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DEVELOPMENT OF AN ANTIFOAM TRACKING SYSTEM AS AN OPTION TO SUPPORT THE MELTER OFF-GAS FLAMMABILITY CONTROL STRATEGY AT THE DWPF

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August 2014

SRNL-STI-2014-00323

SRNL.DOE.GOV

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Printed in the United States of America

Prepared for
U.S. Department of Energy

Keywords: *carbon, oxalate, formate, nitrate, TOC, uncertainty, random, bias*

Retention: *permanent*

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Prepared for the U.S. Department of Energy under contract number DE-AC09-08SR22470.

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Executive Summary

The Savannah River National Laboratory (SRNL) has been working with the Savannah River Remediation (SRR) Defense Waste Processing Facility (DWPF) in the development and implementation of an additional strategy for confidently satisfying the flammability controls for DWPF's melter operation. An initial strategy for implementing the operational constraints associated with flammability control in DWPF was based upon an analytically determined carbon concentration from antifoam. Due to the conservative error structure associated with the analytical approach, its implementation has significantly reduced the operating window for processing and has led to recurrent Slurry Mix Evaporator (SME) and Melter Feed Tank (MFT) remediation.

To address the adverse operating impact of the current implementation strategy, SRR issued a Technical Task Request (TTR) to SRNL requesting the development and documentation of an alternate strategy for evaluating the carbon contribution from antifoam. The proposed strategy presented in this report was developed under the guidance of a Task Technical and Quality Assurance Plan (TTQAP) and involves calculating the carbon concentration from antifoam based upon the actual mass of antifoam added to the process assuming 100% retention.

The mass of antifoam in the Additive Mix Feed Tank (AMFT), in the Sludge Receipt and Adjustment Tank (SRAT), and in the SME is tracked by mass balance as part of this strategy. As these quantities are monitored, the random and bias uncertainties affecting their values are also maintained and accounted for. This report documents:

1) the development of an alternate implementation strategy and associated equations describing the carbon concentration from antifoam in each SME batch derived from the actual amount of antifoam introduced into the AMFT, SRAT, and SME during the processing of the batch.

2) the equations and error structure for incorporating the proposed strategy into melter off-gas flammability assessments.

Sample calculations of the system are also included in this report. Please note that the system developed and documented in this report is intended as an alternative to the current, analytically-driven system being utilized by DWPF; the proposed system is not intended to eliminate the current system.

Also note that the system developed in this report to track antifoam mass in the AMFT, SRAT, and SME will be applicable beyond just Sludge Batch 8. While the model used to determine acceptability of the SME product with respect to melter off-gas flammability controls must be reassessed for each change in sludge batch, the antifoam mass tracking methodology is independent of sludge batch composition and as such will be transferable to future sludge batches.

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LIST OF ABBREVIATIONS

AF	antifoam
AMFT	Additive Mix Feed Tank
DCS	Distributed Control System
DWPF	Defense Waste Processing Facility
JMP	Pronounced “jump”, commercial statistical software from SAS Institute, Inc.
IC	Ion Chromatography
inwc	inches of water column
LFL	Lower Flammability Limit
M_{AF}	Antifoam mass in kg
MFT	Melter Feed Tank
SB8	Sludge Batch 8
SME	Slurry Mix Evaporator
SRAT	Sludge Receipt
SRNL	Savannah River National Laboratory
SRR	Savannah River Remediation, LLC
TOC	Total Organic Carbon
TTQAP	Task Technical and Quality Assurance Plan
TTR	Technical Task Request

1. Introduction

The Savannah River National Laboratory (SRNL) has been working with the Savannah River Remediation (SRR) Defense Waste Processing Facility (DWPF) in the development and implementation of an additional strategy for confidently satisfying the flammability controls for DWPF's melter operation during the processing of Sludge Batch 8 (SB8). The flammability controls for SB8 were developed by Choi [1], who established a framework of operational constraints that include limitations on the carbon concentration from antifoam. The constraints developed by Choi maintain the contributors to melter flammability to less than 60% of the lower flammability limit (LFL) [1]. An initial strategy for implementing these operational constraints in DWPF was documented by Edwards [2] in the form of a decision support system based upon an analytically determined carbon concentration from antifoam. Since the implementation of the decision support system into its operation, the DWPF has identified issues related to the impact of the system on melter feed composition. Due to the conservative error structure associated with the analytical approach underlying the decision support system, implementation of this control has significantly reduced the operating window for processing and has led to recurrent Slurry Mix Evaporator (SME) and Melter Feed Tank (MFT) remediation.

To address the adverse operating impact of the current implementation strategy, SRR issued a Technical Task Request (TTR) to SRNL requesting the development and documentation of an alternate strategy for evaluating the carbon contribution from antifoam [3]. The proposed strategy involves calculating the carbon concentration from antifoam based upon the actual amount of antifoam added to the process assuming 100% retention. The purpose of this report is to document the following:

1) The development of an alternate implementation strategy and associated equations describing the carbon concentration from antifoam in each SME batch derived from the actual amount of antifoam introduced during the processing of the batch.

2) The error structure for incorporation of the proposed strategy into melter off-gas flammability assessments.

This work was conducted under the guidance of the Task Technical and Quality Assurance Plan (TTQAP) provided in reference [4]. Section 2 provides a background discussion of the melter flammability controls and the implementation strategy currently being utilized by DWPF. Section 3 provides the proposed strategy for tracking the antifoam additions to a SME process batch: Tracking these additions is done on a mass basis with the mass of antifoam being monitored in the Additive Mix Feed Tank (AMFT), in the Sludge Receipt and Adjustment Tank (SRAT) and in the SME. Also in Section 3 are the equations utilized to facilitate this tracking of antifoam along with the associated uncertainties. A summary of this work is provided in Section 4.

2. Background

As stated in the Introduction, SRNL supported SRR in the development of a melter off-gas flammability control strategy for SB8 [1]. The strategy relies on a SME batch satisfying three categories of constraints. The contents of the SME batch must have

(1) a nitrate concentration within the interval from 10,000 to 70,000 mg/kg,
(2) a total organic carbon (TOC) concentration below a value tied (via a functional relationship) to the nitrate content of the SME and one of three different amounts of antifoam (728, 894, and 1,017 gallons (gal)) that were used by Choi [1] to frame possible preparations of the SME batch, and
(3) a carbon concentration contributed from antifoam that is below the amount that is allowed by the same amount of antifoam (728, 894, and 1,017 gal) utilized to meet the TOC constraint in (2).

Regardless of the strategy for implementing these constraints into the DWPF operation, uncertainties affecting the strategy must be addressed to ensure a reliable decision for SME acceptability. The current decision support system is driven by analytical measurements of the SME contents: TOC measurements and measurements by Ion Chromatography (IC) of select anions: nitrate, oxalate, and formate. The uncertainties of these measurements were integrated into the decision support system developed by Edwards [2]. In the following sub-sections, the strategy from [2] for utilizing analytical measurements from the SME samples to meet each of these constraints is summarized.

It is probably worth mentioning again: the decision support system developed and documented in this report is intended as an alternative to the current system being utilized by DWPF; the proposed system is not intended to eliminate the current system. Also note that the system developed in this report to track antifoam mass in the AMFT, SRAT, and SME will be applicable beyond just SB8.

2.1 Assessing the Nitrate Content of the SME

Equation 1 and Equation 2 provide the constraints associated with ensuring that the nitrate (NO_3) content is within the interval of 10,000 to 70,000 mg/kg. (See Edwards [2] for the development of these equations.)

Equation 1

$$\left(\overline{NO_3}\cdot 0.9924\right)-10000-3.182\cdot\sqrt{\left(se_{\overline{NO_3}}\right)^2+\left(s_{\delta_{NO_3}}\right)^2}>0$$

and

Equation 2

$$70000-\overline{NO_3}-3.182\cdot\sqrt{\left(se_{\overline{NO_3}}\right)^2+\left(s_{\delta_{NO_3}}\right)^2}>0$$

where $\overline{NO_3}$ represents the average of the NO_3 concentration measurements for the 4 samples of the given SME batch. This average has a 1-sigma random uncertainty of $se_{\overline{NO_3}}$,

0.9924 is the adjustment for a potentially high bias in the nitrate measurements, and

δ_{NO_3} represents the batch-to-batch source of variation affecting the nitrate measurements for a SME batch. For the evaluations of Equation 1 and Equation 2, the value of δ_{NO_3} is taken as zero. Its 1-sigma relative random uncertainty is the batch-to-batch variation and is represented by $s_{\delta_{NO_3}}$, and based upon the discussion provided by Edwards [2], its value is given by 2.73% of the $\overline{NO_3}$ value.

It should be noted that during the processing of SB8, the constraints imposed by these equations must be met for the current strategy and for the proposed strategy discussed in this report.

2.2 Assessing the TOC Content of the SME

During the processing of SB8, the restrictions on the TOC content of the SME will have to be met regardless of whether the current or the proposed antifoam strategy is utilized. The linearized relationship between TOC and NO_3 developed by Edwards [2] is provided in Equation 3.

Equation 3

$$TOC_i = f_i + g_i \cdot (NO_3)$$

where the TOC_i term on the left-hand side of the equation represents the maximum TOC allowed to maintain the system below the 60% LFL based upon a linearized relationship between TOC and NO_3, while the NO_3 term on the right-hand side represents the nitrate content of the melter feed in mg/kg. The values of the f_i and g_i coefficients are given in Table 1 for i=1, 2, and 3, and each value of the i index corresponds to one of the three additions of antifoam (in gallons) for which Choi developed an associated TOC versus NO_3 relationship.

Table 1 Coefficients for the Linear Equations Relating TOC Content to NO_3 Content

i	Antifoam Addition (gal)	f_i	g_i
1	728	8140	0.37
2	894	6550	0.37
3	1,017	5300	0.37

In the discussion provided in [2], no statistically significant bias was indicated for the TOC measurements; while for the nitrate measurements, there was a statistically significant high bias. Thus, to be conservative, an adjustment is made to the nitrate value in Equation 3 for this bias. The direct utilization of the family of equations given by Equation 3 for melter flammability control yields this acceptability equation for a given SME batch during the processing of SB8:

Equation 4

$$D_i = f_i + g_i \cdot 0.9924 \cdot \overline{NO_3} - \overline{TOC} - other_C > 0$$

where D_i is the measurand, it represents the difference in mg/kg between the carbon allowed by Equation 3 and the carbon content of the SME, and its value must be positive,

f_i & g_i are the coefficients from Table 1 corresponding to the appropriate bound on the gallons of antifoam utilized in the preparation of the SME batch (indexed by i) as determined following the approach discussed in Section 2.3 below,

0.9924 is the adjustment for a potential high bias in the nitrate measurements, and

$\overline{NO_3}$ represents the average of the NO_3 concentration measurements in mg/kg for the samples of the given SME batch,

$\overline{TOC}$ represents the average of the TOC concentration measurements in mg/kg for the samples of the given SME batch, and

$other_C$ represents carbon that is present in the SME in a form that is not measured by the TOC analytical protocol. Note, however, that such carbon was included in the determination of the TOC relationship to nitrate given by Equation 3. Its value is associated with the amount of coal in the SME and is bounded by 240 mg/kg [2].

The form of Equation 4 is such that the value of D_i must be positive for acceptability. That is, the amount of TOC allowed by Equation 3 must be greater than the TOC content of the given SME for the given level of antifoam addition (indexed by i), and this must be true with high confidence after accounting for the uncertainties in the measurements used to make this determination.

The values of δ_{NO_3} and δ_{TOC} are zero in the determination of the value of D_i, but including these terms in the equation for D_i allows for their contributions to the variance of D_i to be included in the variance propagation for Equation 4.

Using the estimates of the batch-to-batch variations discussed above, the variance of Equation 4 may be expressed as:

Equation 5

$$\begin{aligned}\mathrm{var}(D_1) = \mathrm{var}(D_2) = \mathrm{var}(D_3) &\approx 0.1369 \cdot (se_{\overline{NO_3}})^2 + 0.1369 \cdot (s_{\delta_{NO_3}})^2 + (se_{\overline{TOC}})^2 + (s_{\delta_{TOC}})^2 \\ &= 0.1369 \cdot (se_{\overline{NO_3}})^2 + 0.1369 \cdot (0.0273 \cdot \overline{NO_3})^2 + (se_{\overline{TOC}})^2 + (0.0272 \cdot \overline{TOC})^2 \\ &= 0.1369 \cdot (se_{\overline{NO_3}})^2 + 0.00010203 \cdot (\overline{NO_3})^2 + (se_{\overline{TOC}})^2 + 0.00073984 \cdot (\overline{TOC})^2\end{aligned}$$

Integrating these estimates of the variance for D_i into the expression of the constraint leads to

Equation 6

$$D_i - t_{(0.05,3)} \cdot (\text{Est. var}(D_i))^{0.5} = D_i - 2.353 \cdot (\text{Est. var}(D_i))^{0.5} > 0$$

where D_i is determined using Equation 4 (for the associated level of antifoam additions),

$t_{(0.05,3)}$ is the upper 5%-tail of the Student's t distribution with 3 degrees of freedom (i.e., 2.353), and

Est. var(D_i) represents the estimate of the variance of D_i, which may be computed using Equation 5 regardless of the antifoam addition being considered.

2.3 Assessing the Carbon Content of the SME Attributable to Antifoam

As described above, the relationship between the maximum allowable TOC content and the NO_3 content of a SME batch was associated with the total amount of antifoam used to prepare the SME batch. Three levels for this total antifoam amount in gallons were established by Choi [1]: 728, 894, and 1,017 gal. As part of his study, Choi developed a relationship between the maximum amount of carbon that would be generated from each of these bounding amounts of antifoam and the NO_3 content of the SME. These relationships are given by:

Equation 7

$$(\text{Antifoam}_{Ci})^2 = h_i + j_i \cdot (NO_3) \Rightarrow \text{Antifoam}_C = \sqrt{h_i + j_i \cdot (NO_3)}$$

where Antifoam_{Ci} represents the maximum amount of carbon in mg/kg that would be generated from antifoam additions for three cases indexed by i: 1 corresponds to 728 gal, 2 to 894 gal, and finally, 3 to 1,017 gal of antifoam, and NO_3 represents the nitrate content of the SME in mg/kg. The h_i and j_i values, which are provided in Table 2, are from the models developed by Choi [1].

Table 2 Coefficients for the Equations Relating Maximum Carbon Content from Antifoam Additions to NO_3 Content

i	Antifoam Addition (gal)	h_i	j_i
1	728	5117745.1	-35.869438
2	894	7884790.5	-55.545316
3	1,017	10373798	-73.602487

Thus, DWPF's melter flammability control strategy requires that SRR confidently estimate an upper bound on the amount of carbon attributable to the antifoam added during the processing of each SME batch, and then use this result to confidently establish one of the three antifoam levels developed by Choi (i.e., for additions of no more than 728, 894, or 1,017 gal) as the upper bound on the gallons of antifoam added during the processing of the SME batch. The importance of this outcome is that it establishes the appropriate TOC to NO_3 relationship that must be used in satisfying Equation 6 as discussed in Section 2.2.

Thus, there is a need to estimate the amount of antifoam that was added during DWPF's preparation of a given SME batch. From the previous section, there is a relationship between the nitrate content of the SME and the limit on the carbon generated by antifoam additions to that SME for three different levels of antifoam addition: 728, 894, and 1,017 gal. Thus, one may estimate the amount of antifoam added during the preparation of a SME batch by estimating an upper bound on the carbon content due to antifoam additions for the prepared SME. During the processing of SB8, this constraint must be met by the proposed as well as by the current implementation strategy at the DWPF.

For the current strategy as documented by Edwards [2], the estimation is conducted by backing out contributions to the measured TOC concentration in the SME from the oxalate and the formate concentrations that are measured in the SME. The resulting adjusted TOC value provides a basis for estimating the amount of carbon in the SME attributable to antifoam. When this estimate is bounded at 95% confidence by accounting for its uncertainty, the resulting bounded amount of carbon attributable to antifoam must be below the carbon allowed by the level of antifoam (i.e., one of the three values:728, 894, or 1,017 gal) that is selected to be appropriate for the given SME batch. For the current strategy, the restriction imposed on the contents of the SME by Equation 7 may be expressed as:

Equation 8

$$M_{Ci} = \sqrt{h_i + j_i \cdot \overline{NO_3}} - \overline{TOC} + f_C \cdot \overline{formate} \cdot 0.9697 + o_C \cdot \overline{oxalate} \cdot 0.9459 > 0$$

where

M_{Ci} is the measurand; it represents the difference between the allowable concentration in mg/kg of carbon from antifoam at a level indexed by i (where i=1 represents 728 gal, 2 represents 894 gal, and 3 represents 1,017 gal) and the estimated amount of carbon attributable to antifoam; and the difference must be positive,

i is used as an index for the level of antifoam addition in gallons, i = 1 (728 gal), 2 (894 gal), and 3 (1,017 gal).

h_i & j_i are the coefficients corresponding to the i^{th} level of antifoam (see Table 2),

$\overline{NO_3}$ represents (as above) the average of the NO_3 concentration measurements in mg/kg for the samples of the given SME batch,

$\overline{TOC}$ is (as above) the average of the TOC measurements in mg/kg for the samples from the given SME batch,

$\overline{formate}$ is the average of the formate measurements in mg/kg for the samples from the SME batch[f],

$\overline{oxalate}$ is the average of the oxalate measurements in mg/kg for the samples from the SME batch[f],

0.9697 is included in the measurement equation to adjust (with a better than 95% confidence) for the potential bias in the formate measurement,

f_C is the conversion factor needed to determine the carbon contributed by the formate content of the SME in mg/kg (i.e., f_C = 0.266807 mg carbon/(kg SME slurry),

o_C is the conversion factor needed to determine the carbon contributed by the oxalate content of the SME in mg/kg (i.e., o_C = 0.27292 mg carbon/(kg SME slurry), and

0.9459 is included in the measurement equation to adjust (with a better than 95% confidence) for the potential bias in the oxalate measurement,

The expanded uncertainty of the estimated difference, M_{Ci}, at 95% confidence is determined by multiplying the square root of the estimated variance of M_{Ci} by an appropriate Student's t statistic. In this case a one-sided confidence statement is needed; so, an upper 5%-tail of the Student's t distribution will be used. Again, utilizing a conservative 3 degrees of freedom for the estimated variance, the t value is 2.353. Thus, at 95% confidence the expanded uncertainty of the difference is 2.353 times the square root of the estimated variance of M_{Ci}. Thus, for the antifoam content of the SME to be acceptable (at 95% confidence), the following constraint must be met:

Equation 9

$$M_{Ci} - 2.353 \cdot (\mathrm{Var}(M_{Ci}))^{0.5} > 0$$

where, for each level of antifoam indexed by i, M_{Ci} is determined from Equation 8 above, and $\mathrm{Var}(M_{Ci})$ is the estimate of the variance of M_{Ci} determined using Equation 10 below (see Edwards [2] for details). The smallest of the three levels of antifoam that meets the constraint imposed by Equation 9 is used to select the appropriate TOC constraint that must also be met (as described in Section 2.2) for an acceptable SME decision during the processing of SB8.

Equation 10

$$\mathrm{var}\left(M_{Ci}\right) \approx 0.25 \cdot [\,j_i \cdot (h_i + j_i \cdot \overline{NO_3})^{-0.5}]^2 \cdot (se_{\overline{NO_3}})^2 + 0.000186323 \cdot [\,j_i \cdot (h_i + j_i \cdot \overline{NO_3})^{-0.5}]^2 \cdot (\overline{NO_3})^2$$
$$+ (se_{\overline{TOC}})^2 + 0.00073984 \cdot (\overline{TOC})^2 + 0.0711876 \cdot (se_{\overline{formate}})^2 + 0.000043431 \cdot (\overline{formate})^2$$
$$+ 0.0744853 \cdot (se_{\overline{oxalate}})^2 + 0.000189956 \cdot (\overline{oxalate})^2$$

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[f] Note the multiplications by 0.9697 and 0.9459. This makes the adjustments for the potential biases in the measured formate and oxalate content, respectively, of the SME.

Note that Equation 8 may be restated in a more generic form that will support the estimation of the carbon concentration from antifoam, AF_C, for either the SRAT, the SME, or the MFT (based upon analytical measurements of TOC, oxalate, and formate samples from the tank in question). This estimation method holds regardless of the sludge batch being processed, and thus, it would be applicable for future sludge batches beyond SB8. The form of the resulting equation is given by:

Equation 11

$$AF_C = \overline{TOC} - f_C \cdot \overline{formate} \cdot 0.9697 - o_C \cdot \overline{oxalate} \cdot 0.9459$$

with an estimated random error variance, based upon the measurement uncertainty information in [2], given by

Equation 12

$$\begin{aligned}\operatorname{var}(AF_C) \approx\ & (se_{\overline{TOC}})^2 + 0.00073984 \cdot (\overline{TOC})^2 + 0.0711876 \cdot (se_{\overline{formate}})^2 + 0.000043431 \cdot (\overline{formate})^2 \\ & + 0.0744853 \cdot (se_{\overline{oxalate}})^2 + 0.000189956 \cdot (\overline{oxalate})^2\end{aligned}$$

Since potential biases in the measurements have been addressed in this approach, there is no additional bias associated with this estimate of AF_C. That is, the AF_C value determined by Equation 11 is unbiased.

3. Discussion of Antifoam Tracking System

During the processing of SB8, the proposed strategy would have to meet the restrictions imposed by Equation 1 through Equation 6 and these rely on analytical measurements of both TOC and NO_3. However, the benefit of the proposed strategy derives from an improvement in the approach of demonstrating that antifoam carbon is bounded by one of the limits imposed by Equation 7 and the set of parameters given in Table 2. Specifically, the proposed strategy relies on tracking the antifoam additions affecting the processing of each SME batch to estimate the carbon concentration from antifoam instead of relying of deriving this estimate from analytical measurements of SME Product samples. Another benefit of this alternate approach is that it will be transferable to future sludge batches. That is, this is a generic approach to estimating the carbon concentration from antifoam in the SME Product and is not specific to SB8.

Such additions occur to the Additive Mix Feed Tank (AMFT), to the SRAT, and to the SME. The role of the AMFT is to be the source of antifoam that is added to the SRAT and to the SME. In addition, the role of the SRAT is to be the source of feed to the SME tank, where this feed would also contain antifoam from additions that were made to the SRAT during routine processing. The SME tank is the hold point in the DWPF process flow at which acceptability decisions regarding the contents of the SME are made. These acceptability decisions are two-fold: (1) Product Composition Control System (PCCS) acceptability, whose criteria are defined by [5] and are not the subject of this report and (2) antifoam carbon acceptability with criteria as defined above from Choi [1].

A tracking system for antifoam mass (M_{AF}) is the proposed, alternative strategy for meeting the second set of acceptability criteria. So, for each of the three tanks listed above (i.e., the AMFT, SRAT, and SME) the goal is to maintain, continuously, the status of the antifoam mass in the tank along with an estimate of the uncertainty of that mass. Two types of uncertainties are to be tracked: (1) random uncertainty and (2) systematic or bias uncertainty. The metric for each source of random uncertainty is an estimated standard deviation for that source while a bound on the bias uncertainty from each contributor to this type of uncertainty is to be estimated at a 95% confidence level. Each type of uncertainty (i.e., random and bias) is to be estimated and maintained separately for each tank (i.e., AMFT, SRAT, and SME) as part of the antifoam tracking system.

With the antifoam status of each tank known, there are several events that have to be processed by the tracking system. These include: an antifoam addition to the AMFT, a transfer of antifoam from the AMFT to the SRAT, a transfer of antifoam from the AMFT to the SME, a transfer of SRAT product from the SRAT to the SME, and, finally, an acceptability decision for a SME Product batch relative to its antifoam content. As each of these events is processed, the impact of the event on the antifoam mass and on its uncertainty (both bias and random) for each of the tanks must be determined.

In addition, the strategy must be able to address any event that leads to the status of the antifoam mass and/or its uncertainty being unavailable or coming into question for any of these tanks. This will require a re-base-lining of the status of the impacted tank utilizing an analytically (e.g., analyses of samples of the tank contents for the SRAT or for the SME, see Equation 11 and Equation 12 above) driven assessment. More on this as each tank is discussed.

A process flow diagram for each tank and each of the events identified above has been prepared during the development of the antifoam tracking system. The software package GUM Workbench Version 2.41.410 [6] was used to support the evaluation of the uncertainties associated with the antifoam tracking system. Specifically, equations utilized to represent the events of the process flow diagram were developed in GUM Workbench and the software was used to generate the partial derivatives required for the determination of uncertainties associated with those equations. This is more fully discussed in the sections that follow.

3.1 Overview of Uncertainty Evaluations

Discussions in [7] were used as the basis of the approach for estimating the uncertainty for the antifoam mass presented in this report. Recall that there are two types of uncertainties being tracked: (1) random and (2) systematic or bias. The metric for each source of random uncertainty is an estimated standard deviation for that source while a bound on each bias uncertainty is to be estimated at a 95% confidence for each contributing source.

The random uncertainty variance of the antifoam mass, M_{AF}, is estimated by appealing to a Taylor Series expansion of the equation providing the value of M_{AF} determined at that processing step. For example, if $M_{AF} = f(x_1,x_2,x_3)$ for some function, f, of three variables x_1, x_2, and x_3, then the estimated random variance of M_{AF} is given by:

Equation 13

$$Variance(M_{AF}) \approx \left(\frac{\partial f}{\partial x_1}\right)^2 \times variance(x_1) + \left(\frac{\partial f}{\partial x_2}\right)^2 \times variance(x_2) + \left(\frac{\partial f}{\partial x_3}\right)^2 \times variance(x_3)$$

where $\left(\frac{\partial f}{\partial x_{.}}\right)$ represents the partial derivative of the function f relative to the variable $x_{.}$ and there is no correlation among the random uncertainties from the variables x_1, x_2, and x_3[♣].

A bound (at 95% confidence) on the bias of the antifoam mass, M_{AF}, is estimated also by appealing to a Taylor Series expansion of the equation providing the value of M_{AF} determined at that processing step. For example, if $M_{AF} = f(x_1,x_2,x_3)$ for some function, f, of three variables x_1, x_2, and x_3 and b_1, b_2, and b_3, respectively, are bounds (all positive) on the estimated bias for these terms, then the estimated variance of M_{AF} is given by:

Equation 14

$$\{bias(M_{AF})\}^2 \approx \left(\frac{\partial f}{\partial x_1}\right)^2 \times (b_1)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \times (b_2)^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 \times (b_3)^2$$

$$+2 \times |\rho_{12}| \times \left|\frac{\partial f}{\partial x_1}\right| \times \left|\frac{\partial f}{\partial x_2}\right| \times b_1 \times b_2 + 2 \times |\rho_{13}| \times \left|\frac{\partial f}{\partial x_1}\right| \times \left|\frac{\partial f}{\partial x_3}\right| \times b_1 \times b_3$$

$$+2 \times |\rho_{23}| \times \left|\frac{\partial f}{\partial x_2}\right| \times \left|\frac{\partial f}{\partial x_3}\right| \times b_2 \times b_3$$

where $\left(\frac{\partial f}{\partial x_{.}}\right)$ represents the partial derivative of the function f relative to the variable $x_{.}$ and ρ_{ij} represents a potential correlation between the biases for variables x_i and x_j. The expression for this equation is provided in a manner to demonstrate how to include the impact of potential correlations among biases. Where there appears to be some likelihood of a correlation between a pair of biases, the absolute values of the partials and a value for the correlation of 1 (i.e., ρ_{ij} =1) will be included in the Taylor Series expansion. This approach will ensure that potential correlations that could lead to increased uncertainty are accounted for in a conservative manner. More will be said regarding potential correlations for biases as necessary in the discussions that follow.

3.2 Tracking Antifoam in the AMFT

The AMFT is the tank where antifoam is prepared for introduction into the DWPF process. Additions of antifoam to the AMFT and transfers of antifoam from the AMFT to the SRAT or to the SME are routine actions during DWPF operations. To support the tracking of antifoam for a SME batch, the M_{AF} value and its uncertainty associated with the AMFT contents are to be maintained at all times. If these values are not known, the contents of the AFMT must be re-base-lined. This requires that the AMFT be emptied and rinsed and its contents reinitialized with a bounded mass of antifoam. This operation will re-establish the value of M_{AF}, and since it is bounding the random and bias uncertainties are set to zero. More of this aspect of the AF tracking system will be discussed below.

With M_{AF} status of the AMFT known, the antifoam tracking system must be capable of handling (1) events involving transfers from the AMFT to either the SRAT or the SME. Handling these events entails

♣ For the situations evaluated in this report, there are no correlations among random uncertainties. This is not the case for the bias uncertainties, as seen in the discussions that follow.

updating the status of the M_{AF} value and its uncertainty in the AMFT as well as updating the status of the M_{AF} and its uncertainty in the receiving tank, and (2) events involving the addition of antifoam to the AMFT.

Exhibit 1 provides a flow diagram for processing an event involving the AMFT. For such an event, there is an initial assessment of the current status of the M_{AF} value and its uncertainty in the AMFT. That is represented by the first decision step in the process flow diagram. If the status is unknown, then the "No" branch is taken out of this decision block and the value of the M_{AF} and its uncertainty must be re-base-lined as indicated in Step 1 of the diagram. If the status is known, the "Yes" branch is taken out of this decision block. With the M_{AF} value and its uncertainty known, the next decision block is evaluated to determine the event that needs to be addressed by the tracking system. There are two events captured in this flow diagram: a) if the AMFT level is low, then an antifoam addition is to be made to the AMFT and the impact of this addition on the M_{AF} value must be determined (indicated as Step 2 in Exhibit 1) or b) if a transfer of antifoam is to be made from the AMFT, then the impact of the change in the level of the contents of the AMFT (determined by information from the bubbler instrumentation in the tank) as the transfer is made must be utilized to provide information on the mass of antifoam for the receiving tank involved in the transfer (indicated as Step 3 in Exhibit 1) and to update the M_{AF} status of the AMFT after the transfer (indicated as Step 4 in Exhibit 1).

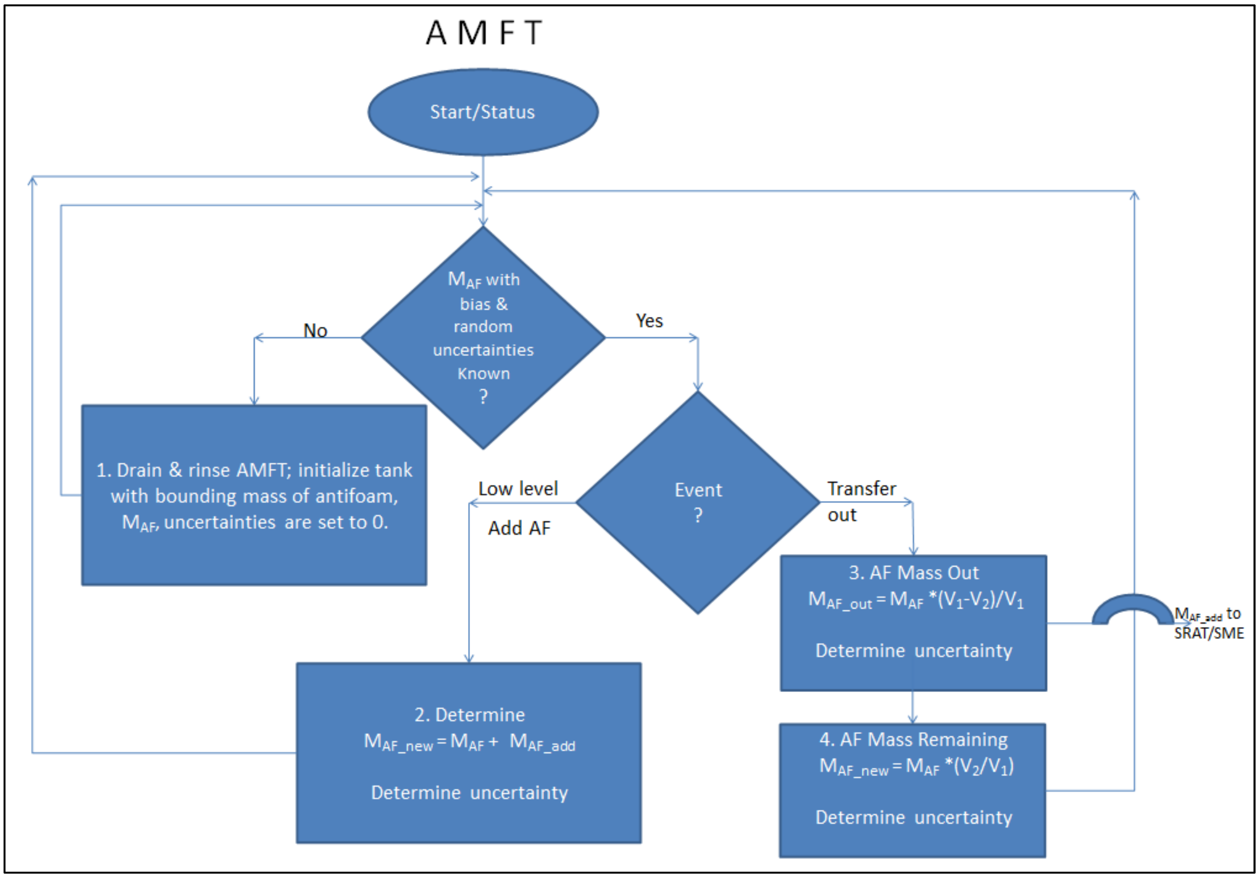

Exhibit 1 Process Flow for Tracking Antifoam Mass in the AMFT

Exhibit 2 provides an overview of the AMFT calculations supporting the antifoam tracking system. Step 1 is executed if the M_{AF} value or its uncertainty is unknown for the AMFT; this results in the re-establishment of a M_{AF} value along with its one-sigma random uncertainty and the bounding bias. The re-base-lining of the value for M_{AF} will utilize a bounding value for this mass, 18.91 kg, and the uncertainties (bias and random) are set to zero. See Appendix A for the development of this bounding mass for Step 1.

AMFT Calculations

1. Initial antifoam makeup or re-base-lining of the AMFT
   Drain and rinse the tank. Add 5-gallon pail of antifoam and dilute with 95 gallons of water. Mass of antifoam, M_{AF}, is bounded by 18.91 kg (See Appendix A). Random uncertainty is zero and bias uncertainty is zero.

2. Adding Antifoam to the AMFT
   Mass of antifoam added, M_{AF_Add}, is bounded by 18.91 kg (See Appendix A). For the M_{AF_Add}, the 1-sigma random uncertainty is zero and the bias uncertainty is zero. The mass of antifoam after the addition is given by:

$$M_{AF\_new} = M_{AF} + M_{AF\_Add}$$

3. Amount of Antifoam Transferred Out of the AMFT
   The amount of antifoam transferred out of the AMFT to the receiving vessel is

$$M_{AF_{out}} = M_{AF} \cdot \frac{(V_1 - V_2)}{V_1}$$

   where V_1 and V_2 are volumes in the AMFT which can be represented in terms of the values for i = 1 (before the transfer) and 2 (after the transfer) of their source instrumentation, LI2614, (See Appendix C) as

$$V_i = 1.1686 + 2.9466 \cdot \frac{LI2614_i}{\rho}$$

   where ρ is the density of the contents of the AMFT (See Appendix D).

4. Amount of Antifoam in the AMFT Following a Transfer Out
   The amount of antifoam remaining in AMFT after a transfer out to a receiving vessel is

$$M_{AF_{new}} = M_{AF} \cdot \frac{V_2}{V_1}$$

   where the volumes V_1 and V_2 are the same as those used in Step 3.

Exhibit 2 AMFT Calculations Supporting the Antifoam Tracking System

3.2.1 AMFT Step 2 Processing

For Step 2, the one-sigma random uncertainty for the new value of M_{AF} (i.e., M_{AF_New} in the Step 2 equation) is the square root of the sum of the variances of the random errors of the two terms on the right-hand side of the Step 2 equation: M_{AF} and M_{AF_Add}. The bias of the M_{AF_New} value is taken as the sum of the biases of the two terms on the right-side of the Step 2 equation. This way of combining the biases (i.e., directly summing) is more conservative than combining them in quadrature (i.e., root sum of squares). Note that the random and bias uncertainties of the addition are zero, since a bounded value for the mass of the addition is utilized. A sample calculation for this step is provided in Exhibit B1 in Appendix B.

3.2.2 AMFT Step 3 Processing

So, consider the impact of a Step 3 event with the AMFT M_{AF} information known. The equation for the M_{AF_out} is given by:

Equation 15

$$M_{AF_{out}} = \frac{M_{AF} \cdot (V_1 - V_2)}{V_1}$$

where the volumes V_1 and V_2 in gallons are intermediary values which are determined from the level instrument LI2614 and the specific gravity, ρ, as described in Exhibit 2 with the two volumes each having an additional random variability that is to be incorporated into the random uncertainty and biases that are to be incorporated in the bias uncertainty of M_{AF_out}.

Following the Taylor's Series expansion approach described above, the estimated variance of M_{AF_out} for Step 3 of Exhibit 1 may be written in terms of the fundamental measurements with δ_1 and δ_2 representing the random uncertainties for V_1 and V_2, respectively, as:

Equation 16

$$Variance\left(M_{AF_{out}}\right) \approx \left(\frac{\partial M_{AF_{out}}}{\partial M_{AF}}\right)^2 \times variance(M_{AF}) + \left(\frac{\partial M_{AF_{out}}}{\partial LI2614_1}\right)^2 \times variance(LI2614_1)$$

$$+ \left(\frac{\partial M_{AF_{out}}}{\partial \rho}\right)^2 \times variance(\rho) + \left(\frac{\partial M_{AF_{out}}}{\partial LI2614_2}\right)^2 \times variance(LI2614_2)$$

$$+ \left(\frac{\partial M_{AF_{out}}}{\partial \delta_1}\right)^2 \times variance(\delta_1) + \left(\frac{\partial M_{AF_{out}}}{\partial \delta_2}\right)^2 \times variance(\delta_2)$$

where the numerical subscripts 1 and 2 represent the before and after, respectively, transfer values for the level instrument LI2614.

Exhibit 3 provides equations developed using GUM Workbench that support the evaluation of M_{AF_out} determined in Step 3. The model equation and associated intermediary values supporting the determination of M_{AF_out} for Step 3 are given in the upper portion of Exhibit 3 and the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_out} value are given in the lower portion of this exhibit.

Equation:

$$V_1=2.9466*(LI2614_1/\rho)+1.1686+\delta_1;$$
$$V_2=2.9466*(LI2614_2/\rho)+1.1686+\delta_2;$$
$$M_{AF\_out}=M_{AF}*(V_1-V_2)/V_1;$$

Partial Derivatives:

$$\partial V_1/\partial LI2614_1 = 2.9466\cdot 1.0/\rho;$$
$$\partial V_1/\partial \rho = 2.9466\cdot(-LI2614_1)/sqr(\rho);$$
$$\partial V_1/\partial \delta_1 = 1.0;$$
$$\partial V_2/\partial \rho = 2.9466\cdot(-LI2614_2)/sqr(\rho);$$
$$\partial V_2/\partial LI2614_2 = 2.9466\cdot 1.0/\rho;$$
$$\partial V_2/\partial \delta_2 = 1.0;$$
$$\partial M_{AF\_out}/\partial LI2614_1 = M_{AF}\cdot\partial V_1/\partial LI2614_1/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial LI2614_1)/sqr(V_1);$$
$$\partial M_{AF\_out}/\partial \rho = M_{AF}\cdot(\partial V_1/\partial \rho + -\partial V_2/\partial \rho)/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial \rho)/sqr(V_1);$$
$$\partial M_{AF\_out}/\partial \delta_1 = M_{AF}\cdot\partial V_1/\partial \delta_1/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial \delta_1)/sqr(V_1);$$
$$\partial M_{AF\_out}/\partial LI2614_2 = M_{AF}\cdot(-\partial V_2/\partial LI2614_2)/V_1;$$
$$\partial M_{AF\_out}/\partial \delta_2 = M_{AF}\cdot(-\partial V_2/\partial \delta_2)/V_1;$$
$$\partial M_{AF\_out}/\partial M_{AF} = (V_1 - V_2)/V_1;$$

Exhibit 3 Equations for Calculating the Standard Deviation of the Random Uncertainty of the M_{AF} Transferred Out of the AMFT

To complete the information necessary to compute the estimate of the random variance of the M_{AF_out}, estimates of the variances of the terms of Equation 16 are needed. These values are provided in Table 3.

Table 3 Terms and Estimated Random Uncertainties Supporting Equation 16

Term/Instrument	Description	1-Sigma Random Uncertainty
M_{AF}	mass of antifoam (kg)	from the AMFT status
LI2614 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 41 inwc span [10];
	Distribution Control System (DCS) deviation limit (inwc)	±0.025 inwc [10]
		Using a uniform distribution, 1-sigma random uncertainty (inwc) is $[(0.41/\sqrt{3})^2+(0.025/\sqrt{3})^2]^{0.5} = 0.2372$
ρ (this value is set to 1)	Specific Gravity of AMFT Material	0.0036♣
δ_1	Calibration uncertainty (see Appendix C for details)	0.637 gallon
δ_2	Calibration uncertainty (see Appendix C for details)	0.637 gallon

♣ See Appendix D for a discussion of the uncertainties associated with the density assessments of the AMFT.

With the information from Exhibit 1, Exhibit 2, and Equation 14 as background, the estimated bias error for M_{AF_out} , determined in Step 3, using the Taylor's Series approach, may be written as:

Equation 17

$$\{bias(M_{AF_{out}})\}^2 \approx \left(\frac{\partial M_{AF_{out}}}{\partial M_{AF}}\right)^2 \times \{bias(M_{AF})\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial LI2614_1}\right)^2 \times \{bias(LI2614_1)\}^2$$

$$+\left(\frac{\partial M_{AF_{out}}}{\partial \rho}\right)^2 \times \{bias(\rho)\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial LI2614_2}\right)^2 \times \{bias(LI2614_2)\}^2$$

$$+\left(\frac{\partial M_{AF_{out}}}{\partial b_1}\right)^2 \times \{bias(b_1)\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial b_2}\right)^2 \times \{bias(b_2)\}^2$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{out}}}{\partial LI2614_1}\right| \times \left|\frac{\partial M_{AF_{out}}}{\partial LI2614_2}\right| \times bias(LI2614_1) \times bias(LI2614_2)$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{out}}}{\partial b_1}\right| \times \left|\frac{\partial M_{AF_{out}}}{\partial b_2}\right| \times bias(b_1) \times bias(b_2)$$

Note that in evaluating Equation 17, the bias for the M_{AF} term, i.e., bias(M_{AF}) term, is provided by the status information for the AMFT prior to the transfer out and that two potential correlations among the biases are introduced into the equation, both represented in a bounding manner. So the approach may be stated as: the b_1 and b_2 terms are the estimated bias in the V_1 and V_2 volumes, respectively, and there is a potential correlation in these biases. Also, there is a potential correlation in the biases for the two LI2614 measurements. Basically, a perfect correlation is assumed and the sign of the correlation (i.e., representing the correlation as positive or negative) is taken as the worst of the two possibilities. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of M_{AF_out} for Step 3 (see the upper portion of Exhibit 4) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_out} value (see the lower portion of Exhibit 4). To complete the information necessary to compute the estimate of the bias of the M_{AF_out}, estimates of the bias terms of Equation 17 are needed. Table 4 provides the details of the bias information needed to complete the estimation of the bias for the M_{AF_out} value. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B2 in Appendix B.

Equation:

$$V_1=2.9466*(LI2614_1/\rho)+1.1686+b_1;$$
$$V_2=2.9466*(LI2614_2/\rho)+1.1686+b_2;$$
$$M_{AF\_out}=M_{AF}*(V_1-V_2)/V_1;$$

Partial Derivatives:

$$\partial V_1/\partial LI2614_1 = 2.9466 \cdot 1.0/\rho;$$

$$\partial V_1/\partial \rho = 2.9466 \cdot (-LI2614_1)/sqr(\rho);$$

$$\partial V_1/\partial b_1 = 1.0;$$

$$\partial V_2/\partial \rho = 2.9466 \cdot (-LI2614_2)/sqr(\rho);$$

$$\partial V_2/\partial LI2614_2 = 2.9466 \cdot 1.0/\rho;$$

$$\partial V_2/\partial b_2 = 1.0;$$

$$\partial M_{AF\_out}/\partial LI2614_1 = M_{AF} \cdot \partial V_1/\partial LI2614_1/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial LI2614_1)/sqr(V_1);$$

$$\partial M_{AF\_out}/\partial \rho = M_{AF} \cdot (\partial V_1/\partial \rho + -\partial V_2/\partial \rho)/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial \rho)/sqr(V_1);$$

$$\partial M_{AF\_out}/\partial b_1 = M_{AF} \cdot \partial V_1/\partial b_1/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial b_1)/sqr(V_1);$$

$$\partial M_{AF\_out}/\partial LI2614_2 = M_{AF} \cdot (-\partial V_2/\partial LI2614_2)/V_1;$$

$$\partial M_{AF\_out}/\partial b_2 = M_{AF} \cdot (-\partial V_2/\partial b_2)/V_1;$$

$$\partial M_{AF\_out}/\partial M_{AF} = (V_1 - V_2)/V_1;$$

Exhibit 4 Equations for Calculating the Bias of the M_{AF} Transferred Out of the AMFT

Table 4 Terms and Estimated Bias Uncertainties Supporting Equation 17

Term/Instrument	Description	Bias Error at 95% Confidence
M_{AF}	mass of antifoam (kg)	bias from the AMFT status information
LI2614 with subscripts 1 and 2	level bubbler values (inwc)	1% of 41 inwc span = 0.41 inwc [10]
ρ (this value is set to 1)	specific gravity of AMFT material	0.0108♣
b_1	calibration uncertainty (see Appendix C for details)	0.232 gallon
b_2	calibration uncertainty (see Appendix C for details)	0.232 gallon

♣ See Appendix D for a discussion of the uncertainties associated with the density assessments of the AMFT.

3.2.3 AMFT Step 4 Processing

Consider the determination of the M_{AF} information after a transfer from the AMFT (Step 4 of Exhibit 1 and Exhibit 2). The M_{AF_new} value is given by:

Equation 18

$$M_{AF_{new}} = \frac{M_{AF} \cdot V_2}{V_1}$$

Using the approach described above, the estimated random uncertainty variance of M_{AF_new} determined for that step may be written as:

Equation 19

$$Variance\left(M_{AF_{new}}\right) \approx \left(\frac{\partial M_{AF_{new}}}{\partial M_{AF}}\right)^2 \times variance(M_{AF}) + \left(\frac{\partial M_{AF_{new}}}{\partial LI2614_1}\right)^2 \times variance(LI2614_1)$$
$$+\left(\frac{\partial M_{AF_{new}}}{\partial \rho}\right)^2 \times variance(\rho) + \left(\frac{\partial M_{AF_{new}}}{\partial LI2614_2}\right)^2 \times variance(LI2614_2)$$
$$+\left(\frac{\partial M_{AF_{new}}}{\partial \delta_1}\right)^2 \times variance(\delta_1) + \left(\frac{\partial M_{AF_{new}}}{\partial \delta_2}\right)^2 \times variance(\delta_2)$$

GUM Workbench was used to develop equations supporting the evaluation of Step 4 and the resulting equations are provided in Exhibit 5. Also, in this exhibit is the set of partial derivatives needed to support the estimation of the variance of the M_{AF_new} value. To complete the information necessary to compute the estimate of the variance of the M_{AF_new}, estimates of the variance terms of Equation 19 are needed. These values are provided in Table 5.

Table 5 Terms and Estimated Random Uncertainties Supporting Equation 19

Term/Instrument	Description	1-Sigma Random Uncertainty
M_{AF}	mass of antifoam (kg)	from the AMFT status
LI2614 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 41 inwc span [10];
	DCS deviation limit (inwc)	±0.025 inwc [10]
		Using a uniform distribution, 1-sigma random uncertainty (inwc) is $[(0.41/\sqrt{3})^2+(0.025/\sqrt{3})^2]^{0.5} = 0.2372$
ρ (this value is set to 1)	specific gravity of AMFT material	0.0036♣
δ_1	calibration uncertainty (see Appendix C for details)	0.637gallon
δ_2	calibration uncertainty (see Appendix C for details)	0.637 gallon

♣ See Appendix D for a discussion of the uncertainties associated with the density assessments of the AMFT.

Equation:

$$V_1=2.9466*(LI2614_1/\rho)+1.1686+\delta_1;$$
$$V_2=2.9466*(LI2614_2/\rho)+1.1686+\delta_2;$$
$$M_{AF\_new}=M_{AF}*V_2/V_1;$$

Partial Derivatives:

$$\partial V_1/\partial LI2614_1 = 2.9466 \cdot 1.0/\rho;$$

$$\partial V_1/\partial \rho = 2.9466 \cdot (-LI2614_1)/sqr(\rho);$$

$$\partial V_1/\partial \delta_1 = 1.0;$$

$$\partial V_2/\partial \rho = 2.9466 \cdot (-LI2614_2)/sqr(\rho);$$

$$\partial V_2/\partial LI2614_2 = 2.9466 \cdot 1.0/\rho;$$

$$\partial V_2/\partial \delta_2 = 1.0;$$

$$\partial M_{AF\_new}/\partial LI2614_1 = (-M_{AF} \cdot V_2 \cdot \partial V_1/\partial LI2614_1)/sqr(V_1);$$

$$\partial M_{AF\_new}/\partial \rho = M_{AF} \cdot \partial V_2/\partial \rho/V_1 + (-M_{AF} \cdot V_2 \cdot \partial V_1/\partial \rho)/sqr(V_1);$$

$$\partial M_{AF\_new}/\partial \delta_1 = (-M_{AF} \cdot V_2 \cdot \partial V_1/\partial \delta_1)/sqr(V_1);$$

$$\partial M_{AF\_new}/\partial LI2614_2 = M_{AF} \cdot \partial V_2/\partial LI2614_2/V_1;$$

$$\partial M_{AF\_new}/\partial \delta_2 = M_{AF} \cdot \partial V_2/\partial \delta_2/V_1;$$

$$\partial M_{AF\_new}/\partial M_{AF} = V_2/V_1;$$

Exhibit 5 Equations for Calculating the Random Uncertainty for the M_{AF} Remaining After the Transfer Out of the AMFT

The estimated bias error for M_{AF_new} , determined in Step 4, may be written as:

Equation 20

$$\left\{bias\left(M_{AF_{new}}\right)\right\}^2 \approx \left(\frac{\partial M_{AF_{new}}}{\partial M_{AF}}\right)^2 \times \{bias(M_{AF})\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI2614_1}\right)^2 \times \{bias(LI2614_1)\}^2$$
$$+ \left(\frac{\partial M_{AF_{new}}}{\partial \rho}\right)^2 \times \{bias(\rho)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI2614_2}\right)^2 \times \{bias(LI2614_2)\}^2$$
$$+ \left(\frac{\partial M_{AF_{new}}}{\partial b_1}\right)^2 \times \{bias(b_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial b_2}\right)^2 \times \{bias(b_2)\}^2$$
$$+2 \times 1 \times \left|\frac{\partial M_{AF_{new}}}{\partial LI2614_1}\right| \times \left|\frac{\partial M_{AF_{new}}}{\partial LI2614_2}\right| \times bias(LI2614_1) \times bias(LI2614_2)$$
$$+2 \times 1 \times \left|\frac{\partial M_{AF_{new}}}{\partial b_1}\right| \times \left|\frac{\partial M_{AF_{new}}}{\partial b_2}\right| \times bias(b_1) \times bias(b_2)$$

Note that in evaluating Equation 20, the bias for the M_{AF} term, i.e., bias(M_{AF}) term, is provided by the status information for the AMFT prior to the transfer out and that two potential correlations among the biases are introduced into the equation, both represented in a bounding manner. So the approach may be stated as: the b_1 and b_2 terms are the estimated biases in the V_1 and V_2 volumes, respectively, and there is a potential correlation in these biases. Also, there is a potential correlation in the biases for the two LI2614 measurements. Basically, a perfect correlation is assumed and the sign of the correlation (i.e., representing the correlation as positive or negative) is taken as the worst of the two possibilities. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of M_{AF_new} for Step 4 (see the upper portion of Exhibit 6) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_new} value (see the lower portion of Exhibit 6). To complete the information necessary to compute the estimate of the bias of the M_{AF_out}, estimates of the bias terms of Equation 20 are needed. Table 6 provides the details of the bias information needed to complete the estimation of the bias for the M_{AF_new} value. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B3 in Appendix B.

	Equation: V_1=2.9466*(LI2614$_1$/ρ)+1.1686+b_1; V_2=2.9466*(LI2614$_2$/ρ)+1.1686+b_2; M_{AF_new}=M_{AF}*V_2/V_1;
	Partial Derivatives: $\partial V_1/\partial LI2614_1 = 2.9466\cdot 1.0/\rho$; $\partial V_1/\partial \rho = 2.9466\cdot(-LI2614_1)/sqr(\rho)$; $\partial V_1/\partial b_1 = 1.0$; $\partial V_2/\partial \rho = 2.9466\cdot(-LI2614_2)/sqr(\rho)$; $\partial V_2/\partial LI2614_2 = 2.9466\cdot 1.0/\rho$; $\partial V_2/\partial b_2 = 1.0$; $\partial M_{AF\_new}/\partial LI2614_1 = (-M_{AF}\cdot V_2\cdot \partial V_1/\partial LI2614_1)/sqr(V_1)$; $\partial M_{AF\_new}/\partial \rho = M_{AF}\cdot \partial V_2/\partial \rho/V_1 + (-M_{AF}\cdot V_2\cdot \partial V_1/\partial \rho)/sqr(V_1)$; $\partial M_{AF\_new}/\partial b_1 = (-M_{AF}\cdot V_2\cdot \partial V_1/\partial b_1)/sqr(V_1)$; $\partial M_{AF\_new}/\partial LI2614_2 = M_{AF}\cdot \partial V_2/\partial LI2614_2/V_1$; $\partial M_{AF\_new}/\partial b_2 = M_{AF}\cdot \partial V_2/\partial b_2/V_1$; $\partial M_{AF\_new}/\partial M_{AF} = V_2/V_1$;

Exhibit 6 Equations for Calculating the Bias for the M_{AF} Remaining After the Transfer Out of the AMFT

Table 6 Terms and Estimated Bias Uncertainties Supporting Equation 20

Term/Instrument	Description	Bias Error at 95% Confidence
M_{AF}	antifoam carbon concentration (mg/kg)	Bias from the AMFT status information
LI2614 with subscripts 1 and 2	Level Bubbler (inwc)	1% of 41 inwc span = 0.41 inwc [10]
ρ (this value is set to 1)	Specific Gravity of AMFT Material	0.0108♣
b_1	Calibration uncertainty (see Appendix C for details)	0.232 gallon
b_2	Calibration uncertainty (see Appendix C for details)	0.232 gallon

♣ See Appendix D for a discussion of the uncertainties associated with the density assessments of the AMFT.

3.3 Tracking Antifoam in the SRAT

The SRAT is a sludge preparation tank where antifoam is added by transfers from the AMFT. To support the tracking of antifoam for a SME batch, the M_{AF} value and its uncertainty (i.e., a one-sigma random uncertainty and a bias uncertainty at 95% confidence) associated with the SRAT contents are to be maintained at all times. If these values are not known, the contents of the SRAT are to be sampled and analyzed to re-baseline the antifoam mass and its uncertainty in this tank. With the values for the M_{AF} and its uncertainty known, the antifoam tracking system must be capable up handling two types of events: (1) an event involving a transfer from the AMFT into the SRAT and (2) an event involving the transfer of SRAT product to the SME. Handling these events entails updating the status of the M_{AF} value and its uncertainty in the SRAT for event types (1) and (2) as well as updating the status of the M_{AF} and its uncertainty in the SME tank to complete the impact of a type (2) event (addressed in the next section).

Exhibit 7 provides a flow diagram at the SRAT level for processing an event involving the SRAT. For any SRAT-related event, there is an initial assessment of the current status of the M_{AF} value and its uncertainty in the SRAT as represented by the first decision step in the event flow diagram. If the M_{AF} status is unknown, then the "No" branch is taken out of this decision block and the value of the M_{AF} and its uncertainty must be re-base-lined as indicated in Step 1 of the diagram. If the status is known, the "Yes" branch is taken out of this decision block. With the M_{AF} value and its uncertainty known for the SRAT, the next decision block is evaluated to determine the type of event that needs to be addressed by the tracking system. Once again there are two types of events captured in this flow diagram: a) if there is a transfer from the AMFT to the SRAT, then an antifoam addition is to be made to the SRAT and the impact of this addition on the M_{AF} value and its uncertainty must be determined (indicated as Step 2 in Exhibit 7) or b) if a transfer of SRAT product is to be made to the SME, then the impact of the change in the level of the contents of the SRAT (determined by information from the bubbler instrumentation in the tank) as the transfer is made must be utilized to provide information on the mass of antifoam for the receiving tank (i.e., the SME) involved in the transfer (indicated as Step 3 in Exhibit 7) and to update the M_{AF} status of the SRAT after the transfer (indicated as Step 4 in Exhibit 7).

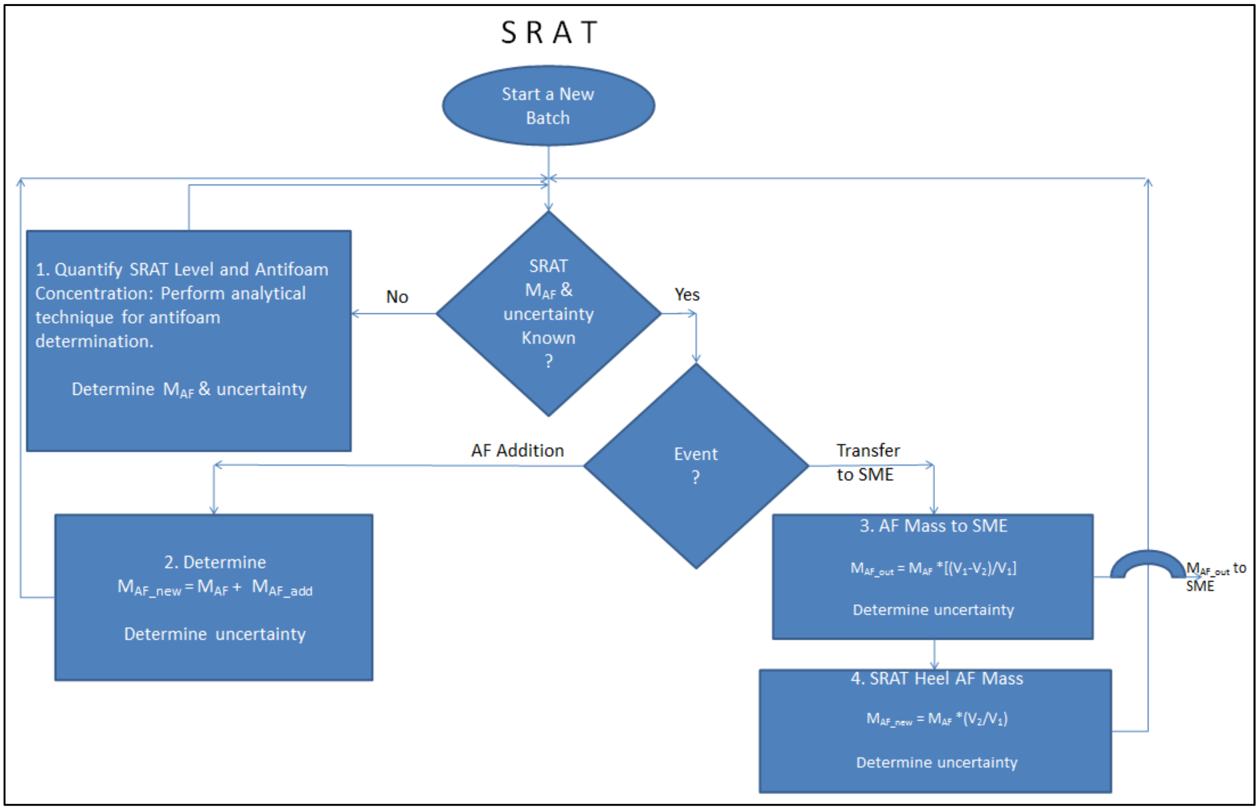

Exhibit 7 Process Flow for Tracking Antifoam Mass in the Sludge Receipt and Adjustment Tank (SRAT)

Exhibit 8 and Exhibit 9 provide an overview of the SRAT calculations supporting the antifoam tracking system. The calculations for Step 1 are executed if the M_{AF} value or its uncertainty is unknown for the SRAT; and these calculations re-establish the M_{AF} value along with its uncertainty.

3.3.1 SRAT Step 2 Processing

For Step 2, the uncertainty is to be updated as follows: the one-sigma random uncertainty for the new value of M_{AF} (i.e., M_{AF_New} in the Step 2 equation) is the square root of the sum of the random variances of the two terms on the right-hand side of the Step 2 equation (M_{AF} and M_{AF_Add}), and the bias for the M_{AF_New} value is simply the sum of the biases of the two terms on the right-hand side of the Step 2 equation. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B4 in Appendix B.

3.3.2 Overview of SRAT Steps 1, 3 and 4 Processing

Step 3 provides the determination of the M_{AF_out} value associated with the transfer from the SRAT to the SME. The role of the bubbler instruments is also indicated as part of the Step 3 calculations. Step 4 provides the mass of the antifoam remaining in the SRAT (i.e., the SRAT heel) after the transfer out has been completed. The calculations supporting each of steps 1, 3 and 4 of the tracking system for the SRAT are covered in turn in the following discussion.

SRAT Calculations

1. Re-base-lining of the SRAT

The SRAT vessel will be re-base-lined using the analytical method for antifoam concentration determination that is currently used. The mass of antifoam at the time of the SRAT sample can be determined by

$$M_{AF} = C_{AF} \cdot V_1 \cdot \rho$$

where the determination of volume, V_1, from the source instrumentation, LI3025 and DI3026, is described below and ρ is calculated from these instruments by

$$\rho = \frac{(LI3025_1 - DI3026_1)}{Sep}$$

and Sep stands for the separation between the two instruments. The Sep value is 47 inches.

The volume in the SRAT can be represented in terms of the value at event, i =1 (before a transfer) or i= 2 (after a transfer), of its source instrumentation LI3025 as:

$$For\ x_{i2} < \left(\frac{LI3025_i}{\rho} + Heel\right) < x_{i1},$$

$$\frac{\frac{LI3025_i}{\rho} + Heel - x_{i2}}{(x_{i1} - x_{i2})} \times (y_{i1} - y_{i2}) + y_{i2}$$

where the Heel is 6.77 inches and there are five sets of x's and y's corresponding to 5 segments within the SRAT. These values are (see reference [5]):

Segment	x_{i1}	x_{i2}	y_{i1}	y_{i2}
1 (lowest)	17.416	0	1000	0
2	78.513	17.416	5175	1000
3	138.28	78.513	9400	5175
4	158.89	138.28	10850	9400
5 (highest)	175.91	158.89	12000	10850

Exhibit 8 SRAT Calculations Supporting the Antifoam Tracking System (part 1 of 2)

2. Antifoam Mass in the SRAT after an Addition from the AMFT
   The mass of antifoam in the SRAT after the AMFT addition (the amount from the AMFT is as determined in AMFT Step 3) is given by:

$$M_{AF\_new} = M_{AF} + M_{AF\_Add}$$

3. Amount of Antifoam Transferred Out of the SRAT to the SME
   The amount of antifoam transferred out of the SRAT to the SME is

$$M_{AF_{out}} = M_{AF} \cdot \frac{(V_1 - V_2)}{V_1}$$

   where V_1 and V_2 are volumes in the SRAT which can be represented in terms of the values for i = 1 (before the transfer) and 2 (after the transfer) of their source instrumentation, LI3025. These values are determined as described in Step 1 above. Note that the density value, ρ, that is used in the determination of V_1 is also used in the determination of V_2.

4. Amount of Antifoam in the SRAT Heel Following a Transfer Out to the SME
   The amount of antifoam remaining in SRAT after a transfer out to a receiving vessel is

$$M_{AF_{new}} = M_{AF} \cdot \frac{V_2}{V_1}$$

   where the V_1 and V_2 are the same volumes from Step 3 above.

Exhibit 9 SRAT Calculations Supporting the Antifoam Tracking System (part 2 of 2)

3.3.3 SRAT Step 1Processing

The equation for Step 1 of Exhibit 8 provides a guide for re-base lining the M_{AF_new} (kg) value for the SRAT. Writing the equation out with more detail to include the appropriate unit conversions yields:

Equation 21

$$M_{AF_{new}} = \frac{3.7854 \cdot C_{AF} \cdot V_1 \cdot \rho}{0.4723 \cdot 1000000}$$

In this equation, C_{AF} represents the carbon concentration (mg/kg) from AF determined from the analytical measurements of the SRAT contents (see Equation 11) with the volume (gal), V_1, and density (assuming units of kg/L), ρ, determined as indicated in the Step 1 description of Exhibit 8 by measurements from instruments LI3025 and DI3026 along with values for the separation (Sep) between the bubblers and the heel (Heel) for LI3025. The value of 3.7854 is a conversion factor with units of L/gal. The value of 0.4723 is a conservative (i.e., bounding on the low side) conversion factor with units of kg of carbon per kg of antifoam[f]. The 1000000 value is a conversion factor with units of mg/kg. Note that ρ and V_1 are intermediary values with the V_1 value having an additional variability described below. Using the Taylor's Series expansion approach described above, the estimated random variance of M_{AF_new} for Step 1 of Exhibit 8 may be expressed in the fundamental measurements as given by:

Equation 22

$$Variance(M_{AF_{new}}) \approx \left(\frac{\partial M_{AF_{new}}}{\partial C_{AF}}\right)^2 \times variance(C_{AF}) + \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_1}\right)^2 \times variance(LI3025_1)$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial DI3026_1}\right)^2 \times variance(DI3026_1) + \left(\frac{\partial M_{AF_{new}}}{\partial \delta_1}\right)^2 \times variance(\delta_1)$$

where all of the estimated variances are for the random uncertainties of the indicated measurements. Specifically, the variance for C_{AF} is estimated from the analyses of the SRAT samples as given by Equation 12 and the variance(δ_1) term represents the variance of the random uncertainty associated with the computed volume, V_1.(see the upper portion of Exhibit 10 for the introduction of the δ_1 term into the model equation for M_{AF_new}). GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_new} for Step 1 (see the upper portion of Exhibit 10) and to document the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_new} value (see the lower portion of Exhibit 10). For example, x_1, x_2, y_1, and y_2 are appropriately selected values (based upon the value of the LI3025 instrument as indicated by $LI3025_1$) for determining volume as indicated in Exhibit 8.To complete the information necessary to compute the estimate of the variance of the M_{AF_new}, estimates of the variance terms of Equation 22 are needed. These values along with a description of the terms of Equation 22 are provided in Table 7.

Table 7 Terms and Estimated Random Uncertainties Supporting Equation 22

Term/Instrument	Description	1-Sigma Random Uncertainty
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	Analytical uncertainty (see Equation 12)
LI3025 with subscript 1	level bubbler value (inwc)	±1% of 231.6 inwc span [11]
	DCS deviation limit	±0.1 inwc [11]
		Using a uniform distribution, 1-sigma random uncertainty (inwc) is $[(2.316/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5}$ = 1.3384 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
	DCS deviation limit	±0.05 inwc [12]
		Using a uniform distribution, 1-sigma random uncertainty (inwc) is $[(1.61/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5}$ = 0.9300 inwc
δ_1	tank calibration uncertainty (see WSRC-TR-92-250 [8][♣])	1-sigma random = 9 gallons

[f] See SRNL E-Notebook O7787-00055-09, Antifoam 747 Basic Data and Acceptance Testing, July 29, 2014.

[♣] The random uncertainty of the tank calibration was estimated in this report for the SRAT and the SME as the total error of the Holledge gauge, 0.25 inch, times the slope of the calibration curve. For the SRAT, the largest slope is 70.696 gal/inch, leading to an estimate of the total (2-sigma) random uncertainty of 70.696 × 0.25 = 17.7 gal, or a 1-sigma random uncertainty of 9 gal.

Equation:

$Sep=47;$
$Heel=6.77;$
$\rho=(LI3025_1-DI3026_1)/Sep;$
$V_1=((((LI3025_1/\rho)+Heel-x_2)*(y_1-y_2))/(x_1-x_2))+y_2+\delta_1;$
$M_{AF\_new}=C_{AF}*V_1*\rho*3.7854/(0.4723*1000000);$

Partial Derivatives:

$\partial\rho/\partial LI3025_1 = 1.0/Sep;$

$\partial\rho/\partial DI3026_1 = (-1.0)/Sep;$

$\partial V_1/\partial LI3025_1 = (y_1 - y_2)\cdot(1.0/\rho + (-LI3025_1\cdot\partial\rho/\partial LI3025_1)/sqr(\rho))/(x_1 - x_2);$

$\partial V_1/\partial DI3026_1 = (y_1 - y_2)\cdot(-LI3025_1\cdot\partial\rho/\partial DI3026_1)/sqr(\rho)/(x_1 - x_2);$

$\partial V_1/\partial x_2 = (y_1 - y_2)\cdot(-1.0)/(x_1 - x_2) + (-(LI3025_1/\rho + Heel - x_2)\cdot(y_1 - y_2)\cdot(-1.0))/sqr(x_1 - x_2);$

$\partial V_1/\partial y_1 = (LI3025_1/\rho + Heel - x_2)/(x_1 - x_2);$

$\partial V_1/\partial y_2 = (LI3025_1/\rho + Heel - x_2)\cdot(-1.0)/(x_1 - x_2) + 1.0;$

$\partial V_1/\partial x_1 = (-(LI3025_1/\rho + Heel - x_2)\cdot(y_1 - y_2))/sqr(x_1 - x_2);$

$\partial V_1/\partial\delta_1 = 1.0;$

$\partial M_{AF\_new}/\partial LI3025_1 = 3.7854\cdot(C_{AF}\cdot V_1\cdot\partial\rho/\partial LI3025_1 + \rho\cdot C_{AF}\cdot\partial V_1/\partial LI3025_1)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial DI3026_1 = 3.7854\cdot(C_{AF}\cdot V_1\cdot\partial\rho/\partial DI3026_1 + \rho\cdot C_{AF}\cdot\partial V_1/\partial DI3026_1)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_2 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial x_2/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_1 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial y_1/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_2 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial y_2/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_1 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial x_1/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial\delta_1 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial\delta_1/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial C_{AF} = 3.7854\cdot\rho\cdot V_1/(0.4723\cdot 1000000.0);$

Exhibit 10 Equations for Re-Base-Lining the M_{AF} of the SRAT with Random Uncertainty[♣]

[♣] In this and in future exhibits that provide the partial derivations associated with a set of GUM Workbench model equations, note that the GUM Workbench software generates the partial derivative for every term that is included in the set of model equations. No effort was made to strip out from the exhibit the partial derivatives for those terms that are considered constants (i.e., terms for which no uncertainties need to be addressed). In this exhibit, the x's and y's are considered constants with no uncertainty.

To complete the updating of the M_{AF} status required for Step 1, the bias for M_{AF_new} determined by Equation 21 must be estimated. A bound (at 95% confidence) on the bias of the antifoam mass, M_{AF_new}, is estimated, as above, by appealing to a Taylor Series expansion of the Equation 21 in the fundamental measurements. Once again, note that ρ and V_1 are intermediary values and the Taylor's Series expansion may be expressed in the fundamental measurements as given by:

Equation 23

$$\{bias(M_{AF_{new}})\}^2 \approx \left(\frac{\partial M_{AF_{new}}}{\partial C_{AF}}\right)^2 \times \{bias(C_{AF})\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_1}\right)^2 \times \{bias(LI3025_1)\}^2$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial DI3026_1}\right)^2 \times \{bias(DI3026_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial \delta_1}\right)^2 \times \{b_1\}^2$$

Note that in evaluating Equation 23, the bias for the C_{AF} term, i.e., bias(C_{AF}) term is estimated to be zero and that there are no correlations among the bias terms in this equation. That is, the analytical estimate of the concentration of carbon from AF is unbiased. Also, the b_1 term is the estimated bias in the volume, V_1, of Equation 21. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_new} for Step 1 (see the upper portion of Exhibit 11) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_new} value (see the lower portion of Exhibit 11). Once again, x_1, x_2, y_1, and y_2 are appropriately selected values (based upon the value of the LI3025 instrument as indicated by $LI3025_1$) for determining volume as indicated in Exhibit 8.To complete the information necessary to compute the estimate of the bias of the M_{AF_new}, estimates of the bias terms of Equation 23 are needed. Table 8 provides the details of the bias information needed to complete the estimation of the bias for the M_{AF_new} value. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B5 in Appendix B.

Table 8 Terms and Estimated Bias Uncertainties Supporting Equation 23

Term/Instrument	Description	Bias Uncertainty at 95% Confidence
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	0
LI3025 with subscript 1	level bubbler value (inwc)	±1% of 231.6 inwc span [11]
		Bias = 2.316 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
		Bias = 1.61 inwc
b_1	tank calibration uncertainty (see WSRC-TR-92-250 [8]♣)	12 gallons
Sep	Separation between bubblers (47 inches)	0.0625 inch [13]
Heel	Tank heel below LI3025 (6.77 inches)	0.0625 inch [13]

♣ The bias in the calibration for the SRAT is taken as the largest value from Table 3d. Rounding up this value is 12 gallons.

Equation:

$\rho=(LI3025_1-DI3026_1)/Sep;$

$V_1=((((LI3025_1/\rho)+Heel-x_2)*(y_1-y_2))/(x_1-x_2))+y_2+b_1;$

$M_{AF\_new}=C_{AF}*V_1*\rho*3.7854/(0.4723*1000000);$

Partial Derivatives:

$\partial\rho/\partial LI3025_1 = 1.0/Sep;$

$\partial\rho/\partial DI3026_1 = (-1.0)/Sep;$

$\partial\rho/\partial Sep = (-(LI3025_1 - DI3026_1))/sqr(Sep);$

$\partial V_1/\partial LI3025_1 = (y_1 - y_2)\cdot(1.0/\rho + (-LI3025_1\cdot\partial\rho/\partial LI3025_1)/sqr(\rho))/(x_1 - x_2);$

$\partial V_1/\partial DI3026_1 = (y_1 - y_2)\cdot(-LI3025_1\cdot\partial\rho/\partial DI3026_1)/sqr(\rho)/(x_1 - x_2);$

$\partial V_1/\partial Sep = (y_1 - y_2)\cdot(-LI3025_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_1 - x_2);$

$\partial V_1/\partial Heel = (y_1 - y_2)/(x_1 - x_2);$

$\partial V_1/\partial x_2 = (y_1 - y_2)\cdot(-1.0)/(x_1 - x_2) + (-(LI3025_1/\rho + Heel - x_2)\cdot(y_1 - y_2)\cdot(-1.0))/sqr(x_1 - x_2);$

$\partial V_1/\partial y_1 = (LI3025_1/\rho + Heel - x_2)/(x_1 - x_2);$

$\partial V_1/\partial y_2 = (LI3025_1/\rho + Heel - x_2)\cdot(-1.0)/(x_1 - x_2) + 1.0;$

$\partial V_1/\partial x_1 = (-(LI3025_1/\rho + Heel - x_2)\cdot(y_1 - y_2))/sqr(x_1 - x_2);$

$\partial V_1/\partial b_1 = 1.0;$

$\partial M_{AF\_new}/\partial LI3025_1 = 3.7854\cdot(C_{AF}\cdot V_1\cdot\partial\rho/\partial LI3025_1 + \rho\cdot C_{AF}\cdot\partial V_1/\partial LI3025_1)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial DI3026_1 = 3.7854\cdot(C_{AF}\cdot V_1\cdot\partial\rho/\partial DI3026_1 + \rho\cdot C_{AF}\cdot\partial V_1/\partial DI3026_1)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Sep = 3.7854\cdot(C_{AF}\cdot V_1\cdot\partial\rho/\partial Sep + \rho\cdot C_{AF}\cdot\partial V_1/\partial Sep)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Heel = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial Heel/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_2 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial x_2/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_1 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial y_1/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_2 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial y_2/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_1 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial x_1/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial b_1 = 3.7854\cdot\rho\cdot C_{AF}\cdot\partial V_1/\partial b_1/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial C_{AF} = 3.7854\cdot\rho\cdot V_1/(0.4723\cdot 1000000.0);$

Exhibit 11 Equations for Re-Base-Lining the M_{AF} of the SRAT with Bias Uncertainty

3.3.4 SRAT Step 3 Processing Linked to a Step 1 Event

Next consider the Step 3 event immediately following a Step 1 re-base lining of the M_{AF} information in the SRAT. The equation for the M_{AF_out} is given by:

Equation 24

$$M_{AF_{out}} = \frac{3.7854 \cdot C_{AF} \cdot \rho \cdot (V_1 - V_2)}{0.4723 \cdot 1000000}$$

Where the density, ρ, and the volumes V_1 and V_2 are intermediary values which are determined from LI3025 and DI3016 as described in Exhibit 8 and Exhibit 9 with the two volumes each having an additional random variability that is to be incorporated to the random uncertainty of M_{AF_out}.

Following the Taylor's Series expansion approach described above, the estimated variance of M_{AF_out} for Step 3 of Exhibit 9 may be written in terms of the fundamental measurements with δ_1 and δ_2 representing the random errors for V_1 and V_2, respectively, as:

Equation 25

$$Variance(M_{AF_{out}}) \approx \left(\frac{\partial M_{AF_{out}}}{\partial C_{AF}}\right)^2 \times variance(C_{AF}) + \left(\frac{\partial M_{AF_{out}}}{\partial LI3025_1}\right)^2 \times variance(LI3025_1)$$

$$+ \left(\frac{\partial M_{AF_{out}}}{\partial DI3026_1}\right)^2 \times variance(DI3026_1) + \left(\frac{\partial M_{AF_{out}}}{\partial \delta_1}\right)^2 \times variance(\delta_1)$$

$$+ \left(\frac{\partial M_{AF_{out}}}{\partial LI3025_2}\right)^2 \times variance(LI3025_2) + \left(\frac{\partial M_{AF_{out}}}{\partial \delta_2}\right)^2 \times variance(\delta_2)$$

GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_out} for Step 3 (see the upper portion of Exhibit 12) and to document the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_out} value (see the lower portion of Exhibit 12 and Exhibit 13). For example, the x_{11}, x_{12}, y_{11}, and y_{12} values; and the x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values (based upon the values of the LI3025 instrument as indicated by $LI3025_1$ and $LI3025_2$, respectively) for determining volume as indicated in Exhibit 8. To complete the information necessary to compute the estimate of the variance of the M_{AF_out}, estimates of the variance terms of Equation 25 are needed. These values are provided in Table 9.

Equation:

ρ=(LI3025$_1$-DI3026$_1$)/Sep;
V$_1$=((LI3025$_1$/ρ+Heel-x$_{12}$)*(y$_{11}$-y$_{12}$)/(x$_{11}$-x$_{12}$))+y$_{12}$+δ_1;
V$_2$=((LI3025$_2$/ρ)+Heel-x$_{22}$)*(y$_{21}$-y$_{22}$)/(x$_{21}$-x$_{22}$)+y$_{22}$+δ_2;
M$_{AF_out}$=3.7854*C$_{AF}$*(V$_1$-V$_2$)*ρ/(0.4723*1000000);

Partial Derivatives:

$\partial\rho/\partial$LI3025$_1$ = 1.0/Sep;

$\partial\rho/\partial$DI3026$_1$ = (-1.0)/Sep;

$\partial\rho/\partial$Sep = (-(LI3025$_1$ - DI3026$_1$))/sqr(Sep);

∂V$_1/\partial$LI3025$_1$ = (y$_{11}$ - y$_{12}$)$\cdot$(1.0/ρ + (-LI3025$_1\cdot\partial\rho/\partial$LI3025$_1$)/sqr($\rho$))/(x$_{11}$ - x$_{12}$);

∂V$_1/\partial$DI3026$_1$ = (y$_{11}$ - y$_{12}$)$\cdot$(-LI3025$_1\cdot\partial\rho/\partial$DI3026$_1$)/sqr($\rho$)/(x$_{11}$ - x$_{12}$);

∂V$_1/\partial$Sep = (y$_{11}$ - y$_{12}$)$\cdot$(-LI3025$_1\cdot\partial\rho/\partial$Sep)/sqr($\rho$)/(x$_{11}$ - x$_{12}$);

∂V$_1/\partial$Heel = (y$_{11}$ - y$_{12}$)/(x$_{11}$ - x$_{12}$);

∂V$_1/\partial$x$_{12}$ = (y$_{11}$ - y$_{12}$)$\cdot$(-1.0)/(x$_{11}$ - x$_{12}$) + (-(LI3025$_1$/ρ + Heel - x$_{12}$)$\cdot$(y$_{11}$ - y$_{12}$)$\cdot$(-1.0))/sqr(x$_{11}$ - x$_{12}$);

∂V$_1/\partial$y$_{11}$ = (LI3025$_1$/ρ + Heel - x$_{12}$)/(x$_{11}$ - x$_{12}$);

∂V$_1/\partial$y$_{12}$ = (LI3025$_1$/ρ + Heel - x$_{12}$)$\cdot$(-1.0)/(x$_{11}$ - x$_{12}$) + 1.0;

∂V$_1/\partial$x$_{11}$ = (-(LI3025$_1$/ρ + Heel - x$_{12}$)$\cdot$(y$_{11}$ - y$_{12}$))/sqr(x$_{11}$ - x$_{12}$);

∂V$_1/\partial\delta_1$ = 1.0;

∂V$_2/\partial$LI3025$_1$ = (y$_{21}$ - y$_{22}$)$\cdot$(-LI3025$_2\cdot\partial\rho/\partial$LI3025$_1$)/sqr($\rho$)/(x$_{21}$ - x$_{22}$);

∂V$_2/\partial$DI3026$_1$ = (y$_{21}$ - y$_{22}$)$\cdot$(-LI3025$_2\cdot\partial\rho/\partial$DI3026$_1$)/sqr($\rho$)/(x$_{21}$ - x$_{22}$);

∂V$_2/\partial$Sep = (y$_{21}$ - y$_{22}$)$\cdot$(-LI3025$_2\cdot\partial\rho/\partial$Sep)/sqr($\rho$)/(x$_{21}$ - x$_{22}$);

∂V$_2/\partial$Heel = (y$_{21}$ - y$_{22}$)/(x$_{21}$ - x$_{22}$);

∂V$_2/\partial$LI3025$_2$ = (y$_{21}$ - y$_{22}$)$\cdot$1.0/ρ/(x$_{21}$ - x$_{22}$);

∂V$_2/\partial$x$_{22}$ = (y$_{21}$ - y$_{22}$)$\cdot$(-1.0)/(x$_{21}$ - x$_{22}$) + (-(LI3025$_2$/ρ + Heel - x$_{22}$)$\cdot$(y$_{21}$ - y$_{22}$)$\cdot$(-1.0))/sqr(x$_{21}$ - x$_{22}$);

∂V$_2/\partial$y$_{21}$ = (LI3025$_2$/ρ + Heel - x$_{22}$)/(x$_{21}$ - x$_{22}$);

∂V$_2/\partial$y$_{22}$ = (LI3025$_2$/ρ + Heel - x$_{22}$)$\cdot$(-1.0)/(x$_{21}$ - x$_{22}$) + 1.0;

∂V$_2/\partial$x$_{21}$ = (-(LI3025$_2$/ρ + Heel - x$_{22}$)$\cdot$(y$_{21}$ - y$_{22}$))/sqr(x$_{21}$ - x$_{22}$);

∂V$_2/\partial\delta_2$ = 1.0;

Exhibit 12 Equations for Analytical M$_{AF_out}$ from the SRAT with Random Uncertainty (part 1 of 2)

Additional Partial Derivatives

$\partial M_{AF\_out}/\partial LI3025_1 = (3.7854 \cdot C_{AF} \cdot (V_1 - V_2) \cdot \partial\rho/\partial LI3025_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot (\partial V_1/\partial LI3025_1 + -\partial V_2/\partial LI3025_1))/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial DI3026_1 = (3.7854 \cdot C_{AF} \cdot (V_1 - V_2) \cdot \partial\rho/\partial DI3026_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot (\partial V_1/\partial DI3026_1 + -\partial V_2/\partial DI3026_1))/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial Sep = (3.7854 \cdot C_{AF} \cdot (V_1 - V_2) \cdot \partial\rho/\partial Sep + \rho \cdot 3.7854 \cdot C_{AF} \cdot (\partial V_1/\partial Sep + -\partial V_2/\partial Sep))/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial Heel = \rho \cdot 3.7854 \cdot C_{AF} \cdot (\partial V_1/\partial Heel + -\partial V_2/\partial Heel)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial x_{12} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial x_{12}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial y_{11} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial y_{11}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial y_{12} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial y_{12}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial x_{11} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial x_{11}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial \delta_1 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial \delta_1/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial LI3025_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial LI3025_2)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial x_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial x_{22})/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial y_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial y_{21})/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial y_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial y_{22})/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial x_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial x_{21})/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial \delta_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial \delta_2)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial C_{AF} = \rho \cdot (V_1 - V_2) \cdot 3.7854/(0.4723 \cdot 1000000.0);$

Exhibit 13 Equations for Analytical M_{AF_out} from the SRAT with Random Uncertainty (part 2 of 2)

Table 9 Terms and Estimated Random Uncertainties Supporting Equation 25

Term/Instrument	Description	1-Sigma Random Uncertainty
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	Analytical uncertainty (see Equation 12)
LI3025 with subscript 1	level bubbler value (inwc)	±1% of 231.6 inwc span [11]
	DCS Deviation Limit	±0.1 inwc [11]
		Using a uniform distribution, 1-sigma random is $[(2.316/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5}$ = 1.3384 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
	DCS deviation limit	±0.05 inwc [12]
		Using a uniform distribution, 1-sigma random is $[(1.61/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5}$ = 0.9300 inwc
δ_1	tank calibration uncertainty (see footnote for Table 7)	1-sigma random = 9 gallons
δ_2	tank calibration uncertainty (see footnote for Table 7)	1-sigma random = 9 gallons

To complete the evaluation of the M_{AF_out} required for Step 3, the bias for M_{AF_out} determined by Equation 24 must be estimated. A bound (at 95% confidence) on the bias of the antifoam mass transferred out of the SRAT, M_{AF_out}, is estimated, as above, by appealing to a Taylor Series expansion of the Equation 24 in the fundamental measurements. Once again, note that ρ, V_1, and V_2 are intermediary values. Also, the V_1 and V_2 values each have a potential bias that is to be included in the evaluation. The Taylor's Series expansion may be expressed in the fundamental measurements and the potential biases (b_1 for V_1 and b_2 for V_2) of the calculated volumes as given by:

Equation 26

$$\{bias(M_{AF_{out}})\}^2 \approx \left(\frac{\partial M_{AF_{out}}}{\partial C_{AF}}\right)^2 \times \{bias(C_{AF})\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial LI3025_1}\right)^2 \times \{bias(LI3025_1)\}^2$$

$$+\left(\frac{\partial M_{AF_{out}}}{\partial DI3026_1}\right)^2 \times \{bias(DI3026_1)\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2$$

$$+\left(\frac{\partial M_{AF_{out}}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial LI3025_2}\right)^2 \times \{bias(LI3025_2)\}^2$$

$$+\left(\frac{\partial M_{AF_{out}}}{\partial b_1}\right)^2 \times \{bias(b_1)\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial b_2}\right)^2 \times \{bias(b_2)\}^2$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{out}}}{\partial LI3025_1}\right| \times \left|\frac{\partial M_{AF_{out}}}{\partial LI3025_2}\right| \times bias(LI3025_1) \times bias(LI3025_2)$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{out}}}{\partial b_1}\right| \times \left|\frac{\partial M_{AF_{out}}}{\partial b_2}\right| \times bias(b_1) \times bias(b_2)$$

Note that in evaluating Equation 26, the bias for the C_{AF} term, i.e., bias(C_{AF}) term is estimated to be zero and that two potential correlations among the biases are introduced in a bounding manner. So the approach may be stated as: The analytical estimate of the concentration of carbon from AF is unbiased and the b_1 and b_2 terms are the estimated biases in the volumes V_1 and V_2, respectively, of Equation 24. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of M_{AF_out} for Step 3 (see the upper portion of Exhibit 14) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_out} value (see the lower portion of Exhibit 14 and Exhibit 15). Once again, the x_{11}, x_{12}, y_{11}, and y_{12} values and the x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values (based upon the value of the LI3025 instrument as indicated by $LI3025_1$ and $LI3025_2$) for determining volumes as indicated in Exhibit 8.To complete the information necessary to compute the estimate of the bias of the M_{AF_out}, estimates of the bias terms of Equation 26 are needed. Table 10 provides the details of the bias information needed to complete the estimation of the bias for the M_{AF_out} value. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B6 in Appendix B.

Equation:

ρ=(LI3025$_1$-DI3026$_1$)/Sep;
V_1=((LI3025$_1$/ρ+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12}))+y_{12}+b_1;
V_2=((LI3025$_2$/ρ)+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22}))+y_{22}+b_2;
M_{AF_out}=3.7854*C_{AF}*(V_1-V_2)*ρ/(0.4723*1000000);

Partial Derivatives:

∂ρ/∂LI3025$_1$ = 1.0/Sep;

∂ρ/∂DI3026$_1$ = (-1.0)/Sep;

∂ρ/∂Sep = (-(LI3025$_1$ - DI3026$_1$))/sqr(Sep);

∂V_1/∂LI3025$_1$ = (y_{11} - y_{12})·(1.0/ρ + (-LI3025$_1$·∂ρ/∂LI3025$_1$)/sqr(ρ))/(x_{11} - x_{12});

∂V_1/∂DI3026$_1$ = (y_{11} - y_{12})·(-LI3025$_1$·∂ρ/∂DI3026$_1$)/sqr(ρ)/(x_{11} - x_{12});

∂V_1/∂Sep = (y_{11} - y_{12})·(-LI3025$_1$·∂ρ/∂Sep)/sqr(ρ)/(x_{11} - x_{12});

∂V_1/∂Heel = (y_{11} - y_{12})/(x_{11} - x_{12});

∂V_1/∂x_{12} = (y_{11} - y_{12})·(-1.0)/(x_{11} - x_{12}) + (-(LI3025$_1$/ρ + Heel - x_{12})·(y_{11} - y_{12})·(-1.0))/sqr(x_{11} - x_{12});

∂V_1/∂y_{11} = (LI3025$_1$/ρ + Heel - x_{12})/(x_{11} - x_{12});

∂V_1/∂y_{12} = (LI3025$_1$/ρ + Heel - x_{12})·(-1.0)/(x_{11} - x_{12}) + 1.0;

∂V_1/∂x_{11} = (-(LI3025$_1$/ρ + Heel - x_{12})·(y_{11} - y_{12}))/sqr(x_{11} - x_{12});

∂V_1/∂b_1 = 1.0;

∂V_2/∂LI3025$_1$ = (y_{21} - y_{22})·(-LI3025$_2$·∂ρ/∂LI3025$_1$)/sqr(ρ)/(x_{21} - x_{22});

∂V_2/∂DI3026$_1$ = (y_{21} - y_{22})·(-LI3025$_2$·∂ρ/∂DI3026$_1$)/sqr(ρ)/(x_{21} - x_{22});

∂V_2/∂Sep = (y_{21} - y_{22})·(-LI3025$_2$·∂ρ/∂Sep)/sqr(ρ)/(x_{21} - x_{22});

∂V_2/∂Heel = (y_{21} - y_{22})/(x_{21} - x_{22});

∂V_2/∂LI3025$_2$ = (y_{21} - y_{22})·1.0/ρ/(x_{21} - x_{22});

∂V_2/∂x_{22} = (y_{21} - y_{22})·(-1.0)/(x_{21} - x_{22}) + (-(LI3025$_2$/ρ + Heel - x_{22})·(y_{21} - y_{22})·(-1.0))/sqr(x_{21} - x_{22});

∂V_2/∂y_{21} = (LI3025$_2$/ρ + Heel - x_{22})/(x_{21} - x_{22});

∂V_2/∂y_{22} = (LI3025$_2$/ρ + Heel - x_{22})·(-1.0)/(x_{21} - x_{22}) + 1.0;

∂V_2/∂x_{21} = (-(LI3025$_2$/ρ + Heel - x_{22})·(y_{21} - y_{22}))/sqr(x_{21} - x_{22});

∂V_2/∂b_2 = 1.0;

Exhibit 14 Equations for Analytical M_{AF_out} from the SRAT with Bias Uncertainty (part 1 of 2)

Additional Partial Derivatives

$\partial M_{AF\_out}/\partial LI3025_1 = (3.7854 \cdot C_{AF} \cdot (V_1 - V_2) \cdot \partial\rho/\partial LI3025_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot (\partial V_1/\partial LI3025_1 + -\partial V_2/\partial LI3025_1))/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial DI3026_1 = (3.7854 \cdot C_{AF} \cdot (V_1 - V_2) \cdot \partial\rho/\partial DI3026_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot (\partial V_1/\partial DI3026_1 + -\partial V_2/\partial DI3026_1))/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial Sep = (3.7854 \cdot C_{AF} \cdot (V_1 - V_2) \cdot \partial\rho/\partial Sep + \rho \cdot 3.7854 \cdot C_{AF} \cdot (\partial V_1/\partial Sep + -\partial V_2/\partial Sep))/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial Heel = \rho \cdot 3.7854 \cdot C_{AF} \cdot (\partial V_1/\partial Heel + -\partial V_2/\partial Heel)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial x_{12} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial x_{12}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial y_{11} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial y_{11}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial y_{12} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial y_{12}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial x_{11} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial x_{11}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial b_1 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_1/\partial b_1/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial LI3025_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial LI3025_2)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial x_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial x_{22})/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial y_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial y_{21})/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial y_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial y_{22})/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial x_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial x_{21})/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial b_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot (-\partial V_2/\partial b_2)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_out}/\partial C_{AF} = \rho \cdot (V_1 - V_2) \cdot 3.7854/(0.4723 \cdot 1000000.0);$

Exhibit 15 Equations for Analytical M_{AF_out} from the SRAT with Bias Uncertainty (part 2 of 2)

Table 10 Terms and Estimated Bias Uncertainties Supporting Equation 26

Term/Instrument	Description	Bias Uncertainty at 95% Confidence
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	0
LI3025 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.6 inwc span [11]
		Bias = 2.316 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
		Bias = 1.61 inwc
b_1	tank calibration uncertainty (see footnote for Table 8)	12 gallons
b_2	tank calibration uncertainty (see footnote for Table 8)	12 gallons
Sep	separation between bubblers (47 inch)	0.0625 inch [13]
Heel	tank heel below LI3025 (6.77 inch)	0.0625 inch [13]

3.3.5 SRAT Step 4 Processing Linked to a Step 1 Event

Next consider the Step 4 event immediately following a Step 1 re-base lining of the M_{AF} information in the SRAT. The equation for the M_{AF_new} for step 4 is given by:

Equation 27

$$M_{AF_{new}} = \frac{3.7854 \cdot C_{AF} \cdot \rho \cdot V_2}{0.4723 \cdot 1000000}$$

where the density, ρ, and the volume V_2 are intermediary values which are determined from LI3025 and DI3016 as described in Exhibit 8 and Exhibit 9 with the volume having an additional random variability that is to be incorporated to the random uncertainty of M_{AF_new}.

Following the Taylor's Series expansion approach described above, the estimated variance of M_{AF_new} for Step 4 of Exhibit 9 may be written in terms of the fundamental measurements with δ_2 representing the random error for V_2 as:

Equation 28

$$Variance\left(M_{AF_{new}}\right) \approx \left(\frac{\partial M_{AF_{new}}}{\partial C_{AF}}\right)^2 \times variance(C_{AF}) + \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_1}\right)^2 \times variance(LI3025_1)$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial DI3026_1}\right)^2 \times variance(DI3026_1) + +\left(\frac{\partial M_{AF_{new}}}{\partial LI3025_2}\right)^2 \times variance(LI3025_2)$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial \delta_2}\right)^2 \times variance(\delta_2)$$

GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_new} for Step 4 (see the upper portion of Exhibit 16) and to document the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_new} value (see the lower portion of Exhibit 16 and Exhibit 17). For example, the x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values (based upon the values of the LI3025 instrument as indicated by $LI3025_2$) for determining volume as indicated in Exhibit 8.To complete the information necessary to compute the estimate of the variance of the M_{AF_new}, estimates of the variance terms of Equation 28 are needed. These values are provided in Table 11.

Equation: ρ=(LI3025$_1$-DI3026$_1$)/Sep; V_2=((LI3025$_2$/ρ)+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22})+y_{22}+δ_2; M_{AF_new}=3.7854*C_{AF}*V_2*ρ/(0.4723*1000000);
Partial Derivatives: $\partial\rho/\partial$LI3025$_1$ = 1.0/Sep; $\partial\rho/\partial$DI3026$_1$ = (-1.0)/Sep; $\partial\rho/\partial$Sep = (-(LI3025$_1$ - DI3026$_1$))/sqr(Sep); $\partial V_2/\partial$LI3025$_1$ = (y_{21} - y_{22})·(-LI3025$_2$·$\partial\rho/\partial$LI3025$_1$)/sqr(ρ)/(x_{21} - x_{22}); $\partial V_2/\partial$DI3026$_1$ = (y_{21} - y_{22})·(-LI3025$_2$·$\partial\rho/\partial$DI3026$_1$)/sqr(ρ)/(x_{21} - x_{22}); $\partial V_2/\partial$Sep = (y_{21} - y_{22})·(-LI3025$_2$·$\partial\rho/\partial$Sep)/sqr(ρ)/(x_{21} - x_{22}); $\partial V_2/\partial$LI3025$_2$ = (y_{21} - y_{22})·1.0/ρ/(x_{21} - x_{22}); $\partial V_2/\partial$Heel = (y_{21} - y_{22})/(x_{21} - x_{22}); $\partial V_2/\partial x_{22}$ = (y_{21} - y_{22})·(-1.0)/(x_{21} - x_{22}) + (-(LI3025$_2$/ρ + Heel - x_{22})·(y_{21} - y_{22})·(-1.0))/sqr(x_{21} - x_{22}); $\partial V_2/\partial y_{21}$ = (LI3025$_2$/ρ + Heel - x_{22})/(x_{21} - x_{22}); $\partial V_2/\partial y_{22}$ = (LI3025$_2$/ρ + Heel - x_{22})·(-1.0)/(x_{21} - x_{22}) + 1.0; $\partial V_2/\partial x_{21}$ = (-(LI3025$_2$/ρ + Heel - x_{22})·(y_{21} - y_{22}))/sqr(x_{21} - x_{22}); $\partial V_2/\partial\delta_2$ = 1.0;

Exhibit 16 Equations for Analytical M_{AF_new} for the SRAT Heel with Random Uncertainty (part 1 of 2)

Additional Partial Derivatives
$\partial M_{AF\_new}/\partial LI3025_1 = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial LI3025_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial LI3025_1)/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial DI3026_1 = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial DI3026_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial DI3026_1)/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial Sep = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial Sep + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial Sep)/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial LI3025_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial LI3025_2/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial Heel = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial Heel/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial x_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial x_{22}/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial y_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial y_{21}/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial y_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial y_{22}/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial x_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial x_{21}/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial \delta_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial \delta_2/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial C_{AF} = \rho \cdot V_2 \cdot 3.7854/(0.4723 \cdot 1000000.0);$

Exhibit 17 Equations for Analytical M_{AF_new} for the SRAT Heel with Random Uncertainty (part 2 of 2)

Table 11 Terms and Estimated Random Uncertainties Supporting Equation 28

Term/Instrument	Description	1-Sigma Random Uncertainty
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	Analytical uncertainty (see Equation 12)
LI3025 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.6 inwc span [11]
	DCS deviation limit	±0.1 inwc [11]
		Using a uniform distribution, 1-sigma random is $[(2.316/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5}$ = 1.3384 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
	DCS deviation limit	±0.05 inwc [12]
		Using a uniform distribution, 1-sigma random is $[(1.61/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5}$ = 0.9300 inwc
δ_2	tank calibration uncertainty (see footnote for Table 7)	1-sigma random = 9 gallons

To complete the evaluation of the M_{AF_new} required for Step 4, the bias for M_{AF_new} determined by Equation 27 must be estimated. A bound (at 95% confidence) on the bias of the antifoam mass remaining in the heel of the SRAT, M_{AF_new}, is estimated, as above, by appealing to a Taylor Series expansion of the Equation 27 in the fundamental measurements. Once again, note that ρ and V_2 are intermediary values. Also, note that the V_2 value has a potential bias that is to be included in the evaluation. The Taylor's Series expansion may be expressed in the fundamental measurements and the potential bias of the calculated volume as given by:

Equation 29

$$\{bias(M_{AF_{new}})\}^2 \approx \left(\frac{\partial M_{AF_{new}}}{\partial C_{AF}}\right)^2 \times \{bias(C_{AF})\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_1}\right)^2 \times \{bias(LI3025_1)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial DI3026_1}\right)^2 \times \{bias(DI3026_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_2}\right)^2 \times \{bias(LI3025_2)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial b_2}\right)^2 \times \{bias(b_2)\}^2$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{new}}}{\partial LI3025_1}\right| \times \left|\frac{\partial M_{AF_{new}}}{\partial LI3025_2}\right| \times bias(LI3025_1) \times bias(LI3025_2)$$

Note that in evaluating Equation 29, the bias for the C_{AF} term, i.e., bias(C_{AF}) term is estimated to be zero and that a potential correlation among the biases for LI3025 values is introduced in a bounding manner. So the approach may be stated as: The analytical estimate of the concentration of carbon from AF is unbiased and the b_2 term is the estimated bias in the volume V_2 of Equation 27. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of M_{AF_new} for Step 4 (see the upper portion of Exhibit 18) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_new} value (see the lower portion of Exhibit 18 and Exhibit 19). Once again, x_{21}, x_{22}, y_{121}, and y_{22} are appropriately selected values (based upon the value of the LI3025 instrument as indicated by $LI3025_2$) for determining volume as indicated in Exhibit 8.To complete the information necessary to compute the estimate of the bias of the M_{AF_new}, estimates of the bias terms of Equation 29 are needed. Table 12 provides the details of the bias information needed to complete the estimation of the bias for the M_{AF_new} value. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B7 in Appendix B.

Equation:

ρ=(LI3025$_1$-DI3026$_1$)/Sep;
V_2=((LI3025$_2$/ρ)+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22})+y_{22}+b_2;
M_{AF_new}=3.7854*C_{AF}*V_2*ρ/(0.4723*1000000);

Partial Derivatives:

$\partial\rho/\partial$LI3025$_1$ = 1.0/Sep;

$\partial\rho/\partial$DI3026$_1$ = (-1.0)/Sep;

$\partial\rho/\partial$Sep = (-(LI3025$_1$ - DI3026$_1$))/sqr(Sep);

$\partial V_2/\partial$LI3025$_1$ = (y_{21} - y_{22})·(-LI3025$_2$·$\partial\rho/\partial$LI3025$_1$)/sqr(ρ)/(x_{21} - x_{22});

$\partial V_2/\partial$DI3026$_1$ = (y_{21} - y_{22})·(-LI3025$_2$·$\partial\rho/\partial$DI3026$_1$)/sqr(ρ)/(x_{21} - x_{22});

$\partial V_2/\partial$Sep = (y_{21} - y_{22})·(-LI3025$_2$·$\partial\rho/\partial$Sep)/sqr(ρ)/(x_{21} - x_{22});

$\partial V_2/\partial$LI3025$_2$ = (y_{21} - y_{22})·1.0/ρ/(x_{21} - x_{22});

$\partial V_2/\partial$Heel = (y_{21} - y_{22})/(x_{21} - x_{22});

$\partial V_2/\partial x_{22}$ = (y_{21} - y_{22})·(-1.0)/(x_{21} - x_{22}) + (-(LI3025$_2$/ρ + Heel - x_{22})·(y_{21} - y_{22})·(-1.0))/sqr(x_{21} - x_{22});

$\partial V_2/\partial y_{21}$ = (LI3025$_2$/ρ + Heel - x_{22})/(x_{21} - x_{22});

$\partial V_2/\partial y_{22}$ = (LI3025$_2$/ρ + Heel - x_{22})·(-1.0)/(x_{21} - x_{22}) + 1.0;

$\partial V_2/\partial x_{21}$ = (-(LI3025$_2$/ρ + Heel - x_{22})·(y_{21} - y_{22}))/sqr(x_{21} - x_{22});

$\partial V_2/\partial b_2$ = 1.0;

Exhibit 18 Equations for Analytical M_{AF_new} for the SRAT Heel with Bias Uncertainty (part 1 of 2)

Additional Partial Derivatives
$\partial M_{AF\_new}/\partial LI3025_1 = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial LI3025_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial LI3025_1)/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial DI3026_1 = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial DI3026_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial DI3026_1)/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial Sep = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial Sep + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial Sep)/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial LI3025_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial LI3025_2/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial Heel = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial Heel/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial x_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial x_{22}/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial y_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial y_{21}/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial y_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial y_{22}/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial x_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial x_{21}/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial b_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial b_2/(0.4723 \cdot 1000000.0);$
$\partial M_{AF\_new}/\partial C_{AF} = \rho \cdot V_2 \cdot 3.7854/(0.4723 \cdot 1000000.0);$

Exhibit 19 Equations for Analytical M_{AF_new} for the SRAT Heel with Bias Uncertainty (part 2 of 2)

Table 12 Terms and Estimated Bias Uncertainties Supporting Equation 29

Term/Instrument	Description	Bias Uncertainty at 95% Confidence
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	0
LI3025 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.6 inwc span [11]
		Bias = 2.316 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
		Bias = 1.61 inwc
b_2	tank calibration uncertainty (see footnote for Table 8)	12 gallons
Sep	separation between bubblers (47 inches)	0.0625 inch [13]
Heel	tank heel below LI3025 (6.77 inches)	0.0625 inch [13]

3.3.6 SRAT Step 3 Processing

Next consider the Step 3 event with the SRAT M_{AF} information available. The equation for the M_{AF_out} is given by:

Equation 30

$$M_{AF_{out}} = \frac{M_{AF} \cdot (V_1 - V_2)}{V_1}$$

where the volumes V_1 and V_2 are intermediary values which are determined from LI3025 and DI3016 as described in Exhibit 8 and Exhibit 9 with the two volumes each having an additional random variability that is to be incorporated into the random uncertainty of M_{AF_out}.

Following the Taylor's Series expansion approach described above, the estimated variance of M_{AF_out} for Step 3 of Exhibit 9 may be written in terms of the fundamental measurements with δ_1 and δ_2 representing the random errors for V_1 and V_2, respectively, as:

Equation 31

$$Variance\left(M_{AF_{out}}\right) \approx \left(\frac{\partial M_{AF_{out}}}{\partial M_{AF}}\right)^2 \times variance(M_{AF}) + \left(\frac{\partial M_{AF_{out}}}{\partial LI3025_1}\right)^2 \times variance(LI3025_1)$$
$$+ \left(\frac{\partial M_{AF_{out}}}{\partial DI3026_1}\right)^2 \times variance(DI3026_1) + \left(\frac{\partial M_{AF_{out}}}{\partial \delta_1}\right)^2 \times variance(\delta_1)$$
$$+ \left(\frac{\partial M_{AF_{out}}}{\partial LI3025_2}\right)^2 \times variance(LI3025_2) + \left(\frac{\partial M_{AF_{out}}}{\partial \delta_2}\right)^2 \times variance(\delta_2)$$

GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_out} for Step 3 (see the upper portion of Exhibit 20) and to document the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_out} value (see the lower portion of Exhibit 20 and Exhibit 21). For example, the x_{11}, x_{12}, y_{11}, and y_{12} values; and the x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values (based upon the values of the LI3025 instrument as indicated by $LI3025_1$ and $LI3025_2$, respectively) for determining volume as indicated in Exhibit 8.To complete the information necessary to compute the estimate of the variance of the M_{AF_out}, estimates of the variance terms of Equation 31 are needed. These values along with a description of the terms of Equation 31 are provided in Table 13.

Equation:

$$\rho=(LI3025_1-DI3026_1)/Sep;$$
$$V_1=((LI3025_1/\rho)+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12})+y_{12}+\delta_1;$$
$$V_2=((LI3025_2/\rho)+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22})+y_{22}+\delta_2;$$
$$M_{AF\_out}=M_{AF}*(V_1-V_2)/V_1;$$

Partial Derivatives:

$$\partial\rho/\partial LI3025_1 = 1.0/Sep;$$

$$\partial\rho/\partial DI3026_1 = (-1.0)/Sep;$$

$$\partial\rho/\partial Sep = (-(LI3025_1 - DI3026_1))/sqr(Sep);$$

$$\partial V_1/\partial LI3025_1 = (y_{11} - y_{12})\cdot(1.0/\rho + (-LI3025_1\cdot\partial\rho/\partial LI3025_1)/sqr(\rho))/(x_{11} - x_{12});$$

$$\partial V_1/\partial DI3026_1 = (y_{11} - y_{12})\cdot(-LI3025_1\cdot\partial\rho/\partial DI3026_1)/sqr(\rho)/(x_{11} - x_{12});$$

$$\partial V_1/\partial Sep = (y_{11} - y_{12})\cdot(-LI3025_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{11} - x_{12});$$

$$\partial V_1/\partial Heel = (y_{11} - y_{12})/(x_{11} - x_{12});$$

$$\partial V_1/\partial x_{12} = (y_{11} - y_{12})\cdot(-1.0)/(x_{11} - x_{12}) + (-(LI3025_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12})\cdot(-1.0))/sqr(x_{11} - x_{12});$$

$$\partial V_1/\partial y_{11} = (LI3025_1/\rho + Heel - x_{12})/(x_{11} - x_{12});$$

$$\partial V_1/\partial y_{12} = (LI3025_1/\rho + Heel - x_{12})\cdot(-1.0)/(x_{11} - x_{12}) + 1.0;$$

$$\partial V_1/\partial x_{11} = (-(LI3025_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12}))/sqr(x_{11} - x_{12});$$

$$\partial V_1/\partial\delta_1 = 1.0;$$

$$\partial V_2/\partial LI3025_1 = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial LI3025_1)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial DI3026_1 = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial DI3026_1)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial Sep = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial Heel = (y_{21} - y_{22})/(x_{21} - x_{22});$$

$$\partial V_2/\partial LI3025_2 = (y_{21} - y_{22})\cdot 1.0/\rho/(x_{21} - x_{22});$$

$$\partial V_2/\partial x_{22} = (y_{21} - y_{22})\cdot(-1.0)/(x_{21} - x_{22}) + (-(LI3025_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22})\cdot(-1.0))/sqr(x_{21} - x_{22});$$

$$\partial V_2/\partial y_{21} = (LI3025_2/\rho + Heel - x_{22})/(x_{21} - x_{22});$$

$$\partial V_2/\partial y_{22} = (LI3025_2/\rho + Heel - x_{22})\cdot(-1.0)/(x_{21} - x_{22}) + 1.0;$$

$$\partial V_2/\partial x_{21} = (-(LI3025_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22}))/sqr(x_{21} - x_{22});$$

$$\partial V_2/\partial\delta_2 = 1.0;$$

Exhibit 20 Equations for Calculating the Random Uncertainty for the M_{AF} Transferred Out of the SRAT (part 1 of 2)

Additional Partial Derivatives

$\partial M_{AF\_out}/\partial LI3025_1 = M_{AF} \cdot (\partial V_1/\partial LI3025_1 + -\partial V_2/\partial LI3025_1)/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial LI3025_1)/sqr(V_1);$

$\partial M_{AF\_out}/\partial DI3026_1 = M_{AF} \cdot (\partial V_1/\partial DI3026_1 + -\partial V_2/\partial DI3026_1)/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial DI3026_1)/sqr(V_1);$

$\partial M_{AF\_out}/\partial Sep = M_{AF} \cdot (\partial V_1/\partial Sep + -\partial V_2/\partial Sep)/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial Sep)/sqr(V_1);$

$\partial M_{AF\_out}/\partial Heel = M_{AF} \cdot (\partial V_1/\partial Heel + -\partial V_2/\partial Heel)/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial Heel)/sqr(V_1);$

$\partial M_{AF\_out}/\partial x_{12} = M_{AF} \cdot \partial V_1/\partial x_{12}/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial x_{12})/sqr(V_1);$

$\partial M_{AF\_out}/\partial y_{11} = M_{AF} \cdot \partial V_1/\partial y_{11}/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial y_{11})/sqr(V_1);$

$\partial M_{AF\_out}/\partial y_{12} = M_{AF} \cdot \partial V_1/\partial y_{12}/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial y_{12})/sqr(V_1);$

$\partial M_{AF\_out}/\partial x_{11} = M_{AF} \cdot \partial V_1/\partial x_{11}/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial x_{11})/sqr(V_1);$

$\partial M_{AF\_out}/\partial \delta_1 = M_{AF} \cdot \partial V_1/\partial \delta_1/V_1 + (-M_{AF} \cdot (V_1 - V_2) \cdot \partial V_1/\partial \delta_1)/sqr(V_1);$

$\partial M_{AF\_out}/\partial LI3025_2 = M_{AF} \cdot (-\partial V_2/\partial LI3025_2)/V_1;$

$\partial M_{AF\_out}/\partial x_{22} = M_{AF} \cdot (-\partial V_2/\partial x_{22})/V_1;$

$\partial M_{AF\_out}/\partial y_{21} = M_{AF} \cdot (-\partial V_2/\partial y_{21})/V_1;$

$\partial M_{AF\_out}/\partial y_{22} = M_{AF} \cdot (-\partial V_2/\partial y_{22})/V_1;$

$\partial M_{AF\_out}/\partial x_{21} = M_{AF} \cdot (-\partial V_2/\partial x_{21})/V_1;$

$\partial M_{AF\_out}/\partial \delta_2 = M_{AF} \cdot (-\partial V_2/\partial \delta_2)/V_1;$

$\partial M_{AF\_out}/\partial M_{AF} = (V_1 - V_2)/V_1;$

Exhibit 21 Equations for Calculating the Random Uncertainty for the M_{AF} Transferred Out of the SRAT (part 2 of 2)

Table 13 Terms and Estimated Random Uncertainties Supporting Equation 31

Term/Instrument	Description	1-Sigma Random Uncertainty
M_{AF}	current AF mass prior to transfer (kg)	based upon SRAT status information
LI3025 with subscript 1	level bubbler value (inwc)	±1% of 231.6 inwc span [11]
	DCS deviation Limit	±0.1 inwc [11]
		Using a uniform distribution, 1-sigma random is $[(2.316/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5}$ = 1.3384 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
	DCS deviation limit	±0.05 inwc [12]
		Using a uniform distribution, 1-sigma random is $[(1.61/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5}$ = 0.9300 inwc
δ_1	tank calibration uncertainty (see footnote for Table 7)	1-sigma random = 9 gallons
δ_2	tank calibration uncertainty (see footnote for Table 7)	1-sigma random = 9 gallons

To complete the evaluation of the M_{AF_out} required for Step 3, the bias for M_{AF_out} determined by Equation 30 must be estimated. A bound (at 95% confidence) on the bias of the antifoam mass transferred out of the SRAT, M_{AF_out}, is estimated, as above, by appealing to a Taylor Series expansion of the Equation 30 in the fundamental measurements. Once again, note that ρ, V_1, and V_2 are intermediary values. Also, the

V_1 and V_2 values each have a potential bias that is to be included in the evaluation. The Taylor's Series expansion may be expressed in the fundamental measurements and the potential biases of the calculated volumes as given by:

Equation 32

$$\{bias(M_{AF_{out}})\}^2 \approx \left(\frac{\partial M_{AF_{out}}}{\partial M_{AF}}\right)^2 \times \{bias(M_{AF})\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial LI3025_1}\right)^2 \times \{bias(LI3025_1)\}^2$$

$$+\left(\frac{\partial M_{AF_{out}}}{\partial DI3026_1}\right)^2 \times \{bias(DI3026_1)\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2$$

$$+\left(\frac{\partial M_{AF_{out}}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial LI3025_2}\right)^2 \times \{bias(LI3025_2)\}^2$$

$$+\left(\frac{\partial M_{AF_{out}}}{\partial b_1}\right)^2 \times \{bias(b_1)\}^2 + \left(\frac{\partial M_{AF_{out}}}{\partial b_2}\right)^2 \times \{bias(b_2)\}^2$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{out}}}{\partial LI3025_1}\right| \times \left|\frac{\partial M_{AF_{out}}}{\partial LI3025_2}\right| \times bias(LI3025_1) \times bias(LI3025_2)$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{out}}}{\partial b_1}\right| \times \left|\frac{\partial M_{AF_{out}}}{\partial b_2}\right| \times bias(b_1) \times bias(b_2)$$

Note that in evaluating Equation 32, two potential correlations among the biases are introduced in a bounding manner. So the approach of may be stated as: the b_1 and b_2 terms are the estimated biases in the volumes V_1 and V_2, respectively, which may be correlated. In addition, the biases of the two LI3025 values ($LI3025_1$ and $LI3025_2$) may also be correlated. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of M_{AF_out} for Step 3 (see the upper portion of Exhibit 22) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_out} value (see the lower portion of Exhibit 22 and Exhibit 23). Once again, the x_{11}, x_{12}, y_{11}, and y_{12} values and the x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values (based upon the values of the LI3025 instrument as indicated by $LI3025_1$ and $LI3025_2$, respectively) for determining volumes as indicated in Exhibit 8. To complete the information necessary to compute the estimate of the bias of the M_{AF_out}, estimates of the bias terms of Equation 32 are needed. These values along with a description of the terms are provided in Table 14. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B8 in Appendix B.

Table 14 Terms and Estimated Bias Uncertainties Supporting Equation 32

Term/Instrument	Description	Bias Uncertainty at 95% Confidence
M_{AF}	Current AF mass prior to transfer (kg)	Based upon SRAT Status information
LI3025 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.6 inwc span [11]
		Bias = 2.316 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
		Bias = 1.61 inwc
b_1	tank calibration uncertainty (see footnote for Table 8)	12 gallons
b_2	tank calibration uncertainty (see footnote for Table 8)	12 gallons
Sep	Separation between bubblers (47 inches)	0.0625 inch [13]
Heel	Tank heel below LI3025 (6.77 inches)	0.0625 inch [13]

Equation:

$$\rho=(LI3025_1-DI3026_1)/Sep;$$
$$V_1=((LI3025_1/\rho)+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12})+y_{12}+b_1;$$
$$V_2=((LI3025_2/\rho)+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22})+y_{22}+b_2;$$
$$M_{AF\_out}=M_{AF}*(V_1-V_2)/V_1;$$

Partial Derivatives:

$$\partial\rho/\partial LI3025_1 = 1.0/Sep;$$

$$\partial\rho/\partial DI3026_1 = (-1.0)/Sep;$$

$$\partial\rho/\partial Sep = (-(LI3025_1 - DI3026_1))/sqr(Sep);$$

$$\partial V_1/\partial LI3025_1 = (y_{11} - y_{12})\cdot(1.0/\rho + (-LI3025_1\cdot\partial\rho/\partial LI3025_1)/sqr(\rho))/(x_{11} - x_{12});$$

$$\partial V_1/\partial DI3026_1 = (y_{11} - y_{12})\cdot(-LI3025_1\cdot\partial\rho/\partial DI3026_1)/sqr(\rho)/(x_{11} - x_{12});$$

$$\partial V_1/\partial Sep = (y_{11} - y_{12})\cdot(-LI3025_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{11} - x_{12});$$

$$\partial V_1/\partial Heel = (y_{11} - y_{12})/(x_{11} - x_{12});$$

$$\partial V_1/\partial x_{12} = (y_{11} - y_{12})\cdot(-1.0)/(x_{11} - x_{12}) + (-(LI3025_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12})\cdot(-1.0))/sqr(x_{11} - x_{12});$$

$$\partial V_1/\partial y_{11} = (LI3025_1/\rho + Heel - x_{12})/(x_{11} - x_{12});$$

$$\partial V_1/\partial y_{12} = (LI3025_1/\rho + Heel - x_{12})\cdot(-1.0)/(x_{11} - x_{12}) + 1.0;$$

$$\partial V_1/\partial x_{11} = (-(LI3025_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12}))/sqr(x_{11} - x_{12});$$

$$\partial V_1/\partial b_1 = 1.0;$$

$$\partial V_2/\partial LI3025_1 = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial LI3025_1)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial DI3026_1 = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial DI3026_1)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial Sep = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial Heel = (y_{21} - y_{22})/(x_{21} - x_{22});$$

$$\partial V_2/\partial LI3025_2 = (y_{21} - y_{22})\cdot 1.0/\rho/(x_{21} - x_{22});$$

$$\partial V_2/\partial x_{22} = (y_{21} - y_{22})\cdot(-1.0)/(x_{21} - x_{22}) + (-(LI3025_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22})\cdot(-1.0))/sqr(x_{21} - x_{22});$$

$$\partial V_2/\partial y_{21} = (LI3025_2/\rho + Heel - x_{22})/(x_{21} - x_{22});$$

$$\partial V_2/\partial y_{22} = (LI3025_2/\rho + Heel - x_{22})\cdot(-1.0)/(x_{21} - x_{22}) + 1.0;$$

$$\partial V_2/\partial x_{21} = (-(LI3025_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22}))/sqr(x_{21} - x_{22});$$

$$\partial V_2/\partial b_2 = 1.0;$$

Exhibit 22 Equations for M_{AF_out} from the SRAT with Bias Uncertainty (part 1 of 2)

Additional Partial Derivatives	
	$\partial M_{AF\_out}/\partial LI3025_1 = M_{AF}\cdot(\partial V_1/\partial LI3025_1 + -\partial V_2/\partial LI3025_1)/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial LI3025_1)/sqr(V_1);$
	$\partial M_{AF\_out}/\partial DI3026_1 = M_{AF}\cdot(\partial V_1/\partial DI3026_1 + -\partial V_2/\partial DI3026_1)/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial DI3026_1)/sqr(V_1);$
	$\partial M_{AF\_out}/\partial Sep = M_{AF}\cdot(\partial V_1/\partial Sep + -\partial V_2/\partial Sep)/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial Sep)/sqr(V_1);$
	$\partial M_{AF\_out}/\partial Heel = M_{AF}\cdot(\partial V_1/\partial Heel + -\partial V_2/\partial Heel)/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial Heel)/sqr(V_1);$
	$\partial M_{AF\_out}/\partial x_{12} = M_{AF}\cdot\partial V_1/\partial x_{12}/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial x_{12})/sqr(V_1);$
	$\partial M_{AF\_out}/\partial y_{11} = M_{AF}\cdot\partial V_1/\partial y_{11}/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial y_{11})/sqr(V_1);$
	$\partial M_{AF\_out}/\partial y_{12} = M_{AF}\cdot\partial V_1/\partial y_{12}/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial y_{12})/sqr(V_1);$
	$\partial M_{AF\_out}/\partial x_{11} = M_{AF}\cdot\partial V_1/\partial x_{11}/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial x_{11})/sqr(V_1);$
	$\partial M_{AF\_out}/\partial b_1 = M_{AF}\cdot\partial V_1/\partial b_1/V_1 + (-M_{AF}\cdot(V_1 - V_2)\cdot\partial V_1/\partial b_1)/sqr(V_1);$
	$\partial M_{AF\_out}/\partial LI3025_2 = M_{AF}\cdot(-\partial V_2/\partial LI3025_2)/V_1;$
	$\partial M_{AF\_out}/\partial x_{22} = M_{AF}\cdot(-\partial V_2/\partial x_{22})/V_1;$
	$\partial M_{AF\_out}/\partial y_{21} = M_{AF}\cdot(-\partial V_2/\partial y_{21})/V_1;$
	$\partial M_{AF\_out}/\partial y_{22} = M_{AF}\cdot(-\partial V_2/\partial y_{22})/V_1;$
	$\partial M_{AF\_out}/\partial x_{21} = M_{AF}\cdot(-\partial V_2/\partial x_{21})/V_1;$
	$\partial M_{AF\_out}/\partial b_2 = M_{AF}\cdot(-\partial V_2/\partial b_2)/V_1;$
	$\partial M_{AF\_out}/\partial M_{AF} = (V_1 - V_2)/V_1;$

Exhibit 23 Equations for M_{AF_out} from the SRAT with Bias Uncertainty (part 2 of 2)

3.3.7 SRAT Step 4 Processing

Next consider the Step 4 event with the SRAT M_{AF} information available. The equation for the M_{AF_new} is given by:

Equation 33

$$M_{AF_{new}} = \frac{M_{AF} \cdot V_2}{V_1}$$

where the volumes V_1 and V_2 are intermediary values which are determined from LI3025 and DI3016 as described in Exhibit 8 and Exhibit 9 with the two volumes each having an additional random variability that is to be incorporated to the random uncertainty of M_{AF_new}.

Following the Taylor's Series expansion approach described above, the estimated variance of M_{AF_new} for Step 4 of Exhibit 9 may be written in terms of the fundamental measurements with δ_1 and δ_2 representing the random errors for V_1 and V_2, respectively, as:

Equation 34

$$Variance\left(M_{AF_{new}}\right) \approx \left(\frac{\partial M_{AF_{new}}}{\partial M_{AF}}\right)^2 \times variance(M_{AF}) + \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_1}\right)^2 \times variance(LI3025_1)$$
$$+ \left(\frac{\partial M_{AF_{new}}}{\partial DI3026_1}\right)^2 \times variance(DI3026_1) + \left(\frac{\partial M_{AF_{new}}}{\partial \delta_1}\right)^2 \times variance(\delta_1)$$
$$+ \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_2}\right)^2 \times variance(LI3025_2) + \left(\frac{\partial M_{AF_{new}}}{\partial \delta_2}\right)^2 \times variance(\delta_2)$$

GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_new} for Step 4 (see the upper portion of Exhibit 24) and to document the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_out} value (see the lower portion of Exhibit 24 and Exhibit 25). For example, the x_{11}, x_{12}, y_{11}, and y_{12} values; and the x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values (based upon the values of the LI3025 instrument as indicated by $LI3025_1$ and $LI3025_2$, respectively) for determining volume as indicated in Exhibit 8.To complete the information necessary to compute the estimate of the variance of the M_{AF_out}, estimates of the variance terms of Equation 34 are needed. These values along with a description of the terms of Equation 34 are provided in Table 15.

Equation:

$$\rho=(LI3025_1-DI3026_1)/Sep;$$
$$V_1=((LI3025_1/\rho)+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12})+y_{12}+\delta_1;$$
$$V_2=((LI3025_2/\rho)+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22})+y_{22}+\delta_2;$$
$$M_{AF\_new}=M_{AF}*V_2/V_1;$$

Partial Derivatives:

$$\partial\rho/\partial LI3025_1 = 1.0/Sep;$$

$$\partial\rho/\partial DI3026_1 = (-1.0)/Sep;$$

$$\partial\rho/\partial Sep = (-(LI3025_1 - DI3026_1))/sqr(Sep);$$

$$\partial V_1/\partial LI3025_1 = (y_{11} - y_{12})\cdot(1.0/\rho + (-LI3025_1\cdot\partial\rho/\partial LI3025_1)/sqr(\rho))/(x_{11} - x_{12});$$

$$\partial V_1/\partial DI3026_1 = (y_{11} - y_{12})\cdot(-LI3025_1\cdot\partial\rho/\partial DI3026_1)/sqr(\rho)/(x_{11} - x_{12});$$

$$\partial V_1/\partial Sep = (y_{11} - y_{12})\cdot(-LI3025_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{11} - x_{12});$$

$$\partial V_1/\partial Heel = (y_{11} - y_{12})/(x_{11} - x_{12});$$

$$\partial V_1/\partial x_{12} = (y_{11} - y_{12})\cdot(-1.0)/(x_{11} - x_{12}) + (-(LI3025_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12})\cdot(-1.0))/sqr(x_{11} - x_{12});$$

$$\partial V_1/\partial y_{11} = (LI3025_1/\rho + Heel - x_{12})/(x_{11} - x_{12});$$

$$\partial V_1/\partial y_{12} = (LI3025_1/\rho + Heel - x_{12})\cdot(-1.0)/(x_{11} - x_{12}) + 1.0;$$

$$\partial V_1/\partial x_{11} = (-(LI3025_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12}))/sqr(x_{11} - x_{12});$$

$$\partial V_1/\partial\delta_1 = 1.0;$$

$$\partial V_2/\partial LI3025_1 = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial LI3025_1)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial DI3026_1 = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial DI3026_1)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial Sep = (y_{21} - y_{22})\cdot(-LI3025_2\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial Heel = (y_{21} - y_{22})/(x_{21} - x_{22});$$

$$\partial V_2/\partial LI3025_2 = (y_{21} - y_{22})\cdot 1.0/\rho/(x_{21} - x_{22});$$

$$\partial V_2/\partial x_{22} = (y_{21} - y_{22})\cdot(-1.0)/(x_{21} - x_{22}) + (-(LI3025_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22})\cdot(-1.0))/sqr(x_{21} - x_{22});$$

$$\partial V_2/\partial y_{21} = (LI3025_2/\rho + Heel - x_{22})/(x_{21} - x_{22});$$

$$\partial V_2/\partial y_{22} = (LI3025_2/\rho + Heel - x_{22})\cdot(-1.0)/(x_{21} - x_{22}) + 1.0;$$

$$\partial V_2/\partial x_{21} = (-(LI3025_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22}))/sqr(x_{21} - x_{22});$$

$$\partial V_2/\partial\delta_2 = 1.0;$$

Exhibit 24 Equations for Calculating the Random Uncertainty for the M_{AF} Heel in the SRAT (part 1 of 2)

Additional Partial Derivatives	
	$\partial M_{AF\_new}/\partial LI3025_1 = M_{AF}\cdot\partial V_2/\partial LI3025_1/V_1 + (-M_{AF}\cdot V_2\cdot\partial V_1/\partial LI3025_1)/sqr(V_1);$
	$\partial M_{AF\_new}/\partial DI3026_1 = M_{AF}\cdot\partial V_2/\partial DI3026_1/V_1 + (-M_{AF}\cdot V_2\cdot\partial V_1/\partial DI3026_1)/sqr(V_1);$
	$\partial M_{AF\_new}/\partial Sep = M_{AF}\cdot\partial V_2/\partial Sep/V_1 + (-M_{AF}\cdot V_2\cdot\partial V_1/\partial Sep)/sqr(V_1);$
	$\partial M_{AF\_new}/\partial Heel = M_{AF}\cdot\partial V_2/\partial Heel/V_1 + (-M_{AF}\cdot V_2\cdot\partial V_1/\partial Heel)/sqr(V_1);$
	$\partial M_{AF\_new}/\partial x_{12} = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial x_{12})/sqr(V_1);$
	$\partial M_{AF\_new}/\partial y_{11} = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial y_{11})/sqr(V_1);$
	$\partial M_{AF\_new}/\partial y_{12} = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial y_{12})/sqr(V_1);$
	$\partial M_{AF\_new}/\partial x_{11} = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial x_{11})/sqr(V_1);$
	$\partial M_{AF\_new}/\partial \delta_1 = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial \delta_1)/sqr(V_1);$
	$\partial M_{AF\_new}/\partial LI3025_2 = M_{AF}\cdot\partial V_2/\partial LI3025_2/V_1;$
	$\partial M_{AF\_new}/\partial x_{22} = M_{AF}\cdot\partial V_2/\partial x_{22}/V_1;$
	$\partial M_{AF\_new}/\partial y_{21} = M_{AF}\cdot\partial V_2/\partial y_{21}/V_1;$
	$\partial M_{AF\_new}/\partial y_{22} = M_{AF}\cdot\partial V_2/\partial y_{22}/V_1;$
	$\partial M_{AF\_new}/\partial x_{21} = M_{AF}\cdot\partial V_2/\partial x_{21}/V_1;$
	$\partial M_{AF\_new}/\partial \delta_2 = M_{AF}\cdot\partial V_2/\partial \delta_2/V_1;$
	$\partial M_{AF\_new}/\partial M_{AF} = V_2/V_1;$

Exhibit 25 Equations for Calculating the Random Uncertainty for the M_{AF} Heel in the SRAT (part 2 of 2)

Table 15 Terms and Estimated Random Uncertainties Supporting Equation 34

Term/Instrument	Description	1-Sigma Random Uncertainty
M_{AF}	current AF mass prior to transfer (kg)	Based upon SRAT Status information
LI3025 with subscript 1	level bubbler value (inwc)	±1% of 231.6 inwc span [11]
	DCS Deviation Limit	±0.1 inwc [11]
		Using a uniform distribution, 1-sigma random is $[(2.316/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5}$ = 1.3384 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
	DCS Deviation Limit	±0.05 inwc [12]
		Using a uniform distribution, 1-sigma random is $[(1.61/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5}$ = 0.9300 inwc
δ_1	tank calibration uncertainty (see footnote for Table 7)	1-sigma random = 9 gallons
δ_2	tank calibration uncertainty (see footnote for Table 7)	1-sigma random = 9 gallons

To complete the evaluation of the M_{AF_new} required for Step 4, the bias for M_{AF_new} determined by Equation 33 must be estimated. A bound (at 95% confidence) on the bias of the antifoam mass for the SRAT heel, M_{AF_new}, is estimated, as above, by appealing to a Taylor Series expansion of Equation 33 in the fundamental measurements. Once again, note that ρ, V_1, and V_2 are intermediary values. Also, the V_1 and V_2 values each have a potential bias that is to be included in the evaluation. The Taylor's Series expansion may be expressed in the fundamental measurements and the potential biases of the calculated volumes as given by:

Equation 35

$$\{bias(M_{AF_{new}})\}^2 \approx \left(\frac{\partial M_{AF_{new}}}{\partial M_{AF}}\right)^2 \times \{bias(M_{AF})\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_1}\right)^2 \times \{bias(LI3025_1)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial DI3026_1}\right)^2 \times \{bias(DI3026_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3025_2}\right)^2 \times \{bias(LI3025_2)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial b_1}\right)^2 \times \{bias(b_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial b_2}\right)^2 \times \{bias(b_2)\}^2$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{new}}}{\partial LI3025_1}\right| \times \left|\frac{\partial M_{AF_{new}}}{\partial LI3025_2}\right| \times bias(LI3025_1) \times bias(LI3025_2)$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{new}}}{\partial b_1}\right| \times \left|\frac{\partial M_{AF_{new}}}{\partial b_2}\right| \times bias(b_1) \times bias(b_2)$$

Note that in evaluating Equation 35, two potential correlations among the biases are introduced in a bounding manner. So the approach may be stated as: the b_1 and b_2 terms are the estimated biases in the volumes V_1 and V_2, respectively, which may be correlated. In addition, the biases of the two LI3025 values ($LI3025_1$ and $LI3025_2$) may also be correlated. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of M_{AF_new} for Step 4 (see the upper portion of Exhibit 26) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_out} value (see the lower portion of Exhibit 26 and Exhibit 27). Once again, the x_{11}, x_{12}, y_{11}, and y_{12} values; and the x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values (based upon the values of the LI3025 instrument as indicated by $LI3025_1$ and $LI3025_2$, respectively) for determining volumes as indicated in Exhibit 8. To complete the information necessary to compute the estimate of the bias of the M_{AF_out}, estimates of the bias terms of Equation 35 are needed. These values along with a description of the terms are provided in Table 16. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B9 in Appendix B.

Table 16 Terms and Estimated Bias Uncertainties Supporting Equation 35

Term/Instrument	Description	Bias Uncertainty at 95% Confidence
M_{AF}	current AF mass prior to transfer (kg)	Based upon SRAT Status information
LI3025 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.6 inwc span [11]
		Bias = 2.316 inwc
DI3026 with subscript 1	density bubbler value (inwc)	±1% of 161.0 inwc span [12]
		Bias = 1.61 inwc
b_1	tank calibration uncertainty (see footnote for Table 8)	12 gallons
b_2	tank calibration uncertainty (see footnote for Table 8)	12 gallons
Sep	Separation between bubblers (47 inches)	0.0625 inch [13]
Heel	Tank heel below LI3025 (6.77 inches)	0.0625 inch [13]

Equation:

ρ=(LI3025$_1$-DI3026$_1$)/Sep;
V_1=((LI3025$_1$/ρ)+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12})+y_{12}+b_1;
V_2=((LI3025$_2$/ρ)+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22})+y_{22}+b_2;
M_{AF_new}=M_{AF}*V_2/V_1;

Partial Derivatives:

∂ρ/∂LI3025$_1$ = 1.0/Sep;

∂ρ/∂DI3026$_1$ = (-1.0)/Sep;

∂ρ/∂Sep = (-(LI3025$_1$ - DI3026$_1$))/sqr(Sep);

∂V_1/∂LI3025$_1$ = (y_{11} - y_{12})·(1.0/ρ + (-LI3025$_1$·∂ρ/∂LI3025$_1$)/sqr(ρ))/(x_{11} - x_{12});

∂V_1/∂DI3026$_1$ = (y_{11} - y_{12})·(-LI3025$_1$·∂ρ/∂DI3026$_1$)/sqr(ρ)/(x_{11} - x_{12});

∂V_1/∂Sep = (y_{11} - y_{12})·(-LI3025$_1$·∂ρ/∂Sep)/sqr(ρ)/(x_{11} - x_{12});

∂V_1/∂Heel = (y_{11} - y_{12})/(x_{11} - x_{12});

∂V_1/∂x_{12} = (y_{11} - y_{12})·(-1.0)/(x_{11} - x_{12}) + (-(LI3025$_1$/ρ + Heel - x_{12})·(y_{11} - y_{12})·(-1.0))/sqr(x_{11} - x_{12});

∂V_1/∂y_{11} = (LI3025$_1$/ρ + Heel - x_{12})/(x_{11} - x_{12});

∂V_1/∂y_{12} = (LI3025$_1$/ρ + Heel - x_{12})·(-1.0)/(x_{11} - x_{12}) + 1.0;

∂V_1/∂x_{11} = (-(LI3025$_1$/ρ + Heel - x_{12})·(y_{11} - y_{12}))/sqr(x_{11} - x_{12});

∂V_1/∂b_1 = 1.0;

∂V_2/∂LI3025$_1$ = (y_{21} - y_{22})·(-LI3025$_2$·∂ρ/∂LI3025$_1$)/sqr(ρ)/(x_{21} - x_{22});

∂V_2/∂DI3026$_1$ = (y_{21} - y_{22})·(-LI3025$_2$·∂ρ/∂DI3026$_1$)/sqr(ρ)/(x_{21} - x_{22});

∂V_2/∂Sep = (y_{21} - y_{22})·(-LI3025$_2$·∂ρ/∂Sep)/sqr(ρ)/(x_{21} - x_{22});

∂V_2/∂Heel = (y_{21} - y_{22})/(x_{21} - x_{22});

∂V_2/∂LI3025$_2$ = (y_{21} - y_{22})·1.0/ρ/(x_{21} - x_{22});

Exhibit 26 Equations for M_{AF_new} for the SRAT Heel with Bias Uncertainty (part 1 of 2)

Additional Partial Derivatives

$\partial V_2/\partial x_{22} = (y_{21} - y_{22})\cdot(-1.0)/(x_{21} - x_{22}) + (-(LI3025_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22})\cdot(-1.0))/sqr(x_{21} - x_{22});$

$\partial V_2/\partial y_{21} = (LI3025_2/\rho + Heel - x_{22})/(x_{21} - x_{22});$

$\partial V_2/\partial y_{22} = (LI3025_2/\rho + Heel - x_{22})\cdot(-1.0)/(x_{21} - x_{22}) + 1.0;$

$\partial V_2/\partial x_{21} = (-(LI3025_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22}))/sqr(x_{21} - x_{22});$

$\partial V_2/\partial b_2 = 1.0;$

$\partial M_{AF\_new}/\partial LI3025_1 = M_{AF}\cdot\partial V_2/\partial LI3025_1/V_1 + (-M_{AF}\cdot V_2\cdot\partial V_1/\partial LI3025_1)/sqr(V_1);$

$\partial M_{AF\_new}/\partial DI3026_1 = M_{AF}\cdot\partial V_2/\partial DI3026_1/V_1 + (-M_{AF}\cdot V_2\cdot\partial V_1/\partial DI3026_1)/sqr(V_1);$

$\partial M_{AF\_new}/\partial Sep = M_{AF}\cdot\partial V_2/\partial Sep/V_1 + (-M_{AF}\cdot V_2\cdot\partial V_1/\partial Sep)/sqr(V_1);$

$\partial M_{AF\_new}/\partial Heel = M_{AF}\cdot\partial V_2/\partial Heel/V_1 + (-M_{AF}\cdot V_2\cdot\partial V_1/\partial Heel)/sqr(V_1);$

$\partial M_{AF\_new}/\partial x_{12} = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial x_{12})/sqr(V_1);$

$\partial M_{AF\_new}/\partial y_{11} = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial y_{11})/sqr(V_1);$

$\partial M_{AF\_new}/\partial y_{12} = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial y_{12})/sqr(V_1);$

$\partial M_{AF\_new}/\partial x_{11} = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial x_{11})/sqr(V_1);$

$\partial M_{AF\_new}/\partial b_1 = (-M_{AF}\cdot V_2\cdot\partial V_1/\partial b_1)/sqr(V_1);$

$\partial M_{AF\_new}/\partial LI3025_2 = M_{AF}\cdot\partial V_2/\partial LI3025_2/V_1;$

$\partial M_{AF\_new}/\partial x_{22} = M_{AF}\cdot\partial V_2/\partial x_{22}/V_1;$

$\partial M_{AF\_new}/\partial y_{21} = M_{AF}\cdot\partial V_2/\partial y_{21}/V_1;$

$\partial M_{AF\_new}/\partial y_{22} = M_{AF}\cdot\partial V_2/\partial y_{22}/V_1;$

$\partial M_{AF\_new}/\partial x_{21} = M_{AF}\cdot\partial V_2/\partial x_{21}/V_1;$

$\partial M_{AF\_new}/\partial b_2 = M_{AF}\cdot\partial V_2/\partial b_2/V_1;$

$\partial M_{AF\_new}/\partial M_{AF} = V_2/V_1;$

Exhibit 27 Equations for M_{AF_new} for the SRAT Heel with Bias Uncertainty (part 2 of 2)

3.4 Tracking Antifoam in the SME

The SME is the hold-point of the antifoam tracking system. It is the contents of the SME that must be shown with high confidence to meet the restrictions imposed in reference [1]. To support the tracking of antifoam for a SME batch, the M_{AF} value and its uncertainty (a 1-sigma random uncertainty and limit on bias at 95% confidence) associated with the SME contents are to be maintained at all times. If these values are not known, the contents of the SME are to be sampled and analyzed to re-baseline the antifoam mass and its uncertainty in this tank. With these values known, the antifoam tracking system must be capable up handling three types of events: (1) an event involving a transfer from the AMFT into the SME, (2) an event involving the transfer of SRAT product to the SME, (3) the event of a determination of acceptability for transfer of the SME material to the Melter Feed Tank (MFT). Handling these events entails updating the status of the M_{AF} value and its uncertainty in the SME as well as confirming that the status of the M_{AF} and its uncertainty in the SME tank meet are restrictions before transferring the SME product to the MFT

Exhibit 28 provides a flow diagram at the SME level for processing an event involving the SME. For any SME-related event, there is an initial assessment of the current status of the M_{AF} value and its uncertainty in the SME. That is represented by the first decision step in the process flow diagram. If the status is unknown, then the “No” branch is taken out of this decision block and the value of the M_{AF} and its uncertainty must be re-base lined as indicated in Step 1 of the diagram. If the status is known, the “Yes” branch is taken out of this decision block. With the M_{AF} value and its uncertainty known, the next decision block is evaluated to determine the type of event that needs to be addressed by the tracking system. Once again, there are three primary events captured in the flow diagram: a) if there is a transfer of SRAT product to the SME, then the impact of the change in the mass of antifoam in the contents of the SME must be determined (indicated as Step 2 in Exhibit 28), b) if there is a transfer from the AMFT, then an antifoam addition is to be made to the SME and the impact of this addition on the M_{AF} value must be determined (indicated as Step 3 in Exhibit 28),and c) if the acceptability of a transfer of SME product to the MFT is to be determined, then the constraints of reference [1] must be met after accounting for appropriate uncertainties (indicated as Step 4 in Exhibit 28) prior to the transfer of material to the MFT. And once acceptability is confirmed and a transfer to the MFT is made there is a need to update the M_{AF} status of the SME after the transfer (indicated as Step 5 in Exhibit 28).

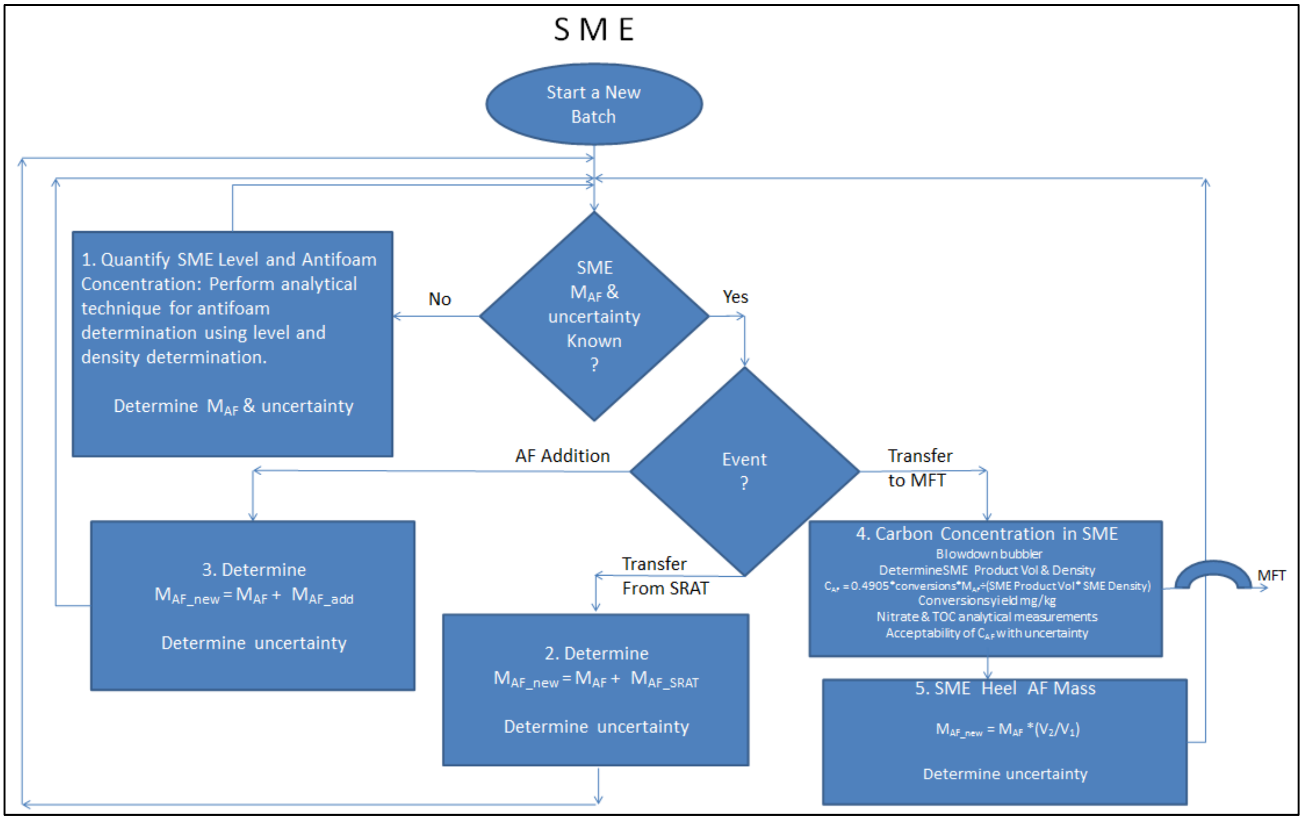

Exhibit 28 Process Flow for Tracking Antifoam Mass in the Slurry Mix Evaporator (SME) Tank

Exhibit 29 and Exhibit 30 provide an overview of the SME calculations supporting the antifoam tracking system. The calculations for Step 1 are executed if the M_{AF} value or its uncertainty is unknown for the SME; this results in the re-establishment of a M_{AF} value along with its uncertainty.

3.4.1 SME Step 2 and Step 3 Processing

For Step 2, the one-sigma uncertainty for the new value of M_{AF} (i.e., M_{AF_new} in the Step 2 equation) is the square root of the sum of the variances of the two terms on the right-hand side of the Step 2 equation: M_{AF} and M_{AF_Add}, while the bias of the new value is simply the sum of the biases of the two terms on the right-hand side of the Step 2 equation. This is also true for Step 3: the one-sigma uncertainty for the new value of M_{AF} (i.e., M_{AF_new} in the Step 3 equation) is the square root of the sum of the variances of the two terms on the right-hand side of the Step 3 equation: M_{AF} and M_{AF_Add} and the bias of the new value is simply the sum of the biases of the two terms on the right-hand side of the Step 3 equation. Sample calculations for Step 2 and Step 3 including the random and bias uncertainties are provided in Exhibit B10 and Exhibit B11, respectively, in Appendix B.

3.4.2 Overview of SME Steps 1, 4 and 5 Processing

Step 4 provides the determination of the acceptability of the SME product for transfer to the MFT. M_{AF_out} value associated with the transfer from the SRAT to the SME. Step 5 provides the mass of the antifoam remaining in the SME (i.e., the SME heel) after the transfer out of the SME to the MFT has been completed. The calculations supporting each of steps 1, 3 and 4 of the tracking system for the SME are covered in turn in the following discussion.

SME Calculations

1. Re-base-lining of the SME
   The SME vessel will be re-base-lined using the analytical method for antifoam concentration determination that is currently used. The mass of antifoam at the time of the SME sample can be determined by

$$M_{AF} = C_{AF} \cdot V_1 \cdot \rho$$

where the determination of volume, V_1, from the source instrumentation, LI3109 and DI3108, is described below and ρ is calculated from these instruments by

$$\rho = \frac{(LI3109_1 - DI3108_1)}{Sep}$$

and Sep stands for the separation between the two instruments. The Sep value is 47 inches.

The volume in the SME can be represented in terms of the value at event, i = 1 (before a transfer) or i = 2 (after a transfer), of its source instrumentation LI3109 as:

$$For\ x_{i2} < \left(\frac{LI3109_i}{\rho} + Heel\right) < x_{i1},$$

$$\frac{\frac{LI3109_i}{\rho} + Heel - x_{i2}}{(x_{i1} - x_{i2})} \times (y_{i1} - y_{i2}) + y_{i2}$$

where the Heel is 6.77 inches and there are four sets of x's and y's corresponding to 4 segments within the SME. These values are (see reference [5]):

Segment	x_{i1}	x_{i2}	y_{i1}	y_{i2}
1 (lowest)	9.8051	0	500	0
2	79.232	9.8051	5240	500
3	125.25	79.232	8500	5240
4 (highest)	175.02	125.25	12000	10850

Exhibit 29 SME Calculations Supporting the Antifoam Tracking System (part 1 of 2)

2. Antifoam Mass in the SME after a Transfer from the SRAT
   The mass of antifoam in the SME after the SRAT transfer (the amount from the SRAT is as determined in SRAT Step 3) is given by:

$$M_{AF\_new} = M_{AF} + M_{AF\_Add}$$

3. Antifoam Mass in the SME after an Addition from the AMFT
   The mass of antifoam in the SME after the AMFT addition (the amount from the AMFT is as determined in AMFT Step 3) is given by:

$$M_{AF\_new} = M_{AF} + M_{AF\_Add}$$

4. SME Acceptability Decision
   The carbon concentration in the SME from antifoam is

$$C_{AF} = \frac{0.4905 \cdot 1000000 \cdot M_{AF}}{\rho \cdot V_1 \cdot 3.7854}$$

   where the density (kg/L), ρ, and the volume V_1 (gal) are determined as described in Step 1 above. The value of 3.7854 is a conversion factor with units of L/gal. The value of 0.4905 is a conservative (i.e., bounding on the high side) conversion factor with units of kg of carbon per kg of antifoam. The 1,000,000 value is a conversion factor with units of mg/kg. The C_{AF} value is assessed against the constraints imposed by [1] with all uncertainties appropriately addressed.

5. Amount of Antifoam in the SME Heel Following a Transfer Out to the Melter Feed Tank (MFT)
   The amount of antifoam remaining in SME after a transfer out to the MFT is

$$M_{AF_{new}} = M_{AF} \cdot \frac{V_2}{V_1}$$

   where V_1 and V_2 are volumes in the SME which can be represented in terms of the values for i = 1 (before the transfer) and 2 (after the transfer) of their source instrumentation, LI3109. These values are determined as described in Step 1 above. Note that the density value, ρ, that is used in the determination of V_1 is also used in the determination of V_2.

Exhibit 30 SME Calculations Supporting the Antifoam Tracking System (part 2 of 2)

3.4.3 SME Step 1 Processing

The equation for Step 1 of Exhibit 29 provides a guide for re-base lining the M_{AF_new} (kg) value for the SRAT. Writing the equation out with more detail to include the appropriate unit conversions yields:

Equation 36

$$M_{AF_{new}} = \frac{3.7854 \cdot C_{AF} \cdot V_1 \cdot \rho}{0.4723 \cdot 1000000}$$

In this equation, C_{AF} represents the carbon concentration (mg/kg) from AF determined from the analytical measurements of the SRAT contents with the volume (gal), V_1, and density (kg/L), ρ, determined as indicated in the Step 1 description of Exhibit 29 by measurements from instruments LI3109 and DI3108 along with values for the separation (Sep) between the bubblers and the heel (Heel) for LI3109. The value of 3.7854 is a conversion factor with units of L/gal. Once again, the value of 0.4723 is a conservative (i.e., bounding on the low side) conversion factor with units of kg of carbon per kg of antifoam. The 1,000,000 value is a conversion factor with units of mg/kg. Note that ρ and V_1 are intermediary values with the V_1 value having an additional variability described below. Using the Taylor's Series expansion approach described above, the estimated random variance of M_{AF_new} for Step 1 of Exhibit 29 may be expressed in the fundamental measurements as given by:

Equation 37

$$Variance\left(M_{AF_{new}}\right) \approx \left(\frac{\partial M_{AF_{new}}}{\partial C_{AF}}\right)^2 \times variance(C_{AF}) + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_1}\right)^2 \times variance(LI3109_1)$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial DI3108_1}\right)^2 \times variance(DI3108_1) + \left(\frac{\partial M_{AF_{new}}}{\partial \delta_1}\right)^2 \times variance(\delta_1)$$

where all of the estimated variances are for the random uncertainties of the indicated measurements. Specifically, the variance for C_{AF} is estimated from the analyses of the SME samples as given by equation 12 and the variance(δ_1) term represents the variance of the random uncertainty associated with the computed volume, V_1.(see the upper portion of Exhibit 31 for the introduction of the δ_1 term into the model equation for M_{AF_new}). GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_new} for Step 1 (see the upper portion of Exhibit 31) and to document the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_new} value (see the lower portion of Exhibit 31). For example, x_{11}, x_{12}, y_{11}, and y_{12} are appropriately selected values (based upon the value of the LI3109 instrument as indicated by $LI3109_1$) for determining volume as indicated in Exhibit 29.To complete the information necessary to compute the estimate of the variance of the M_{AF_new}, estimates of the variance terms of Equation 37 are needed. These values along with a description of the terms are provided in Table 17.

Table 17 Terms and Estimated Random Uncertainties Supporting Equation 37

Term/Instrument	Description	1-Sigma Random Uncertainty
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	Analytical uncertainty (see Equation 12)
LI3109 with subscript 1	level bubbler value (inwc)	±1% of 231.0 inwc span [14]
	DCS Deviation Limit	±0.1 inwc [14]
		Using a uniform distribution, 1-sigma random is $[(2.31/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5} = 1.3349$
DI3108 with subscript 1	density bubbler value (inwc)	±1% of 160.5 inwc span [15]
	DCS Deviation Limit	±0.05 inwc [15]
		Using a uniform distribution, 1-sigma random is $[(1.605/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5} = 0.9271$
δ_1	tank calibration uncertainty (see WSRC-TR-92-250 [8]♣)	1-sigma random = 9 gallons

♣ The random uncertainty of the tank calibration was estimated in this report for the SRAT and the SME as the total error of the Holledge gauge, 0.25 inch, times the slope of the calibration curve. For the SME, the largest slope is 70.847gal/inch, leading to an estimate of the total (2-sigma) random uncertainty of 70.847 × 0.25 = 17.7 gal, or a 1-sigma random uncertainty of 9 gal.

Equation:

$\rho=(LI3109_1-DI3108_1)/Sep;$

$V_1=((LI3109_1/\rho+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12}))+y_{12}+\delta_1;$

$M_{AF\_new}=3.7854*C_{AF}*V_1*\rho/(0.4723*1000000);$

Partial Derivatives:

$\partial\rho/\partial LI3109_1 = 1.0/Sep;$

$\partial\rho/\partial DI3108_1 = (-1.0)/Sep;$

$\partial\rho/\partial Sep = (-(LI3109_1 - DI3108_1))/sqr(Sep);$

$\partial V_1/\partial LI3109_1 = (y_{11} - y_{12})\cdot(1.0/\rho + (-LI3109_1\cdot\partial\rho/\partial LI3109_1)/sqr(\rho))/(x_{11} - x_{12});$

$\partial V_1/\partial DI3108_1 = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial DI3108_1)/sqr(\rho)/(x_{11} - x_{12});$

$\partial V_1/\partial Sep = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{11} - x_{12});$

$\partial V_1/\partial Heel = (y_{11} - y_{12})/(x_{11} - x_{12});$

$\partial V_1/\partial x_{12} = (y_{11} - y_{12})\cdot(-1.0)/(x_{11} - x_{12}) + (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12})\cdot(-1.0))/sqr(x_{11} - x_{12});$

$\partial V_1/\partial y_{11} = (LI3109_1/\rho + Heel - x_{12})/(x_{11} - x_{12});$

$\partial V_1/\partial y_{12} = (LI3109_1/\rho + Heel - x_{12})\cdot(-1.0)/(x_{11} - x_{12}) + 1.0;$

$\partial V_1/\partial x_{11} = (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12}))/sqr(x_{11} - x_{12});$

$\partial V_1/\partial\delta_1 = 1.0;$

$\partial M_{AF\_new}/\partial LI3109_1 = (3.7854\cdot C_{AF}\cdot V_1\cdot\partial\rho/\partial LI3109_1 + \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial LI3109_1)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial DI3108_1 = (3.7854\cdot C_{AF}\cdot V_1\cdot\partial\rho/\partial DI3108_1 + \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial DI3108_1)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Sep = (3.7854\cdot C_{AF}\cdot V_1\cdot\partial\rho/\partial Sep + \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial Sep)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Heel = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial Heel/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_{12} = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial x_{12}/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_{11} = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial y_{11}/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_{12} = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial y_{12}/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_{11} = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial x_{11}/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial\delta_1 = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial\delta_1/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial C_{AF} = \rho\cdot V_1\cdot 3.7854/(0.4723\cdot 1000000.0);$

Exhibit 31 Equations for Re-Base-Lining the M_{AF} of the SME with Random Uncertainty

To complete the updating of the M_{AF} status required for Step 1, the bias for M_{AF_new} determined by Equation 36 must be estimated. A bound (at 95% confidence) on the bias of the antifoam mass, M_{AF}, is estimated, as above, by appealing to a Taylor Series expansion of this equation in the fundamental measurements. Once again, note that ρ and V_1 are intermediary values and that the Taylor's Series expansion may be expressed in the fundamental measurements as given by:

Equation 38

$$\{bias(M_{AF_{new}})\}^2 \approx \left(\frac{\partial M_{AF_{new}}}{\partial C_{AF}}\right)^2 \times \{bias(C_{AF})\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_1}\right)^2 \times \{bias(LI3109_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial DI3108_1}\right)^2 \times \{bias(DI3108_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial \delta_1}\right)^2 \times \{b_1\}^2$$

Note that in evaluating Equation 38, the bias for the C_{AF} term, i.e., bias(C_{AF}) term is estimated to be zero and that there are no correlations among the bias terms in this equation. That is, the analytical estimate of the concentration of carbon from AF is unbiased. Also, the b_1 term is the estimated bias in the volume, V_1, of Equation 37. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_new} for Step 1 (see the upper portion of Exhibit 32) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_new} value (see the lower portion of Exhibit 32). Once again, x_{11}, x_{12}, y_{11}, and y_{12} are appropriately selected values (based upon the value of the LI3109 instrument as indicated by $LI3109_1$) for determining volume as indicated in Exhibit 29. To complete the information necessary to compute the estimate of the bias of the M_{AF_new}, estimates of the bias terms of Equation 38 are needed. Table 18 provides the details of the bias information needed to complete the estimation of the bias for the M_{AF_new} value. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B12 in Appendix B.

Equation:

$\rho=(LI3109_1-DI3108_1)/Sep;$

$V_1=((LI3109_1/\rho+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12}))+y_{12}+b_1;$

$M_{AF\_new}=3.7854*C_{AF}*V_1*\rho/(0.4723*1000000);$

Partial Derivatives:

$\partial\rho/\partial LI3109_1 = 1.0/Sep;$

$\partial\rho/\partial DI3108_1 = (-1.0)/Sep;$

$\partial\rho/\partial Sep = (-(LI3109_1 - DI3108_1))/sqr(Sep);$

$\partial V_1/\partial LI3109_1 = (y_{11} - y_{12})\cdot(1.0/\rho + (-LI3109_1\cdot\partial\rho/\partial LI3109_1)/sqr(\rho))/(x_{11} - x_{12});$

$\partial V_1/\partial DI3108_1 = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial DI3108_1)/sqr(\rho)/(x_{11} - x_{12});$

$\partial V_1/\partial Sep = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{11} - x_{12});$

$\partial V_1/\partial Heel = (y_{11} - y_{12})/(x_{11} - x_{12});$

$\partial V_1/\partial x_{12} = (y_{11} - y_{12})\cdot(-1.0)/(x_{11} - x_{12}) + (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12})\cdot(-1.0))/sqr(x_{11} - x_{12});$

$\partial V_1/\partial y_{11} = (LI3109_1/\rho + Heel - x_{12})/(x_{11} - x_{12});$

$\partial V_1/\partial y_{12} = (LI3109_1/\rho + Heel - x_{12})\cdot(-1.0)/(x_{11} - x_{12}) + 1.0;$

$\partial V_1/\partial x_{11} = (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12}))/sqr(x_{11} - x_{12});$

$\partial V_1/\partial b_1 = 1.0;$

$\partial M_{AF\_new}/\partial LI3109_1 = (3.7854\cdot C_{AF}\cdot V_1\cdot\partial\rho/\partial LI3109_1 + \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial LI3109_1)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial DI3108_1 = (3.7854\cdot C_{AF}\cdot V_1\cdot\partial\rho/\partial DI3108_1 + \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial DI3108_1)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Sep = (3.7854\cdot C_{AF}\cdot V_1\cdot\partial\rho/\partial Sep + \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial Sep)/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Heel = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial Heel/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_{12} = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial x_{12}/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_{11} = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial y_{11}/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_{12} = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial y_{12}/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_{11} = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial x_{11}/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial b_1 = \rho\cdot 3.7854\cdot C_{AF}\cdot\partial V_1/\partial b_1/(0.4723\cdot 1000000.0);$

$\partial M_{AF\_new}/\partial C_{AF} = \rho\cdot V_1\cdot 3.7854/(0.4723\cdot 1000000.0);$

Exhibit 32 Equations for Re-Base-Lining the M_{AF} of the SME with Bias Uncertainty

Table 18 Terms and Estimated Bias Uncertainties Supporting Equation 38

Term/Instrument	Description	Bias Uncertainty at 95% Confidence
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	0
LI3109 with subscript 1	level bubbler value (inwc)	±1% of 231.0 inwc span [14]
		Bias = 2.310 inwc
DI3108 with subscript 1	density bubbler value (inwc)	±1% of 160.5 inwc span [15]
		Bias = 1.605 inwc
b_1	tank calibration uncertainty (see WSRC-TR-92-250 [8]♣)	6 gallons
Sep	separation between bubblers (47 inches)	0.0625 inch [13]
Heel	Tank heel below LI3109 (6.77 inches)	0.0625 inch [13]

3.4.4 SME Step 5 Processing Linked to a Step 1 Event

If a Step 1 effort is conducted for a SME Product as part of the acceptability decision for a transfer to the MFT, then the analytical work and evaluation must be conducted and the acceptability decision made under the direction of reference [2]. If there is a positive outcome from this process and a transfer to the MFT is made, then the following equation provides an estimate of the mass of antifoam that remains in the SME heel, M_{AF_new}, after the transfer has been completed (this corresponds to Step 5 of Exhibit 28):

Equation 39

$$M_{AF_{new}} = \frac{3.7854 \cdot C_{AF} \cdot \rho \cdot V_2}{0.4723 \cdot 1000000}$$

Where the density (kg/L), ρ, and the volume V_2 (gal) are intermediary values which are determined from LI3109 and DI3108 as described in Exhibit 29 and Exhibit 30 with the volume having an additional random variability that is to be incorporated to the random uncertainty of M_{AF_new}. In this equation, C_{AF} represents the carbon concentration (mg/kg) from AF determined from the analytical measurements of the SME contents. The value of 3.7854 is a conversion factor with units of L/gal. The value of 0.4723 is a conservative (i.e., bounding on the low side) conversion factor with units of kg of carbon per kg of antifoam. The 1,000,000 value is a conversion factor with units of mg/kg.

Following the Taylor's Series expansion approach described above, the estimated variance of M_{AF_new} for Step 5 of Exhibit 28 may be written in terms of the fundamental measurements with δ_2 representing the random error for V_2 as:

♣ The bias in the calibration for the SME is taken as the largest value from Table 1d. Rounding up this value is 6 gallons.

Equation 40

$$Variance\left(M_{AF_{new}}\right) \approx \left(\frac{\partial M_{AF_{new}}}{\partial C_{AF}}\right)^2 \times variance(C_{AF}) + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_1}\right)^2 \times variance(LI3109_1)$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial DI3108_1}\right)^2 \times variance(DI3108_1) + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_2}\right)^2 \times variance(LI3109_2)$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial \delta_2}\right)^2 \times variance(\delta_2)$$

GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_new} for Step 5 (see the upper portion of Exhibit 33) and to document the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_new} value (see the lower portion of Exhibit 33 and Exhibit 34). For example, the x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values (based upon the values of the LI3109 instrument as indicated by $LI3109_2$ for determining the volume remaining the Heel of the SME as indicated in Exhibit 28. To complete the information necessary to compute the estimate of the variance of the M_{AF_out}, estimates of the variance terms of Equation 40 are needed. These values along with a description of the terms of the equation are provided in Table 19.

Equation:

ρ=(LI3109$_1$-DI3108$_1$)/Sep;
V_2=((LI3109$_2$/ρ+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22}))+y_{22}+δ_2;
M_{AF_new}=3.7854*C_{AF}*V_2*ρ/(0.4723*1000000);

Partial Derivatives:

$\partial\rho/\partial$LI3109$_1$ = 1.0/Sep;

$\partial\rho/\partial$DI3108$_1$ = (-1.0)/Sep;

$\partial\rho/\partial$Sep = (-(LI3109$_1$ - DI3108$_1$))/sqr(Sep);

$\partial V_2/\partial$LI3109$_1$ = (y_{21} - y_{22})·(-LI3109$_2$·$\partial\rho/\partial$LI3109$_1$)/sqr(ρ)/(x_{21} - x_{22});

$\partial V_2/\partial$DI3108$_1$ = (y_{21} - y_{22})·(-LI3109$_2$·$\partial\rho/\partial$DI3108$_1$)/sqr(ρ)/(x_{21} - x_{22});

$\partial V_2/\partial$Sep = (y_{21} - y_{22})·(-LI3109$_2$·$\partial\rho/\partial$Sep)/sqr(ρ)/(x_{21} - x_{22});

$\partial V_2/\partial$LI3109$_2$ = (y_{21} - y_{22})·1.0/ρ/(x_{21} - x_{22});

$\partial V_2/\partial$Heel = (y_{21} - y_{22})/(x_{21} - x_{22});

$\partial V_2/\partial x_{22}$ = (y_{21} - y_{22})·(-1.0)/(x_{21} - x_{22}) + (-(LI3109$_2$/ρ + Heel - x_{22})·(y_{21} - y_{22})·(-1.0))/sqr(x_{21} - x_{22});

$\partial V_2/\partial y_{21}$ = (LI3109$_2$/ρ + Heel - x_{22})/(x_{21} - x_{22});

$\partial V_2/\partial y_{22}$ = (LI3109$_2$/ρ + Heel - x_{22})·(-1.0)/(x_{21} - x_{22}) + 1.0;

$\partial V_2/\partial x_{21}$ = (-(LI3109$_2$/ρ + Heel - x_{22})·(y_{21} - y_{22}))/sqr(x_{21} - x_{22});

$\partial V_2/\partial\delta_2$ = 1.0;

Exhibit 33 Equations for Analytical M_{AF_new} for SME Heel with Random Uncertainty (part 1 of 2)

Additional Partial Derivatives

$\partial M_{AF\_new}/\partial LI3109_1 = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial LI3109_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial LI3109_1)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial DI3108_1 = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial DI3108_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial DI3108_1)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Sep = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial Sep + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial Sep)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial LI3109_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial LI3109_2/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Heel = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial Heel/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial x_{22}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial y_{21}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial y_{22}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial x_{21}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial \delta_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial \delta_2/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial C_{AF} = \rho \cdot V_2 \cdot 3.7854/(0.4723 \cdot 1000000.0);$

Exhibit 34 Equations for Analytical M_{AF_new} for SME Heel with Random Uncertainty (part 2 of 2)

Table 19 Terms and Estimated Random Uncertainties Supporting Equation 40

Term/Instrument	Description	1-Sigma Random Uncertainty
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	analytical uncertainty (see Equation 12)
LI3109 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.0 inwc span [14]
	DCS Deviation Limit	±0.1 inwc [14]
		Using a uniform distribution, 1-sigma random is $[(2.31/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5} = 1.3349$
DI3108 with subscript 1	density bubbler value (inwc)	±1% of 160.5 inwc span [15]
	DCS Deviation Limit	±0.05 inwc [15]
		Using a uniform distribution, 1-sigma random is $[(1.605/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5} = 0.9271$
δ_2	tank calibration uncertainty (see footnote for Table 17)	1-sigma random = 9 gallons

To complete the evaluation of the M_{AF_new} required for Step 5, the bias for M_{AF_new} determined by Equation 39 must be estimated. A bound (at 95% confidence) on the bias of the antifoam mass remaining in the SME after a transfer out to the MFT, M_{AF_new}, is estimated, as above, by appealing to a Taylor Series expansion of Equation 39 in the fundamental measurements. Once again, note that ρ and V_2 are intermediary values. Also, the V_2 value has a potential bias that is to be included in the evaluation. The Taylor's Series expansion may be expressed in the fundamental measurements and the potential bias as given by:

Equation 41

$$\{bias(M_{AF_{new}})\}^2 \approx \left(\frac{\partial M_{AF_{new}}}{\partial C_{AF}}\right)^2 \times \{bias(C_{AF})\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_1}\right)^2 \times \{bias(LI3109_1)\}^2$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial DI3108_1}\right)^2 \times \{bias(DI3108_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_2}\right)^2 \times \{bias(LI3109_2)\}^2$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial b_2}\right)^2 \times \{bias(b_2)\}^2$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{new}}}{\partial LI3109_1}\right| \times \left|\frac{\partial M_{AF_{new}}}{\partial LI3109_2}\right| \times bias(LI3109_1) \times bias(LI3109_2)$$

Note that in evaluating Equation 41, the bias for the C_{AF} term, i.e., bias(C_{AF}) term, is estimated to be zero and that a potential correlation among a pair of the biases is introduced in a bounding manner. So the approach may be stated as: The analytical estimate of the concentration of carbon from AF is unbiased, the b_2 term is the estimated bias in the V_2 volume, and there is a potential correlation in the biases for the two LI3109 measurements. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of M_{AF_new} for Step 5 (see the upper portion of Exhibit 35) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_new} value (see the lower portion of Exhibit 35 and Exhibit 36). Once again, x_{21}, x_{22}, y_{21}, and y_{22} are appropriately selected values (based upon the value of the LI3109 instrument as indicated by $LI3109_2$) for determining volume as indicated in Exhibit 29. To complete the information necessary to compute the estimate of the bias of the M_{AF_new}, estimates of the bias terms of Equation 41 are needed. Table 20 provides the details of the bias information needed to complete the estimation of the bias for the M_{AF_new} value. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B13 in Appendix B.

Equation:

$\rho=(LI3109_1-DI3108_1)/Sep;$
$V_2=((LI3109_2/\rho+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22}))+y_{22}+b_2;$
$M_{AF\_new}=3.7854*C_{AF}*V_2*\rho/(0.4723*1000000);$

Partial Derivatives:

$\partial\rho/\partial LI3109_1 = 1.0/Sep;$

$\partial\rho/\partial DI3108_1 = (-1.0)/Sep;$

$\partial\rho/\partial Sep = (-(LI3109_1 - DI3108_1))/sqr(Sep);$

$\partial V_2/\partial LI3109_1 = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial LI3109_1)/sqr(\rho)/(x_{21} - x_{22});$

$\partial V_2/\partial DI3108_1 = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial DI3108_1)/sqr(\rho)/(x_{21} - x_{22});$

$\partial V_2/\partial Sep = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{21} - x_{22});$

$\partial V_2/\partial LI3109_2 = (y_{21} - y_{22})\cdot 1.0/\rho/(x_{21} - x_{22});$

$\partial V_2/\partial Heel = (y_{21} - y_{22})/(x_{21} - x_{22});$

$\partial V_2/\partial x_{22} = (y_{21} - y_{22})\cdot(-1.0)/(x_{21} - x_{22}) + (-(LI3109_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22})\cdot(-1.0))/sqr(x_{21} - x_{22});$

$\partial V_2/\partial y_{21} = (LI3109_2/\rho + Heel - x_{22})/(x_{21} - x_{22});$

$\partial V_2/\partial y_{22} = (LI3109_2/\rho + Heel - x_{22})\cdot(-1.0)/(x_{21} - x_{22}) + 1.0;$

$\partial V_2/\partial x_{21} = (-(LI3109_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22}))/sqr(x_{21} - x_{22});$

$\partial V_2/\partial b_2 = 1.0;$

Exhibit 35 Equations for Analytical M_{AF_new} for SME Heel with Bias Uncertainty (part 1 of 2)

Additional Partial Derivatives

$\partial M_{AF\_new}/\partial LI3109_1 = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial LI3109_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial LI3109_1)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial DI3108_1 = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial DI3108_1 + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial DI3108_1)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Sep = (3.7854 \cdot C_{AF} \cdot V_2 \cdot \partial\rho/\partial Sep + \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial Sep)/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial LI3109_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial LI3109_2/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial Heel = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial Heel/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial x_{22}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial y_{21}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial y_{22} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial y_{22}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial x_{21} = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial x_{21}/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial b_2 = \rho \cdot 3.7854 \cdot C_{AF} \cdot \partial V_2/\partial b_2/(0.4723 \cdot 1000000.0);$

$\partial M_{AF\_new}/\partial C_{AF} = \rho \cdot V_2 \cdot 3.7854/(0.4723 \cdot 1000000.0);$

Exhibit 36 Equations for Analytical M_{AF_new} for SME Heel with Bias Uncertainty (part 2 of 2)

Table 20 Terms and Estimated Bias Uncertainties Supporting Equation 41

Term/Instrument	Description	Bias Uncertainty
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	0
LI3109 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.0 inwc span [14]
		Bias = 2.310 inwc
DI3108 with subscript 1	density bubbler (inwc)	±1% of 160.5 inwc span [15]
		Bias = 1.605 inwc
b_2	Tank Calibration uncertainty (see footnote for Table 18)	6 gallons
Sep	Separation between bubblers (47 inches)	0.0625 inch [13]
Heel	Tank heel below LI3109 (6.77 inches)	0.0625 inch [13]

3.4.5 SME Step 4 Processing

The primary purpose of the AF tracking system is to provide an additional method beyond the approach provided in [2] of demonstrating that the SME material meets the constraints imposed by [1] for flammability control. Following the process flow diagram (Exhibit 28) and the supporting calculations (Exhibit 29 and Exhibit 30), if the SME M_{AF} value is known along with the associated uncertainties, then the concentration of carbon associated with this mass of AF, C_{AF}, expressed in mg/kg, is determined by:

Equation 42

$$C_{AF} = \frac{0.4905 \cdot 1000000 \cdot M_{AF}}{\rho \cdot V_1 \cdot 3.7854}$$

where the density (kg/L), ρ, and the volume V_1 (gal) are intermediary values which are determined from LI3109 and DI3108 as described in Exhibit 29 and Exhibit 30 with the volume having an additional random variability that is to be incorporated into the random uncertainty of C_{AF}. The value of 3.7854 is a conversion factor with units of L/gal. The value of 0.4905 is a conservative (i.e., bounding on the high side) conversion factor with units of kg of carbon per kg of antifoam[f]. The 1,000,000 value is a conversion factor with units of mg/kg.

Following the Taylor's Series expansion approach described above, the estimated variance of C_{AF} for Step 4 of Exhibit 28 may be written in terms of the fundamental measurements with δ_1 representing the random error for V_1 as:

Equation 43

$$Variance(C_{AF}) \approx \left(\frac{\partial C_{AF}}{\partial M_{AF}}\right)^2 \times variance(C_{AF}) + \left(\frac{\partial C_{AF}}{\partial LI3109_1}\right)^2 \times variance(LI3109_1)$$

$$+\left(\frac{\partial C_{AF}}{\partial DI3108_1}\right)^2 \times variance(DI3108_1) + \left(\frac{\partial C_{AF}}{\partial \delta_1}\right)^2 \times variance(\delta_1)$$

GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of C_{AF} for Step 4 (see the upper portion of Exhibit 37) and to document the complete set of partial derivatives needed to support the estimation of the variance of the C_{AF} value (see the lower portion of Exhibit 37). For example, the x_{11}, x_{12}, y_{11}, and y_{12} values are appropriately selected values (based upon the values of the LI3109 instrument as indicated by $LI3109_1$ for determining the volume of the SME Product being evaluated for acceptability as indicated in Step 4 of Exhibit 28. To complete the information necessary to compute the estimate of the variance of the C_{AF}, estimates of the variance terms of Equation 43 are needed. These values along with a description of the terms of the equation are provided in Table 21.

[f] See SRNL E-Notebook O7787-00055-09, Antifoam 747 Basic Data and Acceptance Testing, July 29, 2014.

Equation:

ρ=(LI3109$_1$-DI3108$_1$)/Sep;
V_1=((LI3109$_1$/ρ+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12}))+y_{12}+δ_1;
C_{AF}=0.4905*1000000*M_{AF}/(V_1*ρ*3.7854);

Partial Derivatives:

$\partial\rho/\partial$LI3109$_1$ = 1.0/Sep;

$\partial\rho/\partial$DI3108$_1$ = (-1.0)/Sep;

$\partial\rho/\partial$Sep = (-(LI3109$_1$ - DI3108$_1$))/sqr(Sep);

$\partial V_1/\partial$LI3109$_1$ = (y_{11} - y_{12})·(1.0/ρ + (-LI3109$_1$·$\partial\rho/\partial$LI3109$_1$)/sqr(ρ))/(x_{11} - x_{12});

$\partial V_1/\partial$DI3108$_1$ = (y_{11} - y_{12})·(-LI3109$_1$·$\partial\rho/\partial$DI3108$_1$)/sqr(ρ)/(x_{11} - x_{12});

$\partial V_1/\partial$Sep = (y_{11} - y_{12})·(-LI3109$_1$·$\partial\rho/\partial$Sep)/sqr(ρ)/(x_{11} - x_{12});

$\partial V_1/\partial$Heel = (y_{11} - y_{12})/(x_{11} - x_{12});

$\partial V_1/\partial x_{12}$ = (y_{11} - y_{12})·(-1.0)/(x_{11} - x_{12}) + (-(LI3109$_1$/ρ + Heel - x_{12})·(y_{11} - y_{12})·(-1.0))/sqr(x_{11} - x_{12});

$\partial V_1/\partial y_{11}$ = (LI3109$_1$/ρ + Heel - x_{12})/(x_{11} - x_{12});

$\partial V_1/\partial y_{12}$ = (LI3109$_1$/ρ + Heel - x_{12})·(-1.0)/(x_{11} - x_{12}) + 1.0;

$\partial V_1/\partial x_{11}$ = (-(LI3109$_1$/ρ + Heel - x_{12})·(y_{11} - y_{12}))/sqr(x_{11} - x_{12});

$\partial V_1/\partial\delta_1$ = 1.0;

$\partial C_{AF}/\partial$LI3109$_1$ = (-0.4905·1000000.0·M_{AF}·3.7854·(V_1·$\partial\rho/\partial$LI3109$_1$ + ρ·$\partial V_1/\partial$LI3109$_1$))/sqr(V_1·ρ·3.7854);

$\partial C_{AF}/\partial$DI3108$_1$ = (-0.4905·1000000.0·M_{AF}·3.7854·(V_1·$\partial\rho/\partial$DI3108$_1$ + ρ·$\partial V_1/\partial$DI3108$_1$))/sqr(V_1·ρ·3.7854);

$\partial C_{AF}/\partial$Sep = (-0.4905·1000000.0·M_{AF}·3.7854·(V_1·$\partial\rho/\partial$Sep + ρ·$\partial V_1/\partial$Sep))/sqr(V_1·ρ·3.7854);

$\partial C_{AF}/\partial$Heel = (-0.4905·1000000.0·M_{AF}·3.7854·ρ·$\partial V_1/\partial$Heel)/sqr(V_1·ρ·3.7854);

$\partial C_{AF}/\partial x_{12}$ = (-0.4905·1000000.0·M_{AF}·3.7854·ρ·$\partial V_1/\partial x_{12}$)/sqr($V_1$·$\rho$·3.7854);

$\partial C_{AF}/\partial y_{11}$ = (-0.4905·1000000.0·M_{AF}·3.7854·ρ·$\partial V_1/\partial y_{11}$)/sqr($V_1$·$\rho$·3.7854);

$\partial C_{AF}/\partial y_{12}$ = (-0.4905·1000000.0·M_{AF}·3.7854·ρ·$\partial V_1/\partial y_{12}$)/sqr($V_1$·$\rho$·3.7854);

$\partial C_{AF}/\partial x_{11}$ = (-0.4905·1000000.0·M_{AF}·3.7854·ρ·$\partial V_1/\partial x_{11}$)/sqr($V_1$·$\rho$·3.7854);

$\partial C_{AF}/\partial\delta_1$ = (-0.4905·1000000.0·M_{AF}·3.7854·ρ·$\partial V_1/\partial\delta_1$)/sqr($V_1$·$\rho$·3.7854);

$\partial C_{AF}/\partial M_{AF}$ = 0.4905·1000000.0/(V_1·ρ·3.7854);

Exhibit 37 Equations for C_{AF} for SME Product with Random Uncertainty

Table 21 Terms and Estimated Random Uncertainties Supporting Equation 43

Term/Instrument	Description	1-Sigma Random Uncertainty
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	Analytical uncertainty (see Equation 12)
LI3109 with subscript 1	level bubbler value (inwc)	±1% of 231.0 inwc span [14]
	DCS deviation limit	±0.1 inwc [14]
		Using a uniform distribution, 1-sigma random is $[(2.31/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5} = 1.3349$
DI3108 with subscript 1	density bubbler value (inwc)	±1% of 160.5 inwc span [15]
	DCS deviation limit	±0.05 inwc [15]
		Using a uniform distribution, 1-sigma random is $[(1.605/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5} = 0.9271$
δ_1	tank calibration uncertainty (see footnote for Table 17)	1-sigma random = 9 gallons

To complete the evaluation of the C_{AF} required for Step 4, the bias for C_{AF} determined by Equation 42 must be estimated. A bound (at 95% confidence) on the bias of the concentration of carbon from AF in the SME Product, C_{AF}, is estimated, as above, by appealing to a Taylor Series expansion of Equation 42 in the fundamental measurements. Once again, note that ρ and V_1 are intermediary values. Also, the V_1 value has a potential bias that is to be included in the evaluation. The Taylor's Series expansion may be expressed in the fundamental measurements and the potential bias as given by:

Equation 44

$$\{bias(C_{AF})\}^2 \approx \left(\frac{\partial C_{AF}}{\partial M_{AF}}\right)^2 \times \{bias(M_{AF})\}^2 + \left(\frac{\partial C_{AF}}{\partial LI3109_1}\right)^2 \times \{bias(LI3109_1)\}^2$$

$$+ \left(\frac{\partial C_{AF}}{\partial DI3108_1}\right)^2 \times \{bias(DI3108_1)\}^2 + \left(\frac{\partial C_{AF}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2$$

$$+ \left(\frac{\partial C_{AF}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + + \left(\frac{\partial C_{AF}}{\partial b_1}\right)^2 \times \{bias(b_1)\}^2$$

Note that in evaluating Equation 44, the bias for the M_{AF} term, i.e., bias(M_{AF}), is provided by the status information of the SME at the time of the decision for acceptability of the transfer of the SME Product to the MFT. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of C_{AF} for Step 4 (see the upper portion of Exhibit 38) and to document the complete set of partial derivatives needed to support the estimation of the bias of the C_{AF} value (see the lower portion of Exhibit 38). Once again, x_{11}, x_{12}, y_{11}, and y_{12} are appropriately selected values (based upon the value of the LI3109 instrument as indicated by $LI3109_1$) for determining volume as indicated in Exhibit 29. To complete the information necessary to compute the estimate of the bias of the C_{AF}, estimates of the bias terms of Equation 44 are needed. Table 22 provides the details of the bias information needed to complete the estimation of the bias for the C_{AF} value. A sample calculation for this step including the random and bias uncertainties is provided as part of the results in Exhibit B14 in Appendix B.

Equation:

$\rho=(LI3109_1-DI3108_1)/Sep;$
$V_1=((LI3109_1/\rho+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12}))+y_{12}+b_1;$
$C_{AF}=0.4905*1000000*M_{AF}/(V_1*\rho*3.7854);$

Partial Derivatives:

$\partial\rho/\partial LI3109_1 = 1.0/Sep;$

$\partial\rho/\partial DI3108_1 = (-1.0)/Sep;$

$\partial\rho/\partial Sep = (-(LI3109_1 - DI3108_1))/sqr(Sep);$

$\partial V_1/\partial LI3109_1 = (y_{11} - y_{12})\cdot(1.0/\rho + (-LI3109_1\cdot\partial\rho/\partial LI3109_1)/sqr(\rho))/(x_{11} - x_{12});$

$\partial V_1/\partial DI3108_1 = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial DI3108_1)/sqr(\rho)/(x_{11} - x_{12});$

$\partial V_1/\partial Sep = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{11} - x_{12});$

$\partial V_1/\partial Heel = (y_{11} - y_{12})/(x_{11} - x_{12});$

$\partial V_1/\partial x_{12} = (y_{11} - y_{12})\cdot(-1.0)/(x_{11} - x_{12}) + (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12})\cdot(-1.0))/sqr(x_{11} - x_{12});$

$\partial V_1/\partial y_{11} = (LI3109_1/\rho + Heel - x_{12})/(x_{11} - x_{12});$

$\partial V_1/\partial y_{12} = (LI3109_1/\rho + Heel - x_{12})\cdot(-1.0)/(x_{11} - x_{12}) + 1.0;$

$\partial V_1/\partial x_{11} = (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12}))/sqr(x_{11} - x_{12});$

$\partial V_1/\partial b_1 = 1.0;$

$\partial C_{AF}/\partial LI3109_1 = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot(V_1\cdot\partial\rho/\partial LI3109_1 + \rho\cdot\partial V_1/\partial LI3109_1))/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial DI3108_1 = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot(V_1\cdot\partial\rho/\partial DI3108_1 + \rho\cdot\partial V_1/\partial DI3108_1))/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial Sep = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot(V_1\cdot\partial\rho/\partial Sep + \rho\cdot\partial V_1/\partial Sep))/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial Heel = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot\rho\cdot\partial V_1/\partial Heel)/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial x_{12} = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot\rho\cdot\partial V_1/\partial x_{12})/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial y_{11} = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot\rho\cdot\partial V_1/\partial y_{11})/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial y_{12} = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot\rho\cdot\partial V_1/\partial y_{12})/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial x_{11} = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot\rho\cdot\partial V_1/\partial x_{11})/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial b_1 = (-0.4905\cdot 1000000.0\cdot M_{AF}\cdot 3.7854\cdot\rho\cdot\partial V_1/\partial b_1)/sqr(V_1\cdot\rho\cdot 3.7854);$

$\partial C_{AF}/\partial M_{AF} = 0.4905\cdot 1000000.0/(V_1\cdot\rho\cdot 3.7854);$

Exhibit 38 Equations for C_{AF} for SME Product with Bias Uncertainty

Table 22 Terms and Estimated Bias Uncertainties Supporting Equation 44

Term/Instrument	Description	Bias Uncertainty at 95% Confidence
C_{AF} (see Equation 11)	antifoam carbon concentration (mg/kg)	0
LI3109 with subscript 1	level bubbler value (inwc)	±1% of 231.0 inwc span [14]
		Bias = 2.310 inwc
DI3108 with subscript 1	density bubbler value (inwc)	±1% of 160.5 inwc span [15]
		Bias = 1.605 inwc
b_1	tank calibration uncertainty (see footnote for Table 18)	6 gallons
Sep	Separation between bubblers (47 inches)	0.0625 inch [13]
Heel	Tank heel below LI3109 (6.77 inches)	0.0625 inch [14]

The calculations just completed allow for the determination of the C_{AF}, the concentration of carbon in mg/kg, attributable to the AF in the SME Product and its uncertainty (both random and bias). The use of the AF tracking system in the AMFT, SRAT, and SME that have led to the completion of these calculations will be applicable beyond just SB8.

To complete the decision as to the acceptability of the C_{AF} concentration relative to the flammability controls for SB8 involves analytical measurements of samples of the SME Product. Nitrate and TOC measurements must meet the constraints imposed by reference [1]. For nitrate, see the discussion provided in Section 2.1 above.

With the nitrate concentration successfully meeting its constraint, attention turns to TOC. The allowed amount of TOC for a SME batch is determined from the nitrate as described in [1] and summarized in Section 2.2 above. There are three levels of TOC that have been established with each one having its own acceptable level of carbon from AF [1]. The acceptability of each of these levels of TOC must be determined to assess the lowest level that meets the requirements of [1] as implemented in [2]. Section 2.2 above provides a description of these requirements.

There is an acceptable level of carbon from AF for each level of TOC; see [1], [2] and Section 2.3 above. So, an acceptability decision is the determination that the amount of carbon from AF is less than the amount allowed by [2], after accounting for all of the uncertainties involved. Let A_{Ci} represent the allowed carbon for the i^{th} level of TOC and let T_C represent the carbon concentration in the SME Product from the tracking system, then before accounting for uncertainties, acceptability may be defined as:

Equation 45

$$M_{C_i} = A_{C_i} - T_C > 0$$

where i=1, 2, or 3, representing the three levels of TOC developed in [1].

Re-expressing this in terms of the information in Table 2 and the average of the nitrate measurements of the SME Product, $\overline{NO_3}$, yields (for i = 1, 2, or 3):

Equation 46

$$M_{C_i} = \sqrt{h_i + j_i \cdot \overline{NO_3}} - T_C > 0$$

From the discussion leading up to Equation 10 and the approach used there to address the random uncertainties associated with the terms under the radical and letting s_C represent the 1-sigma random uncertainties, associated with the T_C value from the tracking system, the variance of the random errors of M_{Ci} is estimated by:

Equation 47

$$var(M_{C_i}) = 0.25 \cdot (j_i \cdot (h_i + j_i \cdot \overline{NO_3})^{-0.5})^2 \cdot (se_{\overline{NO_3}})^2$$

$$+0.000186323 \cdot (j_i \cdot (h_i + j_i \cdot \overline{NO_3})^{-0.5})^2 \cdot (\overline{NO_3})^2 + (s_C)^2$$

The expanded random uncertainty of the estimated difference, M_{Ci}, at 95% confidence is determined by multiplying the square root of the estimated variance of M_{Ci} by an appropriate Student's t statistic. In this case a one-sided confidence statement is needed; so, an upper 5%-tail of the Student's t distribution will be used. As discussed in Section 2, since the average nitrate value is based upon at least 4 measurements, a conservative 3 degrees of freedom for the estimated variance of M_{Ci} will be used. This leads to a t value of 2.353. Thus, at 95% confidence the expanded random uncertainty of the difference, M_{Ci}, is 2.353 times the square root of the estimated variance of M_{Ci}. To complete the assessment of the impact of uncertainties on the M_{Ci} difference, the bias of the T_C value from the tracking system must be accounted for. Let b_C represent that bias. Then, for the antifoam content of the SME to be acceptable (at 95% confidence), the following constraint must be met:

Equation 48

$$M_{Ci} - b_C - 2.353 \bullet (Var(M_{Ci}))^{0.5} > 0$$

for i = 1, 2, or 3 associated with one of the three values:728, 894, or 1,017 gal that is selected to be appropriate for the given SME batch. A sample of these calculations along with the acceptability decision for the SME Product is provided as part of the results in Exhibit B14 in Appendix B.

3.4.6 SME Step 5 Processing

Once the SME Product is transferred to the MFT, then the following equation provides an estimate of the mass of antifoam that remains in the SME heel, M_{AF_new}, after the transfer has been completed (this corresponds to Step 5 of Exhibit 28):

Equation 49

$$M_{AF_{new}} = \frac{M_{AF} \cdot V_2}{V_1}$$

where the volumes V_1 and V_2 in gallons are intermediary values which are determined from LI3109 and DI3108 as described in Exhibit 29 and Exhibit 30 and with the volumes each having an additional random variability that is to be incorporated to the random uncertainty of M_{AF_new}.

Following the Taylor's Series expansion approach described above, the estimated variance of M_{AF_new} for Step 5 of Exhibit 28 may be written in terms of the fundamental measurements with δ_1 and δ_2 representing the random errors for V_1 and V_2 ,.respectively, as:

Equation 50

$$Variance\left(M_{AF_{new}}\right) \approx \left(\frac{\partial M_{AF_{new}}}{\partial M_{AF}}\right)^2 \times variance(M_{AF}) + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_1}\right)^2 \times variance(LI3109_1)$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial DI3108_1}\right)^2 \times variance(DI3108_1) + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_2}\right)^2 \times variance(LI3109_2)$$

$$+ \left(\frac{\partial M_{AF_{new}}}{\partial \delta_1}\right)^2 \times variance(\delta_1) + \left(\frac{\partial M_{AF_{new}}}{\partial \delta_2}\right)^2 \times variance(\delta_2)$$

GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of M_{AF_new} for Step 5 (see the upper portion of Exhibit 39) and to document the complete set of partial derivatives needed to support the estimation of the variance of the M_{AF_new} value (see the lower portion of Exhibit 39 and Exhibit 40). For example, the x_{11}, x_{12}, y_{11}, and y_{12} values and x_{21}, x_{22}, y_{21}, and y_{22} values are appropriately selected values for determining the volumes V_1 and V_2, respectively, using instrument LI3109 as indicated in Exhibit 28. To complete the information necessary to compute the estimate of the variance of the M_{AF_new}, estimates of the variance terms of Equation 50 are needed. These values along with a description of the terms of the equation are provided in Table 23.

Equation:

$$\rho=(LI3109_1-DI3108_1)/Sep;$$
$$V_1=((LI3109_1/\rho+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12}))+y_{12}+\delta_1;$$
$$V_2=((LI3109_2/\rho+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22}))+y_{22}+\delta_2;$$
$$M_{AF\_new}=M_{AF}*(V_2/V_1);$$

Partial Derivatives:

$$\partial\rho/\partial LI3109_1 = 1.0/Sep;$$

$$\partial\rho/\partial DI3108_1 = (-1.0)/Sep;$$

$$\partial\rho/\partial Sep = (-(LI3109_1 - DI3108_1))/sqr(Sep);$$

$$\partial V_1/\partial LI3109_1 = (y_{11} - y_{12})\cdot(1.0/\rho + (-LI3109_1\cdot\partial\rho/\partial LI3109_1)/sqr(\rho))/(x_{11} - x_{12});$$

$$\partial V_1/\partial DI3108_1 = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial DI3108_1)/sqr(\rho)/(x_{11} - x_{12});$$

$$\partial V_1/\partial Sep = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{11} - x_{12});$$

$$\partial V_1/\partial Heel = (y_{11} - y_{12})/(x_{11} - x_{12});$$

$$\partial V_1/\partial x_{12} = (y_{11} - y_{12})\cdot(-1.0)/(x_{11} - x_{12}) + (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12})\cdot(-1.0))/sqr(x_{11} - x_{12});$$

$$\partial V_1/\partial y_{11} = (LI3109_1/\rho + Heel - x_{12})/(x_{11} - x_{12});$$

$$\partial V_1/\partial y_{12} = (LI3109_1/\rho + Heel - x_{12})\cdot(-1.0)/(x_{11} - x_{12}) + 1.0;$$

$$\partial V_1/\partial x_{11} = (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12}))/sqr(x_{11} - x_{12});$$

$$\partial V_1/\partial\delta_1 = 1.0;$$

$$\partial V_2/\partial LI3109_1 = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial LI3109_1)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial DI3108_1 = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial DI3108_1)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial Sep = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{21} - x_{22});$$

$$\partial V_2/\partial Heel = (y_{21} - y_{22})/(x_{21} - x_{22});$$

$$\partial V_2/\partial LI3109_2 = (y_{21} - y_{22})\cdot 1.0/\rho/(x_{21} - x_{22});$$

$$\partial V_2/\partial x_{22} = (y_{21} - y_{22})\cdot(-1.0)/(x_{21} - x_{22}) + (-(LI3109_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22})\cdot(-1.0))/sqr(x_{21} - x_{22});$$

$$\partial V_2/\partial y_{21} = (LI3109_2/\rho + Heel - x_{22})/(x_{21} - x_{22});$$

$$\partial V_2/\partial y_{22} = (LI3109_2/\rho + Heel - x_{22})\cdot(-1.0)/(x_{21} - x_{22}) + 1.0;$$

$$\partial V_2/\partial x_{21} = (-(LI3109_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22}))/sqr(x_{21} - x_{22});$$

$$\partial V_2/\partial\delta_2 = 1.0;$$

Exhibit 39 Equations for M_{AF_new} for SME Heel with Random Uncertainty (part 1 of 2)

Additional Partial Derivatives

$\partial M_{AF\_new}/\partial LI3109_1 = M_{AF} \cdot (\partial V_2/\partial LI3109_1/V_1 + (-V_2 \cdot \partial V_1/\partial LI3109_1)/sqr(V_1));$

$\partial M_{AF\_new}/\partial DI3108_1 = M_{AF} \cdot (\partial V_2/\partial DI3108_1/V_1 + (-V_2 \cdot \partial V_1/\partial DI3108_1)/sqr(V_1));$

$\partial M_{AF\_new}/\partial Sep = M_{AF} \cdot (\partial V_2/\partial Sep/V_1 + (-V_2 \cdot \partial V_1/\partial Sep)/sqr(V_1));$

$\partial M_{AF\_new}/\partial Heel = M_{AF} \cdot (\partial V_2/\partial Heel/V_1 + (-V_2 \cdot \partial V_1/\partial Heel)/sqr(V_1));$

$\partial M_{AF\_new}/\partial x_{12} = M_{AF} \cdot (-V_2 \cdot \partial V_1/\partial x_{12})/sqr(V_1);$

$\partial M_{AF\_new}/\partial y_{11} = M_{AF} \cdot (-V_2 \cdot \partial V_1/\partial y_{11})/sqr(V_1);$

$\partial M_{AF\_new}/\partial y_{12} = M_{AF} \cdot (-V_2 \cdot \partial V_1/\partial y_{12})/sqr(V_1);$

$\partial M_{AF\_new}/\partial x_{11} = M_{AF} \cdot (-V_2 \cdot \partial V_1/\partial x_{11})/sqr(V_1);$

$\partial M_{AF\_new}/\partial \delta_1 = M_{AF} \cdot (-V_2 \cdot \partial V_1/\partial \delta_1)/sqr(V_1);$

$\partial M_{AF\_new}/\partial LI3109_2 = M_{AF} \cdot \partial V_2/\partial LI3109_2/V_1;$

$\partial M_{AF\_new}/\partial x_{22} = M_{AF} \cdot \partial V_2/\partial x_{22}/V_1;$

$\partial M_{AF\_new}/\partial y_{21} = M_{AF} \cdot \partial V_2/\partial y_{21}/V_1;$

$\partial M_{AF\_new}/\partial y_{22} = M_{AF} \cdot \partial V_2/\partial y_{22}/V_1;$

$\partial M_{AF\_new}/\partial x_{21} = M_{AF} \cdot \partial V_2/\partial x_{21}/V_1;$

$\partial M_{AF\_new}/\partial \delta_2 = M_{AF} \cdot \partial V_2/\partial \delta_2/V_1;$

$\partial M_{AF\_new}/\partial M_{AF} = V_2/V_1;$

Exhibit 40 Equations for M_{AF_new} for SME Heel with Random Uncertainty (part 2 of 2)

Table 23 Terms and Estimated Random Uncertainties Supporting Equation 50

Term/Instrument	Description	1-Sigma Random Uncertainty
M_{AF}	antifoam mass (kg) in SME	Random uncertainty available in status information
LI3109 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.0 inwc span [14]
	DCS deviation limit	±0.1 inwc [14]
		Using a uniform distribution, 1-sigma random is $[(2.31/\sqrt{3})^2+(0.1/\sqrt{3})^2]^{0.5} = 1.3349$
DI3108 with subscript 1	density bubbler value (inwc)	±1% of 160.5 inwc span [15]
	DCS deviation limit	±0.05 inwc [15]
		Using a uniform distribution, 1-sigma random is $[(1.605/\sqrt{3})^2+(0.05/\sqrt{3})^2]^{0.5} = 0.9271$
δ_1	tank calibration uncertainty (see footnote for Table 17)	1-sigma random = 9 gallons
δ_2	Tank Calibration uncertainty (see footnote for Table 17)	1-sigma random = 9 gallons

To complete the evaluation of the M_{AF_new} required for Step 5, the bias for M_{AF_new} determined by Equation 49 must be estimated. A bound (at 95% confidence) on the bias of the antifoam mass remaining in the SME after a transfer out to the MFT, M_{AF_new}, is estimated, as above, by appealing to a Taylor Series expansion of Equation 49 in the fundamental measurements. Once again, note that V_1 and V_2 are intermediary values. Also, the each of these volumes has a potential bias that is to be included in the evaluation. The Taylor's Series expansion may be expressed in the fundamental measurements and the potential bias as given by:

Equation 51

$$\left\{bias\left(M_{AF_{new}}\right)\right\}^2 \approx \left(\frac{\partial M_{AF_{new}}}{\partial M_{AF}}\right)^2 \times \{bias(M_{AF})\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_1}\right)^2 \times \{bias(LI3109_1)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial DI3108_1}\right)^2 \times \{bias(DI3108_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial Sep}\right)^2 \times \{bias(Sep)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial Heel}\right)^2 \times \{bias(Heel)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial LI3109_2}\right)^2 \times \{bias(LI3109_2)\}^2$$

$$+\left(\frac{\partial M_{AF_{new}}}{\partial b_1}\right)^2 \times \{bias(b_1)\}^2 + \left(\frac{\partial M_{AF_{new}}}{\partial b_2}\right)^2 \times \{bias(b_2)\}^2$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{new}}}{\partial LI3109_1}\right| \times \left|\frac{\partial M_{AF_{new}}}{\partial LI3109_2}\right| \times bias(LI3109_1) \times bias(LI3109_2)$$

$$+2 \times 1 \times \left|\frac{\partial M_{AF_{new}}}{\partial b_1}\right| \times \left|\frac{\partial M_{AF_{new}}}{\partial b_2}\right| \times bias(b_1) \times bias(b_2)$$

Note that in evaluating Equation 51, the bias for the M_{AF} term, i.e., bias(M_{AF}) term, is provided by the status information for the SME prior to the transfer to the MFT and that two potential correlations among the biases are introduced into the equation, both represented in a bounding manner. So the approach may be stated as: the b_1 and b_2 terms are the estimated bias in the V_1 and V_2 volumes, respectively, and there is a potential correlation in these biases. Also, there is a potential correlation in the biases for the two LI3109 measurements. GUM Workbench was used to develop the model equation and associated intermediary values supporting the determination of the bias of M_{AF_new} for Step 5 (see the upper portion of Exhibit 41) and to document the complete set of partial derivatives needed to support the estimation of the bias of the M_{AF_new} value (see the lower portion of Exhibit 41 and Exhibit 42). Once again, x_{21}, x_{22}, y_{21}, and y_{22} are appropriately selected values (based upon the value of the LI3109 instrument as indicated by $LI3109_1$ and $LI3109_1$) for determining volumes as indicated in Exhibit 29. To complete the information necessary to compute the estimate of the bias of the M_{AF_new}, estimates of the bias terms of Equation 51 are needed. Table 24 provides the details of the bias information needed to complete the estimation of the bias for the M_{AF_new} value. A sample calculation for this step including the random and bias uncertainties is provided in Exhibit B15 in Appendix B.

Equation:

$$\rho=(LI3109_1-DI3108_1)/Sep;$$
$$V_1=((LI3109_1/\rho+Heel-x_{12})*(y_{11}-y_{12})/(x_{11}-x_{12}))+y_{12}+b_1;$$
$$V_2=((LI3109_2/\rho+Heel-x_{22})*(y_{21}-y_{22})/(x_{21}-x_{22}))+y_{22}+b_2;$$
$$M_{AF\_new}=M_{AF}*(V_2/V_1);$$

Partial Derivatives:

$$\partial\rho/\partial LI3109_1 = 1.0/Sep;$$
$$\partial\rho/\partial DI3108_1 = (-1.0)/Sep;$$
$$\partial\rho/\partial Sep = (-(LI3109_1 - DI3108_1))/sqr(Sep);$$
$$\partial V_1/\partial LI3109_1 = (y_{11} - y_{12})\cdot(1.0/\rho + (-LI3109_1\cdot\partial\rho/\partial LI3109_1)/sqr(\rho))/(x_{11} - x_{12});$$
$$\partial V_1/\partial DI3108_1 = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial DI3108_1)/sqr(\rho)/(x_{11} - x_{12});$$
$$\partial V_1/\partial Sep = (y_{11} - y_{12})\cdot(-LI3109_1\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{11} - x_{12});$$
$$\partial V_1/\partial Heel = (y_{11} - y_{12})/(x_{11} - x_{12});$$
$$\partial V_1/\partial x_{12} = (y_{11} - y_{12})\cdot(-1.0)/(x_{11} - x_{12}) + (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12})\cdot(-1.0))/sqr(x_{11} - x_{12});$$
$$\partial V_1/\partial y_{11} = (LI3109_1/\rho + Heel - x_{12})/(x_{11} - x_{12});$$
$$\partial V_1/\partial y_{12} = (LI3109_1/\rho + Heel - x_{12})\cdot(-1.0)/(x_{11} - x_{12}) + 1.0;$$
$$\partial V_1/\partial x_{11} = (-(LI3109_1/\rho + Heel - x_{12})\cdot(y_{11} - y_{12}))/sqr(x_{11} - x_{12});$$
$$\partial V_1/\partial b_1 = 1.0;$$
$$\partial V_2/\partial LI3109_1 = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial LI3109_1)/sqr(\rho)/(x_{21} - x_{22});$$
$$\partial V_2/\partial DI3108_1 = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial DI3108_1)/sqr(\rho)/(x_{21} - x_{22});$$
$$\partial V_2/\partial Sep = (y_{21} - y_{22})\cdot(-LI3109_2\cdot\partial\rho/\partial Sep)/sqr(\rho)/(x_{21} - x_{22});$$
$$\partial V_2/\partial Heel = (y_{21} - y_{22})/(x_{21} - x_{22});$$
$$\partial V_2/\partial LI3109_2 = (y_{21} - y_{22})\cdot 1.0/\rho/(x_{21} - x_{22});$$
$$\partial V_2/\partial x_{22} = (y_{21} - y_{22})\cdot(-1.0)/(x_{21} - x_{22}) + (-(LI3109_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22})\cdot(-1.0))/sqr(x_{21} - x_{22});$$
$$\partial V_2/\partial y_{21} = (LI3109_2/\rho + Heel - x_{22})/(x_{21} - x_{22});$$
$$\partial V_2/\partial y_{22} = (LI3109_2/\rho + Heel - x_{22})\cdot(-1.0)/(x_{21} - x_{22}) + 1.0;$$
$$\partial V_2/\partial x_{21} = (-(LI3109_2/\rho + Heel - x_{22})\cdot(y_{21} - y_{22}))/sqr(x_{21} - x_{22});$$
$$\partial V_2/\partial b_2 = 1.0;$$

Exhibit 41 Equations for M_{AF_new} for SME Heel with Bias Uncertainty (part 1 of 2)

Additional Partial Derivatives

$\partial M_{AF\_new}/\partial LI3109_1 = M_{AF}\cdot(\partial V_2/\partial LI3109_1/V_1 + (-V_2\cdot\partial V_1/\partial LI3109_1)/sqr(V_1));$

$\partial M_{AF\_new}/\partial DI3108_1 = M_{AF}\cdot(\partial V_2/\partial DI3108_1/V_1 + (-V_2\cdot\partial V_1/\partial DI3108_1)/sqr(V_1));$

$\partial M_{AF\_new}/\partial Sep = M_{AF}\cdot(\partial V_2/\partial Sep/V_1 + (-V_2\cdot\partial V_1/\partial Sep)/sqr(V_1));$

$\partial M_{AF\_new}/\partial Heel = M_{AF}\cdot(\partial V_2/\partial Heel/V_1 + (-V_2\cdot\partial V_1/\partial Heel)/sqr(V_1));$

$\partial M_{AF\_new}/\partial x_{12} = M_{AF}\cdot(-V_2\cdot\partial V_1/\partial x_{12})/sqr(V_1);$

$\partial M_{AF\_new}/\partial y_{11} = M_{AF}\cdot(-V_2\cdot\partial V_1/\partial y_{11})/sqr(V_1);$

$\partial M_{AF\_new}/\partial y_{12} = M_{AF}\cdot(-V_2\cdot\partial V_1/\partial y_{12})/sqr(V_1);$

$\partial M_{AF\_new}/\partial x_{11} = M_{AF}\cdot(-V_2\cdot\partial V_1/\partial x_{11})/sqr(V_1);$

$\partial M_{AF\_new}/\partial b_1 = M_{AF}\cdot(-V_2\cdot\partial V_1/\partial b_1)/sqr(V_1);$

$\partial M_{AF\_new}/\partial LI3109_2 = M_{AF}\cdot\partial V_2/\partial LI3109_2/V_1;$

$\partial M_{AF\_new}/\partial x_{22} = M_{AF}\cdot\partial V_2/\partial x_{22}/V_1;$

$\partial M_{AF\_new}/\partial y_{21} = M_{AF}\cdot\partial V_2/\partial y_{21}/V_1;$

$\partial M_{AF\_new}/\partial y_{22} = M_{AF}\cdot\partial V_2/\partial y_{22}/V_1;$

$\partial M_{AF\_new}/\partial x_{21} = M_{AF}\cdot\partial V_2/\partial x_{21}/V_1;$

$\partial M_{AF\_new}/\partial b_2 = M_{AF}\cdot\partial V_2/\partial b_2/V_1;$

$\partial M_{AF\_new}/\partial M_{AF} = V_2/V_1;$

Exhibit 42 Equations for M_{AF_new} for SME Heel with Bias Uncertainty (part 2 of 2)

Table 24 Terms and Estimated Bias Uncertainties Supporting Equation 51

Term/Instrument	Description	Bias Uncertainty
M_{AF}	Antifoam mass (kg) in SME	Bias available in status information
LI3109 with subscripts 1 and 2	level bubbler values (inwc)	±1% of 231.0 inwc span [14]
		Bias = 2.310 inwc
DI3108 with subscript 1	density bubbler value (inwc)	±1% of 160.5 inwc span [15]
		Bias = 1.605 inwc
b_1	tank calibration uncertainty (see footnote for Table 18)	6 gallons
b_2	tank calibration uncertainty (see footnote for Table 18)	6 gallons
Sep	Separation between bubblers (47 inches)	0.0625 inch [13]
Heel	Tank heel below LI3109 (6.77 inches)	0.0625 inch [13]

4.0 Summary

SRNL has been working with SRR's DWPF in the development and implementation of an additional strategy for confidently satisfying the flammability controls for DWPF's melter operation. An initial strategy for implementing the operational constraints associated with flammability control in DWPF was based upon an analytically determined carbon concentration from antifoam. Due to the conservative error structure associated with the analytical approach, its implementation has significantly reduced the operating window for processing and has led to recurrent SME and MFT remediation.

To address the adverse operating impact of the current implementation strategy, SRR issued a TTR to SRNL requesting the development and documentation of an alternate strategy for evaluating the carbon contribution from antifoam. The proposed strategy presented in this report was developed under the guidance of a TTQAP and involves calculating the carbon concentration from antifoam based upon the actual mass of antifoam added to the process assuming 100% retention.

The mass of antifoam in the AMFT, in the SRAT, and in the SME is tracked by mass balance as part of this strategy. As these quantities are monitored, the random and bias uncertainties affecting their values are also maintained and accounted for. Thus, this report documents:

1) The development of an alternate implementation strategy and associated equations describing the carbon concentration from antifoam in each SME batch derived from the actual amount of antifoam introduced into the AMFT, SRAT, and SME during the processing of the batch.

2) The equations and error structure for incorporating the proposed strategy into melter off-gas flammability assessments.

Sample calculations of the system are also included in this report. Please note that the system developed and documented in this report is intended as an alternative to the current, analytically-driven system being utilized by DWPF; the proposed system is not intended to eliminate the current system

Also note that the system developed in this report to track antifoam mass in the AMFT, SRAT, and SME will be applicable beyond just SB8. While the model used to determine acceptability of the SME product with respect to melter off-gas flammability controls must be reassessed for each change in sludge batch, the antifoam mass tracking methodology is independent of sludge batch composition and as such will be transferable to future sludge batches.

5.0 References

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[13] "Process Tank Bubbler Jumper", M-M0-S-00140 Revision 0.

[14] Moesta, G. (preparer), "Slurry Mix Evaporator Tank Level Scaling Computation," S350-ZZZL-3109, May 8, 1994 (Amended by J-DCF-S-02980 dated July 31, 2014).

[15] "Slurry Mix Evaporator Density Indication Scaling and Setpoint Document," S350-ZZZL-3108, January 30, 2003 (Amended by J-DCF-S-02980 dated July 31, 2014).

Appendix A. Determining a Bounding Mass for Antifoam Additions to the AMFT

The maximum of mass of antifoam could be added when adding a five gallon pail of antifoam is 18.896 kg (might call this 18.9 kg). This bounding mass was developed by filling up an empty, dry pail of antifoam with water and weighing the water added to the pail (18.6597 kg). This was adjusted by multiplying the water weight by the quantity equal to the average density of the last 13 valid antifoam density results + 3 times the standard deviation of these 13 values (see Table A1):

$$18.6597 \times (1.00073 + 3 \times 0.00360) \quad = \quad 18.6597 \times 1.01153 \quad = \quad 18.875$$

This number was adjusted further by dividing by the density of water at 20 Celsius, 0.99835 g/mL).

$$18.875 \div 0.99823 \quad = \quad 18.908 \quad = \quad 18.91$$

Note that this value, 18.91 kg, was conservatively estimated so there is no need for any additional uncertainty (i.e., the random uncertainty is set to zero and the bias is set to zero).

Table A1. Density Measurements of Antifoam Batches[*]

Density of Antifoam and 1:20 antifoam mixture in g/mL at 20 ?C						
Lot	Density	St Dev	Units	1:20 density	St Dev	Units
0811959-11229	0.997		g/mL	0.99817		g/mL
081723-1112	0.997		g/mL	0.99817		g/mL
090422-324	1.001		g/mL	0.99837		g/mL
100106-123	1.00230		g/mL	0.99843		g/mL
100120-125	1.00260		g/mL	0.99845		g/mL
100653-0420	0.999		g/mL	0.99827		g/mL
100897-0525	0.999		g/mL	0.99827		g/mL
101115-0707	1.000		g/mL	0.99832		g/mL
101480-0915	1.000		g/mL	0.99832		g/mL
101820-1110	1.00268		g/mL	0.99845		g/mL
101876-1111	1.00058		g/mL	0.99835		g/mL
110684-0413	1.01075		g/mL	0.99886		g/mL
111128-0613	0.99756		g/mL	0.99820		g/mL
Average	**1.00073**	**0.00360**	**g/mL**	**0.99835**	**0.00018**	**g/mL**
Density of Pure Water at 20?C					0.99823	g/mL

[*] SRNL E-Notebook O7787-00055-09, Antifoam 747 Basic Data and Acceptance Testing, July 29, 2014.

Appendix B. Sample Calculations

Exhibit B1. Example Calculation for AMFT Step 2

$M_{AF\_new} = M_{AF} + M_{AF\_Add}$

AMFT Status Before Event

AMFT		
		1-sigma
	M_{AF}	Uncertainty
	5	0.2
	kg	kg
		0.5
		bias

Addition							
		1-sigma			2	1-sigma	
	M_{AF_add}	Uncertainty	bias		M_{AF_new}	Uncertainty	bias
	18.91	0.0	0		23.91	0.2	0.5
	kg	kg	kg		kg	kg	kg

Exhibit B2. Example Calculation for AMFT Step 3

$$M_{AF_{out}} = \frac{M_{AF} \cdot (V_1 - V_2)}{V_1}$$

AMFT Status Before Event

AMFT		
		1-sigma
	M_{AF}	Uncertainty
	5	0.2
	kg	kg
		0.5
		bias

Transfer to the SRAT or SME

Before Transfer Out		After Transfer Out		
	1-sigma		1-sigma	
$LI2614_1$	Uncertainty	$LI2614_2$	Uncertainty	
30	0.2372	25	0.2372	
inwc	inwc	inwc	inwc	
bias	0.41	bias	0.41	
	inwc		inwc	
	1-sigma			
Density	Uncertainty	bias		
1	0.0036	0.0108		
g/mL (used as a specific gravity)				
	Inches in Tank (before)		30.00	
	Inches in Tank (after)		25.00	
		Gallons	Gallons	
		Before	After	
		V_1	V_2	transfer volume
		89.57	74.83	14.73
		δ_1	δ_2	gal
	1-sigma	0.637	0.637	
		gal	gal	
		b_1	b_2	
	bias	0.232	0.232	
		gal	gal	

Exhibit B2. Example Calculation for AMFT Step 3
(continued)

		Partial Derivatives												
3	1-sigma	∂	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	V_1	V_2	V_1	V_2	V_1	V_2
M_{AF_out}	Uncertainty	∂	M_{AF}	$LI2614_1$	$LI2614_2$	δ_1	δ_2	density	$LI2614_1$	$LI2614_2$	δ_1	δ_2	density	density
0.822	0.0762		0.1645	0.1374	-0.1645	0.0466	-0.0558	-0.0107	2.947	2.947	1.000	1.000	-88.398	-73.665
kg	kg		18.617%	18.273%	26.176%	15.184%	21.751%	0.000%						
		Partial Derivatives												
		∂	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	V_1	V_1	V_2	V_2	V_1	V_2
	bias	∂	M_{AF}	$LI2614_1$	$LI2614_2$	density	b_1	b_2	$LI2614_1$	density	$LI2614_2$	density	b_1	b_2
	0.151		0.164	0.137	-0.164	-0.011	0.047	-0.056	2.947	-88.398	2.947	-73.665	1.000	1.000
	kg													

Exhibit B3. Example Calculation for AMFT Step 4

$$M_{AF_{new}} = \frac{M_{AF} \cdot V_2}{V_1}$$

AMFT Status Before Event

AMFT		
		1-sigma
	M_{AF}	Uncertainty
	5	0.2
	kg	kg
		0.5
		bias

Transfer to the SRAT or SME

Before Transfer Out		After Transfer Out		
	1-sigma		1-sigma	
$LI2614_1$	Uncertainty	$LI2614_2$	Uncertainty	
30	0.2372	25	0.2372	
inwc	inwc	inwc	inwc	
bias	0.41	bias	0.41	
	inwc		inwc	
	1-sigma			
Density	Uncertainty	bias		
1	0.0036	0.0108		
g/mL (used as a specific gravity)				
	Inches in Tank (before)		30.00	
	Inches in Tank (after)		25.00	
		Gallons	Gallons	
		Before	After	
		V_1	V_2	transfer volume
		89.57	74.83	14.73
		δ_1	δ_2	gal
	1-sigma	0.637	0.637	
		gal	gal	
		b_1	b_2	
	bias	0.232	0.232	
		gal	gal	

Exhibit B3. Example Calculation for AMFT Step 4

(continued)

						Partial Derivatives								
4	1-sigma	∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_1	V_2	V_1	V_2	V_1	V_2
M_{AF_new}	Uncertainty	∂	M_{AF}	$LI2614_1$	$LI2614_2$	δ_1	δ_2	density	$LI2614_1$	$LI2614_2$	δ_1	δ_2	density	density
4.1775	0.1807		0.8355	-0.1374	0.1645	-0.0466	0.0558	0.010731	2.9466	2.9466	1.0000	1.0000	-88.3980	-73.6650
kg	kg		85.5%	3.3%	4.7%	2.7%	3.9%	0.000%						
						Partial Derivatives								
		∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_1	V_2	V_1	V_2	V_1	V_2
	bias	∂	M_{AF}	$LI2614_1$	$LI2614_2$	b_1	b_2	density	$LI2614_1$	$LI2614_2$	b_1	b_2	density	density
	0.4364		0.8355	-0.1374	0.1645	-0.0466	0.0558	0.010730869	2.9466	2.9466	1.0000	1.0000	-88.3980	-73.6650
	kg													

Exhibit B4. Example Calculation for SRAT Step 2

$M_{AF\_new} = M_{AF} + M_{AF\_Add}$

SRAT Status Before Event

SRAT			
			1-sigma
Sep	Heel	M_{AF}	Uncertainty
47	6.77	35.210	1.500
0.0625	0.0625	kg	kg
inches	inches		bias
bias	bias		5.25
			kg

Addition		1-sigma			2	1-sigma	
	M_{AF_add}	Uncertainty	bias		M_{AF_new}	Uncertainty	bias
	1.800	0.200	0.5		37.010	1.5133	5.7500
	kg	kg	kg		kg	kg	kg

Exhibit B5. Example Calculation for SRAT Step 1

$$M_{AF_{new}} = \frac{3.7854 \cdot C_{AF} \cdot V_1 \cdot \rho}{0.4723 \cdot 1000000}$$

SRAT Status Unknown Before this Event

	mg/kg	mg/kg	mg/kg
	TOC	Oxalate	Formate
	12852	2667	41883
	11670	2361	41482
	12818	2183	42063
	13129	2690	42065
Avg	12617.25	2475.25	41873.25
Std Dev	646.68	245.86	274.44
Std error	323.34	122.93	137.22
batch % rsd (δ)	2.72%	5.05%	2.47%
C factor	1	0.27292	0.266806
low bias factor	1	0.9495	0.9697

∂	C_{AF}	C_{AF}	C_{AF}	C_{AF}	C_{AF}	C_{AF}	
∂	TOC	δ_{TOC}	Oxalate	$\delta_{oxalate}$	Formate	$\delta_{formate}$	
	1	1	0.27292	0.27292	0.266806	0.266806	
	34.61%	38.99%	0.37%	0.39%	0.44%	25.21%	100.000%

Volume Determination Before Transfer				
	1-sigma		1-sigma	
$LI3025_1$	Uncertainty	$DI3026_1$	Uncertainty	
108.08	1.3384	55.55	0.9300	
bias	2.316		1.61	
	x1	x2	y1	y2
	17.416	0	1000	0
	78.513	17.416	5175	1000
	138.28	78.513	9400	5175
	158.89	138.28	10850	9400
	175.91	158.89	12000	10850
Before Info	138.28	78.513	9400	5175

		1-sigma		
		Uncertainty	bias	
C_{AF}	1142	549.64	0	mg/kg
		1-sigma, δ_1	bias, b_1	
Volume, V_1, Before	6939.4	9.00	12	gal
Density, ρ	1.1177	kg/L		

Exhibit B5. Example Calculation for SRAT Step 1
(continued)

1	1-sigma	∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_1	V_1	ρ	ρ	V_1					
M_{AF_new}	Uncertainty	∂	C_{AF}	$LI3025_1$	$DI3026_1$	δ_1	$LI3025_1$	$DI3026_1$	$LI3025_1$	$DI3026_1$	δ_1					
71.01	34.18		0.062	0.667	-0.020	0.010233	-66.886	130.135	0.021	-0.021	1.000					
kg	kg		99.93%	0.07%	0.0000%	0.0007%						100.00%				
		∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_1	V_1	ρ	ρ	V_1	V_1	ρ	V_1
	bias	∂	C_{AF}	$LI3025_1$	$DI3026_1$	Sep	Heel	δ_1	$LI3025_1$	$DI3026_1$	$LI3025_1$	$DI3026_1$	Sep	Heel	Sep	δ_1
	1.55		0.0622	0.6673	-0.0201	-0.0225	0.7233	0.0102	-66.8856	130.1349	0.0213	-0.0213	145.4465	70.6912	-0.0238	1.0000
	kg															

Exhibit B6. Example Calculation for SRAT Step 3 Linked to Step 1

$$M_{AF_{out}} = \frac{3.7854 \cdot C_{AF} \cdot \rho \cdot (V_1 - V_2)}{0.4723 \cdot 1000000}$$

SRAT Status Unknown --- Sampling the SRAT Product Prior to Transfer to the SME

	mg/kg	mg/kg	mg/kg
	TOC	Oxalate	Formate
	12852	2667	41883
	11670	2361	41482
	12818	2183	42063
	13129	2690	42065
Avg	12617.25	2475.25	41873.25
Std Dev	646.68	245.86	274.44
Std error	323.34	122.93	137.22
batch % rsd (δ)	2.72%	5.05%	2.47%
C factor	1	0.27292	0.266806
low bias factor	1	0.9495	0.9697

∂ / ∂	C_{AF} / TOC	C_{AF} / δ_{TOC}	C_{AF} / Oxalate	C_{AF} / $\delta_{oxalate}$	C_{AF} / Formate	C_{AF} / $\delta_{formate}$	
	1	1	0.27292	0.27292	0.266806	0.266806	
	34.61%	38.99%	0.37%	0.39%	0.44%	25.21%	100.000%

Volume Determination Before Transfer

	1-sigma		1-sigma	
$LI3025_1$	Uncertainty	$DI3026_1$	Uncertainty	
108.08	1.3384	55.55	0.9300	
bias	2.316		1.61	
	x1	x2	y1	y2
	17.416	0	1000	0
	78.513	17.416	5175	1000
	138.28	78.513	9400	5175
	158.89	138.28	10850	9400
	175.91	158.89	12000	10850
Before Info	138.28	78.513	9400	5175

		1-sigma Uncertainty	bias	
C_{AF}	1142	549.64	0	mg/kg
		1-sigma, δ_1	bias, b_1	
Volume, V_1, Before	6939.4	9.00	12	gal
Density, ρ	1.1177	kg/L		

Exhibit B6. Example Calculation for SRAT Step 3 Linked to Step 1

(continued)

	After Transfer Out			
		1-sigma		
	$LI3025_2$	Uncertainty	bias	
	17.45	1.3384	2.316	
		inwc	inwc	
	x1	x2	y1	y2
	17.416	0	1000	0
	78.513	17.416	5175	1000
	138.28	78.513	9400	5175
	158.89	138.28	10850	9400
	175.91	158.89	12000	10850
After Info	78.513	17.416	5175	1000
			1-sigma Uncertainty	
		Volume, after, V_2	δ_2, gal	
		1339.4	9	
			bias, b_2, gal	
			12	

		Partial Derivatives ∂/∂														
3	1-sigma	M_{AF_out} / C_{AF}	M_{AF_out} / $LI3025_1$	M_{AF_out} / $DI3026_1$	M_{AF_out} / $LI3025_2$	M_{AF_out} / δ1	M_{AF_out} / δ2	V_1 / $LI3025_1$	V_1 / $DI3026_1$	V_2 / $LI3025_2$	V_2 / $LI3025_1$	V_2 / $DI3026_1$	ρ / $LI3025_1$	ρ / $DI3026_1$	V_1 / δ1	V_2 / δ2
M_{AF_out} 57.30	Uncertainty 27.597	0.05016	0.61426	0.03294	-0.62562	0.01023	-0.01023	-66.88556	130.13486	61.14023	-20.31024	20.31024	0.02128	-0.02128	1.00000	1.00000
kg	kg	99.817%	0.089%	0.00%	0.09%	0.0011%	0.0011%									

		Partial Derivatives ∂/∂														
	bias	M_{AF_out} / C_{AF}	M_{AF_out} / $LI3025_1$	M_{AF_out} / $DI3026_1$	M_{AF_out} / $LI3025_2$	M_{AF_out} / Sep	M_{AF_out} / Heel	M_{AF_out} / b_1	M_{AF_out} / b_2	V_1 / $LI3025_1$	V_1 / $DI3026_1$	V_2 / $LI3025_2$	V_2 / $LI3025_1$	V_2 / $DI3026_1$	ρ / $LI3025_1$	ρ / $DI3026_1$
	2.883	0.05016	0.61426	0.03294	-0.62562	0.03682	0.02412	0.01023	-0.01023	-66.88556	130.13486	61.14023	-20.31024	20.31024	0.02128	-0.02128
	kg															

V_1 / Sep	V_1 / Heel	V_2 / Sep	V_2 / Heel	V_1 / b_1	V_2 / b_2	ρ / Sep
145.44647	70.69118	22.69994	68.33396	1.00000	1.00000	-0.02378

Exhibit B7. Example Calculation for SRAT Step 4 Linked to Step 1

$$M_{AF_{new}} = \frac{3.7854 \cdot C_{AF} \cdot V_2 \cdot \rho}{0.4723 \cdot 1000000}$$

SRAT Status Unknown --- Sampling the SRAT Product Prior to Transfer to the SME

	mg/kg	mg/kg	mg/kg
	TOC	Oxalate	Formate
	12852	2667	41883
	11670	2361	41482
	12818	2183	42063
	13129	2690	42065
Avg	12617.25	2475.25	41873.25
Std Dev	646.68	245.86	274.44
Std error	323.34	122.93	137.22
batch % rsd (δ)	2.72%	5.05%	2.47%
C factor	1	0.27292	0.266806
low bias factor	1	0.9495	0.9697

∂	C_{AF}	C_{AF}	C_{AF}	C_{AF}	C_{AF}	C_{AF}	
∂	TOC	δ_{TOC}	Oxalate	$\delta_{oxalate}$	Formate	$\delta_{formate}$	
	1	1	0.27292	0.27292	0.266806	0.266806	
	34.61%	38.99%	0.37%	0.39%	0.44%	25.21%	100.000%

	After Transfer Out			
		1-sigma		
	$LI3025_2$	Uncertainty	bias	
	17.45	1.3384	2.316	
		inwc	inwc	
	x1	x2	y1	y2
	17.416	0	1000	0
	78.513	17.416	5175	1000
	138.28	78.513	9400	5175
	158.89	138.28	10850	9400
	175.91	158.89	12000	10850
After Info	78.513	17.416	5175	1000
			1-sigma Uncertainty	
		Volume, after, V_2	δ_2, gal	
		1339.4	9	
			bias, b_2, gal	
			12	

Volume Determination Before Transfer				
	1-sigma		1-sigma	
$LI3025_1$	Uncertainty	$DI3026_1$	Uncertainty	
108.08	1.3384	55.55	0.9300	
bias	2.316		1.61	
	x1	x2	y1	y2
	17.416	0	1000	0
	78.513	17.416	5175	1000
	138.28	78.513	9400	5175
	158.89	138.28	10850	9400
	175.91	158.89	12000	10850
Before Info	138.28	78.513	9400	5175

Exhibit B7. Example Calculation for SRAT Step 4 Linked to Step 1

(continued)

		1-sigma Uncertainty	bias	
C_{AF}	1142	549.64	0	mg/kg
Density, ρ	1.1177	kg/L		

		Partial Derivatives												
4	1-sigma	∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_2	V_2	V_2	ρ	ρ	V_2	V_2	V_2
M_{AF_new}	Uncertainty	∂	C_{AF}	$LI3025_1$	$DI3026_1$	$LI3025_2$	δ_2	$LI3025_2$	$LI3025_1$	$DI3026_1$	$LI3025_1$	$DI3026_1$	Sep	Heel	δ_2
13.71	6.649		0.01200	0.05308	-0.05308	0.62562	0.01023	61.14023	-20.31024	20.31024	0.02128	-0.02128	0.00000	68.33396	1.00000
kg			98.38%	0.01%	0.01%	1.59%	0.0192%								

		Partial Derivatives																
		∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_2	V_2	V_2	ρ	ρ	V_2	V_2	V_2	ρ
	bias	∂	C_{AF}	$LI3025_1$	$DI3026_1$	$LI3025_2$	Sep	Heel	b_2	$LI3025_2$	$LI3025_1$	$DI3026_1$	$LI3025_1$	$DI3026_1$	Sep	Heel	b_2	Sep
	1.5796		0.01200	0.05308	-0.05308	0.62562	-0.02790	0.69923	0.01023	61.14023	-20.31024	20.31024	0.02128	-0.02128	25.77190	68.33396	1.00000	-0.02378
	kg																	

Exhibit B8. Example Calculation for SRAT Step 3

$$M_{AF_{out}} = \frac{M_{AF} \cdot (V_1 - V_2)}{V_1}$$

SRAT Status Before Event

SRAT			
			1-sigma
Sep	Heel	M_{AF}	Uncertainty
47	6.77	35.210	1.500
0.0625	0.0625	kg	kg
inches	inches		bias
bias	bias		5.25
			kg

Before Transfer Out				After Transfer Out			1-sigma
	1-sigma		1-sigma		1-sigma		Uncertainty
$LI3025_1$	Uncertainty	$DI3026_1$	Uncertainty	$LI3025_2$	Uncertainty	V_1	$\delta1$ for V_1
104.02	1.3384	53.389	0.9300	16.975	1.3384	6929.36	9
inwc	Bias	inwc	Bias	inwc	Bias	gal	gal
	2.316		1.61		2.316		bias, b_1
	inwc		inwc		inwc		12
							gal

	x1	x2	y1	y2		1-sigma
	17.416	0	1000	0		Uncertainty
	78.513	17.416	5175	1000	V_2	$\delta2$ for V_2
	138.28	78.513	9400	5175	1349.30	9
	158.89	138.28	10850	9400	gal	gal
	175.91	158.89	12000	10850		bias, b_2
					Density, ρ	12
Before	138.28	78.513	9400	5175	1.0773	gal
After	78.513	17.416	5175	1000	kg/L	

Exhibit B8. Example Calculation for SRAT Step 3

(continued)

						Partial Derivatives												
3	1-sigma	∂	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	V_1	V_1	V_2	V_2	V_2	V_1	V_2	density, ρ	density, ρ
M_{AF_out}	Uncertainty	∂	M_{AF}	$LI3025_1$	$DI3026_1$	$LI3025_2$	δ1	δ2	$LI3025_1$	$DI3026_1$	$LI3025_2$	$LI3025_1$	$DI3026_1$	δ1	δ2	$LI3108_1$	$DI3109_1$
28.354	1.2848		0.805	0.040	0.0253	-0.322	0.00099	-0.0051	-69.196	134.818	63.433	-21.267	21.267	1.000	1.000	0.021	-0.021
kg	kg		88.39%	0.17%	0.03%	11.27%	0.0048%	0.1267%									

						Partial Derivatives												
		∂	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	M_{AF_out}	V_1	V_1	V_2	V_2	V_2	V_1	V_1
	bias	∂	M_{AF}	$LI3025_1$	$DI3026_1$	$LI3025_2$	Sep	Heel	b_1	b_2	$LI3025_1$	$DI3026_1$	$LI3025_2$	$LI3025_1$	$DI3026_1$	Sep	Heel
	4.3108		0.8053	0.0396	0.0253	-0.322	0.027	-0.277	0.001	-0.005	-69.196	134.818	63.433	-21.267	21.267	145.233	70.691
	kg																

V_2	V_2	V_1	V_2	density, ρ	density, ρ	density, ρ
Sep	Heel	b_1	b_2	$LI3108_1$	$DI3109_1$	Sep
22.910	68.334	1.000	1.000	0.021	-0.021	-0.023

Exhibit B9. Example Calculation for SRAT Step 4

$$M_{AF_{new}} = \frac{M_{AF} \cdot V_2}{V_1}$$

SRAT Status Before Event

SRAT			
			1-sigma
Sep	Heel	M_{AF}	Uncertainty
47	6.77	35.210	1.500
0.0625	0.0625	kg	kg
inches	inches		bias
bias	bias		5.25
			kg

Before Transfer Out				After Transfer Out			1-sigma
	1-sigma		1-sigma		1-sigma		Uncertainty
$LI3025_1$	Uncertainty	$DI3026_1$	Uncertainty	$LI3025_2$	Uncertainty	V_1	δ1 for V_1
104.02	1.3384	53.389	0.9300	16.975	1.3384	6929.36	9
inwc	Bias	inwc	Bias	inwc	Bias	gal	gal
	2.316		1.61		2.316		bias, b_1
	inwc		inwc		inwc		12
							gal

	x1	x2	y1	y2		1-sigma
	17.416	0	1000	0		Uncertainty
	78.513	17.416	5175	1000	V_2	δ2 for V_2
	138.28	78.513	9400	5175	1349.30	9
	158.89	138.28	10850	9400	gal	gal
	175.91	158.89	12000	10850		bias, b_2
					Density, ρ	12
Before	138.28	78.513	9400	5175	1.0773	gal
After	78.513	17.416	5175	1000	kg/L	

Exhibit B9. Example Calculation for SRAT Step 4
(continued)

4	1-sigma	Partial Derivatives														
M_{AF_new}	Uncertainty	$\frac{\partial M_{AF\_new}}{\partial M_{AF}}$	$\frac{\partial M_{AF\_new}}{\partial LI3025_1}$	$\frac{\partial M_{AF\_new}}{\partial DI3026_1}$	$\frac{\partial M_{AF\_new}}{\partial LI3025_2}$	$\frac{\partial M_{AF\_new}}{\partial \delta 1}$	$\frac{\partial M_{AF\_new}}{\partial \delta 2}$	$\frac{\partial V_1}{\partial LI3025_1}$	$\frac{\partial V_1}{\partial DI3026_1}$	$\frac{\partial V_2}{\partial LI3025_2}$	$\frac{\partial V_2}{\partial LI3025_1}$	$\frac{\partial V_2}{\partial DI3026_1}$	$\frac{\partial V_1}{\partial \delta 1}$	$\frac{\partial V_2}{\partial \delta 2}$	$\frac{\partial \text{density}, \rho}{\partial LI3108_1}$	$\frac{\partial \text{density}, \rho}{\partial DI3109_1}$
6.8562	0.5263	0.195	-0.040	-0.025	0.322	-0.001	0.005	-69.196	134.818	63.433	-21.267	21.267	1.000	1.000	0.021	-0.021
kg	kg	30.8%	1.0%	0.2%	67.2%	0.0286%	0.7552%									

		Partial Derivatives														
	bias	$\frac{\partial M_{AF\_new}}{\partial M_{AF}}$	$\frac{\partial M_{AF\_new}}{\partial LI3025_1}$	$\frac{\partial M_{AF\_new}}{\partial DI3026_1}$	$\frac{\partial M_{AF\_new}}{\partial LI3025_2}$	$\frac{\partial M_{AF\_new}}{\partial Sep}$	$\frac{\partial M_{AF\_new}}{\partial Heel}$	$\frac{\partial M_{AF\_new}}{\partial b_1}$	$\frac{\partial M_{AF\_new}}{\partial b_2}$	$\frac{\partial V_1}{\partial LI3025_1}$	$\frac{\partial V_1}{\partial DI3026_1}$	$\frac{\partial V_2}{\partial LI3025_2}$	$\frac{\partial V_2}{\partial LI3025_1}$	$\frac{\partial V_2}{\partial DI3026_1}$	$\frac{\partial V_1}{\partial Sep}$	$\frac{\partial V_1}{\partial Heel}$
	1.3247	0.1947	-0.0396	-0.0253	0.3223	-0.0273	0.2773	-0.0010	0.0051	-69.1961	134.8177	63.4334	-21.2672	21.2672	145.2331	70.6912
	kg															

$\frac{\partial V_2}{\partial Sep}$	$\frac{\partial V_2}{\partial Heel}$	$\frac{\partial V_1}{\partial b_1}$	$\frac{\partial V_2}{\partial b_2}$	$\frac{\partial \text{density}, \rho}{\partial LI3108_1}$	$\frac{\partial \text{density}, \rho}{\partial DI3109_1}$	$\frac{\partial \text{density}, \rho}{\partial Sep}$
22.9103	68.3340	1.0000	1.0000	0.0213	-0.0213	-0.0229

Exhibit B10. Example Calculation for SME Step 2

$M_{AF\_new} = M_{AF} + M_{AF\_Add}$

SME Status Before Event

SME			
			1-sigma
Sep	Heel	M_{AF}	Uncertainty
47	6.77	55.02	0.75
0.0625	0.0625	kg	kg
inches	inches		bias
bias	bias		5.25
			kg

AF Addition from SRAT							
		1-sigma			2	1-sigma	
	M_{AF_add}	Uncertainty	bias		M_{AF_new}	Uncertainty	bias
	28.354	1.285	4.311		83.374	1.488	9.561
	kg	kg	kg		kg	kg	kg

Exhibit B11. Example Calculation for SME Step 3

$M_{AF\_new} = M_{AF} + M_{AF\_Add}$

SME Status Before Event

SME			
			1-sigma
Sep	Heel	M_{AF}	Uncertainty
47	6.77	55.02	0.75
0.0625	0.0625	kg	kg
inches	inches		bias
bias	bias		5.25
			kg

AF Addition from AMFT		1-sigma			3	1-sigma	
	M_{AF_add}	Uncertainty	bias		M_{AF_new}	Uncertainty	bias
	14.0	1.2	5.00		69.020	1.396	10.250
	kg	kg	kg		kg	kg	kg

Exhibit B12. Example Calculation for SME Step 1

$$M_{AF_{new}} = \frac{3.7854 \cdot C_{AF} \cdot V_1 \cdot \rho}{0.4723 \cdot 1000000}$$

SME Status Unknown before Event

	mg/kg TOC	mg/kg Oxalate	mg/kg Formate
	11166	2218	36816
	11181	2210	37731
	11023	2246	37157
	11030	2199	37167
Avg	11100	2218.25	37217.75
Std Dev	85.14	20.07	379.08
Std error	42.57	10.04	189.54
batch % rsd (δ)	2.72%	5.05%	2.47%
C factor	1	0.27292	0.266806
low bias factor	1	0.9495	0.9697

∂ / ∂	C_{AF} / TOC	C_{AF} / δ_{TOC}	C_{AF} / Oxalate	C_{AF} / $\delta_{oxalate}$	C_{AF} / Formate	C_{AF} / $\delta_{formate}$
	1	1	0.27292	0.27292	0.266806	0.266806
	1.16%	58.20%	0.00%	0.60%	1.63%	38.41%

Current Method		1-sigma Uncertainty		bias	
C_{AF}	896.13	395.758	mg/kg	0	mg/kg
Volume, V	7648.0	149.02	gal		
Density, ρ	1.2809	0.0346	kg/L		

	Volume Determination					1-sigma
		1-sigma		1-sigma	V_1	Uncertainty
Before	$LI3109_1$	Uncertainty	$DI3108_1$	Uncertainty	Volume	δ_1 for V_1
	136.36	1.3349	76.156	0.9271	7648.0	9
		bias		bias	gal	gal
		2.31		1.605		bias, b_1
		inwc		inwc		6
						gal

	x1	x2	y1	y2
	9.8051	0	500	0
	79.232	9.8051	5240	500
	125.25	79.232	8500	5240
	175.02	125.25	12000	10850
Level Info	125.25	79.232	8500	5240

Exhibit B12. Example Calculation for SME Step 1
(continued)

1	1-sigma	$\frac{\partial M_{AF\_new}}{\partial C_{AF}}$	$\frac{\partial M_{AF\_new}}{\partial LI3108_1}$	$\frac{\partial M_{AF\_new}}{\partial DI3109_1}$	$\frac{\partial M_{AF\_new}}{\partial \delta_1}$	$\frac{\partial V_1}{\partial LI3108_1}$	$\frac{\partial V_1}{\partial DI3109_1}$	$\frac{\partial \rho}{\partial LI3108_1}$	$\frac{\partial \rho}{\partial DI3109_1}$	$\frac{\partial V_1}{\partial \delta_1}$
M_{AF_new}	Uncertainty									
70.36	31.082	0.07852	0.52511	-0.01630	0.00920	-69.95861	125.26335	0.02128	-0.02128	1.00000
kg	kg	99.948%	0.051%	0.000%	0.0007%					

		$\frac{\partial M_{AF\_new}}{\partial C_{AF}}$	$\frac{\partial M_{AF\_new}}{\partial LI3108_1}$	$\frac{\partial M_{AF\_new}}{\partial DI3109_1}$	$\frac{\partial M_{AF\_new}}{\partial Sep}$	$\frac{\partial M_{AF\_new}}{\partial Heel}$	$\frac{\partial M_{AF\_new}}{\partial b_1}$	$\frac{\partial V_1}{\partial LI3108_1}$	$\frac{\partial V_1}{\partial DI3109_1}$	$\frac{\partial \rho}{\partial LI3108_1}$	$\frac{\partial \rho}{\partial DI3109_1}$	$\frac{\partial V_1}{\partial Sep}$	$\frac{\partial V_1}{\partial Heel}$	$\frac{\partial \rho}{\partial Sep}$	$\frac{\partial V_1}{\partial b_1}$
	bias														
	1.2152	0.07852	0.52511	-0.01630	-0.02088	0.65175	0.00920	-69.95861	125.26335	0.02128	-0.02128	160.45435	70.84184	-0.02725	1.00000
	kg														

Exhibit B13. Example Calculation for SME Step 5 Linked to Step 1

$$M_{AF_{new}} = \frac{3.7854 \cdot C_{AF} \cdot V_2 \cdot \rho}{0.4723 \cdot 1000000}$$

SME Status Unknown before Event

	mg/kg TOC	mg/kg Oxalate	mg/kg Formate
	11166	2218	36816
	11181	2210	37731
	11023	2246	37157
	11030	2199	37167
Avg	11100	2218.25	37217.75
Std Dev	85.14	20.07	379.08
Std error	42.57	10.04	189.54
batch % rsd (δ)	2.72%	5.05%	2.47%
C factor	1	0.27292	0.266806
low bias factor	1	0.9495	0.9697

∂ / ∂	C_{AF} / TOC	C_{AF} / δ_{TOC}	C_{AF} / Oxalate	C_{AF} / $\delta_{oxalate}$	C_{AF} / Formate	C_{AF} / $\delta_{formate}$
	1	1	0.27292	0.27292	0.266806	0.266806
	1.16%	58.20%	0.00%	0.60%	1.63%	38.41%

Current Method		1-sigma Uncertainty		bias	
C_{AF}	896.13	395.758	mg/kg	0	mg/kg
Volume, V	7648.0	149.02	gal		
Density, ρ	1.2809	0.0346	kg/L		

	Volume Determination					1-sigma
		1-sigma		1-sigma	V_1	Uncertainty
Before	$LI3109_1$	Uncertainty	$DI3108_1$	Uncertainty	Volume	δ_1 for V_1
	136.36	1.3349	76.156	0.9271	7648.0	9
		bias		bias	gal	gal
		2.31		1.605		bias, b_1
		inwc		inwc		6
						gal

Exhibit B13. Example Calculation for SME Step 5 Linked to Step 1
(continued)

	After Acceptability --- Transfer Out					1-sigma
		1-sigma			After	Uncertainty
After	$LI3109_2$	Uncertainty			V_2	δ_2 for V_2
	23.511	1.3349			1545.9	9
		bias			gal	gal
		2.31				bias, b_2
		inwc				6
						gal

	x1	x2	y1	y2
	9.8051	0	500	0
	79.232	9.8051	5240	500
	125.25	79.232	8500	5240
	175.02	125.25	12000	10850
Level Info	125.25	79.232	8500	5240
After	79.232	9.8051	5240	500

5		Partial Derivatives										
After	1-sigma	∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_2	V_2	V_2	ρ	ρ	V_2
M_{AF_new}	Uncertainty	∂	C_{AF}	$LI3025_1$	$DI3026_1$	$LI3025_2$	δ_2	$LI3025_2$	$LI3025_1$	$DI3026_1$	$LI3108_1$	$DI3109_1$	δ_2
14.222	6.3161		0.01587	0.04474	-0.04474	0.49036	0.00920	53.29949	-20.81464	20.81464	0.02128	-0.02128	1.00000
kg	kg		98.895%	0.009%	0.004%	1.074%	0.0172%						

		Partial Derivatives											
	bias	∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_2	V_2	V_2	ρ
	1.142	∂	C_{AF}	$LI3025_1$	$DI3026_1$	$LI3025_2$	Sep	Heel	b_2	$LI3025_2$	$LI3025_1$	$DI3026_1$	$LI3108_1$
	kg		0.01587	0.04474	-0.04474	0.49036	-0.05731	0.62812	0.00920	53.29949	-20.81464	20.81464	0.02128

ρ	V_2	V_2	V_2	ρ
$DI3109_1$	Sep	Heel	b_2	Sep
-0.02128	26.66222	68.27325	1.00000	-0.02725

Exhibit B14. Example Calculation for SME Step 4

$$C_{AF} = \frac{0.4905 \cdot 1000000 \cdot M_{AF}}{\rho \cdot V_1 \cdot 3.7854}$$

SME Status Before Event

SME			
			1-sigma
Sep	Heel	M_{AF}	Uncertainty
47	6.77	55.02	0.75
0.0625	0.0625	kg	kg
inches	inches		bias
bias	bias		5.25
			kg

Before Acceptability Decision			
Coal	240	mg/kg	
	mg/kg	mg/kg	
	Before	Before	
	TOC	Nitrate	
	9945	20634	
	9958	20047	
	9963	20028	
	9905	20854	
Avg	9942.75	20390.75	
Std Dev	26.29	417.74	
Std error	13.14	208.87	
batch % rsd (δ)	2.72%	2.73%	
C factor	1	0	
low bias factor	1	0.9924	
	Low NO_3	Acceptable	
	High NO_3	Acceptable	

	Volume/Density Determinations			
		1-sigma		1-sigma
Before	$LI3109_1$	Uncertainty	$DI3108_1$	Uncertainty
	126.48	1.3349	65.018	0.9271
		bias		bias
		2.31		1.605
		inwc		inwc
		1-sigma		
Before	Before	Uncertainty		
Density. ρ	Volume, V_1	δ_1 for V_1		
1.3077	6958.4	9		
kg/L	gal	gal		
		bias, b_1		
		6		
		gal		

Exhibit B14. Example Calculation for SME Step 4

(continued)

	x1	x2	y1	y2
	9.8051	0	500	0
	79.232	9.8051	5240	500
	125.25	79.232	8500	5240
	175.02	125.25	12000	10850
Before	125.25	79.232	8500	5240

4		Partial Derivatives									
Before	1-sigma	∂	C_{AF}	C_{AF}	C_{AF}	C_{AF}	V_1	V_1	Density, ρ	Density, ρ	V_1
C_{AF}	Uncertainty	∂	M_{AF}	$LI3109_1$	$DI3108_1$	δ_1	$LI3109_1$	$DI3108_1$	$LI3109_1$	$DI3108_1$	δ_1
783.5	13.628		14.23991	-6.29494	0.19539	-0.11259	-57.30704	111.47980	0.02128	-0.02128	1.00000
mg/kg	mg/kg		61.4%	38.0%	0.0%	0.5529%					

		Partial Derivatives														
		∂	C_{AF}	C_{AF}	C_{AF}	C_{AF}	C_{AF}	C_{AF}	V_1	V_1	Density, ρ	Density, ρ	V_1	V_1	V_1	Density, ρ
	bias	∂	M_{AF}	$LI3109_1$	$DI3108_1$	Sep	Heel	b_1	$LI3109_1$	$DI3108_1$	$LI3109_1$	$DI3108_1$	Sep	Heel	b_1	Sep
	76.166		14.23991	-6.29494	0.19539	0.25551	-7.97639	-0.11259	-57.30704	111.47980	0.02128	-0.02128	145.78238	70.84184	1.00000	-0.02782
	mg/kg															

Acceptability Decision for the SME Product

				Allowed for given NO3						
		C_{AF} Model Coefficients		TOC Model Coefficients						
	Gallons on AF	**h**	**j**	**f**	**g**					
	728	5117745.1	-35.869438	8140	0.37					
	894	7884790.5	-55.545316	6550	0.37					
	1017	10373798	-73.602487	5300	0.37					
		C_{AF}	1-sigma	TOC	1-sigma	**AF Tracking Method Acceptability**				
	Gallons on AF	mg/kg	Uncertainty	mg/kg	Uncertainty		C_{AF}	TOC	Delta C_{AF}	Delta TOC
Allowed C_{AF} & TOC	728	2094.36	5.09	15627.24	219.99	SME C_{AF} Decision	Acceptable	Acceptable	1200.2	4617.3292
	894	2598.50	6.35	14037.24	219.99		Acceptable	Acceptable	1703.2	3027.3292
	1017	2978.76	7.35	12787.24	219.99		Acceptable	Acceptable	2082.4	1777.3292

Exhibit B15. Example Calculation for SME Step 5

$$M_{AF_{new}} = \frac{M_{AF} \cdot V_2}{V_1}$$

SME Status Before Event

SME			
			1-sigma
Sep	Heel	M_{AF}	Uncertainty
47	6.77	55.02	0.75
0.0625	0.0625	kg	kg
inches	inches		bias
bias	bias		5.25
			kg

	Volume/Density Determinations			
		1-sigma		1-sigma
Before	$LI3109_1$	Uncertainty	$DI3108_1$	Uncertainty
	126.48	1.3349	65.018	0.9271
		bias		bias
		2.31		1.605
		inwc		inwc
		1-sigma		
Before	Before	Uncertainty		
Density. ρ	Volume, V_1	δ_1 for V_1		
1.3077	6958.4	9		
kg/L	gal	gal		
		bias, b_1		
		6		
		gal		

	After Acceptability --- Transfer Out			1-sigma
		1-sigma	After	Uncertainty
After	$LI3109_2$	Uncertainty	V_2	δ_2 for V_2
	20.433	1.3349	1359.6	9
		bias	gal	gal
		2.31		bias, b_2
		inwc		6
				gal

Exhibit B15. Example Calculation for SME Step 5
(continued)

	x1	x2	y1	y2
	9.8051	0	500	0
	79.232	9.8051	5240	500
	125.25	79.232	8500	5240
	175.02	125.25	12000	10850
Before	125.25	79.232	8500	5240
After	79.232	9.8051	5240	500

5		Partial Derivatives														
After	1-sigma	∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_1	V_1	V_2	V_2	V_2	V_1	V_2	Density	Density
M_{AF_new}	Uncertainty	∂	M_{AF}	$LI3025_1$	$DI3026_1$	$LI3025_2$	δ_1	δ_2	$LI3025_1$	$DI3026_1$	$LI3025_2$	$LI3025_1$	$DI3026_1$	δ_1	δ_2	$LI3108_1$	$DI3109_1$
10.75	0.5794		0.19538	-0.04871	-0.03499	0.41281	-0.00154	0.00791	-57.30704	111.47980	52.20856	-17.35670	17.35670	1.00000	1.00000	0.02128	-0.02128
kg	kg		6.4%	1.3%	0.3%	90.5%	0.0576%	1.5086%									

		Partial Derivatives															
		∂	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	M_{AF_new}	V_1	V_1	V_2	V_2	V_2	V_1	V_1
	bias	∂	M_{AF}	$LI3025_1$	$DI3026_1$	$LI3025_2$	Sep	Heel	b_1	b_2	$LI3025_1$	$DI3026_1$	$LI3025_2$	$LI3025_1$	$DI3026_1$	Sep	Heel
	1.482		0.19538	-0.04871	-0.03499	0.41281	-0.04575	0.43039	-0.00154	0.00791	-57.30704	111.47980	52.20856	-17.35670	17.35670	145.78238	70.84184
	kg																

V_2	V_2	V_1	V_2	Density	Density	Density
Sep	Heel	b_1	b_2	$LI3108_1$	$DI3109_1$	Sep
22.69739	68.27325	1.00000	1.00000	0.02128	-0.02128	-0.02782

Appendix C. Volume versus Pressure Correlation for the AMFT

The volume versus pressure correlation for the AMFT is developed in this appendix. Exhibit C1 provides an overview of how the data were taken and the how the analyses of the data were conducted [10]. Table C1 shows the data and Exhibit C2 provides the regression statistics from fitting the pressure values to the volume numbers using JMP 11.1.1 [9]. Included in the JMP output are confidence intervals at 95% confidence for inverse predictions (i.e., predictions of volume from pressure values): predictions of mean volumes and predictions of individual volumes.

Table C2 summaries these confidence intervals, which were used to provide insight into possible bias and random uncertainties in the use of the correlation between volume and pressure. The bias is estimated by maximum of the upper limit of the confidence interval for mean predictions minus the mean prediction itself and the mean prediction minus the lower limit of the confidence interval. A column showing these estimated biases is included in Table C2. Over the interval of pressures studied, the largest of these values is 0.232 gal, which is to be used as a bound on the bias of the correlation at 95% confidence.

The random uncertainty is estimated by the maximum of the upper limit of the confidence interval for an individual prediction minus the upper limit of the confidence interval for the mean and of the lower limit of the confidence interval for the mean minus the lower limit of the confidence interval for an individual. These estimated random uncertainties are at 95% confidence, so they are taken to be 2-sigma estimates. A column showing these values at 1-sigma level is provided in Table C1. Over the interval of pressures studied, the largest of these 1-sigma values is 0.637 gal, which is to be used as the 1-sigma random uncertainty of the volume versus pressure correlation for the AMFT.

Exhibit C1. DWPF Procedure Utilized to Generate the Data and to Establish Correlation

Additive Mix Feed Tank (AMFT) Volume-to-Pressure Correlation

A fill and drain experiment was conducted on the AMFT to determine the pressure difference between the level instrumentation outlet and the vapor space of the tank as a function of liquid volume within the tank. The specific gravity of the material was set to a value of unity. The volume in the tank was the controlled quantity in the experiment and was incrementally increased by 3 gallons. The measurement and test equipment (M&TE) used in the determination of the pressure difference had a range of 0-100 inwc with an uncertainty of 0.06% of span. This device was used in parallel with the LI2614 instrumentation, i.e. the mechanical characteristics of the instrument from which the M&TE and the LI2614 received information was identical.

Three experimental runs were conducted. The tank and downstream piping were drained before each run to ensure that the experiment was based on an initially empty system. A calibrated container was filled with water to the 3 gallon mark and then subsequently evacuated to the AMFT. After each 3 gallon addition, the pressure was recorded from the M&TE. After every sixth addition, i.e. 18 gallons, the pressure from the LI2614 was recorded.

The experimental data are plotted in Figure 1 along with the derived correlation. It is important to notice that the derived correlation is only valid for the linear portion of the data. This is appropriate given that the working level in the AMFT will be restricted to the linear portion of the tank. One notices in Figure 1 that the level instrumentation in the tank was not encountered by the liquid level until the volume in the tank was between 3 and 6 gallons. This is inferred from the increase in pressure occurring in this volume range. This information represented the first indication that there may exist some discrepancy between the vendor print for the AMFT, i.e. Drawing Number D-86-241 Rev. 6, and the physical tank. Theoretically, based upon the vendor print tank dimensions, the level instrumentation should not be encountered until the volume of the tank was approximately 10 gallons. A second indication that the tank may have a different geometry is given at the other end of the data range. In this case, one notices that a plateau in the data is encountered which is consistent with an overflow condition. This overflow condition occurs at approximately 168 gallons. The theoretical overflow volume, i.e. that based upon the vendor supplied dimensions, was calculated to occur at 178.5 gallons.

To derive the pressure difference to volume correlation, the data for the three runs were averaged, i.e.

$$\langle \Delta P \rangle_j = \frac{1}{3}\sum_{i=1}^{3} \Delta P_{ij} \qquad (1)$$

Bodey 1

Exhibit C1. DWPF Procedure Utilized to Generate the Data and to Establish Correlation (continued)

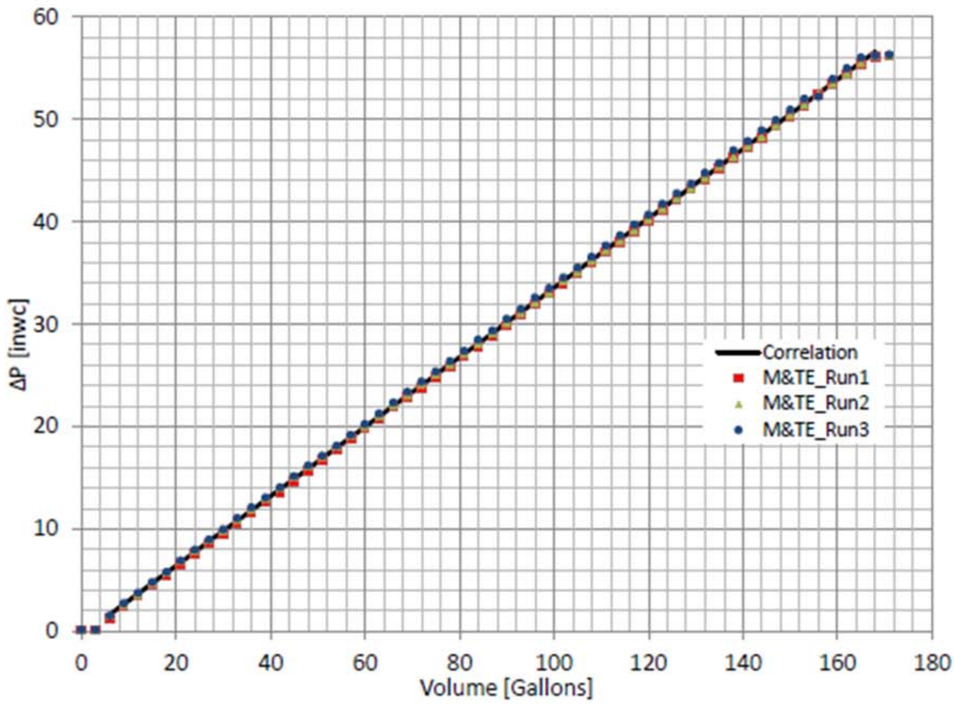


Figure 1. Experimental Determination of Pressure Difference as a Function of Volume and Derived Correlation.

The averaged set, $\langle \Delta P \rangle_j$, was used to establish a correlation between the volume of liquid in the tank and the pressure difference as shown in Figure 2. This exercise results in an affine correlation, i.e.

$$\Delta P(V) = \left(0.3394\frac{inwc}{Gallon}\right)V - 0.3966inwc \qquad (2)$$

In practice, one will be using the level instrumentation to determine the volume in the tank. As a result, Equation 2 must be inverted to arrive at such a relationship.

$$V(\Delta P) = \frac{\Delta P + 0.3966inwc}{0.3394\frac{inwc}{Gallon}} \qquad (3)$$

Bodey 2

Exhibit C1. DWPF Procedure Utilized to Generate the Data and to Establish Correlation (continued)

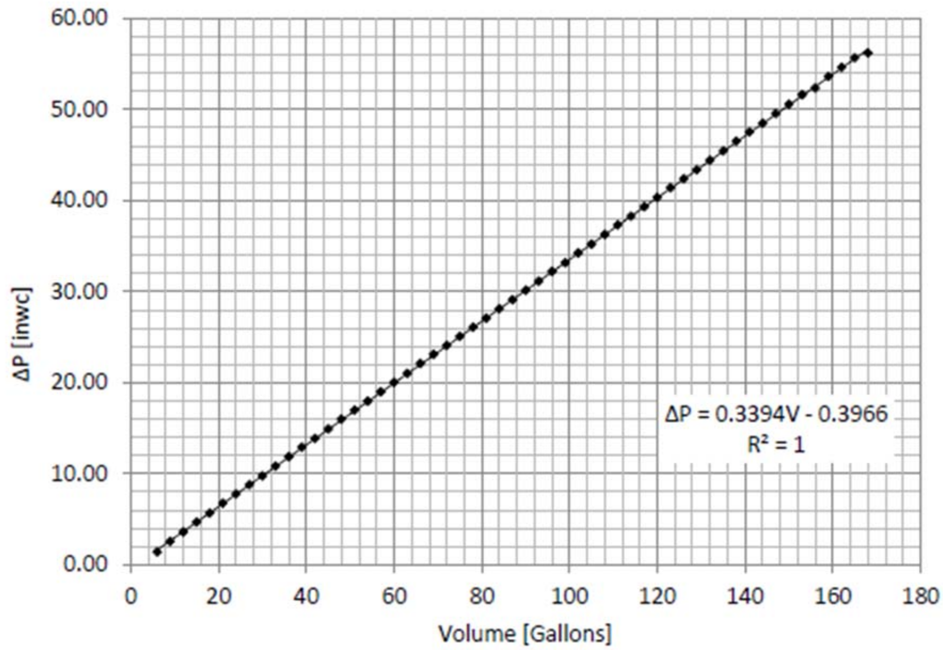


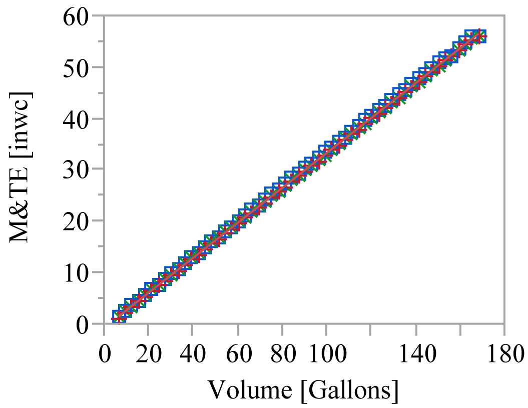
Figure 2. Pressure Difference as a Function of the Volume of the AMFT.

This correlation will be programmed into the DCS to provide an experimentally driven AMFT volume.

Bodey 3

Exhibit C2. Regression Analysis with Inverse Prediction

Response M&TE [inwc]♣
Whole Model
Regression Plot



Summary of Fit

RSquare	0.999788
RSquare Adj	0.999787
Root Mean Square Error	0.236708
Mean of Response	29.12909
Observations (or Sum Wgts)	165

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	43101.143	43101.1	769244.6
Error	163	9.133	0.05603	**Prob > F**
C. Total	164	43110.276		<.0001*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	53	1.1311681	0.021343	0.2934
Pure Error	110	8.0018000	0.072744	**Prob > F**
Total Error	163	9.1329681		1.0000
				Max RSq

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.396606	0.038378	-10.33	<.0001*
Volume [Gallons]	0.3393758	0.000387	877.07	<.0001*

♣ This statistical analysis was conducted using JMP 11.1.1 [7]

Exhibit C2. Regression Analysis with Inverse Prediction

(continued)

Inverse Prediction

Specified M&TE [inwc]	Predicted Volume [Gallons]	Lower 95%	Upper 95%
0.00000	1.1686	0.9472	1.3892
5.00000	15.9016	15.7085	16.0939
10.00000	30.6345	30.4681	30.8003
15.00000	45.3674	45.2248	45.5096
20.00000	60.1003	59.9771	60.2234
25.00000	74.8333	74.7226	74.9439
30.00000	89.5662	89.4588	89.6736
35.00000	104.2991	104.1852	104.4133

Confidence Interval with respect to an expected response

Inverse Prediction

Specified M&TE [inwc]	Predicted Volume [Gallons]	Lower 95%	Upper 95%
25.00000	74.8333	74.7226	74.9439
30.00000	89.5662	89.4588	89.6736
35.00000	104.2991	104.1852	104.4133
40.00000	119.0321	118.9030	119.1614
45.00000	133.7650	133.6150	133.9155
50.00000	148.4979	148.3231	148.6734
55.00000	163.2309	163.0289	163.4336
60.00000	177.9638	177.7331	178.1954

Confidence Interval with respect to an expected response

Inverse Prediction

Specified M&TE [inwc]	Predicted Volume [Gallons]	Lower 95%	Upper 95%
0.00000	1.1686	-0.227	2.5631
5.00000	15.9016	14.511	17.2919
10.00000	30.6345	29.247	32.0215
15.00000	45.3674	43.983	46.7518
20.00000	60.1003	58.717	61.4830
25.00000	74.8333	73.452	76.2149
30.00000	89.5662	88.185	90.9477
35.00000	104.2991	102.917	105.6812

Confidence Interval with respect to an individual response

Inverse Prediction

Specified M&TE [inwc]	Predicted Volume [Gallons]	Lower 95%	Upper 95%
25.00000	74.8333	73.4515	76.2149
30.00000	89.5662	88.1848	90.9477
35.00000	104.2991	102.9172	105.6812
40.00000	119.0321	117.6489	120.4155
45.00000	133.7650	132.3798	135.1507
50.00000	148.4979	147.1099	149.8866
55.00000	163.2309	161.8392	164.6233
60.00000	177.9638	176.5677	179.3608

Confidence Interval with respect to an individual response

Table C1. Measurements of Gallons versus Inches of Water Column

run	Additions	Volume [Gallons]	M&TE [inwc]
1	0	0	0.09
1	1	3	0.09
1	2	6	1.2
1	3	9	2.37
1	4	12	3.46
1	5	15	4.46
1	6	18	5.46
1	7	21	6.47
1	8	24	7.57
1	9	27	8.55
1	10	30	9.54
1	11	33	10.56
1	12	36	11.6
1	13	39	12.64
1	14	42	13.6
1	15	45	14.64
1	16	48	15.7
1	17	51	16.7
1	18	54	17.71
1	19	57	18.77
1	20	60	19.81
1	21	63	20.8
1	22	66	21.87
1	23	69	22.85
1	24	72	23.76
1	25	75	24.78
1	26	78	25.8
1	27	81	26.79
1	28	84	27.8
1	29	87	28.85
1	30	90	29.82
1	31	93	30.87
1	32	96	31.93
1	33	99	32.95
1	34	102	33.94
1	35	105	35
1	36	108	36
1	37	111	37.04
1	38	114	38.03
1	39	117	39.06
1	40	120	40.11
1	41	123	41.19
1	42	126	42.15
1	43	129	43.16
1	44	132	44.15
1	45	135	45.21
1	46	138	46.25
1	47	141	47.25
1	48	144	48.21
1	49	147	49.33
1	50	150	50.35
1	51	153	51.37
1	52	156	52.37
1	53	159	53.41
1	54	162	54.43
1	55	165	55.45
1	56	168	56.16
1	57	171	56.21
2	0	0	0.12
2	1	3	0.11
2	2	6	1.51

Table C1. Measurements of Gallons versus Inches of Water Column

run	Additions	Volume [Gallons]	M&TE [inwc]
2	3	9	2.59
2	4	12	3.58
2	5	15	4.71
2	6	18	5.72
2	7	21	6.89
2	8	24	7.81
2	9	27	8.87
2	10	30	9.79
2	11	33	10.9
2	12	36	11.9
2	13	39	12.99
2	14	42	14
2	15	45	15
2	16	48	16.07
2	17	51	17.06
2	18	54	18.03
2	19	57	19.09
2	20	60	20.03
2	21	63	21
2	22	66	22.1
2	23	69	23.06
2	24	72	24.12
2	25	75	25.07
2	26	78	26.05
2	27	81	27.09
2	28	84	28.1
2	29	87	29.08
2	30	90	30.11
2	31	93	31.09
2	32	96	32.09
2	33	99	33.04
2	34	102	34.2
2	35	105	35.1
2	36	108	36.21
2	37	111	37.2
2	38	114	38.15
2	39	117	39.17
2	40	120	40.22
2	41	123	41.28
2	42	126	42.25
2	43	129	43.24
2	44	132	44.3
2	45	135	45.37
2	46	138	46.3
2	47	141	47.41
2	48	144	48.3
2	49	147	49.4
2	50	150	50.44
2	51	153	51.43
2	52	156	52.45
2	53	159	53.49
2	54	162	54.49
2	55	165	55.55
2	56	168	56.25
2	57	171	56.27
3	0	0	0.12
3	1	3	0.11
3	2	6	1.49
3	3	9	2.72
3	4	12	3.75
3	5	15	4.8

Table C1. Measurements of Gallons versus Inches of Water Column

run	Additions	Volume [Gallons]	M&TE [inwc]
3	6	18	5.76
3	7	21	6.88
3	8	24	7.93
3	9	27	8.93
3	10	30	9.94
3	11	33	11.02
3	12	36	12.07
3	13	39	13.06
3	14	42	14.03
3	15	45	15.12
3	16	48	16.17
3	17	51	17.13
3	18	54	18.1
3	19	57	19.15
3	20	60	20.22
3	21	63	21.23
3	22	66	22.32
3	23	69	23.37
3	24	72	24.37
3	25	75	25.37
3	26	78	26.35
3	27	81	27.36
3	28	84	28.46
3	29	87	29.36
3	30	90	30.5
3	31	93	31.45
3	32	96	32.58
3	33	99	33.5
3	34	102	34.54
3	35	105	35.5
3	36	108	36.56
3	37	111	37.66
3	38	114	38.64
3	39	117	39.69
3	40	120	40.66
3	41	123	41.74
3	42	126	42.77
3	43	129	43.69
3	44	132	44.77
3	45	135	45.71
3	46	138	46.95
3	47	141	47.85
3	48	144	48.9
3	49	147	49.9
3	50	150	50.94
3	51	153	52
3	52	156	52.19
3	53	159	53.92
3	54	162	54.97
3	55	165	56.02
3	56	168	56.2
3	57	171	56.33

Table C2. Bias and 1-Sigma Random Uncertainty for Volume versus Pressure Correlation for the AMFT

Y	Specified M&TE [inwc]	Predicted Volume [Gallons]	Lower 95% for the Line	Upper 95% for the Line	Lower 95% for an Individual	Upper 95% for an Individual	bias (gal)	1-sigma random (gal)
M&TE [inwc]	0.00000	1.1686	0.9472	1.3892	-0.2267	2.5631	0.221	0.587
M&TE [inwc]	5.00000	15.9016	15.7085	16.0939	14.5105	17.2919	0.193	0.599
M&TE [inwc]	10.00000	30.6345	30.4681	30.8003	29.2470	32.0215	0.166	0.611
M&TE [inwc]	15.00000	45.3674	45.2248	45.5096	43.9826	46.7518	0.143	0.621
M&TE [inwc]	20.00000	60.1003	59.9771	60.2234	58.7175	61.4830	0.123	0.630
M&TE [inwc]	25.00000	74.8333	74.7226	74.9439	73.4515	76.2149	0.111	0.636
M&TE [inwc]	30.00000	89.5662	89.4588	89.6736	88.1848	90.9477	0.107	0.637
M&TE [inwc]	35.00000	104.2991	104.1852	104.4133	102.9172	105.6812	0.114	0.634
M&TE [inwc]	40.00000	119.0321	118.9030	119.1614	117.6489	120.4155	0.129	0.627
M&TE [inwc]	45.00000	133.7650	133.6150	133.9155	132.3798	135.1507	0.151	0.618
M&TE [inwc]	50.00000	148.4979	148.3231	148.6734	147.1099	149.8866	0.175	0.607
M&TE [inwc]	55.00000	163.2309	163.0289	163.4336	161.8392	164.6233	0.203	0.595
M&TE [inwc]	60.00000	177.9638	177.7331	178.1954	176.5677	179.3608	0.232	0.583

Appendix D. Determining Random and Bias Uncertainties for AMFT Density Values

Volume determinations made in the AMFT are based on readings provided by LI2614 and utilize the relationship described in Appendix C. As indicated in Exhibit 2, the value from the LI2614 is adjusted for the density (actually the specific gravity) of the AMFT contents. The values from Table A1 in Appendix A indicate that the density of the 1:20 mix of antifoam and water is about 0.99835 g/mL – a value very near 1. The average density of the undiluted antifoam from this same table is 1.00073 g/mL.

In determining the AMFT volume, a specific gravity of 1 will be used in the calculations. The 1-sigma random uncertainty for this value will be taken from the standard deviation of the density measurements for the undiluted antifoam; the random standard deviation associated with the use a value of 1 for the specific gravity is 0.0036. The bounding bias for the specific gravity of the AMFT material is taken to be 3 sigma, which leads to a bias of $3 \times 0.0036 = 0.0108$.

Distribution

SRR			SRNL	
Name:	Location:		Name:	Location:
I.T. Bodey	704-25S		S.L. Marra	773-A
J.D. Ledbetter	704-27S		C.C. Herman	773-A
R.P. Farrow	704-26S		C.J. Coleman	773-A
J.R. Coleman	706-S		C.M. Gregory	773-A
P.J. Ryan	704-S		M.J. Barnes	773-A
E.J. Freed	704-S		E.P. Shine	703-41A
J.M. Bricker	704-30S		M.A. Jones	773-A
J.F. Iaukea	704-27S		M.E. Stone	999-W
R.T. McNew	704-S		D.K. Peeler	999-W
J.W. Ray	704-27S		T.B. Edwards	999-W
R.N. Hinds	704-S		D.P. Lambert	999-W
R.N. Mahannah	704-28S		D.R. Best	999-W
M.T. Feller	704-28S		A.S. Choi	773-42A
H.P. Boyd	704-27S		W.E. Daniel	999-W
H.H. Elder	704-27S		D.H. McGuire	999-W
D.C. Sherburne	704-S		M.K. Harris	703-41A
T.L. Fellinger	766-H			
R.E. Edwards	766-H			