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AEC RESEARCH AND DEVELOPMENT REPORT

A COMPUTER CODE FOR ONE-GROUP FLUX CALCULATIONS IN THREE DIMENSIONS

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Savannah River Laboratory

Aiken, South Carolina

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A COMPUTER CODE FOR ONE-GROUP
FLUX CALCULATIONS IN THREE DIMENSIONS

by

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May 1965

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ABSTRACT

A computer code, 3D-HEXLAX, has been written in FORTRAN II to solve the one-group neutron diffusion equation over a triangular network of points in two- or three-space dimensions by relaxation methods. The network can contain up to 817 grid points in the R- θ plane and up to 16 horizontal levels. Typical computation time on the IBM 704 ranges from 43 to 255 minutes as the number of grid points vary from 3,500 to 13,000.

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A COMPUTER CODE FOR ONE-GROUP FLUX CALCULATIONS IN THREE DIMENSIONS

INTRODUCTION

Numerous studies requiring calculation of the over-all flux distribution in the Savannah River reactors have been made using one-group diffusion theory. Buckling values for the various reactor regions are measured experimentally and are then used in the diffusion equation to calculate the flux distribution. This simplified method has been used successfully to determine control rod positions required for shaping the axial and radial power distribution. At the Savannah River Laboratory one-group diffusion theory codes have been developed with two-space dimensions allowed: the UNCLE code for calculation in the R-Z plane and the HEXLAX code for the R- θ plane.

As reactor loadings have become more varied, there has been an increased need for performing flux calculations in three-space dimensions. This report describes a code, 3D-HEXLAX, that has been developed to meet this need.

DISCUSSION

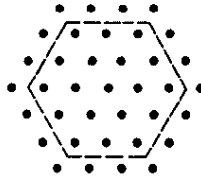
Theory

The requirement is to solve the neutron diffusion equation $\nabla^2 \phi + B^2 \phi = 0$ in both two- and three-space dimensions. Only right cylindrical reactors with either circular or hexagonal cross sections will be considered. If the reactor is infinite in the direction of its axis, then the equation becomes two-dimensional; otherwise it is a three-dimensional problem.

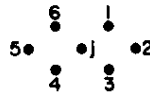
The solution to the diffusion equation is obtained by relaxation over a network of grid points. A finite reactor with hexagonal cross section is assumed. If the reactor has a circular cross section, then it is approximated by truncating the hexagon at its vertices. A number of grid points, N , is chosen for an edge of the hexagon whose length is L_1 . A network is then laid over the hexagonal cross section with a grid spacing $\delta_1 = \frac{L_1}{N-1}$. If N is 2, the grid is

$$\begin{array}{c} \text{---} \delta_1 \text{---} \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \end{array}$$

If N is greater than 2, then all interior points of the grid have six points adjacent to them as point P does above.



All interior points (inside dotted line) have six close neighbors. The central point and its adjacent neighbors are labeled as follows:



$\nabla^2 \phi_j$ may be approximated in the plane by $\frac{2}{3\delta_1^2} \left(\sum_{k=1}^6 \phi_k - 6\phi_j \right)$.

The reactor can be subdivided into L-1 horizontal slices by defining L uniformly spaced planes normal to the axis of the reactor where the first and Lth planes are the planes of the bottom and top of the reactor, respectively. Vertical spacing between planes $\delta_2 = \frac{\text{Height}}{L-1}$. In the vertical dimension, $\nabla^2 \phi = \frac{\partial^2 \phi}{\partial z^2}$ and this is approximated by $\frac{1}{\delta_2^2} (\phi_{i+1} + \phi_{i-1} - 2\phi_{ij})$.

ϕ_{ij} is the reference flux at level i and point j

Since the reactor considered is a right cylinder, then the approximation in three dimensions for $\nabla^2 \phi_{ij}$ is

$$\nabla^2 \phi_{ij} = \frac{2}{3\delta_1^2} \left(\sum_{k=1}^6 \phi_{ik} - 6\phi_{ij} \right) + \frac{1}{\delta_2^2} (\phi_{i+1,j} + \phi_{i-1,j} - 2\phi_{ij}) \quad (1)$$

Then substituting the expression for $\nabla^2 \phi_{ij}$ into the diffusion equation and solving for ϕ_{ij} yield

$$\phi_{ij} = \frac{\delta_2^2 \sum_{k=1}^6 \phi_{ik} + \frac{3}{2}\delta_1^2 (\phi_{i+1,j} + \phi_{i-1,j})}{6\delta_2^2 + 3\delta_1^2 - \frac{3}{2}\delta_1^2 \delta_2^2 B_{ij}^2} \quad (2)$$

where

$$\phi = \text{neutrons/cm}^2/\text{sec}$$

$$B^2 = \text{buckling, cm}^{-2}$$

After each iteration the flux is normalized so that the average flux in the central region of the reactor is approximately 100. The normalization factor, R, is

$$R = \frac{100N_i + 50N_p}{\sum_{i,j} \phi_{ij}}$$

where

N_i = number of interior grid points

N_p = number of peripheral grid points

To speed convergence the difference in flux between successive iterations is extrapolated by a factor γ .

With this extrapolation and with the renormalization factor, the flux equation is

$$\phi_{ij} = R\phi_{ij} + \gamma \left\{ \frac{\delta_2^2 [\phi_{i,j-1} + \phi_{i,j2} + \phi_{i,j1} + R(\phi_{i,j+1} + \phi_{i,j4} + \phi_{i,j3})] + \frac{3}{2}\delta_1^2 [\phi_{i-1,j} + R\phi_{i+1,j}]}{6\delta_2^2 + 3\delta_1^2 - \frac{3}{2}\delta_1^2\delta_2^2(B_{ij}^2 + \Delta B^2)} - R\phi_{ij} \right\}$$

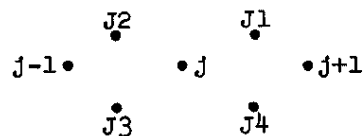
where

R = renormalization factor

γ = extrapolation factor for the flux

J1, J2, J3, and J4 = indices in the plane as shown below

ΔB^2 = buckling that must be added to each grid point to achieve criticality



This is the expression which is used in the code for the three-dimensional solution.

For two-dimensional problems, the expression reduces to

$$\phi_j = R\phi_j + \gamma \left\{ \frac{[\phi_{j-1} + \phi_{j1} + \phi_{j2} + R(\phi_{j+1} + \phi_{j4} + \phi_{j3})]}{6 - \frac{3}{2}\delta_1^2(B_j^2 + \Delta B^2)} - R\phi_j \right\} \quad (4)$$

In the iteration procedure, an extrapolation is made on the change in buckling, ΔB^2 , to speed convergence to critical.

The mesh size has been limited to a maximum of 817 points in the R- θ plane and a maximum of 16 horizontal levels, or a total of 13,072 points. Since each point has a flux and buckling value associated with it, this is the maximum number of points that can be handled by the 32,000-word core storage in the IBM 704 computer.

Computer Code

Input for a problem is supplied on cards. At the outset the program requires a control card which identifies the type of problem to be run and the input to expect.

The code sets the initial flux to be flat. In practice, this appears to be almost as good as any for the first approximation.

The code requires that buckling be supplied for each level at which the buckling is unique. The buckling for each unique level is supplied in part only. Buckling values are supplied for a pie-shaped sector of the hexagon and then these values are used in the other five sectors. If the desired buckling map is not symmetric, then only those nonsymmetric values need to be supplied additionally in the input.

The resultant buckling map is then placed successively in horizontal planes above this plane up to, but not including the next plane which has a distinct buckling map.

For sequential problems, it is possible to change individual buckling values at any grid point at any level. In the case where a symmetric buckling map has been loaded but variations to this map are required then these changes may be accomplished in the same manner.

After the buckling is loaded, the flux values are computed for all of the interior grid points using Equation (3) or (4) for three- or two-dimensional problems. The calculation progresses over the grid by row-order (left to right) for each level, proceeding upward from level 2.

The buckling is adjusted by varying ΔB^2 until the flux converges within prescribed limits. The results are then written on magnetic tape for printing.

Use of the Code

The program allows N, the number of points along an edge of the reference hexagon, to be as great as 17, and the number of vertical grid points, NLVL, must not exceed 16.

Results may be obtained for reactors with either hexagonal or circular cross sections. Appendix A gives in detail the manner in which data are prepared.

Computation time for problems on the IBM 704 depends on the number of iterations required for convergence. The number of iterations required for convergence depends on the value of γ , the flux extrapolation factor, and on XTRAP, the extrapolation factor on the buckling. It takes about 5 hours for 220 iterations when N is 17 and NLVL is 16. Experience indicates that $\gamma = 1.2$ and $400 < XTRAP < 700$ give good convergence over a wide range of problems.

Computation times on an IBM 704 are shown in the following table. The required time varied from 43 to 255 minutes as N varied from 9 to 17, while the number of horizontal levels was 16. These results show that computation times in minutes are given by

$$(\text{number of iterations})(\text{number of non-zero flux points})(1.1 \times 10^{-4})$$

	<u>Problem 1</u>	<u>Problem 2</u>	<u>Problem 3</u>
N	9	13	17
NLVL	16	16	16
γ	1.2	1.2	1.2
XTRAP	700	700	500
No. of grid points	3472	7504	13072
No. of non-zero flux points	2366	5558	10094
No. of iterations	150	161	228
Time, minutes	43	102	255

APPENDIX A - Input Data

The following input is required. For convenience the mnemonics used on the input data form will be used to designate that data.

N	Number of grid points along an edge of the gross hexagon. This number may not exceed 17.
NLVL	Number of grid points in the vertical direction. This number may not exceed 16. (NLVL = 3 for a two-dimensional problem)
δ_1	Horizontal grid spacing in cm.
δ_2	Vertical grid spacing in cm.
CONV	Average absolute deviation allowed per grid point between successive iterations.
γ	Extrapolation factor for the flux.
IDENT	Six character alphameric identifications.
CODE	Two-digit code indicating the type of problem to be run. Twelve types are allowed and are identified below. A refined grid is one which is used to approximate a circular cross section by truncating the corners of the hexagon. A refined grid can be utilized when the number of grid points along an edge of the gross grid is 6, 9, or 17. A gross grid refers to the grid of hexagonal cross section. Types 1 through 6 refers to three-dimensional problems. Code = <ol style="list-style-type: none">1. Refined grid. Normal buckling with no buckling changes.2. Refined grid. Normal buckling load plus buckling changes.3. Refined grid. No normal buckling load. Buckling changes only.4. Gross grid. Normal buckling load with no buckling changes.5. Gross grid. Normal buckling load plus buckling changes.6. Gross grid. Buckling changes only.

Types 7 through 12 refer to two-dimensional problems.

- Code =
- 7. Refined grid. Normal buckling load. No changes.
 - 8. Refined grid. Normal buckling load with changes.
 - 9. Refined grid. Buckling changes only.
 - 10. Gross grid. Normal buckling load. No changes.
 - 11. Gross grid. Normal buckling load with changes.
 - 12. Gross grid. Buckling changes only.

XTRAP Extrapolation factor on the buckling added to each grid point to speed convergence.

NORM Number to which flux will be normalized after convergence and before printout.

NSET The number of levels at which unique buckling maps are introduced plus one.

Input data cards will be described by type.

Card Type 1. One card of this type is required. Data appearing on this card have all been previously described. Decimal locations are indicated for δ_1 , δ_2 , CONV, and γ .

Card Type 2. This type card is required only if a normal buckling map is submitted. None is submitted if only buckling changes are supplied.

The number appearing on the card is the number of entries on the succeeding card (type 3). This is the number of levels at which there is a unique buckling map supplied plus one.

Card Type 3. This type card is likewise required only if a normal buckling map is submitted.

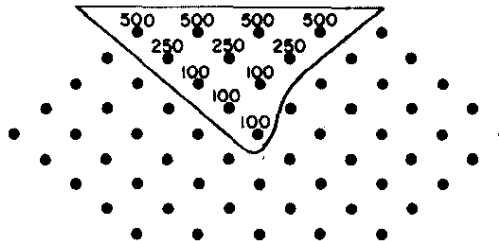
The numbers appearing on the card are the vertical grid numbers at which a new buckling map appears. The first entry is always 02; the last entry is always NLVL. No intermediate numbers are required but if the buckling is not uniform vertically, then each level at which a new map

is to be introduced must be listed in order between these two entries.

Since $NLVL = 3$ for a two-dimensional problem, this card will contain two numbers only, namely 02 and 03. The reason that two-dimensional problems are treated in this fashion is that the buckling is loaded as though it were a three-dimensional problem.

Card Type 4. If a symmetric buckling map is to be loaded, then N cards must be supplied for each level at which a new symmetric buckling map is to be loaded. Buckling must be supplied for the gross grid regardless of whether the grid is gross or refined. To prepare the required N cards, choose row N as your beginning row. This row will have N-1 entries.

To illustrate, let N be 5. Choose a sector of the hexagon as indicated below, where the buckling values are indicated above the grid points.



Buckling values are entered on the input sheet starting with row 5. It will have four entries each equal to 500.0. Row 4 will have three entries each equal to 250.0. Row 3 will have two entries of 100.0. Both row 2 and row 1 will have one entry equal to 100.0.

Card Type 5. This type is required only if modifications to the symmetric buckling map or to the buckling map of the preceding problem

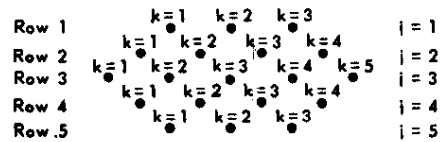
is required. Specific buckling changes are loaded six per card. Each change is designated as follows:

i = level number. Base of reactor is level one.

j = row number. See diagram below.

k = index within row.

B² = buckling value which is to be inserted, 10⁻⁶ cm⁻².



It should be noted that row 1 is the boundary row where the flux is held to zero. Also, the index within a row, k, is always determined with respect to the gross grid.

Buckling changes are terminated by entering on the last card of type 5, i = j = k = 99 for the last buckling entry.

Sample Output

The output for a sample problem is shown on the following pages for a reactor lattice with eight horizontal levels and the radial grid defined by nine points along the edge of the gross hexagon.

APPENDIX B - Sample Problem

PROBLEM IS DP1965 TYPE= 1
 N= 9 NLVL= 8 DELTA1= 12.00 DELTA2= 30.00
 FLUX IS NORMALIZED TO 100.

HEXLAX FLUX
 NO. OF ITER.= 101
 BUCKLING ADDED= 512.46

```

      5. 7. 8. 8. 7. 5.
    5. 9. 13. 15. 16. 15. 13. 9. 5.
  7. 13. 18. 21. 23. 23. 21. 18. 13. 7.
  8. 15. 21. 26. 29. 30. 29. 26. 21. 15. 8.
  8. 16. 23. 29. 34. 36. 36. 34. 29. 23. 16. 8.
  7. 15. 23. 30. 36. 40. 41. 40. 36. 30. 23. 15. 7.
  5. 13. 21. 29. 36. 41. 43. 43. 41. 36. 29. 21. 13. 5.
  9. 18. 26. 34. 40. 43. 45. 43. 40. 34. 26. 18. 9.
  5. 13. 21. 29. 36. 41. 43. 43. 41. 36. 29. 21. 13. 5.
  7. 15. 23. 30. 36. 40. 41. 40. 36. 30. 23. 15. 7.
  8. 16. 23. 29. 34. 36. 36. 34. 29. 23. 16. 8.
  8. 15. 21. 26. 29. 30. 29. 26. 21. 15. 8.
  7. 13. 18. 21. 23. 23. 21. 18. 13. 7.
  5. 9. 13. 15. 16. 15. 13. 9. 5.
    5. 7. 8. 8. 7. 5.
  
```

CONVERGENCE CRIT = .0010 EXTRAPOLATION = 1.20 B SQ. EXTRAP= 700.

PROBLEM NO. DP1965 LEVEL NO. 2

HEXLAX FLUX
 NO. OF ITER.= 101
 BUCKLING ADDED= 512.46

```

      9. 13. 14. 14. 13. 9.
    9. 17. 24. 27. 29. 27. 24. 17. 9.
  13. 24. 32. 39. 42. 42. 39. 32. 24. 13.
  14. 27. 39. 47. 53. 55. 53. 47. 39. 27. 14.
  14. 29. 42. 53. 61. 65. 65. 61. 53. 42. 29. 14.
  13. 27. 42. 55. 65. 71. 73. 71. 65. 55. 42. 27. 13.
  9. 24. 39. 53. 65. 73. 78. 78. 73. 65. 53. 39. 24. 9.
  17. 32. 47. 61. 71. 78. 80. 78. 71. 61. 47. 32. 17.
  9. 24. 39. 53. 65. 73. 78. 78. 73. 65. 53. 39. 24. 9.
  13. 27. 42. 55. 65. 71. 73. 71. 65. 55. 42. 27. 13.
  14. 29. 42. 53. 61. 65. 65. 61. 53. 42. 29. 14.
  14. 27. 39. 47. 53. 55. 53. 47. 39. 27. 14.
  13. 24. 32. 39. 42. 42. 39. 32. 24. 13.
  9. 17. 24. 27. 29. 27. 24. 17. 9.
    9. 13. 14. 14. 13. 9.
  
```

CONVERGENCE CRIT = .0010 EXTRAPOLATION = 1.20 B SQ. EXTRAP= 700.

PROBLEM NO. DP1965 LEVEL NO. 3

HEXLAX FLUX
NO. OF ITER.= 101
BUCKLING ADDED= 512.46

11. 16. 18. 18. 16. 11.
11. 21. 29. 34. 36. 34. 29. 21. 11.
16. 29. 40. 48. 52. 52. 48. 40. 29. 16.
18. 34. 48. 59. 66. 68. 66. 59. 48. 34. 18.
18. 36. 52. 66. 76. 81. 81. 76. 66. 52. 36. 18.
16. 34. 52. 68. 81. 89. 92. 89. 81. 68. 52. 34. 16.
11. 29. 48. 66. 81. 92. 97. 97. 92. 81. 66. 48. 29. 11.
21. 40. 59. 76. 89. 97. 100. 97. 89. 76. 59. 40. 21.
11. 29. 48. 66. 81. 92. 97. 97. 92. 81. 66. 48. 29. 11.
16. 34. 52. 68. 81. 89. 92. 89. 81. 68. 52. 34. 16.
18. 36. 52. 66. 76. 81. 81. 76. 66. 52. 36. 18.
18. 34. 48. 59. 66. 68. 66. 59. 48. 34. 18.
16. 29. 40. 48. 52. 52. 48. 40. 29. 16.
11. 21. 29. 34. 36. 34. 29. 21. 11.
11. 16. 18. 18. 16. 11.

CONVERGENCE CRIT = .0010 EXTRAPOLATION = 1.20 B SQ. EXTRAP= 700.

PROBLEM NO. DP1965 LEVEL NO. 4

HEXLAX FLUX
NO. OF ITER.= 101
BUCKLING ADDED= 512.46

11. 16. 18. 18. 16. 11.
11. 21. 29. 34. 36. 34. 29. 21. 11.
16. 29. 40. 48. 52. 52. 48. 40. 29. 16.
18. 34. 48. 59. 66. 68. 66. 59. 48. 34. 18.
18. 36. 52. 66. 76. 81. 81. 76. 66. 52. 36. 18.
16. 34. 52. 68. 81. 89. 92. 89. 81. 68. 52. 34. 16.
11. 29. 48. 66. 81. 92. 97. 97. 92. 81. 66. 48. 29. 11.
21. 40. 59. 76. 89. 97. 100. 97. 89. 76. 59. 40. 21.
11. 29. 48. 66. 81. 92. 97. 97. 92. 81. 66. 48. 29. 11.
16. 34. 52. 68. 81. 89. 92. 89. 81. 68. 52. 34. 16.
18. 36. 52. 66. 76. 81. 81. 76. 66. 52. 36. 18.
18. 34. 48. 59. 66. 68. 66. 59. 48. 34. 18.
16. 29. 40. 48. 52. 52. 48. 40. 29. 16.
11. 21. 29. 34. 36. 34. 29. 21. 11.
11. 16. 18. 18. 16. 11.

CONVERGENCE CRIT = .0010 EXTRAPOLATION = 1.20 B SQ. EXTRAP= 700.

PROBLEM NO. DP1965 LEVEL NO. 5

HEXLAX FLUX
 NO. OF ITER.= 101
 BUCKLING ADDED= 512.46

9. 13. 14. 14. 13. 9.
 9. 17. 24. 27. 29. 27. 24. 17. 9.
 13. 24. 32. 39. 42. 42. 39. 32. 24. 13.
 14. 27. 39. 47. 53. 55. 53. 47. 39. 27. 14.
 14. 29. 42. 53. 61. 65. 65. 61. 53. 42. 29. 14.
 13. 27. 42. 55. 65. 71. 73. 71. 65. 55. 42. 27. 13.
 9. 24. 39. 53. 65. 73. 78. 78. 73. 65. 53. 39. 24. 9.
 17. 32. 47. 61. 71. 78. 80. 78. 71. 61. 47. 32. 17.
 9. 24. 39. 53. 65. 73. 78. 78. 73. 65. 53. 39. 24. 9.
 13. 27. 42. 55. 65. 71. 73. 71. 65. 55. 42. 27. 13.
 14. 29. 42. 53. 61. 65. 65. 61. 53. 42. 29. 14.
 14. 27. 39. 47. 53. 55. 53. 47. 39. 27. 14.
 13. 24. 32. 39. 42. 42. 39. 32. 24. 13.
 9. 17. 24. 27. 29. 27. 24. 17. 9.
 9. 13. 14. 14. 13. 9.

CONVERGENCE CRIT = .0010 EXTRAPOLATION = 1.20 B SQ. EXTRAP= 700.

PROBLEM NO. DP1965 LEVEL NO. 6

HEXLAX FLUX
 NO. OF ITER.= 101
 BUCKLING ADDED= 512.46

5. 7. 8. 8. 7. 5.
 5. 9. 13. 15. 16. 15. 13. 9. 5.
 7. 13. 18. 21. 23. 23. 21. 18. 13. 7.
 8. 15. 21. 26. 29. 30. 29. 26. 21. 15. 8.
 8. 16. 23. 29. 34. 36. 36. 34. 29. 23. 16. 8.
 7. 15. 23. 30. 36. 40. 41. 40. 36. 30. 23. 15. 7.
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 7. 13. 18. 21. 23. 23. 21. 18. 13. 7.
 5. 9. 13. 15. 16. 15. 13. 9. 5.
 5. 7. 8. 8. 7. 5.

CONVERGENCE CRIT = .0010 EXTRAPOLATION = 1.20 B SQ. EXTRAP= 700.

PROBLEM NO. DP1965 LEVEL NO. 7

500.500.500.500.500.500.
500.500.500.500.500.500.500.500.500.
500.500.500.500.500.500.500.500.500.500.
500.500.500.500.500.500.500.500.500.500.500.
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500.500.500.500.500.500.500.500.500.500.500.500.500.500.500.500.500.500.500.

PROBLEM NO. DP1965 LEVEL NO. 2

[illegible]

PROBLEM NO. DP1965 LEVEL NO. 3

```

HEXLAX B SQD
NO. OF ITER.= 101
BUCKLING ADDED= 512.46

```

[illegible]

CONVERGENCE CRIT = .0010 EXTRAPOLATION = 1.20 B SQ. EXTRAP= 700.

PROBLEM NO. DP1965 LEVEL NO. 4

```

HEXLAX B SQD
NO. OF ITER.= 101
BUCKLING ADDED= 512.46

```

500.500.500.500.500.500.
500.500.500.500.500.500.500.500.
500.500.500.500.500.500.500.500.500.
500.500.500.500.500.500.500.500.500.500.
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500.500.500.500.500.500.500.500.500.500.500.500.500.
500.500.500.500.500.500.500.500.500.500.500.500.500.

CONVERGENCE CRIT = .0010 EXTRAPOLATION = 1.20 B SQ. EXTRAP= 700.

PROBLEM NO. DP1965 LEVEL NO. 5

500.500.500.500.500.500.
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HEXLAX 8 SQD
NO. OF ITER.= 101
BUCKLING ADDED= 512.46

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- 20 -

APPENDIX C - FORTRAN Source Code

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C      3 DIMENSION HEXLAX      J C ENGLISH      MAIN PROGRAM
C
C      N=NUMBER OF GRID POINTS ON AN EDGE
C      NLVL=NUMBER OF VERTICAL GRID SPACES
C      DEL1=GRID CONSTANT IN PLANE
C      DEL2=GRID CONSTANT VERTICAL DIMENSION
C      REN=RENORMALIZATION CONSTANT
C      FCT=EXTRAPOLATION CONSTANT
C      CONV=CONVERGENCE CRITERION
C
C      INITIAL FLUX WILL BE FLAT EVERYWHERE EXCEPT THAT
C      BOUNDARY FLUX WILL BE ZERO. SUCCEEDING PROBLEMS WILL HAVE
C      CONVERGED FLUX FROM PRECEDING PROBLEM AS INITIAL FLUXES.
C
C      DIMENSION O(17,817),B(15,817),K1(17),NDX1(33),NDX2(33),IMP1(6),
1    IMP2(6),IMP3(6),T(6),NDLT(17),NSKP(22),NOEN(22)
C
C      CALL EFM
C
C      DO 11 J=1,17
C      READ 523,(NDLT(I),I=1,J)
11    WRITE TAPE 5,NDLT
C
C      REWIND 5
C      READ 500,(NSKP(I),I=1,22)
C      READ 500,(NOEN(I),I=1,22)
500    FORMAT(12A6)
C
C      1 READ 522,N,NLVL,DEL1,DEL2,CONV,GAM,PROB,NBR,XTRAP,FNRM
522    FORMAT(2I2,2F5.2,F4.4,F3.2,A6,I2,F4.0,F3.0)
C      WRITE OUTPUT TAPE 10,526,PROB,NBR
C      WRITE OUTPUT TAPE 10,527,N,NLVL,DEL1,DEL2
C      WRITE OUTPUT TAPE 10,528,FNRM
526    FORMAT(12H1PROBLEM IS ,A6,6X,6HTYPE= 12)
527    FORMAT ( 4H N= 12,6X, 6HNLVL= ,I2,6X,8HDELTA1= ,F6.2,6X,8HDELTA2=
1    ,F6.2)
528    FORMAT(23H FLUX IS NORMALIZED TO F4.0)
C
C      DO 13 J=1,17
C      READ TAPE 5,NDLT
C      IF(N-J)13,12,13
13    CONTINUE
C
C      12 REWIND 5
C
C      GO TO (2,2,2,8,8,8,2,2,2,8,8,8),NBR
C
C      SET NDLT VECTOR TO ZERO. CASE TO BE RUN IS FOR GROSS GRID.
C      DO 18 J=1,N
18    NDLT(J)=0
C
C      DETERMINE RANGES OF INDICES FOR ROWS OF HEX      NDX1 AND NDX2
C
C      2 KEND=N
C      N1=2
C      N2=N-1
C
C      DO 122 K=1,KEND
C      JEND=N1+N2-2
C      NDX1(K)=N1
C      NDX2(K)=JEND
C      N1=JEND+3
122    N2=N2+1
C
C      N2=N2-2
C      J=KEND
C      KEND=N-1
C
C      DO 123 K=1,KEND
C      JEND=N1+N2-2
C      I=J+K
C      NDX1(I)=N1
C      NDX2(I)=JEND
C      N1=JEND+3
123    N2=N2-1
C
C      N1=N-1
C      N2=2*N
C
C      DO 10 I=1,N1

```

```

      NDX1(I)=NDX1(I)+NCLT(I)
      NDX2(I)=NDX2(I)-NCLT(I)
      N3=N2-1
      NDX1(N3)=NDX1(N3)+NDLT(I)
10  NDX2(N3)=NDX2(N3)-NDLT(I)
C
      NDX1(N)=NDX1(N)+NCLT(N)
      NDX2(N)=NDX2(N)-NCLT(N)
      NTOT=3*N*N-3*N+2
      NTL=NTOT-1
C
C  INITIALIZE FLUX.
23  DO 124 I=1,NTL
      O(I,I)=0.0
124  O(NLVL,I)=0.0
C
      JND=NLVL-1
C
      DO 101 I=2,JND
C
      DO 202 K=1,N
      O(I,K)=0.0
      NN=NTOT-K
202  O(I,NN)=0.0
C
      N1 =1
      N2 =N
C
      DO 102 K=2,N
      N1=NDX1(K)
      JEND=NDX2(K)
C
      THIS DO LOOP SETS ALL THE FLUXES IN A ROW TO BE ZERO IF THEY ARE
      NOT TO BE ITERATED UPON.
      LEND=NDLT(K)+1
C
      DO 22 L=1,LEND
      N2=N1-L
      N3=JEND+L
      O(I,N2)=0.0
22  O(I,N3)=0.0
C
      DO 102 J=N1,JEND
102  O(I,J)=100.
C
      KEND=N-2
C
      DO 103 K=1,KEND
      NN=N+K
      N1=NDX1(NN)
      JEND=NDX2(NN)
      O(I,N1-1)=0.0
      O(I,JEND+1)=0.0
C
      DO 103 J=N1,JEND
103  O(I,J)=100.
C
101  CONTINUE
C
      ALL INITIAL FLUX VALUES SET. OBTAIN BUCKLING OVER GRID.
      READ INPUT DATA. INCLUDES ZERO BOUNDARY POINTS.
C
17  A=NLVL
      NTOT=NTOT-1
      B=NTOT-6*(2*N-3)
      C=6*(N-2)
      REN=(A-2.)*(B*100.+50.*C)
      CNVR=CONV
      CONV=(B+C)*CONV*(A-2.)
      READ BUCKLINGS.
C
      GO TO (21,21,3,21,21,3,21,21,3,21,21,3),NBR
C
21  READ 523, NSET
      READ 523,(K1(I),I=1,NSET)
523  FORMAT(24I2)
      LIM=NSET-1
      DOL1SIM=1,LIM
      NDL1=N+1
      NDL2=N-3
      NX1=1

```



```

      NX2=N-1
      NX3=NX2
      L=K1(IM)-1
C
      DO 116 IN=1,NX2
      READ 524,(B(L,I),I=NX1,NX3)
      NX1=NX1+NDL1
      NDL1=NDL1+1
      NX3=NX1+NDL2
116  NDL2=NDL2-1
C
      READ 524, B(L,NX1)
524  FORMAT(12F6.1)
C
      SPREAD BUCKLING FROM REFERENCE SLICE TO OTHER SLICES.
      M=N-1
      I1=N
      I2=((N-1)*(3*N-2))/2 + 1
      I5= 0
      N1=N
      N2=2*N-1
      I11=I2+2*N-2
      I13=3*N*N-4*N+2
      I12=I13+N-1
      I2SV=I2
      I11SV=I11
      I12SV=I12
C
      DO 114 I=1,M
      I3=N-I
C
      DO 113 J=1,I3
      I4=I5+J
      B(L,I1)=B(L,I4)
      B(L,I2)=B(L,I4)
      B(L,I11)=B(L,I4)
      B(L,I12)=B(L,I4)
      B(L,I13)=B(L,I4)
      I1=I1+N1+J
      I2=I2-N2+J
      I11=I11+N2 -J
      I12=I12 -J
C
113  I13=I13-N1-J
      I13=I12-N1
      N1=N1+1
      I1=I4+N1
      I2=I2SV+I
      I5=I5+N1
      I11=I11SV-I
114  I12=I12SV-I5
      K2=K1(IM+1)-K1(IM)-1
      IF(K2)115,115,118
C
      TRANSFER BUCKLING FROM REFERENCE PLANE INTO ALL PLANES REQUIRED.
118  L2=K1(IM+1)- 2
      L1=K1(IM)
C
      DO 119 L3=L1,L2
      DO 119 L4= 1,NTOT
119  B(L3,L4)= B(L,L4)
C
115  CONTINUE
C
      GO TO (4,3,3,4,3,3,4,3,3,4,3,3),NBR
C
      MAKE SPECIFIED CHANGES TO BUCKLING MAP.
C
      3 READ 525,((IMP1(I),IMP2(I),IMP3(I),T(I)),I=1,6)
525  FORMAT(6(3I2,F6.1))
C
      DO 5 I=1,6
      IF(IMP1(I))6,5,6
      6 IF(IMP1(I)-2)9,4,4
      9 N1=IMP1(I)-1
      IF(N-IMP2(I))104,105,105
104  N2=3*N*N-N+(5*N-IMP2(I)-2)*(IMP2(I)-N-1)
      N2=N2/2+IMP3(I)
      GO TO 106
105  N2=(IMP2(I)-1)*(2*N+IMP2(I)-2)

```

```

      N2=N2/2+IMP3(I)
106  B(N1,N2)=T(I)
      5 CONTINUE
C
      7 GO TO 3
C
      4 N3=NLVL-1
C
      CALL ITRN(N,N3 ,DEL1,DEL2,NDX1,NDX2,B,O,REN,CONV,GAM,ITER,DELB,
INBR,XTRAP)
C
      FMAX=0.0
C
      DO 15 I=2,N3
C
        DO 14 J=1,N1L
          IF (FMAX-O(I,J))16,14,14
16  FMAX=O(I,J)
14  CONTINUE
15  CONTINUE
C
      FCT=FNRM/FMAX
C
      DO 20 I=2,N3
        DO 19 J=1,N1L
19  O(I,J)=O(I,J)*FCT
20  CONTINUE
C
      CALL PRINT (N,N3,B,O,ITER,DELB,CNVR,GAM,PROB,NDX1,NDX2,NDLT,XTRAP,
INSKP,NDEN)
C
      GO TO 1
C
      END(2,0,0,0,1)

```

```

C      3 DIMENSION HEXLAX      J C ENGLISH      ITERATION SUBROUTINE
C
C      SUBROUTINE ITRN(N,N3 ,DEL1,DEL2,NDX1,NDX2,B,O,REN,CONV,GAM,ITER,
1  DELB,NBR,XTRAP)
C
C      DIMENSION NDX1(33),NDX2(33),B(15,817),O(17,817)
C
C      INITIALIZE ITERATION
C      ITER=0
C      R=1.0
C      A1=DEL2*DEL2
C      A2=1.5*DEL1*DEL1
C      A3=6.*A1+2.*A2
C      A4=A1*A2*.000001
C      DELB=0.0
C
C      GO TO (9,9,9,9,9,9,3,3,3,3,3,3),NBR
C
C      3 A2=A2*.000001
C
C      9 SUM = 0.0
C      SUM1=0.0
C
C      DO 16 I=2,N3
C      N2=N+1
C      KEND=N-1
C
C      DO 12 K=2,KEND
C      N1=NDX1(K)
C      JEND=NDX2(K)
C      DO 11 J=N1,JEND
C      J1=J-N2+1
C      J2=J1-1
C      J3=J+N2
C      J4=J3+1
C
C      GO TO (10,10,10,10,10,10,14,14,14,14,14,14),NBR
C
C      10 DEL=GAM*((A1*(O(I,J-1)+O(I,J1)+O(I,J2)+R*(O(I,J+1)+O(I,J3)+
10  O(I,J4))))+A2*(R*O(I+1,J)+O(I-1,J)))/(A3-A4*(B(I-1,J)+DELB))-R*
20  O(I,J))

```

```

C      GO TO 22
C
C      14 DEL=GAM*((O(I,J-1)+O(I,J1)+O(I,J2)+R*(O(I,J+1)+O(I,J3)+O(I,J4)))/
C      1(6.-A2*(B(I-1,J)+CEL8))-R*O(I,J))
C
C      22 O(I,J)=R*O(I,J)+DEL
C      SUM1=SUM1+ABSF(DEL)
C
C      11 SUM=SUM+O(I,J)
C
C      12 N2=N2+1
C
C      N1=NDX1(N)
C      JEND=NDX2(N)
C
C      DO 13 J=N1,JEND
C      J1=J-N2+1
C      J2=J1-1
C      J3=J+N2-1
C      J4=J3+1
C
C      GO TO (17,17,17,17,17,17,18,18,18,18,18,18),NBR
C
C      17 DEL=GAM*((A1*(O(I,J-1)+O(I,J1)+O(I,J2)+R*(O(I,J+1)+O(I,J3)+
C      1O(I,J4)))+A2*(R*C(I+1,J)+O(I-1,J)))/(A3-A4*(B(I-1,J)+CEL8))-R*
C      2O(I,J))
C
C      GO TO 23
C
C      18 DEL=GAM*((O(I,J-1)+O(I,J1)+O(I,J2)+R*(O(I,J+1)+O(I,J3)+O(I,J4)))/
C      1(6.-A2*(B(I-1,J)+CEL8))-R*O(I,J))
C
C      23 O(I,J)=R*O(I,J)+DEL
C      SUM1=SUM1+ABSF(DEL)
C      13 SUM=SUM+O(I,J)
C
C      K2=N+1
C      KEND=2*N-2
C
C      DO 15 K=K2,KEND
C      N2=N2-1
C      N1=NDX1(K)
C      JEND=NDX2(K)
C
C      DO215 J=N1,JEND
C      J1=J-N2
C      J2=J1-1
C      J3=J+N2-1
C      J4=J3+1
C
C      GO TO (19,19,19,19,19,19,21,21,21,21,21,21),NBR
C
C      19 DEL=GAM*((A1*(O(I,J-1)+O(I,J1)+O(I,J2)+R*(O(I,J+1)+O(I,J3)+
C      1O(I,J4)))+A2*(R*C(I+1,J)+O(I-1,J)))/(A3-A4*(B(I-1,J)+CEL8))-R*
C      2O(I,J))
C
C      GO TO 24
C
C      21 DEL=GAM*((O(I,J-1)+O(I,J1)+O(I,J2)+R*(O(I,J+1)+O(I,J3)+O(I,J4)))/
C      1(6.-A2*(B(I-1,J)+CEL8))-R*O(I,J))
C
C      24 O(I,J)=R*O(I,J)+DEL
C      SUM1=SUM1+ABSF(DEL)
C      215 SUM=SUM+O(I,J)
C
C      15 CONTINUE
C
C      16 CONTINUE
C
C      R=REN/SUM
C      DELB=DELB+XTRAP*(R-1.0)
C      ITER=ITER+1
C
C      IF(SUM1-CONV)20,2C,9
C
C      20 RETURN
C      END(2,0,0,0,0)

```

```

C      3 DIMENSION HEXLAX      J C ENGLISH      PRINT SUBROUTINE
C      SUBROUTINE PRINT(N,N3,B,O,ITER,DELB,CNVR,GAM,PROB,NDX1,NDX2,NDLT,
C      1XTRAP,NSKP,NOEN)
C      DIMENSION NDX1(33),NDX2(33),B(15,817),O(17,817),NDLT(17),NSKP(22),
C      1NOEN(22),FMT(3),NVS1(17),NVS2(17)
C      EQUIVALENCE(X1,M1),(X2,M2)
C      CONVERGED.
C      NOUT=10
C      FMT(1)=746060677360
C      FMT(2)=606060260433
C      FMT(3)=003460606060
C      ZERO=770CC0777777
C      NVS1(1)=NSKP(1)
C      NVS2(1)=NOEN(1)
C      NDL=0
C      DO 20 J=3,N
C      NDL=NDL+2*(NDLT(J-1)-NDLT(J))
C      NN=J+NDL-1
C      NVS1(J-1)=NSKP(NN)
C      NVS2(J-1)=NOEN(NN)
C      DO 31 I=2,N3
C      KEND=N
C      WRITE OUTPUT TAPE NOUT,501,ITER,DELB
501  FORMAT(12H1HEXLAX FLUX /14H NC. OF ITER.= 15 /16H BUCKLING ADDED=
C      1F7.2)
C      DO 23 K=2,KEND
C      N1=NDX1(K)
C      JEND=NDX2(K)
C      M1=NVS1(K-1)
C      M2=NVS2(K-1)
C      FMT(1)=FMT(1)*ZERC
C      FMT(1)=FMT(1)+X1
C      FMT(2)=FMT(2)*ZERC
C      FMT(2)=FMT(2)+X2
23  WRITE OUTPUT TAPE NOUT,FMT,(O(I,J),J=N1,JEND)
C      KEND=N-2
C      DO 22 K=1,KEND
C      J=KEND+K+2
C      N1=NDX1(J)
C      JEND=NDX2(J)
C      NY=N-K-1
C      M1=NVS1(NY)
C      M2=NVS2(NY)
C      FMT(1)=FMT(1)*ZERC
C      FMT(1)=FMT(1)+X1
C      FMT(2)=FMT(2)*ZERC
C      FMT(2)=FMT(2)+X2
22  WRITE OUTPUT TAPE NOUT,FMT,(O(I,J),J=N1,JEND)
C      WRITE OUTPUT TAPE NOUT,519
C      WRITE OUTPUT TAPE NOUT,519
C      WRITE OUTPUT TAPE NOUT,520,CNVR,GAM,XTRAP
31  WRITE OUTPUT TAPE NOUT,521,PRCB,I
C      519 FORMAT(1H0)
C      520 FORMAT(20H0CONVERGENCE CRIT = F5.4 ,4X,16HEXTRAPOLATION = F4.2,4X,
C      113HB SQ. EXTRAP= F5.0)
C      521 FORMAT(1H0,20X,12HPROBLEM NC. A6 , 10X,10HLEVEL NO. I2 )
C      701 FORMAT(13H1HEXLAX B SQD/14H NC. OF ITER.= 15 /16H BUCKLING ADDED=
C      1F7.2)
C      DO 26 I=2,N3
C      KEND=N
C      WRITE OUTPUT TAPE NOUT,701,ITER,DELB

```

```

C      DO 27 K=2,KEND
      N1=NDX1(K)
      JEND=NDX2(K)
      M1=NVS1(K-1)
      M2=NVS2(K-1)
C      FMT(1)=FMT(1)*ZERC
      FMT(1)=FMT(1)+X1
C      FMT(2)=FMT(2)*ZERC
      FMT(2)=FMT(2)+X2
C      27 WRITE OUTPUT TAPE NOUT,FMT,(B(I-1,J),J=N1,JEND)
C      KEND=N-2
C      DO 29 K=1,KEND
      J=KEND+K+2
      N1=NDX1(J)
      JEND=NDX2(J)
      NY=N-K-1
      M1=NVS1(NY)
      M2=NVS2(NY)
C      FMT(1)=FMT(1)*ZERC
      FMT(1)=FMT(1)+X1
C      FMT(2)=FMT(2)*ZERC
      FMT(2)=FMT(2)+X2
C      29 WRITE OUTPUT TAPE NOUT,FMT,(B(I-1,J),J=N1,JEND)
C      WRITE OUTPUT TAPE NOUT,519
      WRITE OUTPUT TAPE NOUT,519
      WRITE OUTPUT TAPE NOUT,520,CNVR,GAM,XTRAP
C      26 WRITE OUTPUT TAPE NOUT,521, PROB,I
C      RETURN
      END(2,0,0,0,0)

```