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Burst Pressure Solutions of Thin and Thick-Walled Cylindrical Vessels

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ABSTRACT

Pressure vessels (PVs) are widely used in the energy industry. Accurate burst pressure is critical to structural design and safe operation for both thin and thick-walled PVs. The traditional strength theories utilized a single-parameter material property, such as the yield strength (YS) or the ultimate tensile strength (UTS) to develop failure models for determining the yield or ultimate pressure carrying capacity in the PV design. The UTS-based Barlow formula is a typical burst pressure model developed from the Tresca strength theory that provides the basis for developing regulation rules and failure models for different industry design codes, such as ASME BPVC, ASME B31.3, and ASME B31G, among others. In order to reduce the conservatism of the Tresca strength model, ASME BPVC recently adapted failure models developed from the von Mises strength theory for the PV design and analysis.

It has been commonly accepted that the burst pressure of pipelines depends on the UTS and strain hardening exponent, n, of the pipeline steels. An average shear stress yield theory was thus developed, and the Zhu-Leis solution of burst pressure was obtained as a function of UTS and n for thin-walled line pipes. Experiments showed that the Zhu-Leis solution provides an accurate, reliable prediction of burst pressure for defect-free thin-walled pipes. In order to extend the Zhu-Leis solution to thick-walled cylindrical PVs, this paper defined three new flow stresses, modified the traditional strength theories, and obtained three new burst pressure solutions that are valid for both thin and thick-walled cylindrical vessels.

The proposed flow stresses are able to describe the tensile strength and the plastic flow response of PVs for a strain hardening steel. The associated strength theories were then developed in terms of the Tresca, von Mises and Zhu-Leis yield criteria. From these new strength theories, three burst pressure solutions were obtained for thick-walled cylinders, where the von Mises solution is an upper bound prediction, the Tresca solution is a lower bound prediction, and the Zhu-Leis solution is an intermediate prediction of burst pressure for thick-walled cylinders. Finally, the proposed burst pressure solutions were evaluated and validated by two large datasets of full-scale burst tests for thick-walled tubes and for thin-walled pipes.

Keywords: Burst strength, strength theory, pressure vessel, pipeline, Tresca criterion, von Mises criterion, Zhu-Leis criterion

1. Introduction

Pressure vessels (PVs) and pipelines are important infrastructures used in storage or transportation of large volumes of fluids in the energy, petrochemical, nuclear, and oil and gas industry. Facilities such as nuclear reactors, chemical reactors, plant piping, storage tanks, and transmission pipelines are dependent on these systems for safe and efficient operation. The fluids can be liquids or gases, including crude oil, natural gas, liquified gas, gasoline, hydrogen, carbon dioxide, and other petrochemical or refined products. Many of these fluids are flammable, toxic, corrosive, or explosive. These risk factors, coupled with high internal pressure, make the PVs potentially very dangerous to human life and the environment. Thus, determination of accurate burst pressure is critical to structural design, operation, and integrity management of PVs. This work focuses on end-capped cylindrical vessels subject to internal pressure.

In the PV or pipeline industry, structural design and integrity management require the yield or ultimate strength of steel PVs in consideration. The ultimate strength is characterized by burst pressure that is measured in experiments or predicted using strength theories, empirical formulae, or industrial design codes. While the American Society of Mechanical Engineers (ASME) Boiler and Pressure Vessel Codes (BPVC) [1] is often used in the PV industry, ASME B31.3 Code [2] is usually employed in the pipeline industry. The basic design models in both ASME BPVC and B31.3 Codes were developed based on the Barlow formula [3, 4]. In addition to the traditional strength design criteria, different plastic flow criteria and many other empirical, experimental, or numerical methods have been developed for thin and thick-walled cylindrical vessels. While most large-diameter PVs and line pipes are thin-walled cylindrical vessels with pipe diameter to wall thickness ratio $D/t \geq 20$ [5, 6], many small-diameter piping, tubes, or pipes are thick-walled cylindrical vessels with $D/t < 20$ [6, 7]. Hamada et al. [8] and Christopher et al. [9, 10] delivered technical reviews on burst pressure prediction models for thick-walled tubes or PVs. Law and Bowie [11] discussed the burst pressure prediction models for high-strength, thin-walled line pipes. Zhu and Leis [12] assessed a series of burst pressure prediction

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models for a wide range of pipeline steels in terms of strength theories and plastic flow theories. Zhu [13] performed a comparative study on the traditional strength criteria and the modern plastic flow criteria used in the design and analysis of PVs. More recently, Wang et al. [14] evaluated a set of burst prediction models using their burst test data for structural steels exhibiting a yield plateau.

The literature reviews determined that most of the early burst models were developed from either a simple analysis or an empirical curve fit of experimental data for specific steels of interest, and thus no single model can provide a reliable, accurate prediction of burst pressure for all ductile steels. Some models focused on a conservative lower bound prediction, while other models reflected an upper bound prediction. Many burst prediction models, such as the Barlow formula [3], were obtained using the hoop stress and Tresca strength theory that requires only one-single material strength property, such as yield stress (YS) or ultimate tensile stress (UTS). The effect of plastic flow or strain hardening rate, n, was not considered. For the first time, Steward and Klever [15] reported a set of experimental data that showed a strong effect of plastic flow response on the burst pressure data of PVs for different steels, where the burst data have certain scatters but lies within two bounds. The upper bound was defined by the von Mises solution of burst pressure [16, 17], while the lower bound was defined by the Tresca solution of burst pressure [15]. The averaged result of the von Mises and Tresca predictions provides a good fit to the averaged burst test data, where the von Mises and Tresca solutions were obtained from the flow theory of plasticity and are functions of UTS, n and D/t ratio.

To obtain an intermediate theoretical prediction, Zhu and Leis [18, 19] developed a new multi-axial plastic yield theory that was referred to as average shear stress yield criterion, or simply as the Zhu-Leis criterion. For carbon steels, the Zhu-Leis flow solution agrees well with the full-scale test data of burst pressure on average [12, 19]. Many other investigators [20-23] also validated the Zhu-Leis solution using different full-scale burst pressure test data for a wide range of pipeline steels, including steel grades from Grade A to X120. All validations demonstrated that the Zhu-Leis solution is the best burst pressure prediction for thin-wall line pipes. However, the Zhu-Leis solution was developed only for thin-walled pipes, but is not applicable to thick-walled PVs. In fact, a strict theoretical solution of burst pressure does not yet exit for thick-walled PVs due to the complexity of mathematical calculation. In the 1950s, Faupel [24, 25] proposed an empirical burst model for predicting burst pressure of thick-walled tubes or cylinders based on their large burst test data. At the same time, Svensson [17] performed a detailed theoretical analysis for determining the burst failure of thick-walled cylinders based on the von Mises flow theory, but failed to obtain an exact theoretical solution because of a complicated integration. As such, Svensson [17] obtained an approximate solution in a closed form for predicting the burst pressure of thick-walled PVs. However, this approximate Svensson solution of burst pressure has obtained extensive applications to thick-walled PVs [5, 8, 9, 26, 27] due to the better agreement with burst test data. Nevertheless, a more accurate solution of burst pressure for thick-walled PVs is still desired in the PV and pipeline industry.

This work aims to extend the burst pressure solutions obtained for thin-walled line pipes to thick-walled cylindrical vessels in terms of the Tresca, von Mises and Zhu-Leis flow theories of plasticity. For this end, the traditional strength theories were modified, and three new strength theories were developed by use of a two-parameter based flow stress that is associated with the Tresca, von Mises and Zhu-Leis yield criteria, where the new flow stress was defined as a function of UTS and n. These new strength theories were then used to determine the corresponding burst pressure solutions for thick-walled cylinders as a function of D/t, UTS and n. Two large datasets of full-scale burst pressure tests were employed to evaluate and validate the proposed burst pressure solutions for both thin-walled pipes and thick-walled cylindrical vessels.

2. Existing burst pressure models for cylindrical vessels

As introduced above, extensive investigations were performed for determining the burst pressure of thin and thick-walled PVs. Based on available burst model evaluations, including nine early design formulae by Hamada et al. [8], fifteen empirical analytical models by Christopher et al. [9, 10], twenty-one burst model by Zhu and Leis [12], and eighteen burst models by Wang et al. [14], representative burst pressure models are determined and discussed below for both thin and thick-wall PVs.

2.1 Representative Burst pressure models for thin-walled cylindrical vessels

In the pipeline industry, it is well known that the hoop stress of thin-walled pipes ($D/t \geq 20$) subjected to internal pressure is calculated using the following simple Barlow formula that is the basis for developing the regulation rules of all pressure vessels and piping design codes:

$$\sigma_h = \frac{PD}{2t} \tag{1}$$

where σ_h is the hoop stress of pipe, P is internal pressure, and D is the pipe diameter. In practice, D has been used as inside diameter (ID or D_i), outside diameter (OD or D_o), or mean diameter (MD or D_m). The formula (1) was named after Peter Barlow, an English mathematician. As pointed out by Alicino [4], the current "Barlow formula" was actually obtained by Goodman [28] in 1914 by applying the so-called Barlow theory to thick-walled piping. In terms of the UTS of the pipe steel, the Barlow formula determines the following Barlow strength for thin-walled pipes:

$$P_b = \frac{2t}{D}\sigma_{uts} \tag{2}$$

where P_b is the burst pressure, and σ_{uts} is the UTS of the pipe steel. Zhu and Leis [12] showed that the Barlow formula with the MD predicts an adequate burst pressure for thin-walled pipes made of steels having low strain hardening exponent ($n < 0.1$).

In order to improve the accuracy of the Barlow strength for thin-walled pipes, Zhu and Leis [18, 19] considered the plastic flow rate effect on the burst pressure in terms of the flow theory of plasticity within the large deformation framework. For the thin-walled shell theory, stresses are assumed constant in pipes, and thus MD is used. For a power-law hardening material, three flow solutions of burst pressure were obtained for thin-walled pipes with regard to the Tresca, von Mises and Zhu-Leis criteria:

$$P_{Tb} = \left(\frac{1}{2}\right)^{n+1}\frac{4t}{D}\sigma_{uts} \tag{3}$$

$$P_{Mb} = \left(\frac{1}{\sqrt{3}}\right)^{n+1}\frac{4t}{D}\sigma_{uts} \tag{4}$$

$$P_{Ab} = \left(\frac{2+\sqrt{3}}{4\sqrt{3}}\right)^{n+1}\frac{4t}{D}\sigma_{uts} \tag{5}$$

where the strain hardening exponent n is measured from a simple tensile test or estimated from the YS and UTS. Zhu and Leis [29] provided two simple equations to estimate n from the YS defined at the 0.2% offset strain or the 0.5% total strain. For pipeline carbon steels, n has a typical value ranging from 0.02 to 0.25. Zhu and Leis [12] evaluated twenty-one existing burst prediction models using a large burst test dataset for a wide range of pipeline steels, covering Grade A to X120, and concluded that the Zhu-Leis flow solution in Eq. (5) with the MD predicts the most accurate burst pressure for thin-walled pipes.

2.2 Representative Burst pressure models for thick-walled cylindrical vessels

Historically, many valuable burst tests were carried out for small-diameter, thick-walled tubes or pipes ($D/t < 20$) in various structural steels by many investigators, such as Faupel [24, 25] in 1956, Crossland and Bones [30] in 1958, and Martin and Weng [31] in 1963. Based on the burst test data, different burst pressure models were developed analytically or empirically for thick-walled tubes or PVs. Among them, four often used burst prediction models are selected and discussed next.

- Turner burst pressure model that was proposed by Turner [32] in 1910:

  $$P_b = \sigma_{uts}\ln\left(\frac{D_o}{D_i}\right) \tag{6}$$

  This burst pressure solution is an outcome from the Tresca strength theory in terms of UTS.

- Nadai burst pressure model that was proposed by Nadai [33] in 1931:

  $$P_b = \frac{2}{\sqrt{3}}\sigma_{uts}\ln\left(\frac{D_o}{D_i}\right) \tag{7}$$

  This burst pressure solution is an outcome from the von Mises strength theory in terms of UTS.

- Faupel burst pressure model that was proposed by Faupel [25] in 1956:

  $$P_b = \frac{2}{\sqrt{3}}\sigma_{ys}\left(2 - \frac{\sigma_{ys}}{\sigma_{uts}}\right)\ln\left(\frac{D_o}{D_i}\right) \tag{8}$$

  This burst pressure model is an empirical outcome from the von Mises strength theory in terms of YS and UTS. It was assumed that the burst pressure is proportional to the UTS pressure at a factor of YS/UTS and the yield pressure at another factor of (1-YS/UTS).

- Svensson burst pressure model that was proposed by Svensson [17] in 1958:

$$P_b = \left(\frac{0.25}{n+0.227}\right)\left(\frac{e}{n}\right)^n \sigma_{uts} \ln\left(\frac{D_o}{D_i}\right) \quad (9)$$

where e ≈ 2.71828 is the base of exponential functions. This burst pressure model is an approximate solution from the von Mises flow theory of plasticity that considered the ultimate strength UTS and the plastic flow rate of pressure vessel steels.

Many investigators have showed that the Svensson burst pressure solution agreed fairly well with burst test data for thick-walled vessels in different steels, and thus the Svensson model has obtained wide applications in the PV design [5, 9, 26, 27]. For thick-walled PVs, however, exact solutions of burst pressure that correspond to the burst pressure solutions in Eqs. (3) to (5) for thin-walled PVs have not been developed yet to date. This promotes the present work to adapt the strength theory and to determine more accurate burst pressure solutions for thick-walled PVs.

3. Strength theory and burst pressure prediction of steel vessels

Classical strength theory addresses the plastic yield and ultimate failure of materials in a complex stress state. A comprehensive technical review of strength theories was presented by Yu [34] for many materials under complex stress states. Christensen [35] discussed a comprehensive theory of yield and failure for isotropic materials. In general, two basic strength failure modes were assumed for the metallic PV design: elastic failure and plastic failure. The former is governed by the theory of elasticity, and it is suitable for brittle metals. The latter is governed by the theory of plasticity, and it is suitable for ductile metals. When a ductile material is stretched beyond the elastic limit strength, plastic deformation occurs, and plastic instability of the material may happen eventually. Material properties relevant to the PV failure are the YS and UTS. Due to the multiaxial stress state, the PV failure is controlled by a combination of all stress components, but not by a single stress component in the PV. A strength theory has been formulated in terms of the three principal stresses (σ_1, σ_2, σ_3) and can predict failure for any given stress state in an isotropic material. Three classical strength theories have been often used in the PV or pipeline design.

3.1 *Classical strength theories*

3.1.1 *The maximum principal stress theory (Rankine theory)*

This strength theory was proposed by Rankine [36] and used to describe elastic failure for brittle materials, such as cast irons at the elastic limit strength. It is assumed that rupture will occur when the maximum principal stress (σ_1) in a solid reaches the UTS (or σ_{uts}) of the material determined from the uniaxial testing (this failure criterion will not be used in this work).

$$\sigma_1 = \sigma_{uts} \quad (10)$$

3.1.2 *The maximum shear stress theory (Tresca theory)*

This strength theory was proposed by Tresca [37] and used to describe failure of ductile materials, such as carbon steels at the plastic yield. It is assumed that failure will occur in a solid if the maximum shear stress (τ_{max}) in a solid reaches the shear yield strength (τ_{ys}) of the material determined from the uniaxial testing.

$$\tau_{max} = \max\left(\frac{|\sigma_1-\sigma_2|}{2}, \frac{|\sigma_2-\sigma_3|}{2}, \frac{|\sigma_3-\sigma_1|}{2}\right) = \tau_{ys} \quad (11a)$$

or

$$\sigma_T = max(|\sigma_1 - \sigma_2|,\ |\sigma_2 - \sigma_3|,\ |\sigma_3 - \sigma_1|) = \sigma_{ys} \quad (11b)$$

where σ_T is the Tresca effective stress, σ_{ys} is the tensile yield strength of material, and $\tau_{ys} = \frac{\sigma_{ys}}{2}$.

3.1.3 *The maximum distortion energy theory (von Mises theory)*

The von Mises theory is also known as the distortion energy theory [38] and is used to characterize the plastic yield failure of ductile steels, such as carbon steels. It is assumed that failure will occur when the distortion energy per unit volume in a solid due to applied stresses reaches the distortion energy per unit volume of the material at the yield point from the uniaxial tensile testing.

$$\tau_M = \sqrt{\frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} = \tau_{ys} \quad (12)$$

or

$$\sigma_M = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} = \sigma_{ys} \quad (13)$$

where τ_M is the von Mises effective shear stress, σ_M is the von Mises effective stress, and $\tau_{ys} = \frac{\sigma_{ys}}{\sqrt{3}}$.

3.1.4 *Average shear stress theory (Zhu-Leis theory)*

In addition to three classical strength theories discussed above, Zhu [19] recently proposed an average shear stress strength theory used to predict plastic failure for ductile steels. The Zhu-Leis criterion [6] is an intermediate yield criterion that lies between the Tresca and von Mises criteria. Average shear stress (τ_A) was defined as the average of the maximum shear stress (τ_{max}) and the von Mises effective shear stress (τ_M). Accordingly, the Zhu-Leis effective stress, σ_A, can be expressed as a combined function of the Tresca effective stress σ_T and the von Mises effective stress σ_M. It is assumed that failure will occur if the average shear stress in a solid reaches the average shear strength of the material. Using the effective stress σ_A, this yield criterion is expressed as:

$$\sigma_A = \frac{1}{2+\sqrt{3}}\left[\sqrt{3}\sigma_T + 2\sigma_M\right] = \sigma_{ys} \quad (14)$$

It is well known that the yield failure defined at the plastic yield is very conservative to define ultimate burst failure for ductile steels, particularly for high strain hardening steels, where the UTS is much larger than the YS [13]. To reduce the conservatism of the classical yield strength theories, the UTS has been used to replace the YS in the Tresca, von Mises and Zhu-Leis criteria [13]. Then the UTS-based failure criterion can be used to characterize plastic collapse of ductile steel PVs. In addition, a flow stress failure criterion is often used to describe plastic collapse of line pipes in the pipeline industry.

3.2. *Strength solutions of burst pressure for thin-walled pipes*

From the UTS-based Tresca, von Mises and Zhu-Leis criteria and the flow stress criterion for ductile steels, four simple analytical solutions of burst pressure were obtained for defect-free, thin-walled pipes (D/t ≥ 20) [13] as:

(1) Tresca strength solution of burst pressure (Barlow formula, Eq. (2))

$$P_0 = \frac{2t}{D}\sigma_{uts} \quad (15)$$

(2) von Mises strength solution of burst pressure

$$P_{M0} = \frac{4t}{\sqrt{3}D}\sigma_{uts} \quad (16)$$

(3) Zhu-Leis strength solution of burst pressure

$$P_{A0} = \left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)\frac{t}{D}\sigma_{uts} \quad (17)$$

(4) Flow stress-based failure pressure

$$P_f = \frac{2t}{D}\sigma_{flow} \quad (18)$$

where D is the pipe MD (or conservatively OD), t is the wall thickness, and σ_{flow} is the flow stress that was initially defined as $\sigma_{flow} = \sigma_{ys} + 10\ ksi$ [39] for pipeline steels and later changed to be an averaged material strength $\sigma_{flow} = \frac{1}{2}(\sigma_{ys} + \sigma_{uts})$ [29]. If $\sigma_{flow} = \sigma_{uts}$, Eq. (18) reduces to Eq. (15) that has been often used to determine the plastic failure pressure or the minimum required wall thickness in pipeline design.

Figure 1 compares the just-noted four strength solutions of burst pressure for thin-walled pipes with the full-scale burst test data that were collected by Zhu and Leis [19] from open literature for thin-walled pipes, where all burst pressures are normalized by the Barlow strength in Eq. (2) with use of MD. Also included is the linear curve fit of the test data. This full-scale test dataset covers a wide range of pipeline steels from Grade A to X120 as well as oil-well casing tube steels H40 to Q125 used in offshore exploration. Note that the reference numbers marked in Fig. 1 were given by Zhu and Leis [19]. From this figure, the following observations are made:

1) The three UTS-based strength solutions do not consider the strain hardening rate, while the flow stress-based burst pressure solution does consider the plastic flow effect.
2) The von Mises strength solution is an absolute upper bound prediction that is higher than all test data.
3) The Zhu-Leis strength solution determines a near upper bound that is slightly lower than the upper bound of the test data.
4) The Tresca strength solution is an overall averaged prediction of all test data. It is lower than the linear curve fit of the test data when $n < 0.1$, and higher than the linear curve fit when $n > 0.1$.
5) The flow stress failure criterion determines a lower bound prediction of test data and thus serves as a conservative criterion. It has been widely applied in pipeline design and integrity management.

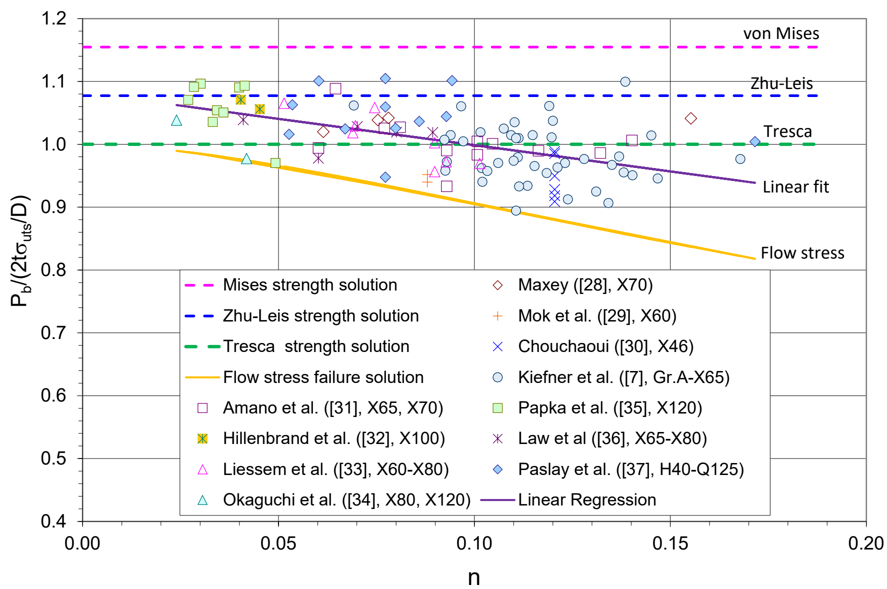


Figure 1. Comparison of four strength solutions of burst pressure with full-scale burst data

In summary, the UTS-based strength theories do not consider the plastic flow response effect and thus cannot accurately predict burst pressure. In contrast, the flow stress failure criterion does consider the plastic flow effect and determines a lower bound prediction of burst pressure for thin-walled pipes.

3.3. *Plastic flow solutions of burst pressure for thin-walled pipes*

As discussed above, the UTS-based strength theories provide only a single-parameter strength solution of burst pressure as a function of D/t and UTS, as evident in Eqs. (15) to (17) in reference to the Tresca, von Mises and Zhu-Leis strength theories. On the other hand, the flow stress solution in Eq. (18) describes the effect of the UTS and plastic flow response in terms of the YS/UTS ratio on the burst pressure in a conservative, lower bound manner. In order to obtain a more accurate burst pressure prediction, Zhu and Leis [19] employed the flow theory of plasticity and the large deformation theory to consider the plastic flow effect. For a power-law hardening material, they obtained three flow solutions of burst pressure for thin-walled pipes (D/t ≥ 20) with regard to the Tresca, von Mises, and Zhu-Leis criteria as given in Eqs (3) to (5).

It is noted that both the strength and flow solutions of burst pressure discussed above were obtained in terms of the thin-wall shell theory, with the assumption that the axial and hoop stresses are uniformly distributed through the cross-sections of a cylinder. Accordingly, the mean diameter or MD should be used generally. The full-scale test data shown in Fig. 1 are utilized again for validating the three flow solutions in Eqs (3) to (5). Figure 2 compares these flow solutions with the full-scale burst test data, where both predicted and measured burst pressures are normalized using the MD-based Barlow strength. The normalized flow stress solution is also included in this figure for comparison. It is observed from this figure that:

1) All three flow solutions of burst pressure consider the plastic flow effect and are functions of D/t, UTS, and n.
2) The von Mises flow solution is an upper bound prediction of the burst test data.
3) The Tresca flow solution is a lower bound prediction of the burst test data. For low strain hardening steels with n < 0.1, the Tresca flow solution and the flow stress solution are comparable. For high strain hardening steels with n > 0.1, the Tresca flow solution gradually deviates from the flow stress solution, and becomes less conservative.
4) The Zhu-Leis flow solution is comparable to the linear curve fit result, and thus is the best prediction of the burst test data on average.

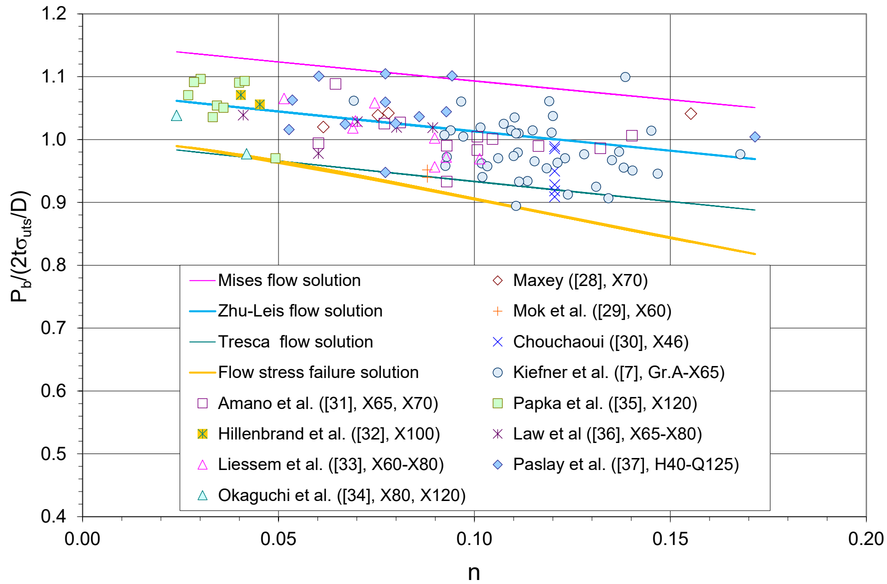


Figure 2. Comparison of the four flow solutions of burst pressure with full-scale test data

3.4. *Two-parameter flow stress*

Because the three flow solutions of burst pressure are able to describe the plastic flow effect, the corresponding flow stresses are thus defined from these flow solutions. A general expression of the effective stress in a thin-walled pipe for the three yield criteria was obtained as [18, 19]:

$$\sigma_{eff} = \frac{1}{K}\frac{PD}{2t} \tag{19}$$

where K is a yield criterion-dependent constant:

$$K = \begin{cases} 1, & \text{for Tresca yield criterion} \\ \frac{2}{\sqrt{3}}, & \text{for von Mises yield criterion} \\ \frac{1}{2}+\frac{1}{\sqrt{3}}, & \text{for Zhu-Leis yield criterion} \end{cases} \tag{20}$$

Using the burst solutions in Eqs (3) to (5), the critical stress at the burst failure is determined for the three yield criteria as:

$$\sigma_c = \left(\frac{K}{2}\right)^n \sigma_{uts} \tag{21}$$

where σ_c is a new flow stress in a function of both UTS and n. For the perfectly plastic material, $n \to 0$, and $\sigma_c = \sigma_{uts}$. As such, the physical meaning of Eq. (21) is that the proposed flow stress σ_c is a yield criterion dependent material strength.

3.5. ***New strength theories***

With the new flow stress defined in Eq. (21), the power-law hardening material can be approximated as an elastic-perfectly plastic material. Accordingly, a material will remain in the elastic condition if the effective stress in the solid is less than the flow stress. Otherwise, the strength theory assumes that "plastic yielding" or ultimate failure will occur if the effective stress in the solid reaches the new flow stress. Therefore, using the new flow stresses, the traditional one-parameter strength theories can be modified as two-parameter strength theories in the following formats.

(1) Tresca two-parameter strength criterion

$$\sigma_T = \sigma_T^c = \left(\frac{1}{2}\right)^n \sigma_{uts} \tag{22}$$

(2) von Mises two-parameter strength criterion

$$\sigma_M = \sigma_M^c = \left(\frac{1}{\sqrt{3}}\right)^n \sigma_{uts} \tag{23}$$

(3) Zhu-Leis two-parameter strength criterion

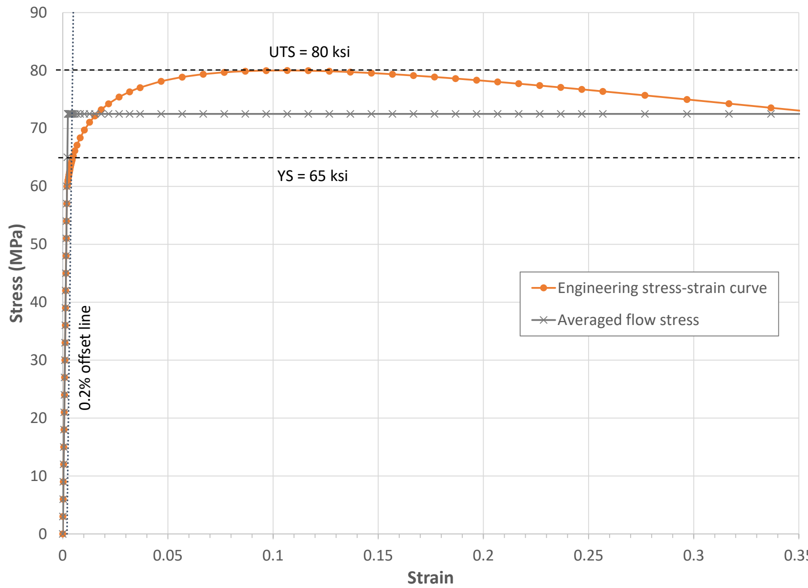
$$\sigma_A = \sigma_A^c = \left(\frac{2+\sqrt{3}}{4\sqrt{3}}\right)^n \sigma_{uts} \tag{24}$$

If the strain hardening effect is negligibly small or n = 0, Eqs. (22), (23), and (24) reduce to Eqs (15), (16), and (17), respectively for the Tresca, von Mises and Zhu-Leis strength theories. As a result, the traditional strength theory becomes a special case of the newly proposed strength theory for each yield criterion.

To illustrate the differences between the traditional flow stress and the newly defined flow stresses, Figure 3 shows an example of an engineering stress-strain curve for pipeline steel grade X65, where the material properties are YS = 65 ksi (448 MPa), UTS = 80 ksi (552 Mpa), and the 0.2% offset line are included. In particular, Figure 3(a) shows the full range of the engineering stress-strain curve and the conventional averaged flow stress, and Figure 3(b) shows a partial range of the engineering stress-strain curve and the three newly defined two-parameter flow stresses: Tresca flow stress, von Mises flow stress, and Zhu-Leis flow stress for X65 pipeline steel. As shown in Fig. 3, for the X65 pipeline steel, the Tresca flow stress is $\sigma_T^c = 74.658$ ksi (514.767 Mpa), the von Mises flow stress is $\sigma_M^c = 75.735$ ksi (522.193 Mpa), the Zhu-Leis flow stress is $\sigma_A^c = 75.213$ ksi (518.594 Mpa), and the averaged flow stress is $\sigma_{flow} = 72.501$ ksi (499.894 Mpa). Note that the averaged value of the Tresca and von Mises flow stresses is 75.196 ksi (518.480 MPa), which is nearly identical to the Zhu-Leis flow stress of 75.213 ksi (518.594), demonstrating that the Zhu-Leis flow stress is an averaged result of the Tresca and von Mises stresses.

Each of the four flow stresses defines a nominal material strength of the material, and then the power-law stress-strain curve of the material is approximated as an elastic-perfectly plastic stress-strain curve. Accordingly, when the effective stress in a solid is less than the flow stress, the material deformation is treated as elastic, and the solid is safe. However, when the effective stress equals to the flow stress, the material deformation is treated as plastic, and "plastic yielding" or burst failure occurs in the solid. Figure 3(b) shows that the newly proposed Tresca, von Mises and Zhu-Leis flow stresses are comparable to each other and lie between the conventional flow stress and the UTS of the X65 pipeline steel. Accordingly, it is expected that the three new flow stress criteria will predict higher and thus less conservative burst pressure values in comparison to that predicted by the conventional averaged flow stress criterion.

(a)



(b)

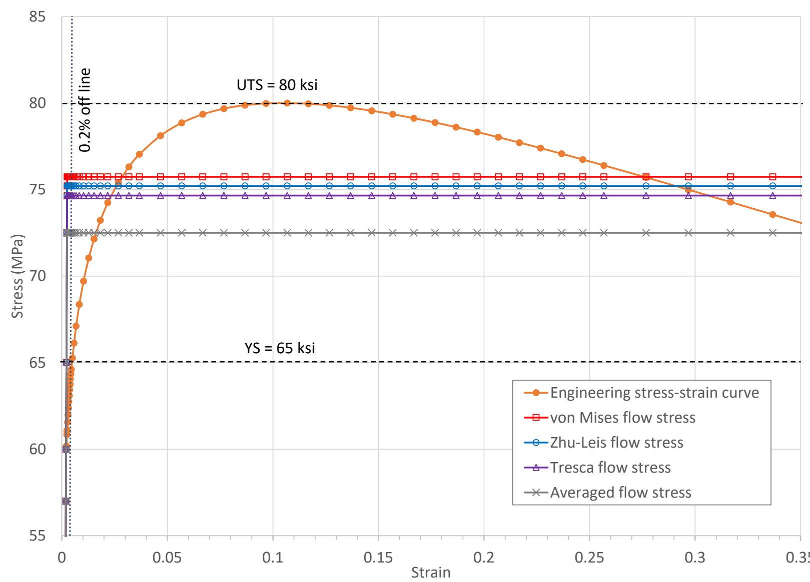


Figure 3. Engineering stress-strain curve and flow stresses for pipeline steel X65: (a) full-range stress-strain curve with average flow stress, and (b) partial range stress-strain curve with three newly defined flow stress

4. Determination of burst pressure for thick-walled pressure vessels

Many elastic or elastic-plastic stress analyses, or the limit load analyses are publicly available for thick-walled cylinders under internal pressure, such as those by Ayob et al. [40] and Barashkov and Shevchenko [41]. As a result, the detailed stress and strain analyses of an end-capped, thick-walled cylinder will not be presented in this work. While hoop and axial stresses in a thin-walled cylinder ($D/t \geq 20$) are considered as constant through the wall thickness, these stresses in a thick-walled cylinder ($D/t < 20$) are not constant and vary through the wall thickness.

To derive a thick-walled burst pressure solution, consider an infinitesimally small stress element in a pressurized cylinder using a polar coordinate system (r, θ), as illustrated in Figure 4. In this figure $\sigma_{\theta\theta}$ is the hoop stress of the cylinder, σ_{rr} is the radial stress, $d\sigma_{rr}$ is the differential radial stress, dr is an infinitesimally small increment of the circular radius, and $d\theta$ is an infinitesimally small angular increment. From the force equilibrium along the radial direction of the stress element, the following stress equilibrium equation is obtained:

$$\sigma_{\theta\theta} - \sigma_{rr} - r\frac{d\sigma_{rr}}{dr} = 0 \tag{25}$$

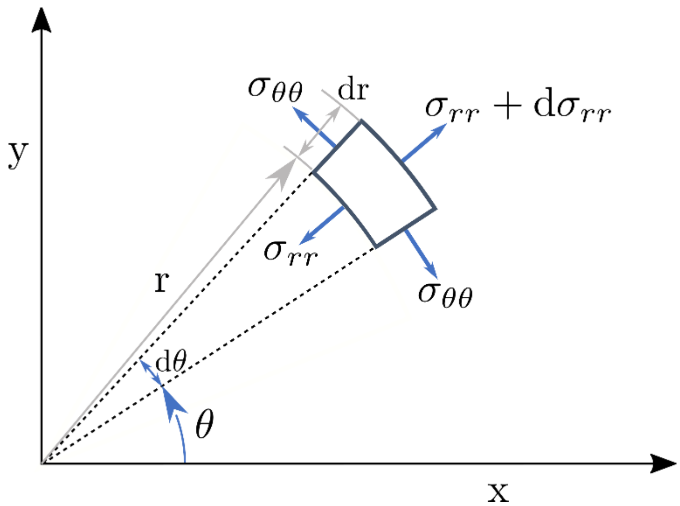


Figure 4. Scheme of differential stress element in a cylinder

For an elastic-perfectly plastic material, the entire cross-section of the cylinder wall will plastically yield at the limit load. In this case, using the proposed new strength theories, the following general yield condition is obtained in terms of Tresca, von Mises or Zhu-Leis criterion:

$$\sigma_{eff} = \frac{1}{K}(\sigma_{\theta\theta} - \sigma_{rr}) = \sigma_c \tag{26}$$

Substitution of Eq. (26) into Eq. (25) determines:

$$\frac{d\sigma_{rr}}{dr} = \frac{K\sigma_c}{r} \tag{27}$$

Integration of Eq. (27) obtains:

$$\sigma_{rr} = K\sigma_c \ln(r) + C \tag{28}$$

where C is an integration constant to be determined by the following stress boundary conditions:

$$\sigma_{rr}(r = R_i) = -P$$
$$\sigma_{rr}(r = R_o) = 0 \tag{29}$$

in which R_i is the pipe inside radius, and R_o is the pipe outside radius. From Eqs. (28) and (29), the burst pressure of the thick-walled cylinder at the "plastic yield" is determined as:

$$P_b = 2\left(\frac{K}{2}\right)^{n+1} \sigma_{uts} ln\left(\frac{D_0}{D_i}\right) \tag{30}$$

Particularly, from Eq. (30), the burst pressure solution of a thick-walled cylinder for the Tresca, von Mises and Zhu-Leis criterion is expressed separately as:

$$P_{Tb} = 2\left(\frac{1}{2}\right)^{n+1} \sigma_{uts} ln\left(\frac{D_0}{D_i}\right) \tag{31}$$

$$P_{Mb} = 2\left(\frac{1}{\sqrt{3}}\right)^{n+1} \sigma_{uts} ln\left(\frac{D_0}{D_i}\right) \tag{32}$$

$$P_{Ab} = 2\left(\frac{2+\sqrt{3}}{4\sqrt{3}}\right)^{n+1} \sigma_{uts} ln\left(\frac{D_0}{D_i}\right) \tag{33}$$

When the strain hardening exponent n approaches zero, the power-law material approximates an elastic-perfectly plastic material, and the three flow solutions of burst pressure in Eqs. (31) to (33) become the corresponding UTS-based strength solutions for thick-walled PVs. In particular, Eq. (31) reduces to Eq. (6), and Eq. (32) reduces to Eq. (7). As a result, the Turner burst model is identical to the Tresca strength solution, and the Nadai burst model is identical to the von Mises strength solution for thick-walled PVs.

In general, the flow solutions of burst pressure in Eqs. (31) to (33) are applicable to both thick and thin-walled PVs. For a thin-walled cylinder, the $\frac{D_0}{D_i}$ ratio approaches 1, and the logarithmic function $\ln(D_o/D_i)$ can be approximated by the following Taylor series using the cylinder OD or MD:

$$\ln(D_o/D_i) = 2t/D_o + 0.5(2t/D_o)^2 + O(2t/D_o)^3 \tag{34}$$

or

$$\ln(D_o/D_i) = 2t/D_m + O(t/D_m)^3 \tag{35}$$

where $O(2t/D_o)^3$ and $O(t/D_m)^3$ are the higher-order terms of a polynomial with the degree of 3 and higher. Comparison of Eq. (34) with Eq. (35) shows that for thin-walled pipes, $2t/D_m$ is more accurate than $2t/D_o$ when used to approximate the logarithmic function of $ln(D_o/D_i)$. This answers why the MD should be used in all thin-walled burst pressure models. Figure 5 compares the four geometry terms of $ln(D_o/D_i)$, $2t/D_m$, $2t/D_o$, and $2t/D_i$ for a wide range of the D_o / D_i ratios. At the engineering definition of thin-walled pipes with the minimum D/t ratio of 20, the thin-walled geometry terms of $2t/D_m$, $2t/D_o$, and $2t/D_i$ have an error of -0.1%, -5.1% and +5.5%, respectively compared to the thick-walled geometry term. In general, 2t / D_i overestimates the logarithmic function, 2t / D_o underestimates the logarithmic function, and 2t / D_m is close to the logarithmic function over the wide range of D_o / D_i ratios. At D/t = 7, 2t / D_m has an error less than -1% compared to the logarithmic function. As a result, the OD-based Barlow burst strength is commonly used in the pipeline industry because of its conservative prediction of burst strength.

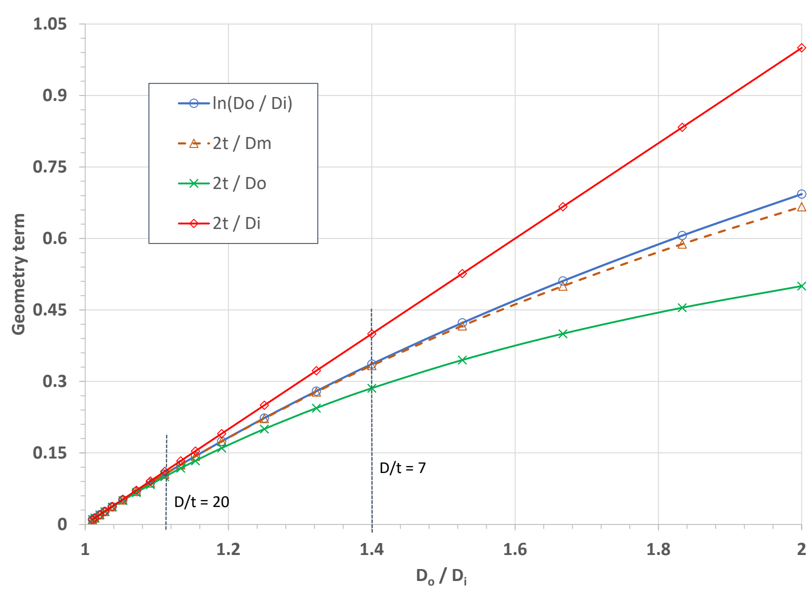


Figure 5. Comparison of four geometry terms of $\ln(D_o/D_i)$, 2t / D_m, 2t / D_o, and 2t / D_i

Using the first-order approximation of the logarithmic function in Eq. (34) or (35), Eqs. (31) - (33) for thick walls reduce to Eqs. (3) - (5) for thin walls. In general, for thin-walled line-pipes ($D_o/t \geq 20$), the error from the second term in Eq. (34) is less than 5.1% of the first term. Therefore, for thin-walled pipes, the burst pressure solutions in Eqs. (3) - (5) with use of OD have an error less than 5.1% in comparison to the burst pressure solutions in Eqs. (31) - (33). In contrast, the burst pressure solutions in

Eqs. (3) – (5) with use of the MD have a comparable accuracy to the burst pressure solutions in Eqs. (31) – (33), and their differences are very small (< 0.1%) for thin-walled PVs ($D/t \geq 20$), as shown in Fig. 5.

5. Experimental validation of burst pressure solutions

This section aims to evaluate the four representative burst pressure models in Eq. (6) to Eq. (9) and the proposed burst pressure solutions in Eqs (31) to (33) with regard to the newly proposed Tresca, von Mises and Zhu-Leis strength theories for thick-walled PVs. Two large datasets of burst pressure tests are employed here for evaluating these burst prediction models for thick-walled cylindrical PVs. One dataset contains burst test data for small diameter, thick-walled tubes, and the other dataset contains burst test data for large diameter, thin-walled line pipes.

5.1 Validation with thick-walled burst tests

5.1.1 Experimental burst dataset for thick-walled tubes (D/t<5)

Faupel [25] conducted nearly one hundred burst pressure tests over seven years and in 1956 reported the burst pressure test data for small diameter, thick-walled pressure tubes that were constructed of various metals, including plain carbon steels, stainless steels, gun steels, low alloy steels, weld steels, aluminum, and bronze. From these burst tests, thirty burst test data for plain carbon steels were selected and used in this work for evaluating the burst prediction models for thick-walled tubes, pipes or cylinders. Note that the test data of burst pressure obtained by Faupel [25] were also reported by Christopher et al. [9] in their Table 5 for various steels.

Figure 6 shows the thirty burst test data that were obtained for small diameter, very thick pressure tubes with small D/t ratios in the range of $2.4 < D/t < 4.7$. The tube materials for these 30 burst tests were low, medium, or high carbon steels designated as AISI 1025, AISI 1030, AISI 3130, AISI 3320, AISI 4130, AISI 4140, and AISI 4340. These carbon steels have the yield strengths in the range of 244 to 1076 MPa and the tensile strengths in the range of 459 to 1119 MPa. The strain hardening exponent n of the tube materials is less than 0.25.

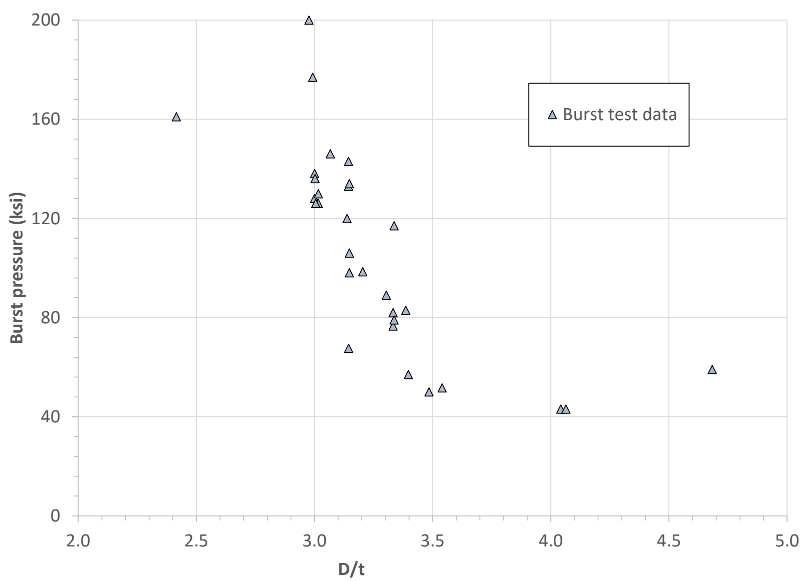


Figure 6. Thirty burst test data for thick-walled pressure tubes with $2.4 < D/t < 4.7$

5.1.2 Burst pressure versus strain hardening exponent

Figure 7 compares the burst pressure predictions with measured burst pressure data that were obtained by Faupel [25], as shown in Fig. 6 for thick-walled pressure tubes, where the y-axis represents the burst pressure normalized by the Tresca strength solution, $P_b = \sigma_{uts} \ln(D_o/D_i)$, for thick-walled PVs, and the x-axis denotes the material strain hardening exponent n. The burst pressure predictions were obtained from four representative models for thick-walled PVs, that is: the Turner model (i.e., Tresca strength solution) in Eq. (6); the Nadai model (i.e., von Mises strength solution) in Eq. (7); the Faupel empirical solution in Eq. (8); the Svensson approximate solution in Eq. (9); and the three newly proposed flow solutions including the Tresca flow solution in Eq. (31), the von Mises flow solution in Eq. (32), and the Zhu-Leis flow solution in Eq. (33). From Fig. 7, the following observations are obtained:

1) The von Mises strength solution is independent of n and provides an absolute upper bound prediction of the burst data.
2) The Tresca strength solution is independent of n but provides a good burst pressure prediction, particularly for $n < 0.1$.
3) The von Mises flow solution is an upper bound prediction of the burst data for all n values.
4) The Tresca flow solution is a lower bound prediction of the burst data for all n values.
5) The Faupel empirical model provides a comparable result to the Mises strength solution for low hardening strain materials with $n < 0.06$, and then its prediction decreases with n in a nearly linear trend. For most ductile steels with $n < 0.15$, the Faupel solution varies around the von Mises flow solution, and thus it overestimates the burst data. For high strain hardening steels with $0.15 < n < 0.22$, the Faupel solution seems to give an adequate prediction of burst data.
6) The Svensson approximate model predicts outcomes that are smaller than and nearly parallel to the Mises flow solutions for all n values. However, the Svensson predictions still overestimate most burst data.
7) The Zhu-Leis flow solution agrees the averaged burst data for all n values, and thus is the best prediction model on average.

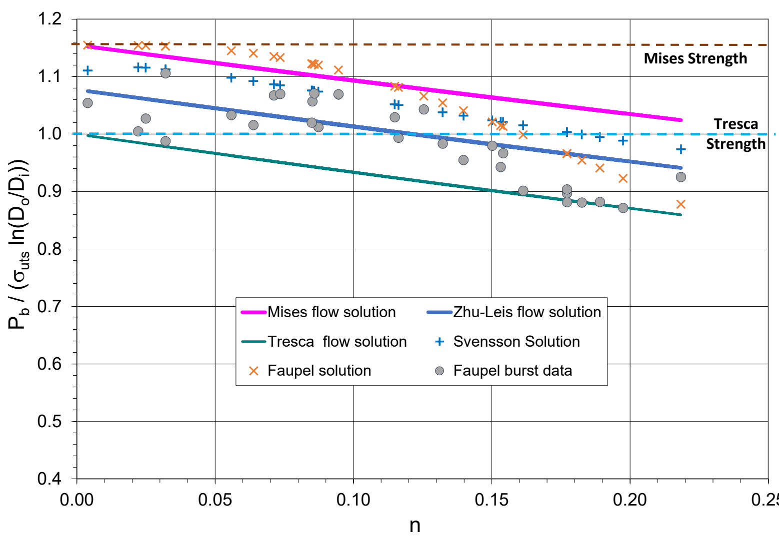


Figure 7. Comparison of burst pressure predictions with burst test data for thick-walled tubes as a function of n

5.1.3 Burst pressure versus tube D/t ratio (D/t < 5)

Figure 8 replots all predictions and test data shown in Fig. 7 as a function of D/t instead of n and compares the burst pressure predictions with measured burst pressure data for thick-walled pressure tubes, where the y-axis represents the burst pressure normalized by the Tresca strength solution, $P_b = \sigma_{uts} \ln(D_o/D_i)$, for thick-walled PVs. From Fig. 8, the similar observations to Fig. 7 are made.

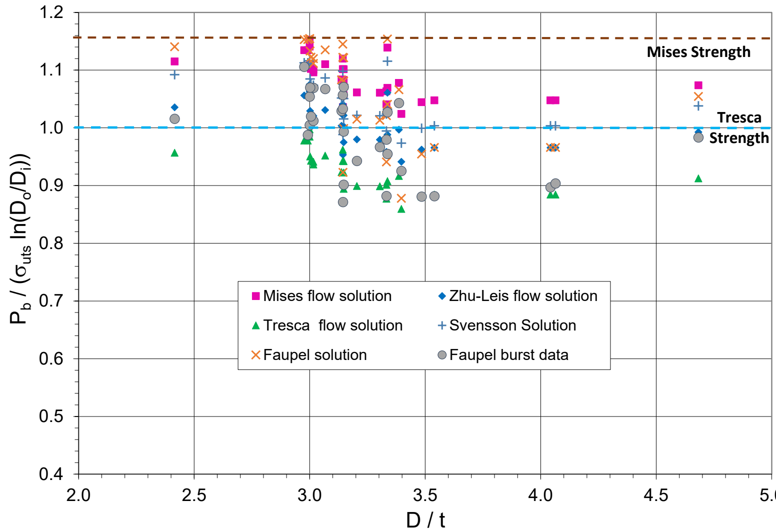


Figure 8. Comparison of burst pressure predictions with burst test data for thick-walled tubes as a function of D/t ratio

5.1.4 Direct comparison of burst pressure predictions with test data

Figure 9 shows direct comparison of burst pressure predictions with measured burst pressure data for thick-walled pressure tubes, where the prediction models include Svensson approximate model, Faupel empirical model, von Mises flow solution, Tresca flow solution and Zhu-Leis flow solution. Also included in the figure are an upper bound line (+15%) and a lower bound line (-15%) of the scatter plot. Again, from Figure 9, it is observed that 1) the von Mises flow solution, Svensson model and Faupel model predict comparable predictions but overestimate the burst data; 2) the Tresca flow solution underestimates the burst data; 3) the Zhu-Leis flow solution provides the best predictions that match well with the burst data on average.

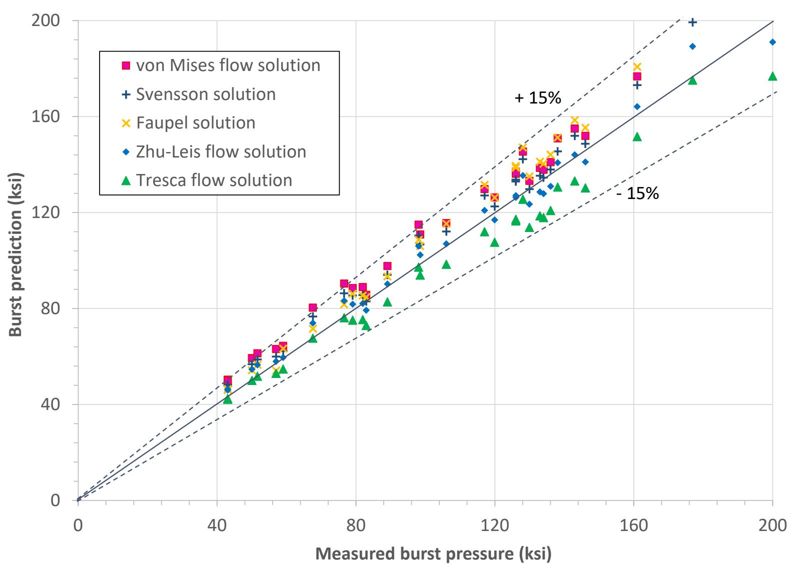


Figure 9. Direct comparison of burst pressure predictions with burst test data for thick-walled tubes

From the above model evaluations, it is concluded that for thick-walled PVs, 1) The Nadai model (i.e., von Mises strength solution), Faupel model, and Svensson model generally overpredict the burst data; 2) The Turner model (i.e., Tresca strength solution) determines overall good predictions of all burst data; 3) The Mises solution is an upper bound prediction; 4) The Tresca solution is a lower bound solution, and 5) The Zhu-Leis solution predicts the closest results with the burst data on average.

5.2 Validation with thin-walled burst tests

5.2.1 Experimental burst dataset for thin-walled pipes

The full-scale burst data shown in Fig. 1 for line pipes and casing tubes are reused here to evaluate four representative models in Eqs. (6) to (9) and three flow models in Eqs. (31) to (33). Figure 9 shows the burst test data from 109 full-scale burst tests for line pipes and casing tubes in a variety of carbon steels. Among these full-scale burst tests, 97 burst tests were conducted for large diameter, thin-walled line pipes with large D/t ratios (20 < D/t < 120), and 12 burst tests were carried out for small diameter, thick-walled casing tubes with small D/t ratios (6 < D/t ≤ 20). As a result, this large dataset contains burst data for thin and thick-walled PVs with a large range of D/t ratios (6 < D/t < 120). Accordingly, this dataset is adequate for assessing the thin-wall flow solutions of burst pressure in Eqs. (3) to (5) and the thick-wall flow solutions of burst pressure in Eqs (31) to (32).

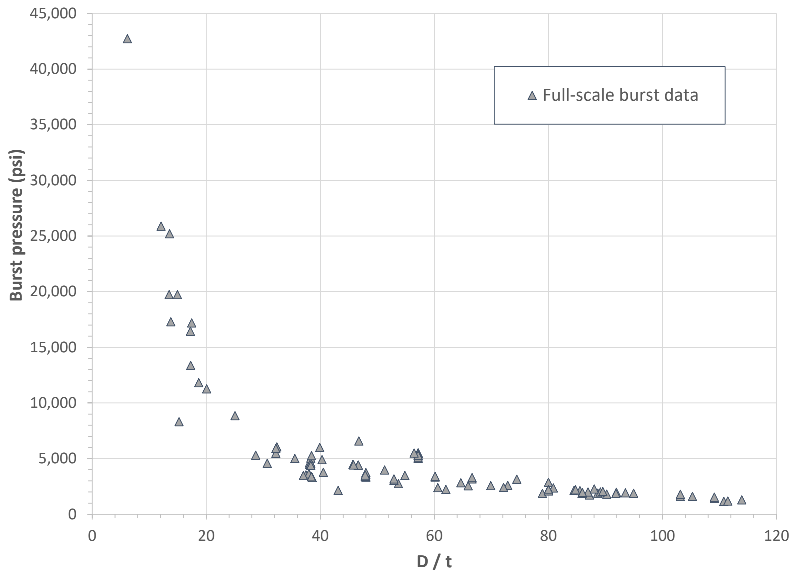


Figure 10. Full-scale burst test data for thin-wall line pipes

5.2.2 Burst pressure versus strain hardening exponent

Figure 11 compares the three theoretical predictions with the measured burst pressures as a function of n, where all test data of burst pressures are normalized by three reference strengths P_0: the OD-Barlow strength, $P_0 = 2\sigma_{uts}\, t/OD$, the MD-Barlow strength, $P_0 = 2\sigma_{uts}\, t/MD$, and the thick-wall Tresca strength $P_0 = \sigma_{uts} \ln(D_0/D_i)$. For thin-walled PVs, the three theoretical burst solutions are determined from Eqs. (3) to (5) and normalized by the MD-Barlow strength. For thick-walled PVs, the three theoretical burst solutions are determined from Eqs. (31) to (33) and normalized by the thick-wall Tresca strength. It is seen that all three normalized flow solutions are a function of n only and remain the same for the thin-wall and thick-wall theories. However, the normalized full-scale burst data have different responses. From Fig. 11, we observe that:

1) The burst test data normalized by the OD-Barlow strength are higher than the other two sets of normalized test data, and several test data points are much higher than the von Mises flow solution of burst pressure. This implies that the OD-based flow solutions tend to be overly conservative, particularly for thick-walled pipes.

2) The burst test data normalized by the MD-Barlow strength are comparable or nearly identical to the full-scale test data normalized by the thick-wall Tresca strength.

3) The Zhu-Leis thin-wall solution with use of the MD is comparable to the Zhu-Leis thick-wall solution.

As a result, it is concluded that the MD-based Zhu-Leis burst pressure solution is more accurate in comparison to the OD-based Zhu-Leis burst pressure solution for thin-walled pipes. This observation is consistent with the implication from Eqs. (34) and (35). Moreover, the MD-based Zhu-Leis burst solution for thin-walled pipes has a similar or comparable accuracy to the Zhu-Leis burst pressure solution for thick-walled cylinders, as shown in Fig. 5.

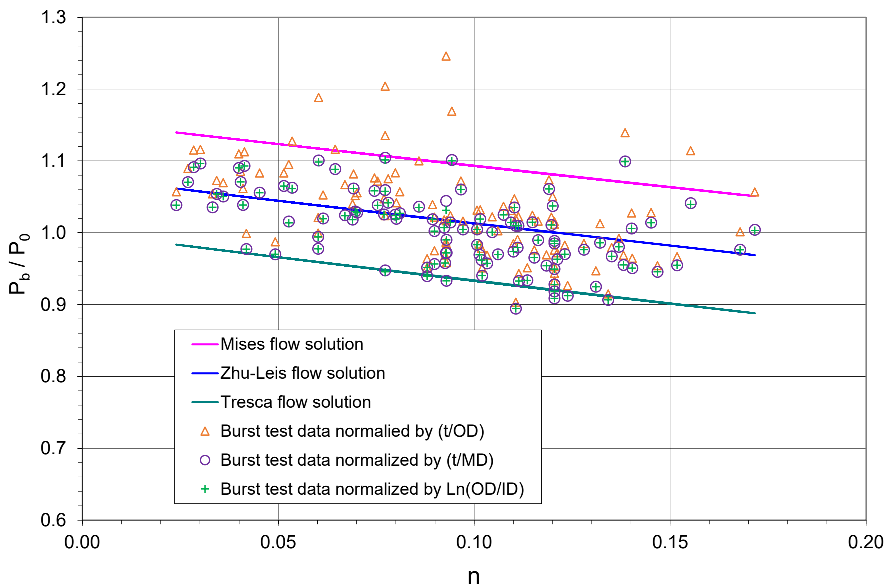


Figure 11. Comparison of predicted and measured burst pressures for thin-walled pipes as a function of n

5.2.3 Burst pressure versus the D/t ratio

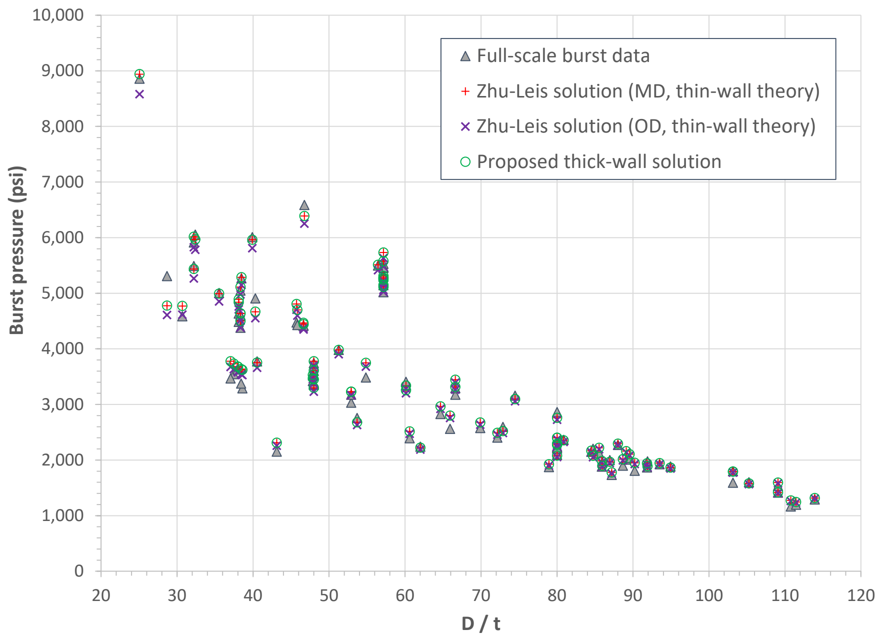
Figures 12(a) and 12(b) compares the burst pressure predictions of the Zhu-Leis flow solutions from the thin-wall and thick-wall theories with the measured burst data for a large range of D/t ratios, for thin-walled pipes ($20 < D/t < 120$) and thick-walled tubes ($5 < D/t \leq 20$), respectively. This figure reveals that:

(1) For thin-walled pipes with $20 < D/t < 120$, as shown in Fig. 12(a), all burst pressures are less than 9,000 psi (62.05 MPa). The Zhu-Leis flow solution for the thin-wall theory is nearly identical to the Zhu-Leis flow solution for the thick-wall theory.

(2) For intermediate or thick-walled vessels with $5 < D/t \leq 20$, as shown in Fig. 12 (b), all burst pressures are larger than 5,000 psi (34.47 MPa). The MD-based Zhu-Leis flow solution for thin-walled pipes is either comparable or nearly identical to the proposed Zhu-Leis flow solution for thick-walled cylinders.

(3) In contrast, the OD-based Zhu-Leis flow solutions developed for thin-walled pipes is significantly lower than the MD-based Zhu-Leis thin-wall solution and the Zhu-Leis thick-wall solution for intermediate and thick-walled cylinders with D/t < 30. Moreover, the difference increases with a decreasing D/t ratio.

(a)



(b)

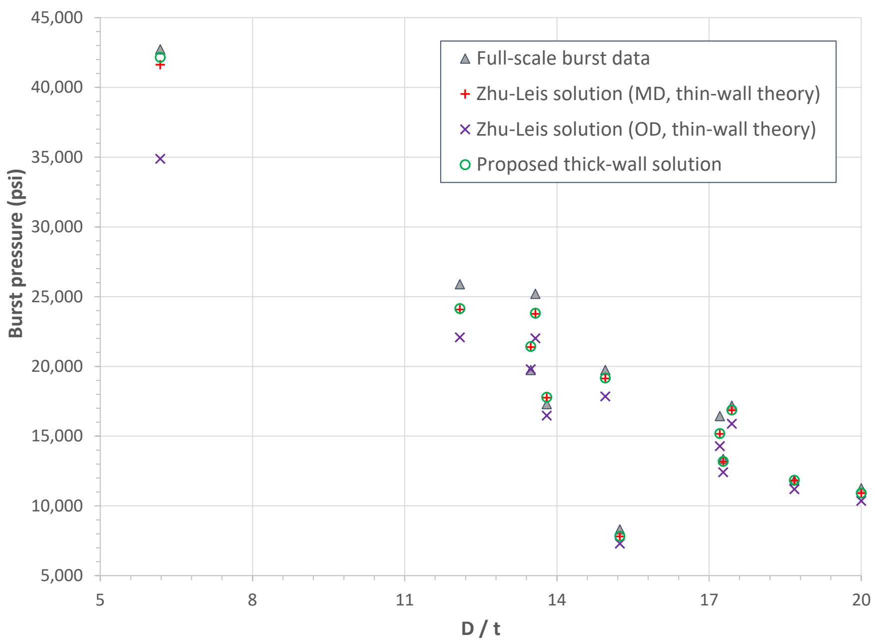


Figure 12. Comparison of predicted and measured burst pressures for line pipes and casing tubes as a function of D/t: (a) thin-walled line pipes with 20< D/t <120, and (b) thick-walled casing tubes with 5< D/t <20

5.2.4 Direct comparison of burst pressure predictions with burst data

Figure 13 directly compares the Zhu-Leis burst pressure predictions from the thin-wall and thick-wall theories with the measured burst pressure data. For thin-walled pipes, the Zhu-Leis flow solution is determined from Eq. (5) with use of the MD and OD. For thick-walled pipes, the Zhu-Leis flow solution is determined from Eq. (33). Also included in this figure are an upper bound line (+15%) and a lower bound line (-15%) of the scatter plot. From Fig. 13, the following observations are made:

1) For low burst strength when $P_b < 7{,}000$ psi (48.26 MPa), the wall thickness corresponds to thin-walled pipes with $D/t > 20$. The three Zhu-Leis burst predictions are nearly identical to the measured burst pressure data.
2) For intermediate burst strengths with a range of 7,000 psi (48.26 MPa) < P_b< 20,000 psi (137.9 MPa), the wall thickness corresponds to the intermediate-walled pipes with $13.5 < D/t < 20$. The OD-based Zhu-Leis thin-wall solution is less than the Zhu-Leis thick-wall solution, while the MD-based Zhu-Leis thin-wall solution is nearly identical to the Zhu-Leis thick-wall solution.
3) For high burst strength, when $P_b > 20{,}000$ psi (137.9 MPa), the wall thickness corresponds to thick-walled PVs with $D/t < 13.5$. The OD-based Zhu-Leis thin-wall solution is significantly less than the Zhu-Leis thick-wall solution, while the MD-based Zhu-Leis thin-wall solution remains comparable to the Zhu-Leis thick-wall solution.

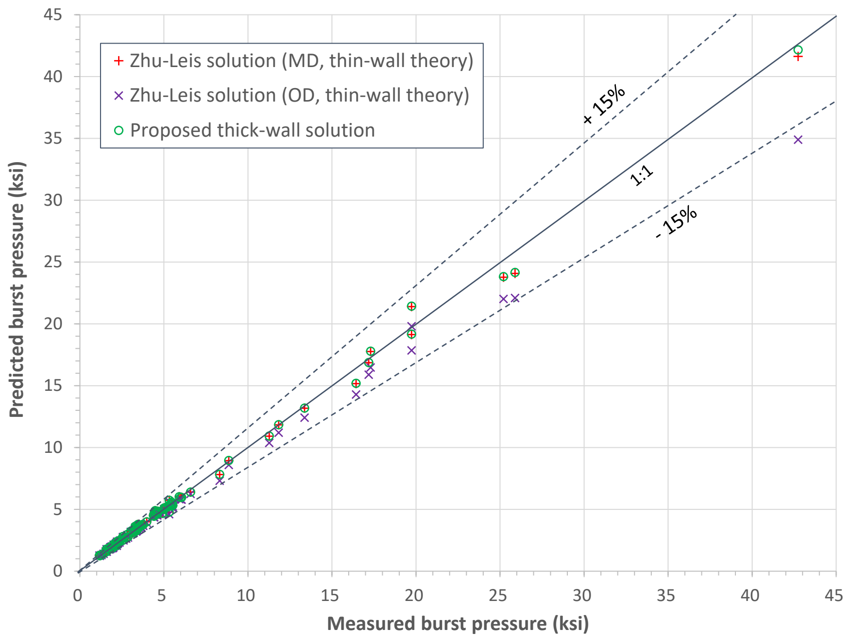


Figure 13. Comparison of predicted burst pressures by thin-wall and thick-wall theories with measured burst data

In summary, the model evaluations made above confirms that 1) the proposed Zhu-Leis flow solution for thick-walled PVs can accurately predict burst pressure for both thin and thick-walled cylinders with small to large D/t ratios; 2) the Zhu-Leis flow solution developed for thin-walled pipes with the use of the MD can predict accurate burst pressure for most D/t ratios, and its burst predictions are comparable to those from the Zhu-Leis flow solution for thick-walled PVs; and 3) the Zhu-Leis flow solution for thin-walled pipes with use of OD is accurate only for thin-walled pipes with D/t > 30.

6. Conclusions

This paper identified four representative burst pressure models often used in the design and analysis of thick-walled PVS, that is, the Turner model, the Nadai model, the Faupel model, and the Svensson model. However, these burst models were shown to predict less accurate burst pressure for thick-walled tubes, pipes or PVs. Recently, the Zhu-Leis flow solution was developed which can predict more accurate burst pressure for large diameter, thin-walled pipes. In order to extend the Zhu-Leis thin-wall solution to thick-walled PVs, three new flow stresses were defined for describing the tensile strength and the plastic flow response of a strain hardening material. Accordingly, three new strength theories were developed in terms of the Tresca, von Mises and Zhu-Leis yield criteria, and then the associated burst pressure solutions were obtained for thick-walled cylinders. After that, two large datasets of full-scale burst tests were employed to evaluate and validate four representative thick-wall burst models and three proposed flow solutions of burst pressure for thick-walled PVs in comparison with the Zhu-Leis thin-wall solution. From these results, the following conclusions are obtained:

1) Based on the two-parameter material properties, three new flow stresses were defined in terms of the Tresca, von Mises, and Zhu-Leis yield criteria, and thus these new flow stresses are a function of UTS and n.
2) With use of the new flow stresses, three new strength theories, i.e., Tresca, von Mises and Zhu-Leis strength theories were developed for predicting the burst failure of thick-walled cylinders.
3) From the three newly proposed strength theories, three burst pressure solutions were obtained for thick-walled cylinders. This includes the von Mises flow solution that is an upper bound prediction of burst pressure for thick-walled cylinders, the Tresca flow solution that is a lower bound prediction of burst pressure for thick-walled cylinders, and the Zhu-Leis flow solution that is an intermediate prediction of burst pressure for thick-walled cylinders.
4) The Zhu-Leis flow solution of burst pressure proposed for thick-walled PVs was validated by two large datasets of full-scale burst tests, and this theoretical solution matches well with the burst test data for both thin and thick-walled vessels on average.
5) The OD-based Zhu-Leis thin-wall solution of burst pressure is accurate and comparable to the new Zhu-Leis thick-wall solution when applied to pipes with $D/t > 30$. However, when applied to pipes with $D/t<30$, particularly for $D/t < 20$, a significant difference between the two theoretical models appears.
6) The MD-based Zhu-Leis thin-wall solution of burst pressure is nearly identical to the proposed Zhu-Leis thick-wall solution for both thin and thick-walled PVs with the D/t ratio >10.
7) The Nadai, Faupel, and Svensson models overpredict burst pressure solutions that are close to the von Mises flow solution for thick-walled PVs. Thus, these burst models should not be used generally in the design or integrity assessment of PVs.
8) The Turner model (or Tresca strength solution) can adequately predict burst pressure for most thick-walled PVs, particularly for low to intermediate strain hardening materials with $n < 0.1$. Thus, the Turner model can be used for simple design or analysis of thick-walled cylinders.

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