UNCLASSIFIED

DP - 159

Physics

AEC Research and Development Report

IBM SUB-ROUTINES II

IRREGULAR BESSEL FUNCTIONS

by

J. C. English

Theoretical Physics Division

May 1956

E. I. du Pont de Nemours & Co.
Explosives Department - Atomic Energy Division
Technical Division - Savannah River Laboratory

Printed in USA. Price $0.25

External Copies should be OTS

Available from the U. S. Atomic Energy Commission,
Technical Information Extension,
P. O. Box 1001, Oak Ridge, Tennessee.

Please direct to the same address inquiries covering the procurement of other classified AEC reports.

253649 ✓

UNCLASSIFIED

DP - 159

PHYSICS

IBM SUB-ROUTINES II

Irregular Bessel Functions

by

Julius C. English
Theoretical Physics Division

May 1956

E. I. du Pont de Nemours & Co.
Explosives Department - Atomic Energy Division
Technical Division - Savannah River Laboratory

Printed for
The United States Atomic Energy Commission
Contract AT(07-2)-1

ABSTRACT

A sub-routine is given for the IBM 650 to calculate the following functions: ln x, $Y_n(x)$ and $K_n(x)$ for n = 0, 1, 2, and 3. The circular and hyperbolic sines and cosines, and the Bessel functions $J_n(x)$ and $I_n(x)$ for n = 0, 1, 2, and 3 are also available from this routine.

External Distribution according to
TID-4500 (11th Ed.)

INTERNAL DISTRIBUTION

No. of Copies		
(5)	AEC, SROO	Aiken, S. C.
(1)	R. M. Evans - B. H. Mackey	Wilmington AED
(1)	J. E. Cole - M. H. Smith - J. B. Tinker	"
(1)	H. Worthington	"
(1)	"W" File	"
(1)	S. I. Winde	Engineering Department
(1)	J. A. Monier	Savannah River Plant
(1)	W. P. Overbeck	"
(1)	PRD File	"
(1)	M. H. Wahl - C. W. J. Wende - W. M. Heston	Savannah River Laboratory
(1)	G. Dessauer	"
(1)	R. R. Haefner	"
(1)	J. L. Crandall	"
(1)	E. J. Hennelly	"
(1)	L. M. Wheatley	"
(1)	J. C. English	"
(15)	TIS File	"
(1)	TIS File Record Copy	"

TABLE OF CONTENTS

IBM SUB-ROUTINES II

Irregular Bessel Functions

INTRODUCTION

The Bessel functions $Y_n(x)$ and $K_n(x)$ are frequently needed in the calculation of physical problems. If the range of the argument is too large, it is less convenient to use the table look-up feature of the IBM Magnetic Drum Calculator (650) than to calculate these functions by means of their series expansions. In the calculation of these Bessel functions the natural logarithm of x is needed. This report describes the programming to obtain ln x, $Y_n(x)$, and $K_n(x)$.

SUMMARY

A program was written for the IBM 650 to compute ln x, and the irregular Bessel functions $Y_n(x)$ and $K_n(x)$ for n = 0, 1, 2, and 3. The argument for these functions is loaded into storage in fixed decimal form but the function is available in fixed or floating decimal form. This program utilizes a sub-routine devised by W. V. Baxter[1] to compute the regular Bessel functions $J_n(x)$ and $I_n(x)$. These functions are independently available from the program. Circular and hyperbolic sines and cosines are also available from the present sub-routine.[1] The routine uses 449 storage locations.

DISCUSSION

The following series expansions for $Y_n(x)$ and $K_n(x)$ were used:[2]

$$\pi Y_n(x) = 2\left(\gamma + \ln \frac{x}{2}\right) J_n(x) - \sum_{r=0}^{n-1} \frac{(n-r-1)!}{r!} \left(\frac{x}{2}\right)^{-n+2r}$$

$$- \sum_{r=0}^{\infty} \frac{(-1)^r}{r!(n+r)!} \left(\frac{x}{2}\right)^{n+2r} \left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{r} + \frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{n+r}\right]$$

$$K_n(x) = (-1)^{n+1} \left(\gamma + \ln \frac{x}{2}\right) I_n(x) + \frac{1}{2} \sum_{r=0}^{n-1} \frac{(-1)^r (n-r-1)!}{r!} \left(\frac{x}{2}\right)^{-n+2r}$$

$$+ (-1)^n \frac{1}{2} \sum_{r=0}^{\infty} \frac{1}{r!\,(n+r)!} \left(\frac{x}{2}\right)^{n+2r} \left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{r} + \frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{n+r}\right]$$

where γ is Euler's constant. A routine was developed to calculate $Y_n(x)$ and $K_n(x)$ from these expansions. Whenever any term of the infinite sum becomes less than 10^{-8} the subsequent terms are neglected.

The series expansions for $Y_n(x)$ and $K_n(x)$ require the natural logarithm of $\frac{x}{2}$, $J_n(x)$, and $I_n(x)$. The natural logarithm of x was calculated from the following expansion:

$\ln x = 2\left(U + \frac{1}{3}U^3 + \frac{1}{5}U^5 + \ldots\right)$ where $U = \frac{x-1}{x+1}$. This routine is valid for $0.0086 \leq x < 100$. $J_n(x)$ and $I_n(x)$ are calculated by a sub-routine (hereafter called WB-1) developed at the Savannah River Laboratory.[1] The functions calculated by WB-1 are also available with the present routine.

The routine to obtain $Y_n(x)$ and $K_n(x)$ contains 449 orders and normally occupies storages 0000 through 0448. It may be translated m positions to locations m through (0448 + m). To use the routine:

1. The argument of the function must be in the form xx.xxxxxxxx in storage (0004 + m).

2. A value of n must be in the form 000000000n in storage (0033 + m).

3. An exit instruction must be placed in storage 1971.

4. The fixed decimal answer will be found in the lower accumulator in the form xx.xxxxxxxx and the floating decimal answer in the upper accumulator in the form YYx.xxxxxxx.

5. The following functions may be calculated by means of this sub-routine.

Function	n	Point of entry	Range of argument
sin x	1	0001	$\|x\| < 100$
cos x	-1	0001	$\|x\| < 100$
sinh x	1	0011	$\|x\| < 5.29$
cosh x	-1	0011	$\|x\| < 5.29$
$I_0(x)$	0	0031	$\|x\| < 6.32$
$I_1(x)$	1	0031	$\|x\| < 6.52$
$I_2(x)$	2	0031	$\|x\| < 6.77$
$I_3(x)$	3	0031	$\|x\| < 7.15$
$J_0(x)$	0	0101	$\|x\| < 7.82$
$J_1(x)$	1	0101	$\|x\| < 9.62$
$J_2(x)$	2	0101	$\|x\| < 8.94$
$J_3(x)$	3	0101	$\|x\| < 7.82$

(Continued on following page)

Function	n	Point of entry	Range of argument
ln x	...	0446	$0.0086 \leq x < 100$
$Y_0(x)$	0	0420	$0.021 \leq x \leq 6.30$
$Y_1(x)$	1	0420	$0.021 \leq x \leq 6.46$
$Y_2(x)$	2	0420	$0.21 \leq x \leq 6.64$
$Y_3(x)$	3	0420	$0.55 \leq x \leq 6.98$
$K_0(x)$	0	0214	$0.021 \leq x \leq 5.20$
$K_1(x)$	1	0214	$0.021 \leq x \leq 5.30$
$K_2(x)$	2	0214	$0.21 \leq x \leq 5.57$
$K_3(x)$	3	0214	$0.55 \leq x \leq 5.98$

The argument in 0004 and the value of n remain unchanged by the sub-routine.

The programs are listed in the order in which they are performed. They are also listed in the order in which they are stored for convenience in punching. The routine WB-1 described in DP-124[1] has been altered for use in this program. These revised orders are listed in storage sequence.

J. C. English

J. C. English
Theoretical Physics Division

BIBLIOGRAPHY

1. Baxter, W. V. E. I. du Pont de Nemours & Co. DP-124, IBM Sub-Routine I, July 1955.

2. British Association Mathematical Tables, Volume VI, Bessel Functions, Part I. pp. 174 and 264. Cambridge: University Press (1950).

IBM PROGRAMS FOR KEY-PUNCHING

PROBLEM: WB-1 Sub-Routine in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0000	16	0093	0047	
0001	67	0004	0009	
0002	69	0005	0008	
0003	69	0108	0061	
0004	00	0000	0000	
0005	24	0069	0122	
0006	00	0000	0002	
0007	24	0013	0066	
0008	24	0106	0003	
0009	16	0012	0017	
0010	31	0001	0118	
0011	69	0014	0018	
0012	06	2831	8531	
0013	00	0000	0000	
0014	65	8003	0034	
0015	31	0001	0040	
0016	69	0004	0007	
0017	46	0020	0009	
0018	24	0069	0016	
0019	65	0013	0118	
0020	15	0012	0067	
0021	65	0013	0068	
0022	24	0021	0002	
0023	00	0000	0000	
0024	64	0006	0010	
0025	30	0007	0138	
0026	60	8001	0082	
0027	67	0039	0044	
0028	67	0039	0043	
0029	19	0060	0092	
0030	65	0033	0037	
0031	69	0014	0107	
0032	31	0001	0137	
0033	00	0000	0000	
0034	20	0039	0100	
0035	20	0139	0143	
0036	21	0280	0053	
0037	46	0049	0050	
0038	16	0041	0145	
0039	00	0000	0000	
0040	20	0045	0048	
0041	00	0000	0003	
0042	60	8002	0115	
0043	35	0001	0099	
0044	35	0001	0051	
0045	00	0000	0000	
0046	60	0077	0064	
0047	45	0027	0028	
0048	65	0033	0038	
0049	69	0052	0055	

PROBLEM: WB-1 Sub-Routine in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0050	69	0013	0105	
0051	64	0006	0015	
0052	01	0000	0000	
0053	69	0586	0585	
0054	20	0120	0129	
0055	24	0058	0021	
0056	65	0023	0137	
0057	24	0060	0063	
0058	00	0000	0000	
0059	20	0013	0066	
0060	00	0000	0000	
0061	24	0119	0030	
0062	31	0008	0083	
0063	24	0116	0119	
0064	10	0120	0065	
0065	21	0120	0074	
0066	69	0019	0022	
0067	10	0004	0076	
0068	35	0001	0024	
0069	66	8003	0034	
0070	15	0060	0125	
0071	24	0119	0123	
0072	65	0033	0087	
0073	69	0080	0084	
0074	60	0142	0096	
0075	65	0013	0068	
0076	46	0088	0059	
0077	00	0000	0000	
0078	24	0021	0073	
0079	00	0000	0000	
0080	24	0069	0072	
0081	19	0116	0062	
0082	19	8001	0102	
0083	45	0086	0091	
0084	24	0106	0111	
0085	21	0142	0140	
0086	20	0116	0070	
0087	45	0090	0095	
0088	11	0004	0109	
0089	24	0142	0046	
0090	16	0093	0097	
0091	60	0058	0029	
0092	31	0008	0113	
0093	00	0000	0001	
0094	60	8002	0081	
0095	65	0052	0137	
0096	10	0077	0085	
0097	45	0000	0056	
0098	01	0000	8000	
0099	64	0006	0032	

PROBLEM: WB-1 Sub-Routine in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0100	69	0103	0106	
0101	69	0004	0104	
0102	35	0002	0069	
0103	66	8003	0034	
0104	24	0013	0021	
0105	24	0058	0021	
0106	24	0069	0072	
0107	24	0069	0101	
0108	60	0006	0131	
0109	66	8002	0059	
0110	64	0139	0094	
0111	69	0052	0057	
0112	60	0023	0127	
0113	69	0117	0071	
0114	00	0000	0051	
0115	15	0079	1971	
0116	00	0000	0000	
0117	60	0093	0131	
0118	20	0023	0026	
0119	60	0093	0131	
0120	00	0000	0000	
0121	47	0126	0126	
0122	69	0075	0078	
0123	20	0079	0132	
0124	15	0130	0134	
0125	20	0060	0046	
0126	21	0130	0144	
0127	19	0045	0025	
0128	31	0001	0137	
0129	10	0033	0089	
0130	00	0000	0000	
0131	24	0077	0054	
0132	60	8002	0141	
0133	61	8002	0136	
0134	31	0002	0042	
0135	11	0114	0124	
0136	10	0114	0124	
0137	20	0058	0111	
0138	64	0041	0128	
0139	00	0000	0000	
0140	19	0120	0035	
0141	36	0000	0121	
0142	00	0000	0000	
0143	65	0039	0110	
0144	46	0146	0133	
0145	45	0098	0112	
0146	61	8002	0135	
0147	69	0213	0445	
0148	60	0151	0155	
0149	10	0169	0183	

PROBLEM: Sub-Routine for Irregular Bessel Functions in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0042	60	8002	0115	
0098	01	0000	3000	
0115	15	0079	1971	
0121	47	0126	0126	
0123	20	0079	0132	
0124	14	0130	0134	
0126	21	0130	0144	
0132	60	8002	0141	
0133	61	8002	0136	
0134	31	0002	0042	
0135	11	0114	0124	
0136	10	0114	0124	
0141	36	0000	0121	
0144	46	0146	0133	
0146	61	8002	0135	
0147	69	0213	0445	
0148	60	0151	0155	
0149	10	0169	0183	
0150	11	0156	0184	
0151	00	0000	0000	
0152	60	0175	0161	
0153	00	0000	0000	Not Used
0154	20	0175	0185	
0155	11	0158	0163	
0156	02	0000	0000	
0157	46	0162	0165	
0158	10	0000	0000	
0159	35	0001	0149	
0160	00	0000	0000	
0161	19	8001	0190	
0162	67	8003	0174	
0163	46	0166	0167	
0164	60	0151	0181	
0165	61	0170	0177	
0166	60	0212	0173	
0167	69	0170	0176	
0168	60	0151	0159	
0169	01	0000	0000	
0170	23	0258	5093	
0171	00	0000	0000	
0172	24	0189	0186	
0173	11	0151	0157	
0174	21	0160	0192	
0175	00	0000	0000	
0176	24	0160	0164	
0177	21	0160	0168	
0178	60	0175	0179	
0179	19	0182	0205	
0180	19	0189	0208	
0181	30	0001	0149	

PROBLEM: Sub-Routine for Irregular Bessel Functions in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0182	00	0000	0000	
0183	21	0204	0150	
0184	14	0204	0154	
0185	30	0001	0200	
0186	60	0189	0193	
0187	30	0000	0194	
0188	30	0001	0195	
0189	01	0000	0000	
0190	21	0182	0186	
0191	00	0000	0000	
0192	60	0151	0149	
0193	10	0156	0211	
0194	60	8002	0203	
0195	10	0191	0196	
0196	21	0191	0197	
0197	69	0171	0172	
0198	65	0191	0199	
0199	15	8001	0201	
0200	20	0191	0152	
0201	15	0160	0202	
0202	69	0169	0206	
0203	44	0207	0198	
0204	00	0000	0000	
0205	60	8003	0180	
0206	24	0189	0209	
0207	60	0175	0188	
0208	14	0171	0210	
0209	30	0001	0123	
0210	20	0175	0187	
0211	21	0171	0178	
0212	00	1000	0000	
0213	30	0001	0123	
0214	69	0257	0215	
0215	24	0267	0216	
0216	69	0258	0217	
0217	24	0123	0031	
0218	69	0023	0219	
0219	24	0151	0220	
0220	69	0260	0221	
0221	24	0209	0148	
0222	60	8002	0223	
0223	19	0259	0224	
0224	35	0001	0225	
0225	65	8003	0226	
0226	15	8001	0227	
0227	20	0268	0228	
0228	60	0267	0229	
0229	10	0262	0230	
0230	44	0448	0253	
0231	61	0039	0232	

PROBLEM: Sub-Routine for Irregular Bessel Functions in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0232	21	0269	0233	
0233	69	0039	0234	
0234	24	0270	0235	
0235	60	0263	0236	
0236	14	0023	0237	
0237	20	0271	0238	
0238	60	0033	0239	
0239	44	0240	0336	
0240	11	0264	0241	
0241	44	0242	0328	
0242	11	0264	0243	
0243	44	0244	0316	
0244	11	0264	0245	
0245	44	0098	0287	
0246	24	0276	0340	
0247	24	0319	0231	
0248	00	0000	0000	Not Used
0249	03	0000	0000	
0250	01	0000	0000	
0251	01	5000	0000	
0252	01	8333	3333	
0253	69	0286	0254	
0254	24	0319	0424	
0255	11	0250	0320	
0256	06	0000	0000	
0257	20	0000	0000	
0258	20	0259	0218	
0259	00	0000	0000	
0260	15	0261	0222	
0261	05	7721	5665	
0262	31	4159	2654	
0263	00	0100	0000	
0264	00	0000	0001	
0265	00	0000	0000	Not Used
0266	00	0000	0000	Not Used
0267	20	0000	0000	
0268	00	0000	0000	
0269	00	0000	0000	
0270	00	0000	0000	
0271	00	0000	0000	
0272	00	0000	0000	Not Used
0273	00	0000	0000	
0274	00	0000	0000	
0275	00	0000	0000	
0276	00	0000	0000	
0277	00	0000	0000	
0278	00	0000	0000	
0279	00	0000	0000	
0280	00	0000	0000	
0281	00	0000	0001	

PROBLEM: Sub-Routine for Irregular Bessel Functions in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0282	00	0000	0000	
0283	00	0000	0000	
0284	30	0002	0285	
0285	20	0283	0246	
0286	10	0250	0320	
0287	60	0271	0288	
0288	19	8001	0289	
0289	35	0002	0290	
0290	60	8003	0291	
0291	19	0271	0292	
0292	35	0002	0293	
0293	60	8003	0294	
0294	10	8001	0295	
0295	21	0276	0296	
0296	19	0269	0297	
0297	35	0001	0298	
0298	64	0257	0299	
0299	15	0276	0300	
0300	20	0276	0301	
0301	60	0023	0302	
0302	30	0001	0303	
0303	64	0257	0304	
0304	15	0276	0305	
0305	20	0276	0306	
0306	60	0256	0307	
0307	30	0001	0308	
0308	21	0273	0309	
0309	60	0023	0310	
0310	19	0269	0311	
0311	35	0002	0312	
0312	67	8003	0313	
0313	20	0274	0314	
0314	69	0252	0315	
0315	24	0275	0347	
0316	60	0271	0317	
0317	19	8001	0318	
0318	35	0002	0319	
0319	11	0250	0320	
0320	21	0276	0321	
0321	60	0257	0322	
0322	30	0002	0323	
0323	21	0273	0324	
0324	67	0269	0325	
0325	20	0274	0326	
0326	69	0251	0327	
0327	24	0275	0349	
0328	69	0271	0329	
0329	24	0276	0330	
0330	60	0250	0331	
0331	21	0275	0332	

PROBLEM: Sub-Routine for Irregular Bessel Functions in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0332	30	0001	0333	
0333	21	0273	0334	
0334	69	0023	0335	
0335	24	0274	0347	
0336	67	0269	0337	
0337	60	8002	0338	
0338	10	8001	0339	
0339	21	0280	0429	
0340	60	0257	0341	
0341	30	0001	0342	
0342	21	0277	0343	
0343	24	0278	0344	
0344	24	0279	0345	
0345	69	0249	0346	
0346	24	0275	0372	
0347	69	0264	0348	
0348	24	0281	0351	
0349	61	0264	0350	
0350	21	0281	0351	
0351	60	0274	0352	
0352	30	0003	0353	
0353	14	0273	0354	
0354	60	8002	0355	
0355	19	0275	0356	
0356	35	0002	0357	
0357	21	0280	0358	
0358	24	0282	0359	
0359	69	0250	0360	
0360	24	0278	0361	
0361	69	0275	0362	
0362	24	0279	0363	
0363	60	0033	0364	
0364	35	0008	0419	
0365	10	0250	0366	
0366	21	0277	0367	
0367	60	0263	0368	
0368	64	0277	0369	
0369	15	0250	0370	
0370	15	0275	0371	
0371	20	0275	0372	
0372	60	0282	0373	
0373	30	0002	0374	
0374	14	0277	0375	
0375	60	8002	0376	
0376	19	0270	0377	
0377	14	0278	0378	
0378	60	8002	0379	
0379	30	0002	0380	
0380	14	0279	0381	
0381	60	8002	0382	

PROBLEM: Sub-Routine for Irregular Bessel Functions in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0382	19	0275	0383	
0383	35	0002	0384	
0384	21	0282	0385	
0385	30	0000	0386	
0386	44	0387	0405	
0387	60	0282	0388	
0388	10	0280	0389	
0389	21	0280	0390	
0390	69	0275	0391	
0391	24	0279	0392	
0392	60	0278	0393	
0393	10	0250	0394	
0394	21	0278	0395	
0395	10	0283	0396	
0396	21	0277	0397	
0397	60	0263	0398	
0398	64	0277	0399	
0399	15	0275	0400	
0400	20	0275	0401	
0401	60	0263	0402	
0402	64	0278	0403	
0403	15	0275	0404	
0404	20	0275	0372	
0405	60	0267	0406	
0406	10	0262	0407	
0407	44	0408	0412	
0408	60	0268	0409	
0409	11	0280	0410	
0410	19	0281	0411	
0411	15	0276	0415	
0412	65	0276	0413	
0413	15	0280	0414	
0414	16	0268	0415	
0415	60	8002	0416	
0416	30	0001	0417	
0417	14	0267	0418	
0418	69	0428	0444	
0419	21	0283	0365	
0420	61	0262	0421	
0421	21	0267	0422	
0422	69	0258	0423	
0423	24	0123	0101	
0424	61	0039	0425	
0425	21	0269	0426	
0426	69	0039	0427	
0427	24	0270	0235	
0428	20	0079	0132	
0429	61	0264	0430	
0430	21	0281	0431	
0431	60	0267	0432	

PROBLEM: Sub-Routine for Irregular Bessel Functions in Storage Sequence

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0432	10	0262	0433	
0433	44	0434	0436	
0434	69	0438	0435	
0435	24	0440	0440	
0436	69	0439	0437	
0437	24	0440	0440	
0438	60	0280	0441	
0439	61	0280	0441	
0440	61	0280	0441	
0441	21	0282	0442	
0442	21	0280	0443	
0443	60	0033	0284	
0444	24	0123	0147	
0445	24	0209	0123	
0446	69	0004	0447	
0447	24	0151	0148	
0448	69	0255	0247	

PROBLEM: Sub-Routine for Irregular Bessel Functions

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
Initial Steps for $Y_n(x)$				
0420	61	0262	0421	
0421	21	0267	0422	$-\pi \rightarrow 267$
0422	69	0258	0423	
0423	24	0123	0101	Exit instruction = 20 0259 0218 → 123
				Enter sub-routine to calculate $J_n(x)$
Initial Steps for $K_n(x)$				
0214	69	0257	0215	
0215	24	0267	0216	$2 \rightarrow 267$
0216	69	0258	0217	
0217	24	0123	0031	Exit instruction = 20 0259 0218 → 123
				Enter sub-routine to compute $I_n(x)$
0218	69	0023	0219	$x/2$
0219	24	0151	0220	$x/2 \rightarrow 151$
0220	69	0260	0221	
0221	24	0209	0148	Exit instruction = 15 0261 0222 → 209
				Enter sub-routine to compute $\ln(x/2)$
0222	60	8002	0223	$[\gamma + \ln (x/2)]$
0223	19	0259	0224	$[\gamma + \ln (x/2)]$ $[I_n(x)$ or $J_n(x)]$
0224	35	0001	0225	
0225	65	8003	0226	
0226	15	8001	0227	
0227	20	0268	0228	$2[\gamma + \ln (x/2)][I_n(x)$ or $J_n(x)] \rightarrow 268$
0228	60	0267	0229	2 or $-\pi$
0229	10	0262	0230	$2 + \pi$ or 0
0230	44	0448	0253	$K_n(x)$ or $Y_n(x)$?
0448	69	0255	0247	$K_n(x)$
0247	24	0319	0231	11 0250 0320 → 319
0231	61	0039	0232	
0232	21	0269	0233	$-(x/2)^2 \rightarrow 269$
0233	69	0039	0234	
0234	24	0270	0235	$(x/2)^2 \rightarrow 270$
0253	69	0286	0254	$Y_n(x)$
0254	24	0319	0424	10 0250 0320 → 319
0424	61	0039	0425	
0425	21	0269	0426	$-(x/2)^2 \rightarrow 269$
0426	69	0039	0427	
0427	24	0270	0235	$(x/2)^2 \rightarrow 270$
0235	60	0263	0236	1
0236	14	0023	0237	$2/x$
0237	20	0271	0238	$2/x \rightarrow 271$
0238	60	0033	0239	n

PROBLEM: Sub-Routine for Irregular Bessel Functions

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0239	44	0240	0336	n = 0?
0240	11	0264	0241	n ≠ 0
0241	44	0242	0328	n = 1?
0242	11	0264	0243	n ≠ 1
0243	44	0244	0316	n = 2?
0244	11	0264	0245	n ≠ 2
0245	44	0098	0287	n = 3?
0098	01	0000	3000	n ≠ 3 Stop
Calculation of Finite Sum for n = 0				
0336	67	0269	0337	$(x/2)^2$
0337	60	8002	0338	
0338	10	8001	0339	$2(x/2)^2$
0339	21	0280	0429	$2(x/2)^2 \rightarrow 280$
0429	61	0264	0430	-1
0430	21	0281	0431	$-1 \rightarrow 281$
0431	60	0267	0432	2 or $-\pi$
0432	10	0262	0433	$2 + \pi$ or 0
0433	44	0434	0436	$K_0(x)$ or $Y_0(x)$?
0434	69	0438	0435	$K_0(x)$
0435	24	0440	0440	60 0280 0441 → 440
0436	69	0439	0437	$Y_0(x)$
0437	24	0440	0440	61 0280 0441 → 440
0440	60	0280	0441	± first term (This order set up)
0441	21	0282	0442	± previous term → 282
0442	24	0280	0443	± partial infinite sum → 280
0443	60	0033	0284	n
0284	30	0002	0285	
0285	20	0283	0246	n → 283
0246	24	0276	0340	n → 276
0340	60	0257	0341	2
0341	30	0001	0342	
0342	21	0277	0343	2 → 277
0343	24	0278	0344	2 → 278
0344	24	0279	0345	2 → 279
0345	69	0249	0346	
0346	24	0275	0372	3 → 275
Calculation of Finite Sum for n = 1				
0328	69	0271	0329	
0329	24	0276	0330	2/x → 276
0330	60	0250	0331	
0331	21	0275	0332	$\sum_1^1 1/k = 1 \rightarrow 275$
0332	30	0001	0333	

PROBLEM: Sub-Routine for Irregular Bessel Functions

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0333	21	0273	0334	1! → 273
0334	69	0023	0335	
0335	24	0274	0347	x/2 → 274
0347	69	0264	0348	
0348	24	0281	0351	1 → 281
0351	60	0274	0352	$(x/2)^n$
0352	30	0003	0353	
0353	14	0273	0354	$1/n!\ (x/2)^n$
0354	60	8002	0355	
0355	19	0275	0356	
0356	35	0002	0357	
0357	21	0280	0358	Partial infinite Σ → 280
0358	24	0282	0359	Previous term → 282
0359	69	0250	0360	
0360	24	0278	0361	1 → 278
0361	69	0275	0362	
0362	24	0279	0363	Σ → 279
0363	60	0033	0364	
0364	35	0008	0419	
0419	21	0283	0365	n → 283
0365	10	0250	0366	
0366	21	0277	0367	n + 1 → 277
0367	60	0263	0368	
0368	64	0277	0369	1/(n+1)
0369	15	0250	0370	1 + 1/(n+1)
0370	15	0275	0371	$\left(\sum_{}^{r} 1/k + \sum^{n+r} 1/k\right)$
0371	20	0275	0372	→ 275
Calculation of Finite Sum for n = 2				
0316	60	0271	0317	2/x
0317	19	8001	0318	$(2/x)^2$
0318	35	0002	0319	SL 2
0319	11	0250	0320	$(2/x)^2 \pm 1$
0320	21	0276	0321	$[(2/x)^2 \pm 1]$ → 276
0321	60	0257	0322	2
0322	30	0002	0323	SR 2
0323	21	0273	0324	2! → 273
0324	67	0269	0325	$(x/2)^2$
0325	20	0274	0326	$(x/2)^2$ → 274
0326	69	0251	0327	
0327	24	0275	0349	$\sum^{2} 1/k$ → 275
0349	61	0364	0350	
0350	21	0281	0351	-1 → 281
Calculation of Finite Sum for n = 3				
0287	60	0271	0288	2/x
0288	19	8001	0289	$(2/x)^2$
0289	35	0002	0290	
0290	60	8002	0291	
0291	19	0271	0292	$(2/x)^3$

PROBLEM: Sub-Routine for Irregular Bessel Functions

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0292	35	0002	0293	
0293	60	8003	0294	$2(2/x)^3$
0294	10	8001	0295	
0295	21	0276	0296	$2(2/x)^3 \rightarrow 276$
0296	19	0269	0297	$\pm\ 2(2/x)$
0297	35	0001	0298	
0298	64	0257	0299	$\pm\ 2/x$
0299	15	0276	0300	$2(2/x)^3 \pm (2/x)$
0300	20	0276	0301	$2(2/x)^3 \pm (2/x) \rightarrow 276$
0301	60	0023	0302	$x/2$
0302	30	0001	0303	
0303	64	0257	0304	$1/2\ (x/2)$
0304	15	0276	0305	
0305	20	0276	0306	$[2(2/x)^3 \pm 2/x + 1/2\ (x/2)] \rightarrow 276$
0306	60	0256	0307	$3! \rightarrow$ UA
0307	30	0001	0308	
0308	21	0273	0309	$3! \rightarrow 273$
0309	60	0023	0310	$x/2$
0310	19	0269	0311	$\pm\ (x/2)^3$
0311	35	0002	0312	
0312	67	8003	0313	$(x/2)^3$
0313	20	0274	0314	$(x/2)^3 \rightarrow 274$
0314	69	0252	0315	
0315	24	0275	0347	$\sum^{3} 1/k \rightarrow 275$
Iterative Procedure for Infinite Sum				
0372	60	0282	0373	Previous Term = P.T.
0373	30	0002	0374	
0374	14	0277	0375	$P.T./(n+r)$
0375	60	8002	0376	
0376	19	0270	0377	$\pm\ (x/2)^2\ [P.T./(n+r)]$
0377	14	0278	0378	$\pm\ (x/2)^2\ [P.T./r(n+r)]$
0378	60	8002	0379	
0379	30	0002	0380	
0380	14	0279	0381	$\pm\ (x/2)^2\ P.T./[r(n+r)\Sigma_{old}]$
0381	60	8002	0382	
0382	19	0275	0383	$\pm\ (x/2)^2\ P.T.\Sigma_{new}/[r(n+r)\Sigma_{old}] =$ New Term
0383	35	0002	0384	
0384	21	0282	0385	new term $\rightarrow$ 282
0385	30	0000	0386	
0386	44	0387	0405	Is last term = 0?
0387	60	0282	0388	NO
0388	10	0280	0389	
0389	21	0280	0390	Partial infinite Sum $\rightarrow$ 280
0390	69	0275	0391	
0391	24	0279	0392	Σ_{new} becomes $\Sigma_{old} \rightarrow 270$
0392	60	0278	0393	
0393	10	0250	0394	

PROBLEM: Sub-Routine for Irregular Bessel Functions

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0394	21	0278	0395	new r → 278
0395	10	0283	0396	
0396	21	0277	0397	new n + r → 277
0397	60	0263	0398	
0398	64	0277	0399	
0399	15	0275	0400	$\sum_{n}$ ——— $\sum_{n+r}$
0400	20	0275	0401	$(\sum 1/k + \sum 1/k)_{new} \rightarrow 275$
0401	60	0263	0402	
0402	64	0278	0403	
0403	15	0275	0404	
0404	20	0275	0372	new Σ → 275 calculate next term
0405	60	0267	0406	Last term = 0
0406	10	0262	0407	
0407	44	0408	0412	$K_n(x)$ or $Y_n(x)$?
0408	60	0268	0409	$2\{\gamma + \ln x/2\} I_n(x)$ $K_n(x)$
0409	11	0280	0410	$2\{\gamma + \ln x/2\} I_n(x)$ - infinite Σ^n
0410	19	0281	0411	$\pm[2(\gamma + \ln x/2)\ I_n(x)$ - infinite Σ]
0411	15	0276	0415	$2 K_n(x)$
0412	65	0276	0413	add finite sum $Y_n(x)$
0413	15	0280	0414	finite Σ + infinite
0414	16	0268	0415	fin. Σ + infin. $\Sigma + 2(\gamma + \ln x/2) J_n(x)$
Conversion of Function to Floating Decimal Form				
0415	60	8002	0416	$2 K_n(x)$ or $-\pi Y_n(x)$
0416	30	0001	0417	
0417	14	0267	0418	$K_n(x)$ or $Y_n(x)$
0418	69	0428	0444	
0444	24	0123	0147	20 0079 0132 → 123
0147	69	0213	0445	
0445	24	0209	0123	30 0001 0123 → 209
0123	20	0079	0132	$K_n(x)$ or $Y_n(x)$ → 79
0132	60	8002	0141	$K_n^n(x)$ or $Y_n^n(x)$ → UA
0141	36	0000	0121	Shift and count
0121	47	0126	0126	Reset overflow circuit if tripped
0126	21	0130	0144	$K_n(x)$ or $Y_n(x)$ → 130
0144	46	0146	0133	Is function negative?
0146	61	8002	0135	+ count → UA YES
0135	11	0114	0124	- 51 + count → UA
0133	61	8002	0136	- count NO
0136	10	0114	0124	51 - count → UA

PROBLEM: Sub-Routine for Irregular Bessel Functions

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0124	15	0130	0134	(51-count) + [$K_n(x)$ or $Y_n(x)$]
0134	31	0002	0042	SR 2 and Round
0042	60	8002	0115	Floated $K_n(x)$ or $Y_n(x)$
0115	15	0079	1971	Floated function in UA; fixed decimal function in LA Go to exit instruction
0446	69	0004	0447	Special entry for ln
0447	24	0151	0148	argument → 151
(Constants and Storage Assignments)				
0130	00	0000	0000	$K_n(x)$ or $Y_n(x)$
0213	30	0001	0123	
0249	03	0000	0000	
0250	01	0000	0000	
0251	01	5000	0000	
0252	01	8333	3333	
0255	11	0250	0320	
0256	06	0000	0000	
0257	20	0000	0000	
0258	20	0259	0218	
0259	00	0000	0000	$I_n(x)$ or $J_n(x)$
0260	15	0261	0222	
0261	05	7721	5665	γ
0262	31	4159	2654	π
0263	00	0100	0000	
0264	00	0000	0001	
0267	00	0000	0000	2 or $-\pi$
0268	00	0000	0000	$2(\gamma + \ln x/2)[I_n(x)$ or $J_n(x)]$
0269	00	0000	0000	$-(x/2)^2$
0270	00	0000	0000	$(x/2)^2$
0271	00	0000	0000	$2/x$
0273	00	0000	0000	n!
0274	00	0000	0000	$(x/2)^n$
0275	00	0000	0000	$(\sum 1/k + \sum 1/k)_{new}$
0276	00	0000	0000	finite sum
0277	00	0000	0000	n + r
0278	00	0000	0000	r
0279	00	0000	0000	$(\sum 1/k + \sum 1/k)_{old}$
0280	00	0000	0000	Infinite sum
0281	00	0000	0000	± 1
0282	00	0000	0000	Previous term
0283	00	0000	0000	n
0286	10	0250	0320	
0428	20	0079	0132	
0438	60	0280	0441	
0439	61	0280	0441	

PROBLEM: Sub-Routine for Natural Logarithm

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0148	60	0151	0155	x/2 ≡ U → UA
0155	11	0158	0163	U - 10
0163	46	0166	0167	U > 10?
0166	60	0212	0173	0.1 U < 10
0173	11	0151	0157	0.1 - U
0157	46	0162	0165	U < 0.1?
0167	69	0170	0176	U > 10
0176	24	0160	0164	ln 10 → 160
0164	60	0151	0181	U
0181	30	0001	0149	0.1 U
0165	61	0170	0177	U < 0.1
0177	21	0160	0168	- ln 10 → 160
0168	60	0151	0159	U
0159	35	0001	0149	
0162	67	8003	0174	0.1 < U < 10
0174	21	0160	0192	0 → 160
0192	60	0151	0149	U
0149	10	0169	0183	U + 1
0183	21	0204	0150	U + 1 → 204
0150	11	0156	0184	U - 1
0184	64	0204	0154	
0154	20	0175	0185	(U-1)/(U+1) = Z → 175
0185	30	0001	0200	
0200	20	0191	0152	Z → 191
0152	60	0175	0161	
0161	19	8001	0190	Z^2
0190	21	0182	0186	Z^2 → 182
0186	60	0189	0193	n → UA
0193	10	0156	0211	n + 2 → UA
0211	21	0171	0178	n + 2 → 171
0178	60	0175	0179	Previous term = P.T. → UA
0179	19	0182	0205	(P.T.)Z^2
0205	60	8003	0180	
0180	19	0189	0208	(P.T.)Z^2 n
0208	14	0171	0210	(P.T.)Z^2 n/(n+2)
0210	20	0175	0187	(P.T.)Z^2 n/(n+2) = New Term → 175
0187	30	0000	0194	Shift permits ln to be more or less accurate
0194	60	8002	0203	New Term → UA
0203	44	0207	0198	New Term = 0?

PROBLEM: Sub-Routine for Natural Logarithm

LOCATION	OP.	DATA	INSTRUCTION	REMARKS
0207	60	0175	0188	New Term NO
0188	30	0001	0195	
0195	10	0191	0196	New Term + Σ_{old} → UA
0196	21	0191	0197	New Term + Σ_{old} = Σ_{new} → 191
0197	69	0171	0172	
0172	24	0189	0186	N + 2 becomes N for next pass → 189
0198	65	0191	0199	Σ → LA New Term = 0
0199	15	8001	0201	2Σ → LA
0201	15	0160	0202	2Σ + 0 or 2Σ ± ln 10
0202	69	0169	0206	
0206	24	0189	0209	1 → 189 Resets 189
0209	15	0261	0222	γ + ln x/2 → LA Exit instruction
(Constants and Storage Assignments)				
0151	00	0000	0000	Argument U as xx.xxxxxxxx
0156	02	0000	0000	2
0158	10	0000	0000	10
0160	00	0000	0000	0 or ± ln 10 as x.xxxxxxxxx
0169	01	0000	0000	1
0170	23	0258	5093	ln 10
0171	00	0000	0000	N + 2 as xx.xxxxxxxx
0175	00	0000	0000	Term n as .xxxxxxxxxx
0182	00	0000	0000	$Z^2 = [(U-1)/(U+1)]^2$ as .xxxxxxxxxx
0189	01	0000	0000	n as xx.xxxxxxxx
0191	00	0000	0000	Σ as x.xxxxxxxxx
0204	00	0000	0000	U + 1
0212	00	1000	0000	0.1

OSR 25-5
(REV. 10-55)

DUPONT
REG. U.S. PAT. OFF.
ESTABLISHED 1802

E. I. DU PONT DE NEMOURS & COMPANY
INCORPORATED
SAVANNAH RIVER PLANT
AIKEN, SOUTH CAROLINA
(TEL. & TEL. ADDRESS, AUGUSTA, GA.)

EXPLOSIVES DEPARTMENT
ATOMIC ENERGY DIVISION

DP-159 File Record Copy

June 6, 1957

TO: Recipients of DP-159, "IBM Sub-Routines II - Irregular Bessel Functions," by J. C. English.

Correction Notice

The instruction listed for storage location 0184 on page 11 should be 64 0204 0154. It is listed correctly on page 23 in the program analysis.

TECHNICAL DIVISION
SAVANNAH RIVER LABORATORY

J. C. English

J. C. English
Theoretical Physics Division

OSR 25-5
(REV. 10-55)

DU PONT
REG. U.S. PAT. OFF.
ESTABLISHED 1802

E. I. DU PONT DE NEMOURS & COMPANY
INCORPORATED
SAVANNAH RIVER PLANT
AIKEN, SOUTH CAROLINA
(TEL. & TEL. ADDRESS, AUGUSTA, GA.)

EXPLOSIVES DEPARTMENT
ATOMIC ENERGY DIVISION

June 6, 1957

TO: Recipients of DP-159, "IBM Sub-Routines II - Irregular Bessel Functions," by J. C. English.

Correction Notice

The instruction listed for storage location 0184 on page 11 should be 64 0204 0154. It is listed correctly on page 23 in the program analysis.

TECHNICAL DIVISION
SAVANNAH RIVER LABORATORY

J. C. English

J. C. English
Theoretical Physics Division

DP- 159

RECORD COPY CHECK LIST

Internal Distribution ✓

AEC Approval ✓

External Distribution ✓

Record copy to P.S. _____

Record copy from P.S. _____

Record copy to J.C.A. _____

Date Internal copies mailed 6-4-56

Date External copies mailed 6-29-56

Railway Express (Motor Freight) 7-3-56

Publication Form to Oak Ridge ✓

The following items are to be bound with the Record Copy:

	Secret	Confidential	Unclassified
Approval Sheet			✓
Internal Distribution			✓
External Distribution (M-3679)(TID-4500)			✓
Publication Form			✓
AEC Approval Letter			✓

JUN 29 '56

TELEPHONE
AUGUSTA, GA.
4-6872

TELEGRAM ADDRESS
AUGUSTA, GA.

UNITED STATES
ATOMIC ENERGY COMMISSION
SAVANNAH RIVER OPERATIONS OFFICE
P. O. BOX A
AIKEN, SOUTH CAROLINA

IN REPLY REFER TO:

J C Andrews

T:LTP:ven

June 27, 1956

Mr. Hood Worthington, Director
Technical Division - AED
Explosives Department
E. I. du Pont de Nemours & Company
Wilmington 98, Delaware

Dear Mr. Worthington:

Please refer to your letter of May 28, 1956, to Mr. H. L. Kilburn, Director, Technical and Production Division, transmitting the following proposed publication for classification and patent review:

DP-159, "IBM SUB-ROUTINES II, Irregular Bessel Functions," by J. C. English

This office has reviewed this report both as to classification and as to patent aspects and agrees that it may be released for standard external distribution.

Dr. M. H. Wahl's office has been notified of this action by telephone.

Very truly yours,

Joel V. Levy
Assistant Director for Technical
Technical and Production Division

cc: Dr. M. H. Wahl,
Savannah River Laboratory

PUBLICATION FORM

In accordance with the Atomic Energy Act of 1946 (Public Law 585) the U. S. Atomic Energy Commission makes every effort to assure the proper dissemination of information. However, in order not to jeopardize acceptance of material for publication, distribution of a document will be limited or withheld if requested, in accordance with the stipulations under section 3, below.

This form, appropriately filled out, should be submitted to the Technical Information Service, Oak Ridge, with each unclassified document issued as a result of work done with AEC funds and with each document submitted for declassification.

TO: United States Atomic Energy Commission
Technical Information Service
P.O. Box 62, Oak Ridge, Tenn.
Attention: Cataloging Branch

1. Subject document:

Title: IBM SUB-ROUTINES II
Irregular Bessel Functions

Author: J. C. English

Document Date: May 1956 Document Number: DP-159

This document submitted under Contract No. AT (07-2)-1

is (unclassified) (~~being submitted for declassification~~)

2. The information in subject document is contained in project report number _____. Contents of project report and of subject document (are, are not) identical.

3. *Disposition of subject document is intended to be as follows:

☐ a. To be submitted for publication in _____

☐ b. To be given orally at _____

☐ c. To be used for research, development, or manufacturing (no formal distribution).

☒ d. Distribution in accordance with TID-4500 (Standard Distribution Lists for USAEC Nonclassified Research and Development Reports) by issuing installation.

e. Distribution by TIS

is permitted to		is requested to	
☐	official recipients (TID-4500)	☐	official recipients (TID-4500)
☒	public by sale	☐	public by sale

* When journal publication is indicated, the TIS normally will not reproduce for public distribution since such a distribution might prejudice acceptance of material for journal publication. Dual publication by a journal and the TIS is normally considered an unjustifiable expenditure of public funds.

Revised: June 1953

f. No formal distribution should be made, for the following reason:

_____

_____

4. In the event that large scale reproduction of the report is made by the Technical Information Service, it is requested that ____ copies of the reproduced document (maximum of twenty-five copies) be made available to the author for distribution to those especially interested in the subject field. These should be addressed to:

Name: _____

Address: _____

_____

5. ☐ Revised copy in form suitable for distribution is forthcoming.

6. Correspondence relative to publication, disposition or content of subject document should be addressed to:

Name: James W. Hill

Address: Savannah River Laboratory

Aiken, South Carolina

7. Remarks:

James Hill

Name and Title

6/25/56

Date

NOTES

1. All nonclassified documents are reviewed for indexing in the card catalog and for inclusion in Nuclear Science Abstracts.

2. Official distribution refers to distribution of documents by the Technical Information Service to other Atomic Energy Commission installations, cooperating Government agencies, and AEC depository libraries. Public distribution refers to the distribution of documents by the TIS to the Office of Technical Services, Dept. of Commerce, Washington, D. C., for sale to the public on a nonprofit basis.

EX-1176-AX

DU PONT
ESTABLISHED 1802
E. I. DU PONT DE NEMOURS & COMPANY
INCORPORATED
SAVANNAH RIVER PLANT
AIKEN, SOUTH CAROLINA
(TEL. & TEL. ADDRESS, AUGUSTA, GA.)

EXPLOSIVES DEPARTMENT
ATOMIC ENERGY DIVISION

REC-D15--15

CC: F. A. Robertson - SROO
~~M. H. Wahl - C. W. J. Wende -~~
~~J. C. Andrews~~ - TIS File
G. Deesauer
S. A. McNeight - L. C. Evans

approved

Date May 28, 1956

Mr. H. L. Kilburn, Director (2)
Technical and Production Division
Savannah River Operations Office
U. S. Atomic Energy Commission
Post Office Box A
Aiken, South Carolina

Dear Mr. Kilburn:

PROPOSED PUBLICATION - DP-150

Attached for review as to classification and patent matters are two copies of an unclassified report

IBM SUB-ROUTINES II
Irregular Bessel Functions
by J. C. English

which we propose to release for standard external distribution.

To facilitate the release of this report, it would be appreciated if you would telephone your comments to M. H. Wahl's office and send a confirming letter to me with a copy to M. H. Wahl.

If further clarification or technical information are needed to aid in your patent review, we suggest you contact

G. Deesauer
Physics Section
Savannah River Laboratory

If there is a continued interest in pursuing the patentability of a development, I would be happy to supply the additional information required, such as, appropriate references and the name of the person responsible for the development.

ATOMIC ENERGY DIVISION

SWOR

Hood Worthington, Director
Technical Division

LCE:ev

OSR 25-5 (REV. 10-55)

TECHNICAL DIVISION
SAVANNAH RIVER LABORATORY

Report Data Sheet

Report Number DP-159 S C (U) Approved by S.W.O'Rear

Author J. C. English

Title IBM Sub-Routines II
Inregular Bessel Functions

Category ~~Mathematics~~ Physics Approved by S.W.O'Rear

Abstract _____ Approved by S.W.O'Rear

	Author	Supv.	Div. Hd.	Sec. Dir.	Lab. Dir.
Manuscript Approval	J. C. English	R. Hafner	G. Dessauer	4/26	C.J.W. for A.R.M. 5/2
Reproduction Copy Approval	JCE	.....	G. Dessauer	5/17	.....
Printed Copy Approval	JCE 5/25	.....	G.D. 5/25	.....	.....
Recommended for Journal Publication	.....	.....	_____	_____	_____

_____ Required ..... Optional

TO BE FILLED OUT BY TIS

MS:pp. Typed 8 Figures _____ Tables _____ Total pp. 26

Direct Master _____ Photo Offset _____ Carbon Ribbon _____ Fabric _____

Paper Stock _____ Cover _____

No. Copies: Internal 36 Int. Extra 40 TID-4500 (11 Unlimited) 700 M-3679 _____

Other ( ) _____ Total Copies 776

FD:pp. Typed _____ Figures _____ Tables _____ Total pp. 24

Doc. Date May 1956 Received 5/4/56 Int. Issue _____ Ext. Issue _____

Remarks —

OSR-24-A40

DP-159
External Distribution
Mailed 6/29/56

PHYSICS

This category includes physical and mathematical studies of interest to the AEC program.

No. of Copies	Distribution
1	Alco Products, Inc.
1	Argonne Cancer Research Hospital
6	Argonne National Laboratory
1	Armed Forces Special Weapons Project, Washington
5	Armed Services Technical Information Agency, Dayton
2	Army Chemical Center
1	Army Medical Research Laboratory
1	Army Medical Service Graduate School (Department of Biophysics)
~~2~~	~~Atomic Energy Commission, Isotopes Extension~~
1	Atomic Energy Commission, Patent Branch
3	Atomic Energy Commission, Technical Library
2	Atomics International
2	Battelle Memorial Institute
4	Bettis Plant
1	Boeing Airplane Company
4	Brookhaven National Laboratory
1	Brush Beryllium Company
1	Bureau of Ships (Code 590)
2	Carnegie Institute of Technology
1	Chicago Patent Group
1	Columbia University (Dr. Failla)
1	Columbia University (Dr. Hassialis)
2	Columbia University (Dr. Havens)
1	Combustion Engineering, Inc.
1	Committee on Atomic Casualties (APO-182)
2	Consolidated Vultee Aircraft Corporation
1	Department of Army, Office of Ordnance Research
1	Department of Army, Office of Quartermaster General
1	Department of Navy (Code 811)
1	Division of Raw Materials, Denver
1	Dow Chemical Company, Rocky Flats
~~4~~	~~duPont Company, Aiken~~
~~1~~	~~duPont Company, Wilmington~~
1	Frankford Arsenal
2	General Electric Company (ANPP)
4	General Electric Company, Richland
3	Goodyear Atomic Corporation
1	Hawaii Marine Laboratory
2	Iowa State College
2	Kirtland Air Force Base
3	Knolls Atomic Power Laboratory
2	Lockheed Aircraft Corporation
2	Los Alamos Scientific Laboratory
1	Massachusetts Institute of Technology (Dr. Hardy)

No. of Copies	Distribution
1	Mound Laboratory
1	National Advisory Committee for Aeronautics, Cleveland
2	National Bureau of Standards, Atomic Energy Project
1	National Bureau of Standards (Library)
1	National Lead Company of Ohio
1	Naval Medical Research Institute
3	Naval Research Laboratory
1	New Brunswick Laboratory
1	New York Operations Office
1	New York University (Dr. Bomberg)
2	Nuclear Development Corporation of America
1	Nuclear Metals, Inc. (Dr. Kaufmann)
1	Oak Ridge Institute of Nuclear Studies
6	Oak Ridge National Laboratory
1	Office of Naval Research
1	Ordnance Department
1	Pennsylvania State University (Blanchard)
6	Phillips Petroleum Company
1	Princeton University
2	Public Health Service
1	Radioisotopes Laboratory
1	RAND Corporation
1	Sandia Corporation
1	Signal Corps Center
1	Union Carbide Nuclear Company, C-31 Plant
2	Union Carbide Nuclear Company, K-25 Plant
1	United Aircraft Corporation
1	U. S. Geological Survey, Denver
1	U. S. Geological Survey, Menlo Park
1	U. S. Geological Survey, Naval Gun Factory
1	U. S. Geological Survey, Washington
1	U. S. Naval Postgraduate School
2	U. S. Naval Radiological Defense Laboratory
1	U. S. Patent Office
1	UCLA Medical Research Laboratory
2	University of California Radiation Laboratory, Berkeley
4	University of California Radiation Laboratory, Livermore
1	University of Chicago, Air Force Radiation Laboratory
1	University of Rochester, Atomic Energy Project
2	University of Rochester (Dr. Marshak)
1	University of Utah
2	University of Washington (Dr. Manley)
1	Vitro Corporation of America, 120 Wall St., N. Y.
1	Walter Kidde Nuclear Laboratories, Inc.
1	Walter Reed Army Medical Center
1	Watertown Arsenal
1	Weil, Dr. George L.
2	Western Reserve University
1	Yale University (Gregory Breit)

PHYSICS (Cont'd.)

No. of Copies	Distribution
1	Yale University (Dr. Wadey)
250	Technical Information Extension, Oak Ridge
300*	Office of Technical Services, Washington
707	

DP-159

IBM Sub-routines II

DATE	TIS SUPV.	TIS EDIT	TIS TYPE	TIS PROOF.	TIS DWG	AUTHOR PROOF.	TIS CORR.	AUTHOR CORR.	TIS MAKE-UP	AUTHOR APPR.	TIS PRINT.	AUTHOR APPR.	TIS BIND	TIS PROC.
5/4	X	X												
5/5		X				X								
5/7			X			X								
5/8			X											
5/9			X			X								
5/10			X											
5/11			X	X										
5/14				X		X								
5/15							X	X						
5/16							X	X		X				
5/17	X							X		X				
5/18	X						X							
5/21							X							
5/22											X			
5/23											X			
5/24											X			
5/25												X		
5/28														X
5/29														X
5/30														X
5/31														X
6/1														X
Est. issue 6/15														
Est. issue 6/25													X	
	1	1	4	1		(1)	2	(1)		(2)	3	(1)		6

18 WD

23 TD

NOTE: Numbers refer to notes on the other side of this sheet.