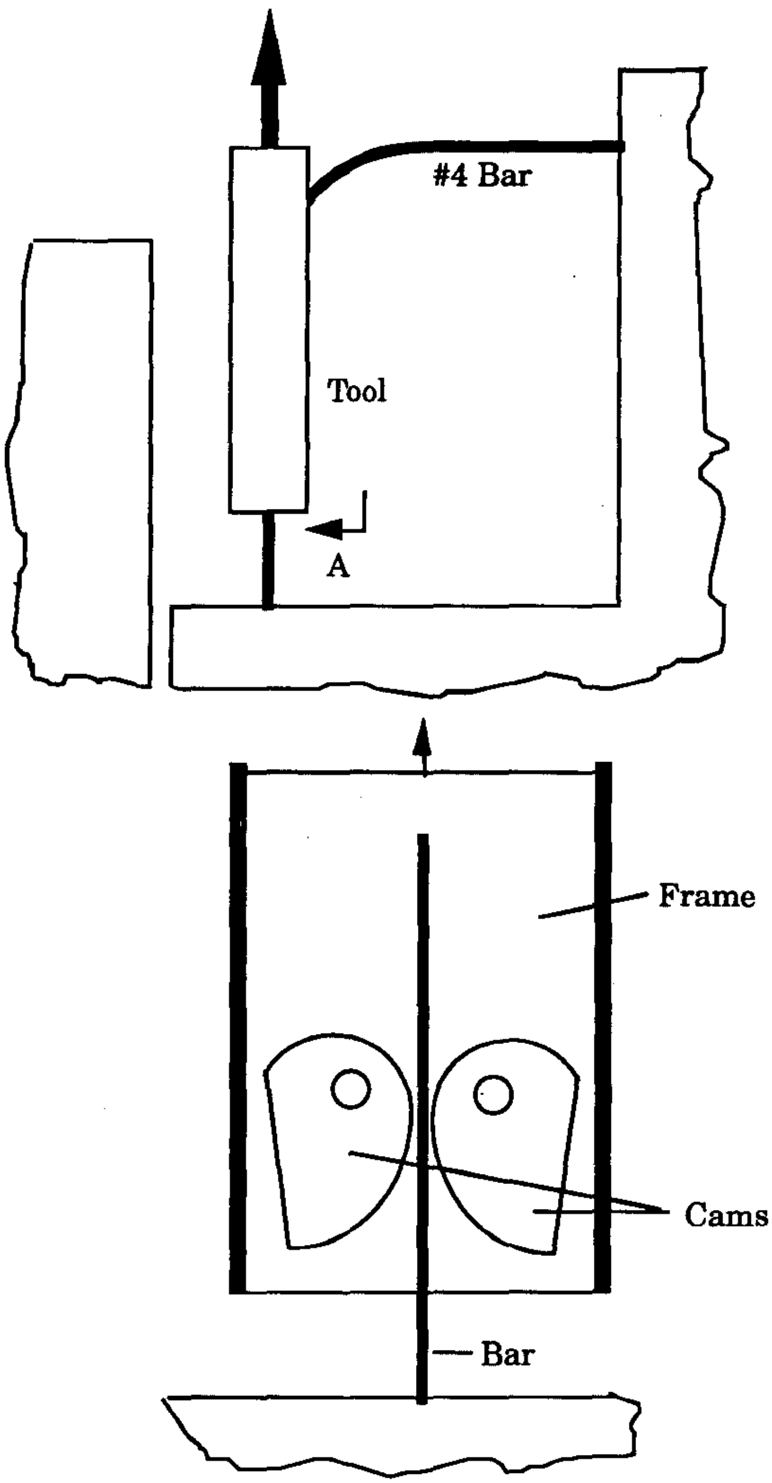


Figure 11 Eccentric Cam Tool Used to Lift Plug

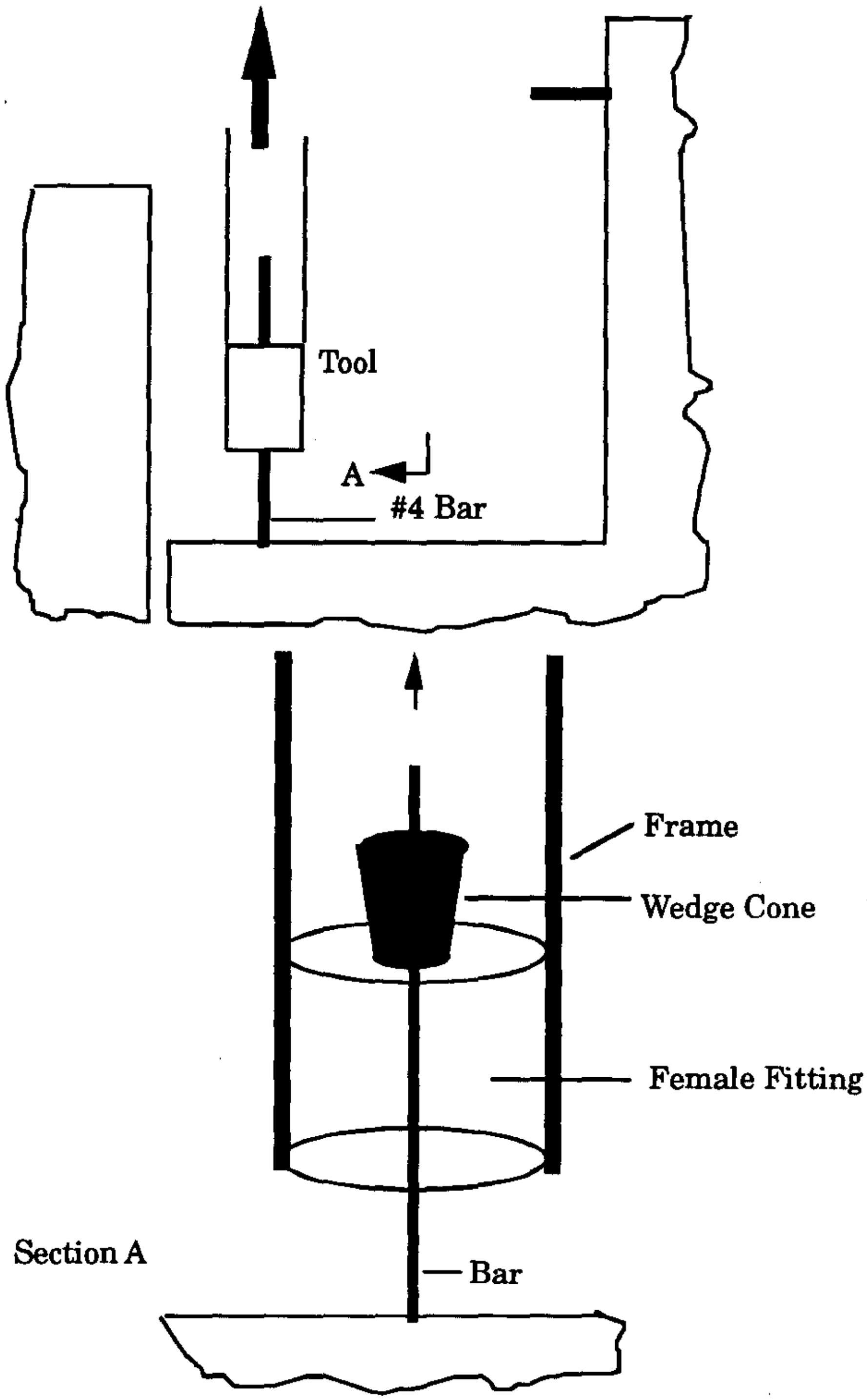
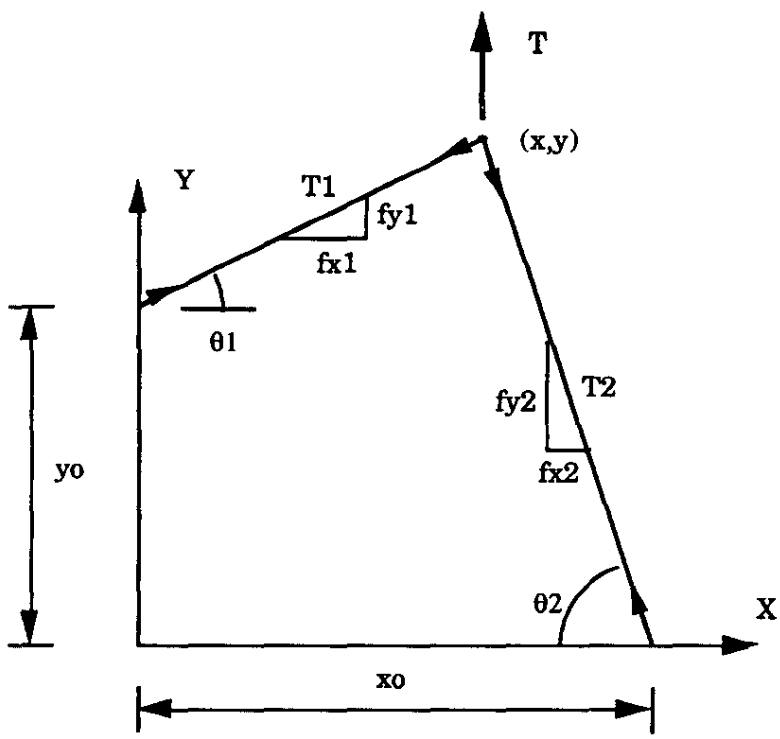


Figure 12 Prestressing Wedge Cone Used to Lift Plug

Ultimate Capacity of the Lifting Bar Neglecting Strain Limitations

■ As Built Geometry

The geometry and nomenclature for the two bar truss is shown in the following figure. Let T1 be the tensile force in bar #1, T2 be the tensile force in bar #2 and T is the total applied load. The point (x,y) is the location of the applied load.



Assume that the length of the bar remains the same or that the change in the bar's length is small compared to it's initial length.

```
xo=5.25;
yo=6.25;
r=1.75;
lo=xo-r+yo-r+Pi/2 r //N
```

```
10.7489
```

```
l1=Sqrt[x^2+(y-yo)^2];
```

```
l2=Sqrt[(x-xo)^2+y^2];
```

Solve for y, given x

```
soly=NSolve[l1+l2==lo,{y}];
```

```
y1=y/.soly[[1,1]]
```

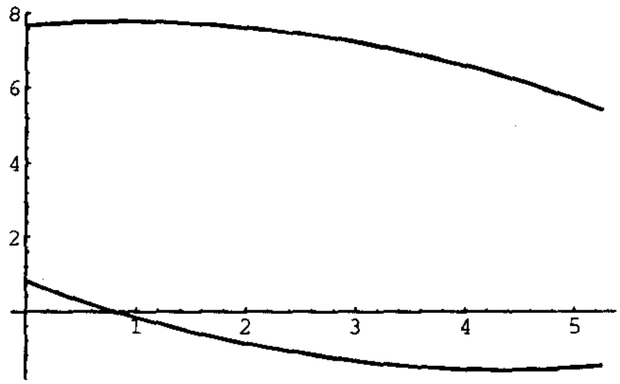
```
4.36053 (0.974944 - 0.0983952 x +
    0.225431 Sqrt[6.99753 - x] Sqrt[1.74753 + x])
```

```
y2=y/.soly[[2,1]]
```

```
4.36053 (0.974944 - 0.0983952 x -
    0.225431 Sqrt[6.99753 - x] Sqrt[1.74753 + x])
```

Plot the two roots of the solution to determine which root is valid

```
p1=Plot[{y1,y2},{x,0,xo}]
```



```
-Graphics-
```

y1, the upper solution, is valid.

□ Equilibrium Equations

Ultimate force/bar

```
tu=90  0.2
```

```
18.
```

Concrete capacity

```
Vmax=1.2;
```

Bar forces as a function of x and y

```
theta1:=ArcTan[(y-yo)/x];
fx1:= t1 Cos[theta1];
fy1:= t1 Sin[theta1];

theta2:=ArcTan[y/(xo-x)];
fx2:= t2 Cos[theta2];
fy2:= t2 Sin[theta2];

t:=fy1+fy2;
```

Solve for the ultimate load, given that
1) the concrete capacity cannot be exceeded,
2) the ultimate strength of any bar cannot be exceeded,
3) the horizontal component of the bar forces are in equilibrium, and
4) the geometrical constraint defined above.

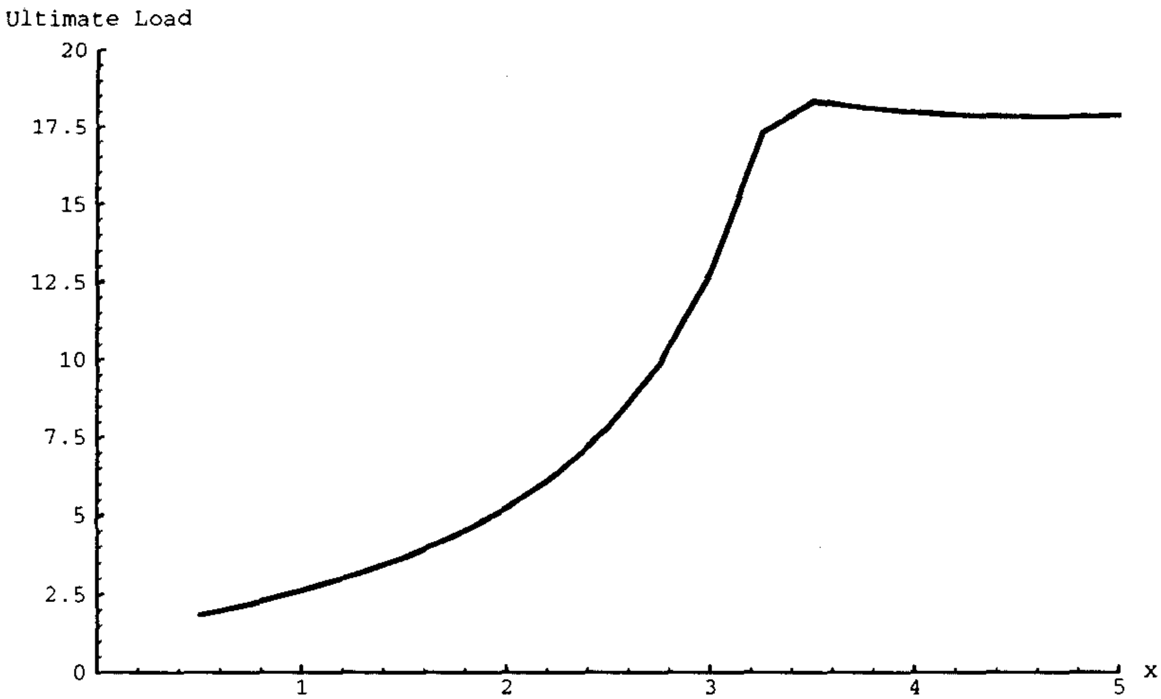
```
ans=
Module[{a},
    ans={};
    Do[ Clear[alpha];
        x=a;                        (* define geometry     *)
        y=y1;
        t2=tu;                      (* assume bar 2 breaks first*)
        t1=alpha t2;
        alpha=alpha /.
            FindRoot[
                fx2==fx1,           (* sum fx=0, solve for alpha*)
                {alpha,0,2}
                ][[1]];
        If[t1>tu,                   (* scale results if bar 1       *)
            t2=tu/alpha ;           (* breaks first                 *)
            t1=tu];
        If[fy1>Vmax,                (* scale results if the         *)
            reduct=Vmax/fy1;        (* concrete capacity is         *)
            t1=t1 reduct;           (* exceeded                     *)
            t2=t2 reduct];

        ans=Append[ans,             (* output results               *)
            {x,y,t1,fx1,fy1,t2,fx2,fy2,t}];

        ,{a,.5,xo-.1,0.25}];
        Clear[x,y];
        ans];
TableForm[N[ans,4],TableSpacing->{0,2},
    TableHeadings->
    {None,{"x","y","t1","fx1","fy1","t2","fx2","fy2","t"}}]
```

x	y	t1	fx1	fy1	t2	fx2	fy2	t
0.5	7.793	1.261	0.3888	1.2	0.747	0.3888	0.6379	1.838
0.75	7.812	1.331	0.576	1.2	1.154	0.576	1.	2.2
1.	7.813	1.425	0.768	1.2	1.607	0.768	1.412	2.612
1.25	7.795	1.544	0.9708	1.2	2.126	0.9708	1.892	3.092
1.5	7.761	1.691	1.191	1.2	2.738	1.191	2.465	3.665
1.75	7.712	1.872	1.437	1.2	3.476	1.437	3.166	4.366
2.	7.647	2.095	1.718	1.2	4.392	1.718	4.042	5.242
2.25	7.568	2.374	2.048	1.2	5.558	2.048	5.167	6.367
2.5	7.475	2.727	2.449	1.2	7.093	2.449	6.656	7.856
2.75	7.368	3.187	2.952	1.2	9.188	2.952	8.701	9.901
3.	7.246	3.807	3.613	1.2	12.18	3.613	11.64	12.84
3.25	7.111	4.686	4.53	1.2	16.73	4.53	16.11	17.31
3.5	6.961	4.478	4.389	0.8913	18.	4.389	17.46	18.35
3.75	6.796	3.921	3.88	0.5647	18.	3.88	17.58	18.14
4.	6.615	3.356	3.342	0.3051	18.	3.342	17.69	17.99
4.25	6.418	2.773	2.771	0.1096	18.	2.771	17.79	17.9
4.5	6.203	2.161	2.16	-0.02233	18.	2.16	17.87	17.85
4.75	5.97	1.505	1.502	-0.08864	18.	1.502	17.94	17.85
5.	5.715	0.7912	0.7867	-0.08419	18.	0.7867	17.98	17.9

```
{xa,ya,t1a,fx1a,fy1a,t2a,fx2a,fy2a,ta}=Transpose[ans];
```

Plot results: x vs ultimate load capacity

```
p1=ListPlot[Transpose[{xa,ta}],PlotJoined->True,
    PlotRange->{{0,5},{0,20}},AxesLabel->{"x","Ultimate Load"}]
```




```
-Graphics-
```

■ Geometry After Spalling

Assume that 4" of concrete above the horizontal reinforcing bar has broken off. Recalculate the ultimate capacity. Let x1 be the length of bar that has been uncovered by spalling concrete.

Assume that the length of the bar does not change

```
x1=4;
xo=5.25;
yo=6.25;
r=1.75;
lo=x1+xo-r+yo-r+Pi/2 r //N

14.7489

l1=Sqrt[(x+x1)^2+(y-yo)^2];

l2=Sqrt[(x-xo)^2+y^2];
```

Solve for y, given x

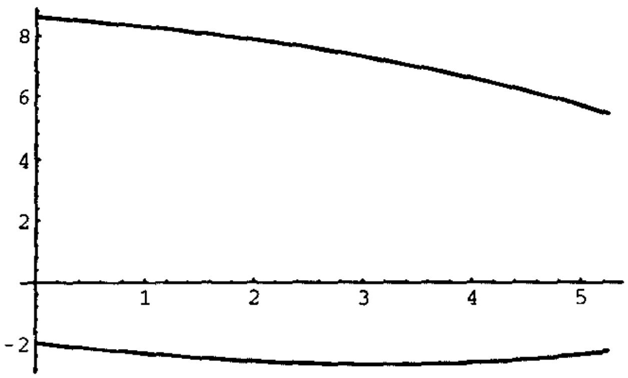
```
soly=NSolve[l1+l2==lo,{y}];

y1=y/.soly[[1,1]]

8.49503 (0.391695 - 0.0381328 x +
    0.0937681 Sqrt[7.30458 - x] Sqrt[6.05458 + x])
```

```
y2=y/.soly[[2,1]]
```

```
8.49503 (0.391695 - 0.0381328 x -
    0.0937681 Sqrt[7.30458 - x] Sqrt[6.05458 + x])
```

Plot the two roots of the solution to determine which root is valid

```
p1=Plot[{y1,y2},{x,0,xo}]
```




```
-Graphics-
```

y1, the upper solution, is valid.

◻ Equilibrium Equations

Ultimate force/bar

```
tu=90  0.2
```

```
18.
```

Concrete capacity

```
Vmax=1.2;
```

Bar forces as a function of x and y

```
theta1:=ArcTan[(y-yo)/(x1+x)];
fx1:= t1 Cos[theta1];
fy1:= t1 Sin[theta1];

theta2:=ArcTan[y/(xo-x)];
fx2:= t2 Cos[theta2];
fy2:= t2 Sin[theta2];

t:=fy1+fy2;
```

Solve for the ultimate load, given that
1) the concrete capacity cannot be exceeded,
2) the ultimate strength of any bar cannot be exceeded,
3) the horizontal component of the bar forces are in equilibrium, and
4) the geometrical constraint defined above.

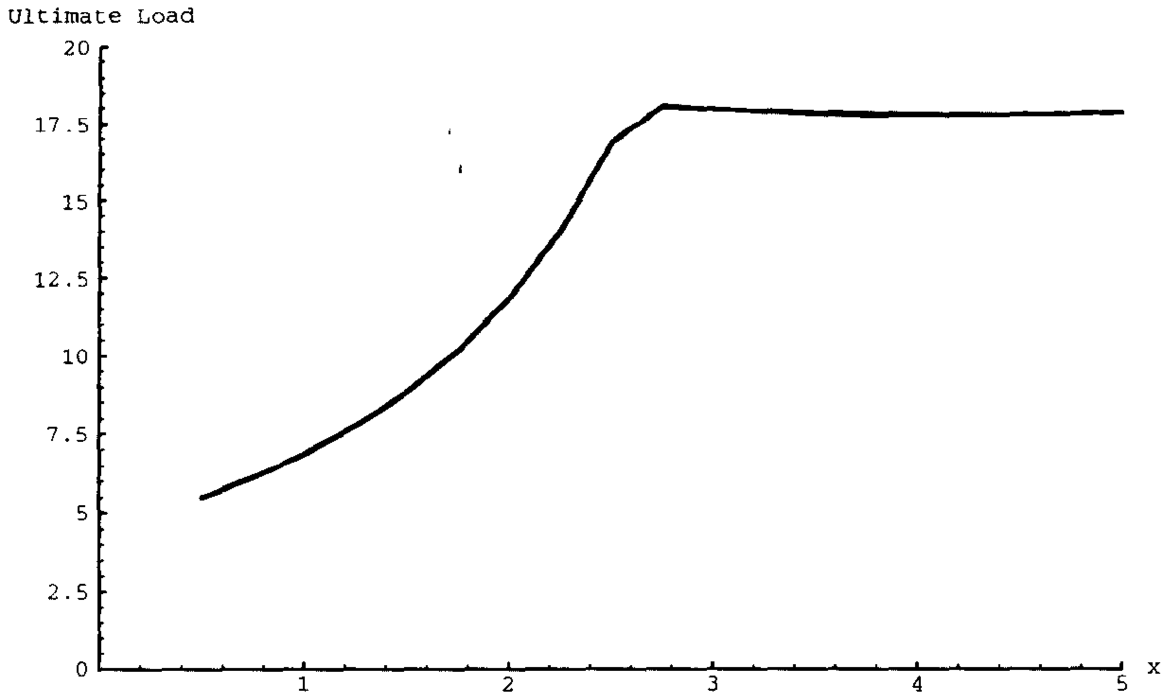
```
ans=
Module[{a},
     ans={};
     Do[  Clear[alpha];
          x=a;                          (* define geometry       *)
          y=y1;
          t2=tu;                        (* assume bar 2 breaks first*)
          t1=alpha t2;
          alpha=alpha /.
               FindRoot[
                    fx2==fx1,           (* sum fx=0, solve for alpha*)
                    {alpha,0,2}
                    ][[1]];
          If[t1>tu,                     (* scale results if bar 1       *)
               t2=tu/alpha ;            (* breaks first                 *)
               t1=tu];
          If[fy1>Vmax,                  (* scale results if the         *)
               reduct=Vmax/fy1;         (* concrete capacity is         *)
               t1=t1 reduct;            (* exceeded                     *)
               t2=t2 reduct];

          ans=Append[ans,               (* output results               *)
               {x,y,t1,fx1,fy1,t2,fx2,fy2,t}];

          ,{a,.5,xo-.1,0.25}];
          Clear[x,y];
          ans];
TableForm[N[ans,4],TableSpacing->{0,2},
     TableHeadings->
     {None,{"x","y","t1","fx1","fy1","t2","fx2","fy2","t"}}]
```

x	y	t1	fx1	fy1	t2	fx2	fy2	t
0.5	8.485	2.697	2.416	1.2	4.946	2.416	4.316	5.516
0.75	8.404	2.905	2.646	1.2	5.605	2.646	4.942	6.142
1.	8.316	3.143	2.904	1.2	6.382	2.904	5.683	6.883
1.25	8.22	3.416	3.198	1.2	7.309	3.198	6.572	7.772
1.5	8.116	3.734	3.536	1.2	8.431	3.536	7.654	8.854
1.75	8.005	4.11	3.931	1.2	9.813	3.931	8.991	10.19
2.	7.886	4.561	4.4	1.2	11.55	4.4	10.68	11.88
2.25	7.759	5.112	4.969	1.2	13.78	4.969	12.85	14.05
2.5	7.624	5.801	5.675	1.2	16.73	5.675	15.73	16.93
2.75	7.481	5.799	5.705	1.04	18.	5.705	17.07	18.11
3.	7.329	5.345	5.283	0.8141	18.	5.283	17.21	18.02
3.25	7.167	4.877	4.838	0.6121	18.	4.838	17.34	17.95
3.5	6.996	4.389	4.368	0.4346	18.	4.368	17.46	17.9
3.75	6.815	3.879	3.869	0.2822	18.	3.869	17.58	17.86
4.	6.623	3.342	3.338	0.1558	18.	3.338	17.69	17.84
4.25	6.42	2.771	2.77	0.057	18.	2.77	17.79	17.84
4.5	6.204	2.16	2.16	-0.01179	18.	2.16	17.87	17.86
4.75	5.974	1.502	1.501	-0.04742	18.	1.501	17.94	17.89
5.	5.728	0.7861	0.7848	-0.04549	18.	0.7848	17.98	17.94

```
{xa,ya,t1a,fx1a,fy1a,t2a,fx2a,fy2a,ta}=Transpose[ans];
```

Plot results: x vs ultimate load capacity

```
p3=ListPlot[Transpose[{xa,ta}],PlotJoined->True,
     PlotRange->{{0,5},{0,20}},AxesLabel->{"x","Ultimate Load"}]
```

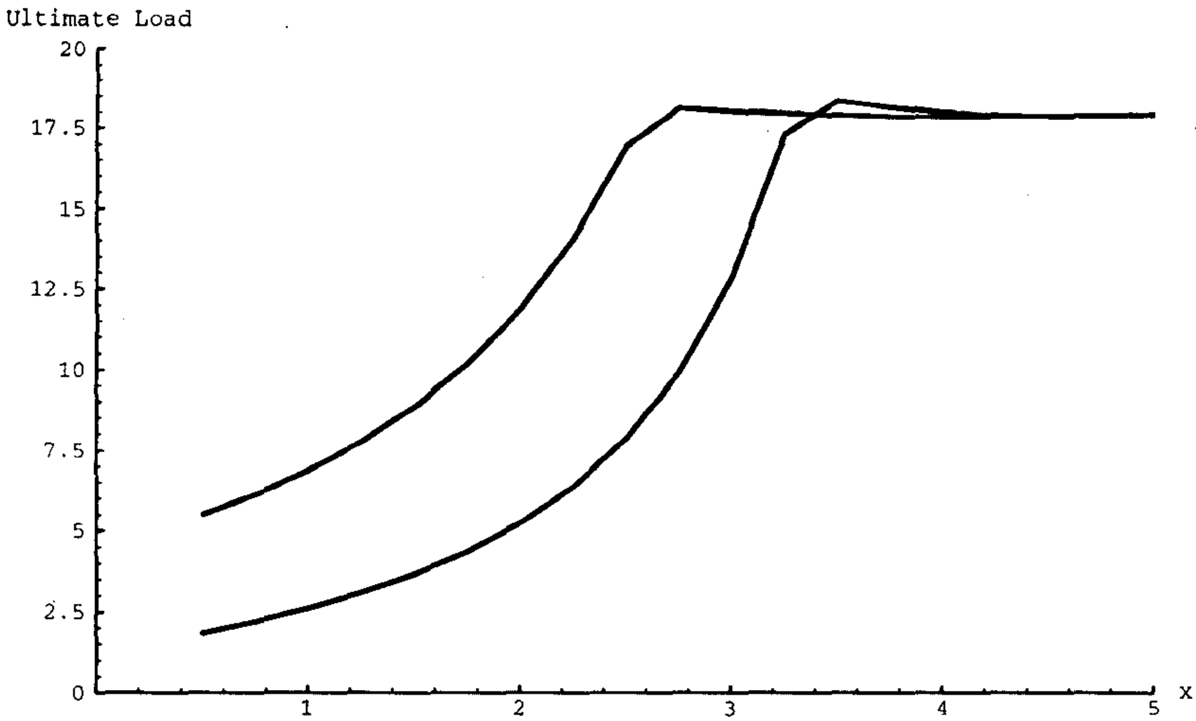


-Graphics-

■ Compare Solutions

Compare ultimate capacity before and after spalling 4" (the lower curve is the load capacity before spalling)

```
Show[p1,p3]
```



-Graphics-

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