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MACHINE COMPUTATION OF BESSEL FUNCTIONS

by

J. C. English
Theoretical Physics Division

November 1954

E. I. du Pont de Nemours & Co.
Explosives Department — Atomic Energy Division
Technical Division — Savannah River Laboratory

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ABSTRACT

Details are given for wiring the electronic calculator panel of the IBM Card Programmed Calculator to yield the Bessel functions $I_n(x)$ and $K_n(x)$ for n = 0, 1, 2, and 3.

TABLE OF CONTENTS

MACHINE COMPUTATION OF BESSEL FUNCTIONS

INTRODUCTION

In reactor physics calculations that employ the Bessel functions $I_n(x)$ and $K_n(x)$, it is convenient to approximate the values of these functions by expanding them in series on the IBM Card Programmed Calculator (CPC). Considerable time can be saved if a special panel is wired for the electronic calculator of the CPC to perform the series expansions as well as the additional operations that are required to complete the computations.

The present report describes a panel of this type that was developed at the Savannah River Laboratory.

SUMMARY

A panel was designed to compute the Bessel functions $I_n(x)$ and $K_n(x)$ for n = 0, 1, 2, and 3. These functions, plus certain auxiliary operations, are required to compute neutron flux distributions by the P_3 approximation to transport theory.

The panel, designed for the electronic calculator of the IBM Card Programmed Calculator, Model II, is used in conjunction with the Heising general purpose accounting machine panel. The latter was designed by William Heising of IBM's Washington Technical Computing Bureau.

After the Bessel functions are obtained they can be used without changing the 605 panel, provided additional functions are available from the 605 panel. These additional functions are listed below. The first three functions and the square root of A are calculated in floating decimal form. A, B, and D are the three factors used by the calculator. These factors are available from a card or from storage.

$A + D$

$A \times B + D$

$A \div B + D$

$A + D$ (No left shift)

$A \times B + D$ (No left shift)

$A \div B + D$ (No left shift)

$\sqrt{A}$

$\frac{1}{2} \ln A$

"No left shift" means that the CPC does not shift the sum to the left after addition but maintains the sum in fixed decimal form. The "Left shift" operation drops any zeros appearing in the positions of the most significant digits.

DISCUSSION

The neutron flux in a heterogeneous cylindrical cell of concentric regions is calculable by means of the P_3 spherical harmonic approximation to the transport equation[2]. A system of linear equations results from the application of the boundary conditions. This system may be solved by matrix methods; the matrix elements can be computed by means of the special 605 panel. A sample matrix element is

$\frac{k}{\Sigma_t}\left(\frac{3\Sigma_t\Sigma_a}{k^2} - 1\right) I_3\,(kr)$ where Σ_t, Σ_a, and r are given numbers and k is calculated from cell parameters.

The object of developing a special 605 board to yield $I_n(x)$ and $K_n(x)$ was to have these functions available from the CPC and to be able to use them without changing the panel in any calculations normally encountered. These Bessel functions were approximated by calculating their series expansion. This board can compute Bessel functions for arguments less than 3.5. Twelve terms of the series expansion give sufficient accuracy for Savannah River Laboratory calculations.

It is known that

$$I_n(x) = \frac{x^n}{2^n n!}\left[1 + \frac{x^2}{2^2 1!(n+1)} + \frac{x^4}{2^4 2!(n+1)(n+2)} + \frac{x^6}{2^6 3!(n+1)(n+2)(n+3)} + \ldots\right]$$

$$K_n(x) = (-1)^{n+1}\left(\gamma + \ln\frac{x}{2}\right)I_n(x) + \frac{1}{2}\sum_{r=0}^{n-1}\frac{(-1)^r(n-r-1)!}{r!}\left(\frac{x}{2}\right)^{-n+2r} + (-1)^n\frac{1}{2}\sum_{r=0}^{\infty}\frac{1}{r!(n+r)!}\left(\frac{x}{2}\right)^{n+2r}\left[1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{r} + 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n+r}\right]$$

where γ is Euler's constant.

The board was designed to give the functions

$$\phi_n(x) \equiv n!\left(\frac{2}{x}\right)^n I_n(x) = 1 + \frac{x^2}{2^2 1!(n+1)} + \frac{x^4}{2^4 2!(n+1)(n+2)}$$

$$+ \frac{x^6}{2^6 3!(n+1)(n+2)(n+3)} + \dots ;$$

$$\Psi_n(x) \equiv 1 + \sum_{r=1}^{\infty} \frac{1}{r!(n+r)!} \left(\frac{x}{2}\right)^{2r} \left[1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r}\right] ;$$

and

$$\eta_n(x) \equiv \sum_{r=0}^{\infty} \frac{1}{r!(n+r)!} \left(\frac{x}{2}\right)^{2r} \left[1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n+r}\right] .$$

The latter two functions lead to $K_n(x)$ by means of the relation given on Page 11. $I_0(x)$ is the only Bessel function obtained directly; $I_n(x)$ for $n \neq 0$ requires additional card programming.

To obtain $K_n(x)$, $I_n(x)$ must be calculated or be available from storage. Likewise the board must yield ln x.

The 605 panel was designed to compute the following functions. The chain used, the 605 calculate selectors transferred, and the operation code for Columns 9 and 10 for each function are also listed.

CHAIN A

	Function	Column 9 Code	Calculate Selectors Transferred
1.	A + D	-	----
2.	A x B + D	1	1,9
3.	A ÷ B + D	2	2,3
4.	A + D (No left shift)	3	11
5.	A x B + D (No left shift)	1 3	1,9,11
6.	A ÷ B + D (No left shift)	2 3	2,3,11

CHAIN B

	Function	Columns 9 - 10 Code	Calculate Selectors Transferred
7.	Ψ_0 (x)	40	1,4,9
8.	Ψ_1 (x)	41	4,12

CHAIN B (Continued)

	Function	Columns 9 - 10 Code	Calculate Selectors Transferred
9.	$\Psi_2(x)$	42	4,13
10.	$\Psi_3(x)$	43	4,14,17
11.	$\eta_1(x)$	71	6,7,12
12.	$\eta_2(x)$	72	6,7,13
13.	$\eta_3(x)$	73	6,7,14,17

CHAIN C

	Function	Columns 9 - 10 Code	Calculate Selectors Transferred
14.	$\sqrt{A}$	4 5	4,5
15.	$\frac{1}{2}\ln A$	86	6,7,8,15
16.	$I_o(x)$	00	1,9,16
17.	$\frac{2}{x} I_1(x)$	01	12,16
18.	$2\left(\frac{2}{x}\right)^2 I_2(x)$	02	13,16
19.	$6\left(\frac{2}{x}\right)^3 I_3(x)$	03	14,16,17

$K_n(x)$ can be expressed in terms of functions defined above:

$$K_o(x) = -\left[\gamma + \ln\frac{x}{2}\right] I_o(x) + \left[\Psi_o(x) - 1\right]$$

$$K_1(x) = \left[\gamma + \ln\frac{x}{2}\right] I_1(x) + \frac{1}{2}\left(\frac{x}{2}\right)^{-1} - \frac{1}{2}\left(\frac{x}{2}\right)\left[\Psi_1(x) - 1 + \eta_1(x)\right]$$

$$K_2(x) = -\left[\gamma + \ln\frac{x}{2}\right] I_2(x) + \frac{1}{2}\left[-1 + \left(\frac{x}{2}\right)^{-2}\right] + \frac{1}{2}\left(\frac{x}{2}\right)^2 \left[\frac{1}{2}\left(\Psi_2(x) - 1\right) + \frac{1.5}{2}\eta_2(x)\right]$$

$$K_3(x) = \left[\gamma + \ln\frac{x}{2}\right] I_3(x) + \frac{1}{2}\left[\frac{1}{2}\left(\frac{x}{2}\right) - \left(\frac{x}{2}\right)^{-1} + 2\left(\frac{x}{2}\right)^{-3}\right]$$

$$-\frac{1}{2}\left(\frac{x}{2}\right)^3 \left[\frac{1}{6}\left(\Psi_3(x)-1\right) + \frac{1.833333}{6}\eta_3(x)\right]$$

$K_n(x)$ is obtained by card programming the above equations.

Card columns 11-19, 21-29, and 31-39 are reserved for factors A, B, and D, respectively, for the Heising general purpose boards.

Functions 1-6 are coded in exactly the same manner as they would be if the Heising general purpose boards were used.

Functions 7-13, and 16-19 require that factor A be $\frac{x}{2}$ in fixed decimal form with Heising exponent 50. Here x is the argument of the Bessel function.

Functions 7-13 and 15-19 require that factor B be 1000000 with arbitrary exponent.

Operations 7-10 require that factor D be xxxxxxxNN where xxxxxxx* $= \sum_{k=1}^{NN} \frac{1}{k}$ and NN is the number of terms to be kept in the approximation to the infinite series. Operations 11-13 require that factor D be of the same form as for operations 7-10 but here xxxxxxx $= \sum_{k=1}^{NN+n} \frac{1}{k}$, where n is the order of the particular Bessel function.

Factor A for operation 14 can be any non-negative number in floating decimal form but factor D must be 9999999 --. Factor B is arbitrary.

To obtain $\frac{1}{2}\ln x$, form $\frac{x-1}{x+1}$ and place in fixed decimal form (with Heising exponent of 50); this is factor A. Factor D must be NN- - - - - - - X where NN is odd and the power of the last term to be retained in the series expansion.

For operations 16-19, factor D must be xxxxxxxNN where xxxxxxx is arbitrary and NN is even and one less than the number of terms to be retained in the series for $I_n(x)$.

* xxxxxxx represents the seven digit value of the sum indicated. The decimal point follows the first digit.

In the wiring scheme for the 605 panel shown later the following abbreviations are used:

P.S.	Program step
R.	Reset counter
R.O.	Read out of counter
R.I.	Read into counter
R.R.	Read out and reset counter
F12	Read out of factor storages 1 and 2 coupled, when in read out column.
	Read into factor storages 1 and 2 when in read in column.
D.E.	Digit emit
Z.T.	Zero test
B.T.	Balance test
GSIPU	Group Suppress 1 pick-up
GSIDO	Group Suppress 1 drop out
+	Read into counter plus
-	Read into counter minus

The wiring details for the 605 panel are given below:

CHAIN A

P.S.	Suppression	R.O.	R.I.		
1.					
2.					
3.	NT if 1N and 3N	M.Q.	+*		*- if 2 T
4.	NT if 1N and 3N	D.E.5	+*	in 2	*- if 1 T
5.	NT if 1N and 3N	G1	+		
6.	NT if 1N and 3N	R.R.	G1		
7.	NT if 1N and 3N	F12	+	*	*in 4 if 2 T

CHAIN A (Continued)

P.S.	Suppression	R̂.O.	R.I.		
8.	NT if 1N and 3N	R.O.*	(F12-out 6)*		*8-1 F34 if 2T *8-2 Divide if 2T
9.	NT if 1N and 3N	R.R.	M.Q.*		*F12 if 2T
10.	NT if 1N and 3N	F34*	X*	*	*10-1 F12 if 3T *10-2 + if 3T *10-3 in 6 if 3T
11.	NT if 1N and 3N	F12*	M.Q.*		*11-1 M.Q. if 3T *11-2 F12 if 2T
12.	NT if 1N and 3N	R.O.*	(F12-out 6)*	R	*12-1 Split wire to 8-1 *12-2 Split wire to 8-2
13.	NT if 1N and 3N	F34*	X*	*	*13-1 Split wire to 10-1 *13-2 Split wire to 10-2 *13-3 Split wire to 10-3
14.	NT if 1N and 3N	F12*	+		*Split wire to 11-1
15.	NT if 1N and 3N	R.O.	F34	out 3	
16.	NT if 1N and 3N	R.O.	F12	out 2	
17.	None	D.E.1	M.Q.	in 5	
18.	None	M.Q.-Z.T.	R.	in 5	
19.	NT if 1N and 3N S on Z	D.E.1	+*		*Split wire to 3-2
20.	NT if 1N and 3N S on Z if 1T S on NZ if 1N	F34	F12		
21.	None	G1	+		
22.	None	R.O.	G1		

CHAIN A (Continued)

P.S.	Suppression	R.O.	R.I.	
23.	None	G2	-	B.T.
24.	S on +	G2	G1	
25.	S on +	R.R.	G2	
26.	S on +	G2	-	
27.	None	D.E.4	ZT	
28.	S on NZ	D.E.4	+	GS1PU
29.	None	D.E.2	ZT	
30.	S on NZ	D.E.2	+	GS2PU
31.	None	D.E.1	ZT	R
32.	S on -	G34	+	
33.	S on +	F12	+	
34.	GS1	R.R.	F34	out 5
35.	GS1	F34	+	
36.	GS2	R.R.	F34	out 3
37.	GS2	F34	+	
38.	S on Z	R.R.	F34	out 2
39.	S on Z	F34	+	
40.	S on -	F12	+	
41.	S on +	G34	+	
42.	None	R.O.	F12	GS1PU
43.	None	F12	F34	in 5
44.	None	M.Q.- Z.T.	R.	GS2PU
45.	S on NZ if 11N S on NT if 11T	F34	F12	GS1DO
46.	None	F12	+	

CHAIN A (Continued)

P.S.	Suppression	R.O.	R.I.	
47.	None	F12	F34	in 3
48.	None	M.Q.-Z.T.	R.	in 3
49.	S on NZ if 11N S on NT if 11T	F34	F12	GS2DO
50.	None	F12	+	
51.	None	R.O.	F34	out 2
52.	None	M.Q.-Z.T.	R.	in 4
53.	S on Z	F34	F12	
54.	None	G1	+	
55.	GS1	D.E.4	-	
56.	GS2	D.E.2	-	
57.	S on Z	D.E.1	+	
58.	None	R.R.	G1	
59.	None	F12	+	in 4
60.	None	G1	$\frac{1}{2}$ adjust	

CHAIN B

P.S.	Suppression	R.O.	R.I.	
1.	GS4	G2	-	
2.	GS4	R.R.	G1	
3.	GS4	F12	+	
4.	GS4	R.R.	G2	out 3
5.	None	G2	M.Q.	GS4PU
6.	None	F34	X	
7.	None	R.R.	F34	out 5

CHAIN B (Continued)

P.S.	Suppression	R.O.	R.I.	
8.	None	G1	-	in 3
9.	GS1	D.E.n	+	in 3 GS2PU
10.	None	R.R.	F12	
11.	None	F34	+	
12.	None	F12	Divide	
13.	None	R.R.	F34	
14.	None	F34	+	in 4
15.	None	M.Q.	F34	in 4
16.	None	F12	Divide	R
17.	None	M.Q.	+	
18.	None	F34	+	
19.	GS1	R.R.	F34	out 4
20.	GS1	G34	X	
21.	GS1	F34	M.Q.	
22.	GS1	R.R.	F34	out 4
23.	GS1	F34	+	
24.	GS1	G34	X	
25.	GS2	G34	Divide	
26.	GS2	R.R.	F12	
27.	GS2	F12	+	in 6
28.	GS2	M.Q.	F12	in 6
29.	GS2	G34	Divide	R.
30.	GS2	M.Q.	+	
31.	GS2	F12	+	
32.	GS2	D.E.1	M.Q.	in 5

CHAIN B (Continued)

P.S.	Suppression	R.O.	R.I.	
33.	GS2	M.Q.	+	in 3
34.	GS1	R.R.	F34	out 5
35.	GS2	R.R.	F34	
36.	SNT if 7N GS1	D.E.n	-	
37.	GS1	G1	+	
38.	GS1	R.R.	G1	
39.	GS1	D.E.1	+	in 2
40.	GS1	G1	Divide	
41.	GS1	R.R.	F12	
42.	GS1	F12	+	in 6
43.	GS1	M.Q.	F12	in 6
44.	GS1	G1	Divide	R.
45.	GS1	M.Q.	+	
46.	GS1	F12	+	
47.	GS1	G34	+	
48.	GS1	R.R.	G34	
49.	None	G1	+	
50.	SNT if 7N GS1	D.E.n	+	
51.	GS2	D.E.1	+	
52.N*	None	R.O.	G1	
52.T	None	F34	+	
53.N	GS1	D.E.1	+	B.T. R.
53.T	None	R.O.	F12	out 2
54.N	None		GS1PU	

CHAIN B (Continued)

P.S.	Suppression	R.O.	R.I.	
54.T	None	D.E.1	M.Q.	in 5
55.N	GS2	GS1DO		
55.T	None	M.Q.	Z.T.	in 4 R.
56.N	S on -	D.E.1	M.Q.	in 4
56.T	S on NZ	F34	+	in 4
57.N	S on -	M.Q.	G34	in 4
57.T	S on Z	F12	+	in 4
58.N	None			
58.T	S on Z	D.E.1	$\frac{1}{2}$ adjust	
59.N	S on -	Prog. P.U.	Repeat Delay	P.U. Repeat Sels. 5,7,8,9,10
59.T	None	D.E.5	$\frac{1}{2}$ adjust	in 2
60.	None	Program Repeat		GS2DO

CHAIN C

P.S.	Suppression	R.O.	R.I.	
1.	GS4 NT if 16N	G2	-	
2.	GS4 NT if 16N	R.R.	G2	
3.	GS4 NT if 16N	F12	M.Q.	
4.	GS4 NT if 16N	F12	X	
5.	GS4 NT if 16N	R.R.	G34	out 6

* 52N means that this program step operates when the program repeat selectors are normal.

CHAIN C (Continued)

P.S.	Suppression	R.O.	R.I.		
6.	GS4 NT if 16N	F12	+		
7.	GS4 NT if 16N	R.R.	M.Q.	out 6	
8.	GS4 NT if 16N	F12	X		
9.	GS4 NT if 16N	G34	+		
10.	GS4 NT if 16N	R.R.	F12	out 2	
11.	NT if 4N	G1	+		
12.	NT if 4N	D.E.5	+	in 2	
13.	NT if 4N	D.E.2	Divide		
14.	NT if 4N	M.Q.	G2		
15.	NT if 4N	D.E.1	Z.T.	R.	
16.	NT if 4T	G2*	+	GS4PU	*G34 if 15T
17.	NT if 4T	D.E.n*	-*	*	*17-1 D.E.2 if 15T *17-2 + if 15T *17-3 GS2PU
18.	NT if 4T	R.R.	M.Q.	*	*out 6 if 15T
19.	NT if 4T	G-2*	X*		*19-1 F34 if 15T *19-2 X- if 15T
20.	NT if 4T	R.R.	G34*		*F34 if 15T
21.	NT if 16N	G34	+	in 5	
22.	NT if 16N	R.R.	G34		
23.	NT if 4N NZ if 4T	F12	+	in 2	

CHAIN C (Continued)

P.S.	Suppression	R.O.	R.I.		
24.	GS1 if 15T Z if 4T	F12 if 4T F34 if 15T F34 if 16T	+	in 3 if 4T in 2 if 15T	
25.	GS1 if 15T	G34	Divide		
26.	GS1 if 15T	R.R.	F34		
27.	GS1 if 15T	F34	+	in 6	
28.	GS1 if 15T	M.Q.	F34	*	*in 3 if 4T
29.	GS1 if 15T	G34	Divide	Reset	
30.	GS1 if 15T	M.Q.	+	*	*in 3 if 4T
31.	GS1 if 15T	F34	+	in 6	
32.	NT if 4N	D.E.5	M.Q.	in 3	
33.	NT if 4N	G34	-	in 3	
34.	NT if 4N	R.R.	F34	out 3	
35.	NT if 4N	F34	X	B.T.	
36.	NT if 4N	G34	+	in 4	
37.	NT if 4N	G2	$\frac{1}{2}$ adjust		
38.	NT if 15N GS2 if 15T	F34	+		
39.	NT if 4T	R.O.	M.Q.		
40.	NT if 4T	R.R.	F34	out 6	
41.	NT if 4T	D.E.1	+	*	*in 6 if 15T
42.	NT if 4T	G2*	+	B.T.	*G34 if 15T
43.	NT if 4T	R.R.	G2*		*G34 if 15T
44.	NT if 4T	F12	X		
45.	NT if 4T	F34	M.Q.		
46.	NT if 4T	R.R.	F34	out 6	
47.	NT if 4T	F34	+		

CHAIN C (Continued)

P.S.	Suppression	R.O.	R.I.		
48.	NT if 4T	F12	X		GSIPU
49.	NT if 15N GS2 if 15T	R.R.	F34		
50.	NT if 15N GS2 if 15T	F34	-	in 2	
51.	NT if 4T	D.E.1	M.Q.	in 5	
52.	NT if 4T GS2 if 15T	M.Q.	+	in 5	GS1DO
53.	NT if 4T S on +	R.O.	F34	out 3	
54.	NT if 4T S on -	R.O.	G34	out 4	
55.	NT if 4T	M.Q.	Z.T.	in 6	R
56.	NT if 4T S on NZ	F34	+	in 4	
57.	NT if 4T S on Z	G34	+	in 4	
58.	NT if 4T S on Z	D.E.1	+		
59.	NT if 4T	D.E.5	+	in 2	
60.	S on +	R.R.	*	out 4	*G34 if 4T
		Program Repeat		GS2DO	

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