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NEW STRENGTH THEORY AND ITS APPLICATION TO DETERMINE BURST PRESSURE OF THICK-WALL PRESSURE VESSELS

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ABSTRACT

The classical Tresca and von Mises strength theories have been utilized extensively for pressure vessel and pipeline design. For pressure vessel design, ASME B&PV code was developed using both Tresca and von Mises strength theories, where the yield strength (YS) was used for an elastic design and the ultimate tensile strength (UTS) was used for an elastic-plastic design. For pipeline design, ASME B31.3, B31G, or other codes were developed using the Tresca strength theory coupled with the YS or a flow stress. The flow stress was introduced to reduce over conservatism from the YS and avoid overestimation from the UTS for many steels.

It has been widely accepted that the burst strength of pipelines depends on the UTS and strain hardening rate, n, of the ductile steel. The average shear stress yield theory was thus developed, and the associated burst pressure solution was obtained as a function of UTS and n. Experiments showed that the Zhu-Leis solution provides a reliable prediction of the burst pressure for defect-free thin-wall pipes. In order to extend the Zhu-Leis solution to thick-wall pressure vessels, this work modified the traditional strength theories and obtained new burst pressure solutions for thick-wall pressure vessels.

Three new flow stresses were proposed to describe the tensile strength and plastic flow response for a strain hardening material. The associated strength theories were then developed in terms of the Tresca, von Mises and Zhu-Leis yield criteria. From these new strength theories, three burst pressure solutions were obtained for thick-wall cylinders, where the von Mises solution is an upper bound prediction, Tresca solution is a lower bound prediction, and the Zhu-Leis solution is an averaged prediction of burst pressure for thick-wall vessels. Subsequently, the proposed burst solutions were validated by a large dataset of full-scale burst tests.

KEYWORDS: Burst strength, strength theory, pressure vessel, pipeline, Tresca, von Mises, Zhu-Leis criterion

1. INTRODUCTION

Pressure vessels (PVs) and pipelines are critical national infrastructure and are widely utilized in storage or transportation of large volumes of fluids in the energy, petrochemical, and oil and gas industry. Facilities such as nuclear reactors, chemical reactors, plant piping, storage tanks, and transmission pipelines are dependent on these systems for safe and efficient operation. The fluids can be liquids or gases, including crude oil, natural gas, liquified gas, gasoline, hydrogen, carbon dioxide, and other petrochemical or refined products. Many of these fluids are flammable, toxic, corrosive, or explosive. These risk factors, coupled with high internal pressure, make the PVs potentially very dangerous to human life and the environment. Thus, the structural design, manufacture, construction, operation, and integrity management of PVs and pipelines are very important.

In the PV or pipeline industry, structural design and integrity management require the yield or failure strength of metallic PVs in consideration. The failure strength is characterized by the burst pressure and is either measured or predicted using strength theories, empirical formulae, or industrial design codes. While the American Society of Mechanical Engineers (ASME) Boiler and Pressure Vessel (B&PV) Code [1] is often used in the PV industry, ASME B31.3 Code [2] is frequently employed in the pipeline industry. The basic design models developed in both ASME B&PV and B31.3 Codes were based on the Barlow formula [3]. In addition to the traditional strength design methods, there are several plastic flow criteria and many other analytical, empirical, experimental, or numerical methods that have been developed. Christopher et al. [4] reviewed the burst pressure prediction models for thick-wall PVs. Law and Bowie [5] discussed the burst pressure prediction for high-strength line pipes or thin-wall cylindrical PVs. Zhu and Leis [6] evaluated a series of burst pressure prediction models for different pipeline steels in terms of strength theories and plastic flow theories. More recently, Zhu [7] performed a comparative study on the

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traditional strength criteria and the modern plastic flow criteria used in the design and analysis of PVs.

The literature reviews determined that most of the early models were developed from either a simple analysis or an empirical curve fit of experimental data for specific steels of interest, and thus no single model provided a robust, accurate prediction of the burst pressure for all ductile steels. Some models focused on a conservative lower bound prediction, while other models reflected an upper bound prediction. Many models, such as the Barlow formula [3], were obtained using the hoop stress and Tresca strength theory that requires only one material strength property, such as yield stress (YS) or ultimate tensile stress (UTS). The effect of plastic flow or strain hardening rate, n, was not considered. Steward and Klever [8] reported a set of experimental data that showed a strong effect of plastic flow response on the burst pressure of PVs for different steels in a big scatter within two bounds. The upper bound is defined by the von Mises solution of burst pressure [9, 10], while the lower bound is defined by the Tresca solution of burst pressure [8]. The averaged result of these two bound predictions provides a better fit. The von Mises and Tresca solutions were obtained from the flow theory of plasticity, and are functions of UTS, n and cylinder diameter to thickness ratio D/t.

To obtain an averaged prediction, Zhu and Leis [11, 12] developed a new multi-axial plastic yield theory that was referred to as average shear stress yield criterion, or simply as the Zhu-Leis criterion. For carbon steels, the Zhu-Leis flow solution of burst pressure agrees well with the test data on average [6, 12]. Many other investigators [13-16] also validated the Zhu-Leis solution using different full-scale burst pressure data for a wide range of pipeline steels, including steel grades from Grade A to X120. All validations demonstrated that the Zhu-Leis solution is the best burst pressure prediction for thin-wall line pipes. However, the Zhu-Leis solution was developed only for thin-wall pipes, but was not applicable to thick-wall PVs. In fact, a strict theoretical solution of burst pressure does not exist yet for thick-wall PVs due to the complexity of mathematical calculation. In the late 1950s, Svensson [10] performed a detailed theoretical analysis for determining the burst failure of thick-wall cylinders based on the von Mises flow theory, but failed to obtain an exact theoretical solution because of a complicated integration. Instead, a semi-empirical solution was proposed in a closed-form for thick-wall PVs. In spite of its approximation, the Svensson solution of burst pressure has obtained extensive applications to thick-wall PVs. Nevertheless, a more accurate solution of burst pressure for thick-wall PVs is still desired in the PV industry.

So motivated, this work attempts to modify the traditional strength theories and develop new strength theories utilizing a two-parameter flow stress associated with the Tresca, von Mises and Zhu-Leis yield criteria, where the new flow stress is proposed as a function of the UTS and n. These new strength theories are used to determine the corresponding burst pressure solutions for thick-wall cylinders as a function of D/t, UTS and n. A large dataset of full-scale burst pressure tests is employed to evaluate and validate the proposed burst pressure solutions for both thin-wall pipes and thick-wall vessels.

2. STRENGTH THEORY FOR METALLIC MATERIALS

Classical strength theory addresses plastic yield and failure of materials in a complex stress state. A comprehensive technical review of strength theories was presented by Yu [17] for a variety of materials under complex stress states. Christensen [18] discussed a comprehensive theory of yield and failure for isotropic materials. In general, two basic strength failure modes were assumed for the metallic PV design: elastic failure and plastic failure. The former is governed by the theory of elasticity and suitable for brittle metals, and the latter is governed by the theory of plasticity and suitable for ductile metals. When a ductile material is stretched beyond the elastic limit strength, plastic deformation occurs, and plastic instability of the material may happen. Material properties relevant to the PV failure are the YS and UTS. Due to multiaxial stress state, the PV failure is controlled by a combination of all stress components but not by a single stress component in the vessel. The strength theories have been formulated in terms of the three principal stresses (σ_1, σ_2, σ_3) and can predict failure for any given stress state in an isotropic material. Three classical strength theories are often used in the PV design.

2.1 Classical strength theories

2.1.1 The maximum principal stress theory (Rankine theory)

This strength theory was proposed by Rankine [19] and used to describe elastic failure for brittle materials, such as cast irons at the elastic limit strength. It is assumed that rupture will occur when the maximum principal stress (σ_1) in a solid reaches the UTS (σ_{uts}) of the material determined from the uniaxial testing.

$$\sigma_1 = \sigma_{uts} \tag{1}$$

2.1.2 The maximum shear stress theory (Tresca theory)

This strength theory was proposed by Tresca [20] and used to describe failure of ductile materials like carbon steels at the plastic yield. It is assumed that failure will occur in a solid if the maximum shear stress (τ_{max}) in a solid reaches the shear yield strength (τ_{ys}) of the material determined from the uniaxial testing.

$$\tau_{max} = \max\left(\frac{|\sigma_1-\sigma_2|}{2}, \frac{|\sigma_1-\sigma_2|}{2}, \frac{|\sigma_1-\sigma_2|}{2}\right) = \tau_{ys} \tag{2}$$

or

$$\sigma_T = max(|\sigma_1 - \sigma_2|,\ |\sigma_2 - \sigma_3|,\ |\sigma_3 - \sigma_1|\,) = \sigma_{ys} \tag{3}$$

where σ_T is the Tresca effective stress, σ_{ys} is the tensile yield strength of material, and $\tau_{ys} = \sigma_{ys}/2$.

2.1.3 The maximum distortion energy theory (von Mises theory)

The Von Mises theory is also known as the distortion energy theory [21] and is used to characterize the plastic yield failure of ductile steels, such as carbon steels. It is assumed that failure will occur when the distortion energy per unit volume in a solid due to applied stresses reaches the distortion energy per unit volume of the material at the yield point from the uniaxial tensile testing.

$$\tau_M = \sqrt{\frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} = \tau_{ys} \quad (4)$$

or

$$\sigma_M = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} = \sigma_{ys} \quad (5)$$

where τ_M is the von Mises effective shear stress, σ_M is the von Mises effective stress, and $\tau_{ys} = \sigma_{ys}/\sqrt{3}$.

2.1.4 Average shear stress theory (Zhu-Leis theory)

In addition to the classical strength theories discussed above, Zhu [7] recently proposed an average shear stress strength theory used to predict plastic failure for ductile steels. The Zhu-Leis criterion [6] is an intermediate yield criterion that lies between the Tresca and von Mises criteria. Average shear stress (τ_A) was defined as the average of the maximum shear stress (τ_{max}) and the von Mises effective shear stress (τ_M). Accordingly, the Zhu-Leis effective stress σ_A can be expressed as a combined function of the Tresca effective stress σ_T and the von Mises effective stress σ_M. It is assumed that failure will occur if the average shear stress in a solid reaches the average shear strength of the material. Using the effective stress σ_A, this yield criterion is expressed as:

$$\sigma_A = \frac{1}{2+\sqrt{3}}\left[\sqrt{3}\sigma_T + 2\sigma_M\right] = \sigma_{ys} \quad (6)$$

It is well known that the yield failure defined at the plastic yield is too conservative for ductile steels, particularly for high strain hardening steels, where the UTS is much larger than the YS. To reduce the conservatism of the classical yield strength theories, the UTS has replaced the YS in the Tresca, von Mises and Zhu-Leis criteria. Then the UTS-based failure criterion was used to describe plastic collapse of ductile steel PVs. In addition, a flow stress failure criterion is often used to describe the plastic collapse of line pipes in the pipeline industry.

2.2. Strength solutions of burst pressure of pipelines

From the UTS-based Tresca, von Mises and Zhu-Leis criteria and the flow stress criterion, four simple analytical solutions of burst pressure were obtained for defect-free, thin-wall pipes or cylinders [7] as:

(1) Tresca strength failure pressure

$$P_0 = \frac{2t}{D}\sigma_{uts} \quad (7)$$

(2) von Mises strength failure pressure

$$P_{M0} = \frac{4t}{\sqrt{3}D}\sigma_{uts} \quad (8)$$

(3) Zhu-Leis strength failure pressure

$$P_{A0} = \left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)\frac{t}{D}\sigma_{uts} \quad (9)$$

(4) Flow stress-based failure pressure (Tresca criterion)

$$P_f = \frac{2t}{D}\sigma_{flow} \quad (10)$$

where D is the pipe outside diameter (OD) or mean diameter (MD=D-t), t is the wall thickness, and σ_{flow} is the flow stress that was initially defined as $\sigma_{flow} = \sigma_{ys} + 10\ ksi$ [22] for pipeline steels and later changed to be an averaged material strength $\sigma_{flow} = (\sigma_{ys} + \sigma_{uts})/2$. If $\sigma_{flow} = \sigma_{uts}$, Eq. (10) reduces to Eq. (7). This equation is also referred to as the Barlow formula [3] in the pipeline industry, and has been often used to determine the plastic failure pressure or the minimum required wall thickness in the pipeline design.

Figure 1 compares the just-noted four strength solutions of burst pressure with the full-scale burst test data that were collected by Zhu and Leis [6] from public literature for thin-wall pipes, where all burst pressures are normalized by the Barlow strength in Eq. (7) with use of MD. Also included is the linear curve fit of those test data. The full-scale test dataset covers a wide range of pipeline steels from Grade A to X120 as well as oil-well casing steels H40 to Q125 used in offshore drilling. The reference numbers marked in Fig. 1 were given by Zhu and Leis [6]. From this figure, one can have the following observations:

1) The three UTS-based strength solutions are independent of the strain hardening rate, while the flow stress-based failure pressure can describe the plastic flow effect.
2) The von Mises strength solution is an absolute upper bound prediction that is higher than all of the test data.
3) The Zhu-Leis strength solution is a near upper bound that is slightly lower than the upper bound of the test data.
4) The Tresca strength solution is an overall averaged prediction of all test data. It is lower than the linear curve fit of the test data when $n < 0.1$, and higher than the linear curve fit when $n > 0.1$.
5) The flow stress failure criterion determines a lower bound prediction of the test data and thus serves as a conservative criterion. It has been widely applied in pipeline design and integrity management.

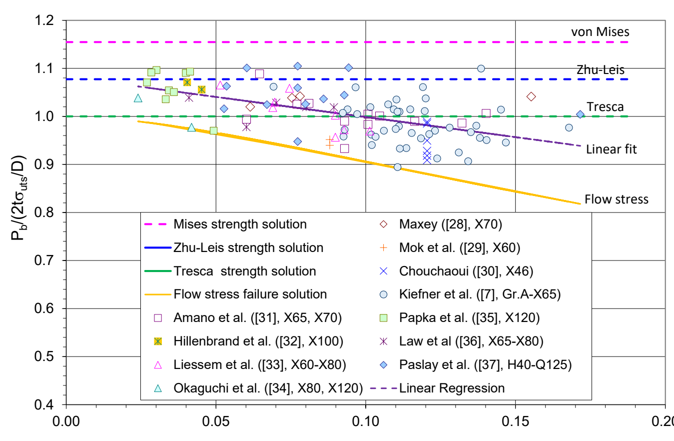

Figure 1. Comparison of four strength solutions of burst pressure with full-scale burst data

In summary, the UTS-based strength theories do not consider the plastic flow response and thus cannot accurately predict burst pressure. In contrast, the flow stress failure criterion can consider the plastic flow effect and determine a lower bound prediction of burst pressure for thin-wall pipes.

2.3. Two-parameter flow stress

As discussed above, the UTS-based strength theories can only determine the single-parameter strength solution of failure pressure as a function of D/t and UTS, as evident in Eqs (7) to (9) in reference to the Tresca, von Mises and Zhu-Leis strength theories. On the other hand, the flow stress solution can describe the UTS and plastic flow effect on the burst pressure in a conservative, lower bound manner. In order to obtain a more accurate burst pressure prediction, Zhu and Leis [6] adapted the flow theory of plasticity within the large deformation framework to consider the plastic flow effect. For a power-law hardening material, three flow solutions of burst pressure were obtained for thin-wall pipes with regard to the Tresca, von Mises and Zhu-Leis criteria as:

$$P_T = \left(\frac{1}{2}\right)^{n+1} \frac{4t}{D} \sigma_{uts} \quad (11)$$

$$P_M = \left(\frac{1}{\sqrt{3}}\right)^{n+1} \frac{4t}{D} \sigma_{uts} \quad (12)$$

$$P_A = \left(\frac{2+\sqrt{3}}{4\sqrt{3}}\right)^{n+1} \frac{4t}{D} \sigma_{uts} \quad (13)$$

where n is the strain hardening exponent that is measured from a simple tensile test or estimated from the YS and UTS. Zhu and Leis [22] provided two simple equations to estimate n from the YS defined at the 0.2% offset strain and the 0.5% total strain. For carbon steels n has a typical value ranging from 0.02 to 0.25.

It is noted that both the strength and flow solutions of burst pressure discussed above were obtained in terms of the thin-wall shell theory, with the assumption that the axial and hoop stresses are uniformly distributed on the cross-sections of a cylinder. For conservatism, the pipe diameter in Eqs (7) to (13) is usually taken as OD in the PV or pipeline design. However, MD should be utilized in those equations for more accurate prediction of burst pressure in the integrity assessment of PVs and pipelines.

The full-scale test data shown in Fig. 1 are utilized for validating the three burst solutions in Eqs (11) to (13). Figure 2 compares those flow solutions with full-scale burst test data, where both predicted and measured burst pressures are normalized using the MD-based Barlow strength. The normalized flow stress solution is also included in this figure for comparison. It can be observed from this figure that:

1) The three flow solutions of burst pressure consider the plastic flow effect, and are functions of D/t, UTS, and n.
2) The von Mises flow solution is an upper bound prediction of burst pressure test data.
3) Tresca flow solution is a lower bound prediction of burst pressure test data. For low strain hardening steels with n<0.1, the Tresca flow solution and the flow stress solution are comparable. For high strain hardening steels with n>0.1, the Tresca flow solution gradually deviates from the flow stress solution, and thus is less conservative.
4) The Zhu-Leis flow solution is comparable to the linear curve fit result, and thus serves as the best prediction of the burst test data on average.

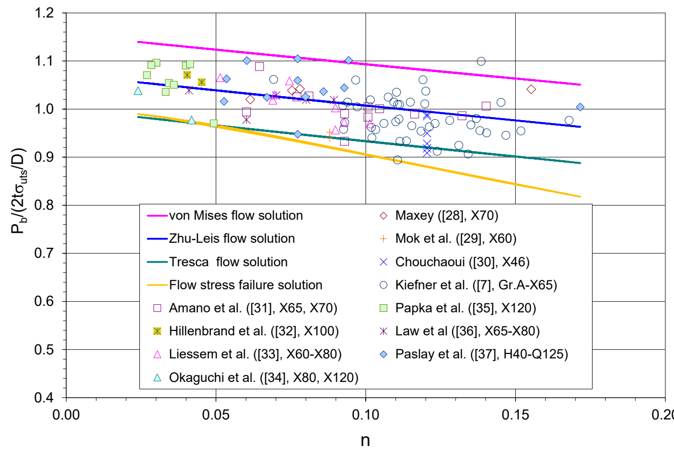

Figure 2. Comparison of three flow solutions of burst pressure with full-scale test data

Because the three flow solutions describe the plastic flow trend, three new flow stresses are thus defined from the three flow solutions for burst pressure. A general expression of the nominal effective stress in a thin-wall pipe for the three yield criteria is written as:

$$\sigma_{eff} = \frac{1}{K}\frac{PD}{2t} \quad (14)$$

where D and t are the nominal pipe diameter and wall thickness. K is a yield criterion-dependent constant:

$$K = \begin{cases} 1, & \text{for Tresca yield criterion} \\ \frac{2}{\sqrt{3}}, & \text{for von Mises yield criterion} \\ \frac{1}{2}+\frac{1}{\sqrt{3}}, & \text{for Zhu-Leis yield criterion} \end{cases} \quad (15)$$

From Eqs (11) to (14), one obtains the critical stress at the burst failure for the three yield criteria as:

$$\sigma_{eff}^{c} = \left(\frac{K}{2}\right)^{n} \sigma_{uts} \quad (16)$$

where σ_{eff}^{c} is the new flow stress that is a function of both UTS and n. For the perfectly plastic material, $n \to 0$, and $\sigma_{eff}^{c} = \sigma_{uts}$.

2.4. New strength theories

With the new flow stress defined in Eq. (16), the power-law hardening material can be approximated as an elastic-perfectly plastic material. Accordingly, a material will remain in the elastic condition if the effective stress in the solid is less than the flow stress. Otherwise, the strength theory assumes that failure will occur if the effective stress in the solid reaches the new flow stress. Thus, using the new flow stresses, the traditional one-parameter strength theories can be modified as new two-parameter strength theories in the following formats.

(1) Tresca two-parameter strength criterion

$$\sigma_T = \sigma_T^c = \left(\frac{1}{2}\right)^n \sigma_{uts} \tag{17}$$

(2) Von Mises two-parameter strength criterion

$$\sigma_M = \sigma_M^c = \left(\frac{1}{\sqrt{3}}\right)^n \sigma_{uts} \tag{18}$$

(3) Zhu-Leis two-parameter strength criterion

$$\sigma_A = \sigma_A^c = \left(\frac{2+\sqrt{3}}{4\sqrt{3}}\right)^n \sigma_{uts} \tag{19}$$

If the strain hardening effect is not considered, n=0 and Eqs (17), (18), and (19) reduce to Eqs (3), (5), and (6), respectively for the Tresca, von Mises and Zhu-Leis strength theories. As a result, the traditional strength theory is a special case of the proposed new strength theory for each yield criterion.

To illustrate the differences between the traditional and new flow stresses, Fig. 3 shows an example of engineering stress-strain curves for pipeline steel X65, where the material properties YS=65 ksi and UTS=80 ksi, and the 0.2% offset line are included. Also included are four flow stress lines: the Tresca flow stress $\sigma_T^c = 74.658$ ksi, the von Mises flow stress $\sigma_M^c = 75.735$ ksi, the Zhu-Leis flow stress $\sigma_A^c = 75.213$ ksi, and the averaged flow stress $\sigma_{flow} = 72.501$ ksi. Each of those flow stresses defines a nominal material strength, and then the power-law stress-strain curve is approximated as an elastic-perfectly plastic stress-strain curve. Accordingly, the material deformation will be elastic when the effective stress in a solid is less than the flow stress, but plastic when the stress is equal to the flow stress.

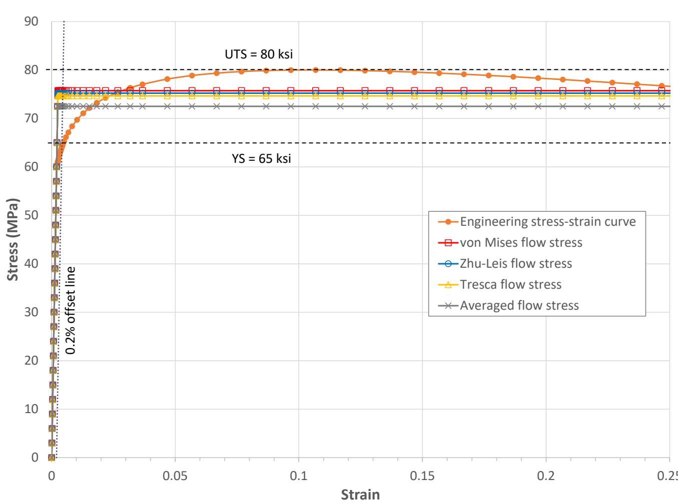

Figure 3. Engineering stress-strain curve and flow stresses for pipeline steel X65

Figure 3 shows that the proposed Tresca, von Mises and Zhu-Leis flow stresses are comparable to each other and lie between the averaged flow stress and UTS. Accordingly, it is expected that these new flow stress criteria will predict higher and thus less conservative burst pressure in comparison to the averaged flow stress criterion.

3. DETERMINATION OF BURST PRESSURE FOR THICK-WALL PRESSURE VESSELS

Many elastic or elastic-plastic stress analyses and limit load analyses are publicly available for thick-wall cylinders under internal pressure, such as those by Ayob et al. [24] and Barashkov and Shevchenko [25]. Thus, the detailed stress and strain analyses will not be presented in this work.

Consider an infinitely small stress element in a pressurized cylinder in a polar coordinate system (r, θ), as illustrated in Fig. 4. In this figure $\sigma_{\theta\theta}$ is the hoop stress of the cylinder, σ_{rr} is the radial stress, $d\sigma_{rr}$ is the incremental radial stress, dr is an infinitely small increment of the circular radius, and dθ is an infinitely small increment of the circular angle. From the force equilibrium along the radial direction of the stress element, the following stress equilibrium equation is obtained:

$$\sigma_{\theta\theta} - \sigma_{rr} - r\frac{d\sigma_{rr}}{dr} = 0 \tag{20}$$

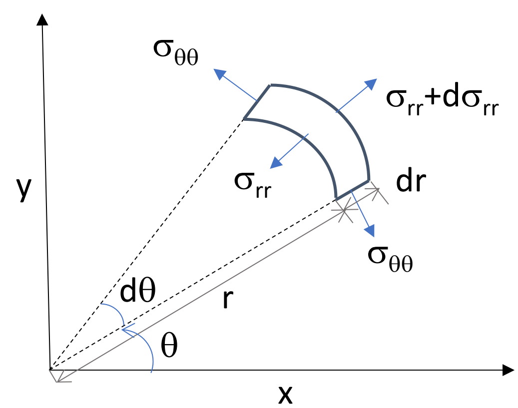

Figure 4. Scheme of stress element in a cylinder

For an elastic-perfectly plastic material, the entire cross-section of the cylinder wall will plastically yield at the limit load. In this case, using the proposed new strength theories, the following general yield condition is obtained in terms of Tresca, von Mises or Zhu-Leis criterion:

$$\sigma_{\theta\theta} - \sigma_{rr} = K\sigma_{eff}^c \tag{21}$$

Substitution of Eq. (21) into Eq. (20) determines:

$$\frac{d\sigma_{rr}}{dr} = \frac{K\sigma_{eff}^c}{r} \tag{22}$$

Integration of Eq. (22) obtains:

$$\sigma_{rr} = K\sigma_{eff}^{c}\ln(r) + C \quad (23)$$

where C is an integration constant to be determined by the following stress boundary conditions:

$$\sigma_{rr}(r = R_i) = -P$$
$$\sigma_{rr}(r = R_0) = 0 \quad (24)$$

where P is the internal pressure, $R_i=D_i/2$, $R_0=D_0/2$, D_i is the inside diameter, and D_0 is the outside diameter. From Eqs (23) and (24), the burst pressure is determined as:

$$P_b = 2\left(\frac{K}{2}\right)^{n+1}\sigma_{uts}ln\left(\frac{D_0}{D_i}\right) \quad (25)$$

Particularly, the burst pressure solution of a thick-wall cylinder for the Tresca, von Mises and Zhu-Leis criterion is expressed separately as:

$$P_T = 2\left(\frac{1}{2}\right)^{n+1}\sigma_{uts}ln\left(\frac{D_0}{D_i}\right) \quad (26)$$

$$P_M = 2\left(\frac{1}{\sqrt{3}}\right)^{n+1}\sigma_{uts}ln\left(\frac{D_0}{D_i}\right) \quad (27)$$

$$P_A = 2\left(\frac{2+\sqrt{3}}{4\sqrt{3}}\right)^{n+1}\sigma_{uts}ln\left(\frac{D_0}{D_i}\right) \quad (28)$$

For a thin-wall cylinder, the D_0/D_i ratio approaches to 1 and an approximate Taylor series is obtained as $\ln(D_0/D_i) = 2t/D_0 + 0.5(2t/D_0)^2$. Eqs (26) - (28) reduce to Eqs (11) - (13). In general, $D_0/t>20$ for a thin-wall pipe, and the error of the second term is less than 5% of the first term. Therefore, for thin-wall pipes, the burst pressure solutions in Eqs (11) - (13) have an error less than 5% in comparison to the burst pressure solutions in Eqs (26) - (28).

4. Experimental validation of burst pressure solutions

This section evaluates the proposed burst pressure solutions in Eqs (26), (27) and (28) with regard to the new Tresca, von Mises and Zhu-Leis strength theories for thick-wall PVs. The evaluation is made through comparison with the full-scale burst data shown in Fig. 1 for a variety of carbon steels. Detailed inspection indicates that most of the full-scale burst tests were conducted for thin-wall line pipes with $D/t > 20$. Also included in the dataset are 12 thick-wall casing tubes with $6 < D/t < 20$. As a result, this dataset can be used to evaluate the D/t ratio effect on burst failure.

Figure 7 compares the three theoretical predictions with the measured burst pressures as a function of n, where all burst pressures are normalized by three reference strengths P_0: the OD-Barlow strength, $P_0 = 2\sigma_{uts}t/OD$, the MD-Barlow strength, $P_0 = 2\sigma_{uts}t/MD$, and the thick-wall Tresca strength $P_0 = \sigma_{uts}\ln(D_0/D_i)$. It is noted that all three normalized flow solutions are a function of n only and remain the same for the thin-wall and thick-wall theories. However, the three sets of normalized full-scale burst data are different. From Fig. 7, the following observations are made:

1) The burst test data normalized by the OD-Barlow strength are higher than the other two sets of normalized test data, and several test data points are much higher than the von Mises burst solution. This means that the OD-based flow solutions tend to be overly conservative.
2) The burst test data normalized by the MD-Barlow strength are comparable or nearly identical to the full-scale test data normalized by the thick-wall Tresca strength.

As a result, it is concluded that the MD-based Zhu-Leis burst pressure solutions is more accurate in comparison to the OD-based Zhu-Leis burst pressure solution for thin-wall pipes. Moreover, the MD-based Zhu-Leis burst solution for thin-wall pipes has a similar or comparable accuracy as does the Zhu-Leis burst pressure solution for thick-wall cylinders.

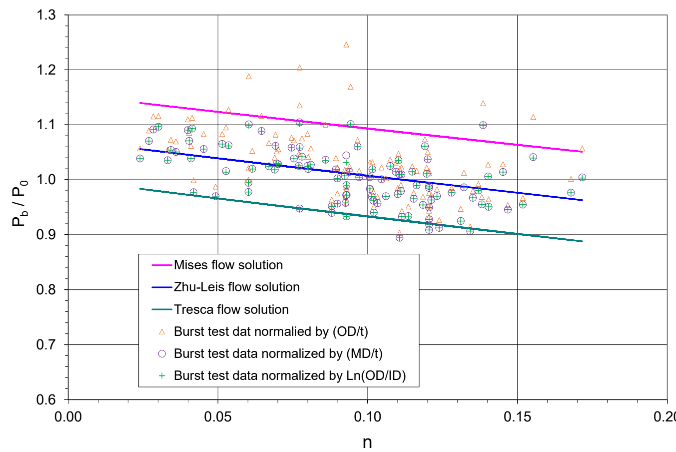

Figure 7. Comparison of predicted and measured burst pressures in the function of n

Figure 8 compares the Zhu-Leis burst pressure solutions from the thin-wall and thick-wall theories with the measured burst data as a function of D/t. The figure shows that:

(1) For thin-wall pipes with $D/t > 20$, all burst pressures are near to or less than 5,000 psi (34.47 MPa). The Zhu-Leis burst pressure solution for the thin-wall theory is nearly equivalent to that for the thick-wall theory.
(2) For intermediate or thick-wall vessels with $D/t < 20$, all burst pressures are larger than 5,000 psi (34.47 MPa). The MD-based Zhu-Leis solution for thin-wall pipes is either comparable or nearly identical to the Zhu-Leis solution for thick-wall cylinders.
(3) In contrast, the OD-based Zhu-Leis solutions for thin-wall pipes is significantly lower than those for the thick-wall cylinders with D/t<10.

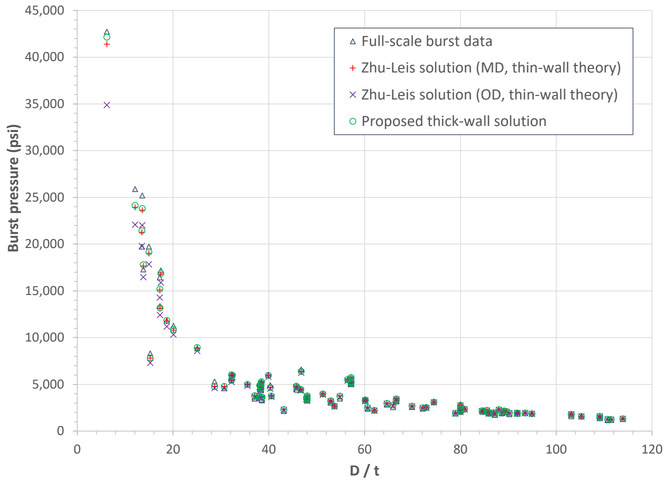

Figure 8. Comparison of predicted and measured burst pressures in the function of D/t

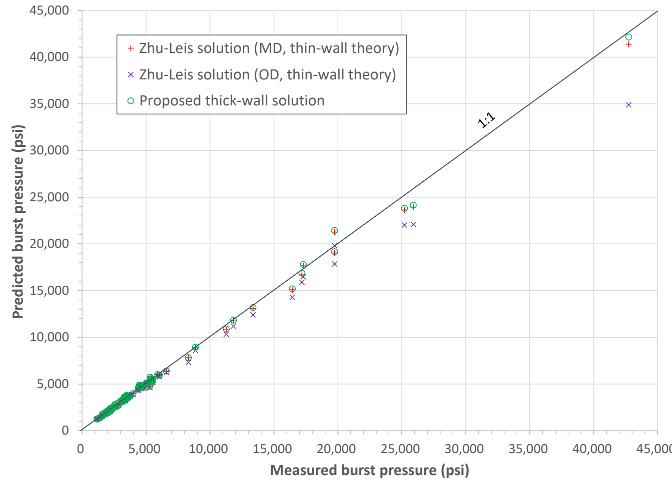

Figure 9. Comparison of predicted burst pressures by thin-wall and thick-wall theories with measured burst data

Figure 9 compares the Zhu-Leis burst pressure predictions from the thin-wall and thick-wall theories with the measured burst pressure data. From this figure, the following observations are obtained:

1) For the low burst strength when P_b<7,500 psi (51.71 MPa), the wall thickness corresponds to thin-wall pipes with D/t>20. The three Zhu-Leis burst predictions are nearly identical to the measured burst pressure data.
2) For the intermediate burst strength with a range of 7,500 psi (51.71 MPa) < P_b < 20,000 psi (137.9 MPa), the wall thickness corresponds to the intermediate-wall pipes with $13.5 < D/t < 20$. The OD-based Zhu-Leis burst pressure solution for thin-wall pipes is less than the Zhu-Leis solution for thick-wall cylinders, while the MD-based Zhu-Leis solution for thin-wall pipes is nearly identical to the Zhu-Leis solution for thick-wall cylinders.
3) For the high burst stremgth when P_b>20,000 psi (137.9 MPa), the wall thickness corresponds to thick-wall vessels with D/t<13.5. The OD-based Zhu-Leis burst solution for thin-wall pipes is significantly less than the Zhu-Leis burst pressure solution for thick-wall cylinders, while the MD-based Zhu-Leis burst solution for thin-wall pipes remains comparable to the Zhu-Leis solution for thick-wall PVs.

5. CONCLUSIONS

This paper proposed three new flow stresses for describing the tensile strength and the plastic flow response of a strain hardening material. Three new strength theories were then developed in terms of the Tresca, von Mises and Zhu-Leis yield criteria, and the associated burst pressure solutions were obtained for thick-wall cylinders. After that, the proposed burst solutions were evaluated in comparison with the Zhu-Leis solution for thin-wall pipes and a large dataset of full-scale burst pressure tests. The results and conclusions are summarized as follows.

(1). Three new flow stresses were proposed as a function of UTS and n in terms of the Tresca, von Mises and Zhu-Leis yield criteria.

(2). With the new flow stresses, three new strength theories, i.e., Tresca, von Mises and Zhu-Leis strength theories were developed for predicting the burst failure of thick-wall cylinders.

(3). From the three newly proposed strength theories, three burst pressure solutions were obtained for thick-wall cylinders. Those include the von Mises burst solution that is an upper bound prediction of burst pressure for thick-wall cylinders, the Tresca burst solution that is a lower bound prediction of burst pressure for thick-wall cylinders, and the Zhu-Leis burst solution that is an intermediate prediction of burst pressure for thick-wall cylinders.

(4). The Zhu-Leis burst pressure solutions were validated by a large dataset of full-scale burst tests and match well with the test data for thin-wall and thick-wall vessels on average.

(5). The OD-based Zhu-Leis burst pressure solutions for thin-wall pipes are comparable to the proposed Zhu-Leis burst pressure solutions for thick-wall PVs when the wall thickness ratio D/t>20. However, when D/t<20, a significant difference between them exists for intermediate and thick-wall cylinders.

(6). The MD-based Zhu-Leis burst solutions for thin-wall pipes are nearly identical or comparable to the proposed Zhu-Leis burst solutions for intermediate and thick-wall cylinders.

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