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METHODS OF ANALYSIS TO DETERMINE SUBCRITICAL REACTIVITY FROM THE PULSED NEUTRON EXPERIMENT

P. B. PARKS

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SUBCRITICAL REACTIVITY FROM THE
PULSED NEUTRON EXPERIMENT**

by

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ABSTRACT

The published methods for the deduction of reactivity from pulsed neutron experiments on subcritical reactors are reviewed. Each method is categorized as inherently yielding a result that is either spatially independent or spatially dependent. The spatially independent results are formally identical with the static reactivity; the result does not depend, in principle, on the location of either the pulsed neutron source or the neutron detector during data collection. The spatially dependent results only approximate the static reactivity; the results are affected, in varying degrees, by the locations of the source and detector. Among the techniques yielding spatially independent results are the Space-Time method of Parks and Stewart and the Inhour method of Preskitt et al. Spatially dependent results are obtained with the Sjöstrand, Gozani, and Garelis-Russell methods which are examined with and without the kinetic distortion corrections given by Becker and Quisenberry. Intercomparisons of all methods are made with reference to pulsed neutron experiments on both unreflected and reflected reactors. Recommendations are made concerning the best choice of method under the various experimental conditions that are likely to be encountered.

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INTRODUCTION

The main function of the pulsed neutron experiment on a multiplying system is to provide a measure of the shutdown reactivity. In the past few years, several analytical methods have been introduced for deducing the reactivity from measured data. The purpose of this report is to review the broad range of methods that can be used, and to show where these methods can be applied without introduction of errors due to violations of assumptions. In broad perspective, and in detail where necessary, answers to the following questions are developed:

- What are the methods available for analyzing pulsed neutron experiments?
- What are the relative accuracies of the various methods under varying conditions?
- Under what conditions are each of the methods appropriate?

In developing the answers to these questions, only those papers which contribute to a basic theoretical understanding of pulsed analysis are reviewed.

The pulsed neutron experiment consists of recording, at one or more points in the reactor, the flux variation in time during and after one or more extraneous source bursts. The data can be analyzed by several different methods, some of which yield spatially independent results that are formally identical with the static reactivity. Other methods, however, yield results which are only approximations to the static reactivity. These methods have spatially dependent results that are influenced with varying sensitivities by the positions of the pulsed neutron source and detector.

Before reviewing the methods of analysis, certain terms commonly appearing in the literature will be defined qualitatively.

Prompt Neutron Harmonics

When a subcritical multiplying system is undisturbed by any external source, the existing neutron distribution arising from spontaneous fission and lattice multiplication will be found in a time-invariant condition. Suppose the lattice is exposed to an extraneous point source with strength much greater than the spontaneous fission source, the usual condition for pulsing of

a subcritical system. After some elapsed time, a new neutron distribution will be assumed with a strong peak at the location of the extraneous source. If the delayed neutron precursors did not exist, all neutrons created by source multiplication would be "prompt neutrons." The transition of the prompt neutron distribution from the initial to the distorted extraneous source dependent distribution would be rapid, at rates governed by the prompt neutron generation time. After the point source was removed, the prompt distribution would rapidly decay back toward the initial distribution.

At the start of the prompt neutron decay following removal of the point source, the amplitude of the postsource distribution will be much greater than the presource amplitude. The flux shape will undergo rather rapid shifting at the start of the decay but will eventually settle on a single shape known as the "fundamental mode." This shape will be maintained for some time until the amplitude falls sufficiently for the initial flux shape to again become dominant.

The behavior of the prompt neutron flux during the transition from the initial to the point source-induced distribution and then to the fundamental mode following point source removal is said to be influenced by "prompt neutron harmonics." The phrase merely denotes the distortion of the flux shape away from the fundamental mode at all times other than during fundamental mode decay.

Delayed Neutron Harmonics

The delayed neutrons do not behave in the same manner as the prompt neutrons because of the long precursor lifetimes. The bulk of the precursors are formed at the very times the prompt neutron amplitude is peaking and most distorted. The postsource delayed neutrons initially will have a distorted flux shape; that is, the delayed shape exhibits "delayed neutron harmonics." The delayed shape will eventually tend toward the fundamental shape, but only very slowly.

Kinetic Distortion

The fundamental mode of delayed neutrons is closely the same as the static eigenfunction found in a k_{eff} search. However, under certain conditions, the prompt neutron fundamental flux shape is not the same as the static eigenfunction. When this condition exists, "kinetic distortion" is said to be present. Kinetic distortion (the nonequivalence of the prompt and delayed modes) commonly arises with reflected reactors and is particularly a problem with D_2O or graphite reflectors where the absorption cross section is very small.

All the distortion mechanisms described previously will be shown to have little or no effect on the validity of reactivities deduced by a method of analysis, called the Space-Time¹ method. Nearly the same accuracy can be obtained but at far less computing expense by an approximation to the Space-Time method, called the Inhour² method. The early Simmons-King³ method is shown to be a variant of the Inhour method. All of these methods yield spatially independent results even though using space-dependent models.

The methods of Sjöstrand,⁴ Gozani,⁵ and Garelis-Russell⁶--often called conventional area analysis methods--yield spatially dependent results. All are affected in varying degrees by prompt and delayed harmonics and kinetic distortion. The kinetic distortion derivation of Becker and Quisenberry⁷ will be the starting point for these discussions.

METHODS YIELDING SPATIALLY INDEPENDENT RESULTS

SPACE-TIME METHOD

Description

The pulsed neutron experiment does not directly measure reactivity. Only the neutron flux response to an external source is measured, and the reactivity is deduced, by various procedures, from the observed prompt and delayed neutron behavior.

The Space-Time method of Parks and Stewart¹ takes the straightforward approach that a correct calculation of the observed prompt neutron response automatically assures the correct calculation of the static eigenvalue, or reactivity. Two basic assumptions are implicit in this method:

- Discrepancies between initially calculated and measured responses may exist, but these discrepancies are assumed to result from small errors in the input parameters of the calculation and not from errors in the method of calculation. The input parameters are to be altered, or normalized, to force agreement.
- Once normalization has produced a set of input parameters that lead to a correct calculation of the prompt response, the same calculational model can be used to compute the static subcritical reactivity. In short, the only purpose for pulsed neutron experiments is to provide a measurable flux response against which the input parameters, that determine the calculated reactivity, can be tested and altered, as necessary, to force agreement between the measured and calculated responses.

Application to a Reflected Lattice

Pulsing a large reflected reactor creates all the distortion mechanisms (prompt and delayed harmonics and kinetic distortion) that make analysis for subcritical reactivity difficult. For that reason, a set of pulsing experiments on a reflected reactor in the large Process Development Pile (PDP) has been chosen for demonstration of the Space-Time method.

Figures 1 and 2 show the vertical and radial schematics of the experimental arrangement for the reflected lattice. All fuel, target, and control assemblies were removed from the ring immediately surrounding the central core to form a heavy water

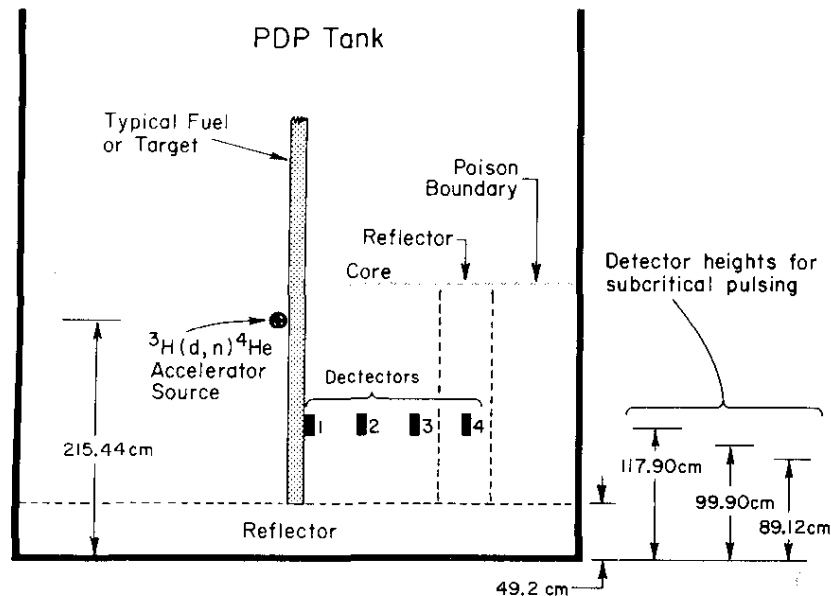


FIGURE 1 Experimental Arrangement for the Reflected Lattice in the PDP - Vertical Display

reflector. The next ring retained its control and target assemblies, but the fuel assemblies were removed. The outermost region was filled with target assemblies in an irregular pattern. All of the area outside the reflector can be considered a poison boundary.

Four different measurements were made on this lattice: one critical at 241.50 cm and three subcritical at lower water heights. The top extrapolation distance of 2.4 cm was established by a vertical gold pin irradiation. The pulsing experiments were performed at successively lower water heights to provide data with increasing subcriticality. Figures 1 and 2 also show the locations of the $^3\text{H}(\text{d},\text{n})^4\text{He}$ source, the neutron detectors, and the water heights for the three experiments. These detector positions were estimated to be at the midplane of the static flux distributions.

The unnormalized cross sections for the core and inner poison boundary ring were computed with the "supercell" option of RAHAB.⁸ The cross sections were homogenized over the entire seven-assembly cluster of control cell, three drivers or vacancies, and three targets. The unnormalized cross sections for the reflector and the irregular outer poison boundary ring were computed with the cell option of RAHAB.

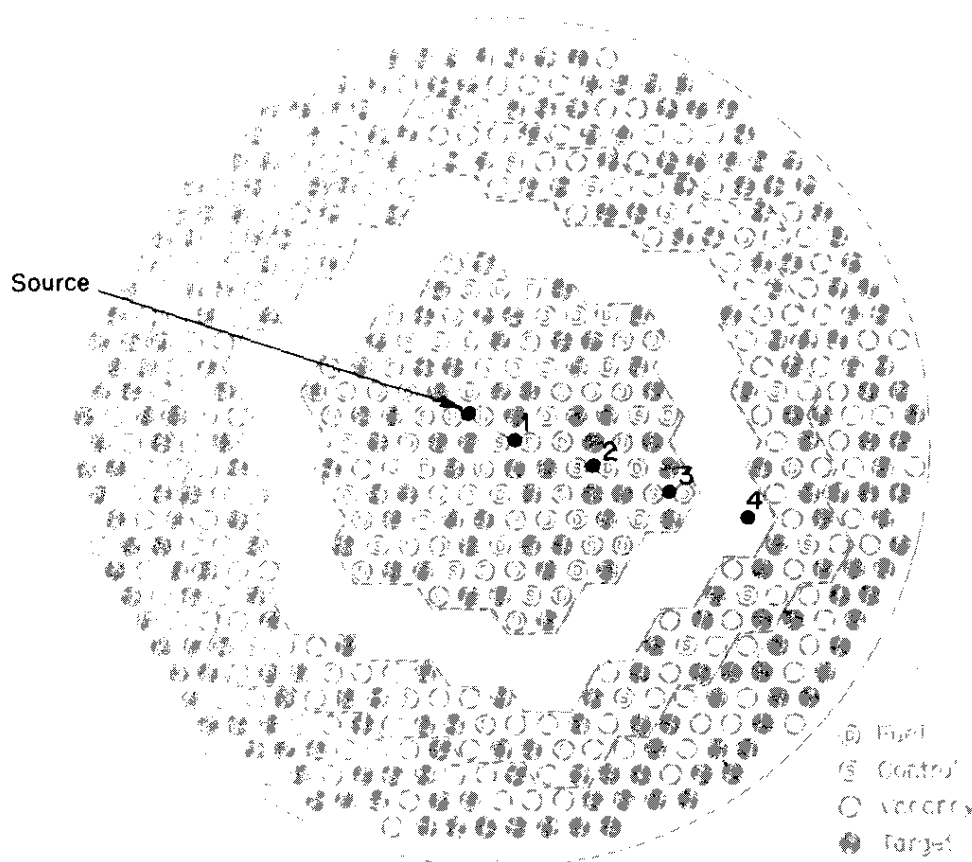


FIGURE 2 Experimental Arrangement for the Reflected Lattice in the PDP - Radial Display

The source distributions for the three pulsing experiments were estimated with DOT.⁹ These estimates include two sources of error due to the limited calculational tools. First, the reactor geometry is hexagonal in the plane, and the closest approximation with DOT is cylindrical (RZ) geometry. Second, the source was not precisely on the axis of the reactor; the DOT calculations require a source centered about the axis. These errors affect only the flux buildup and initial die-away in the source-induced harmonic region; the fundamental mode decay is unaffected.

Three direct solution space-time codes using few-group diffusion theory are available at the Savannah River Laboratory with options for performing time-dependent reactor response calculations:

- WIGLE¹⁰ one dimension - slab
 sphere
 cylinder (B_1^2 input)
- DISCOTHEQUE¹ two dimensions - plane (XY, B_1^2 input)
 cylinder (RZ)
- TRIMHX¹¹ two dimensions - plane (hex, B_1^2 input)
 three dimensions - multilayered planes (hex, z)

Geometrically accurate mockups of most reactors can be obtained with the two- and three-dimensional codes.

The Space-Time calculations were performed with TRIMHX with the top axial boundary extended to include the measured vertical extrapolation distance. The data from the subcritical experiment at water height of 210.73 cm were arbitrarily chosen as the basis of comparison in normalizing the cross sections. The calculated eigenvalue with the unnormalized cross sections was 0.97620. Figure 3 shows the unnormalized calculated response compared to the measured data. The production cross sections, $\nu\Sigma_f$, of all multiplying materials were then increased uniformly by 1.31% causing k_{eff} to increase to 0.98885; the resulting good fit is shown in Figure 4. The uniform normalization is, of course, arbitrary. In the absence of any criterion for region-dependent normalization and assuming a well-known reactor composition, the uniform assumption is judged to be adequate.

No significant difference with the assumed source energy (thermal or fast) was found; the influence of source energy on this highly multiplying lattice appeared to be slight. Part of the discrepancy between calculation and experiment in the source buildup region (Figure 4) may be due to the use of a centered source in the calculations.

The same normalized cross section set, derived by fitting to the experiment at 210.73 cm, was used for the calculations at 174.71- and 153.13-cm water heights. The comparisons between the experimental and calculated data are shown in Figures 5 and 6, respectively. The eigenvalues were 0.96957 at 174.71 cm and 0.95026 at 153.13 cm. Again, the source buildup region has been poorly calculated. Moreover, it is questionable whether a fundamental mode was even reached in the experiment at 153.13 cm.

These problems were solved with a 60° symmetry assumption, 3 mesh points per cell, 20 axial mesh points, and a 6-group delayed neutron structure. The computer core used was 750K bytes. The time-step duration varied slightly in the different calculations. For the calculation of the 210.73-cm lattice, 83 minutes of CPU time on an IBM 360/195 facility were required to solve a 61 time-step problem with time-step durations varying from 0.1 to 25 msec.

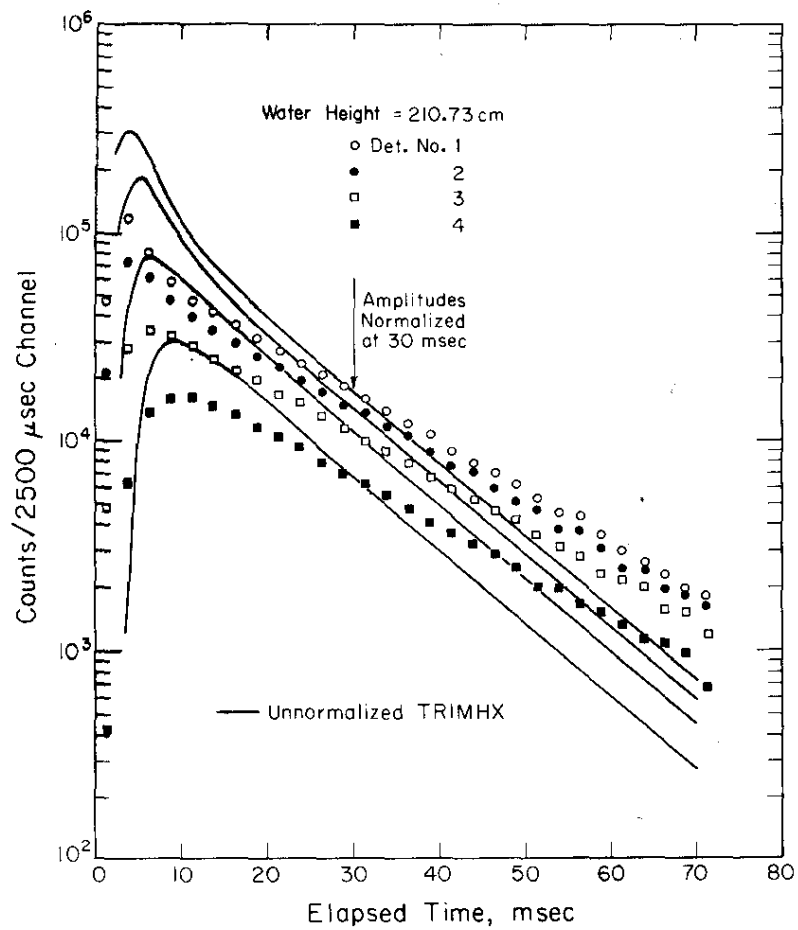


FIGURE 3 Measured and Calculated Prompt Neutron Responses in the Reflected Lattice at 210.73-cm Water Height - Unnormalized Diffusion Parameters

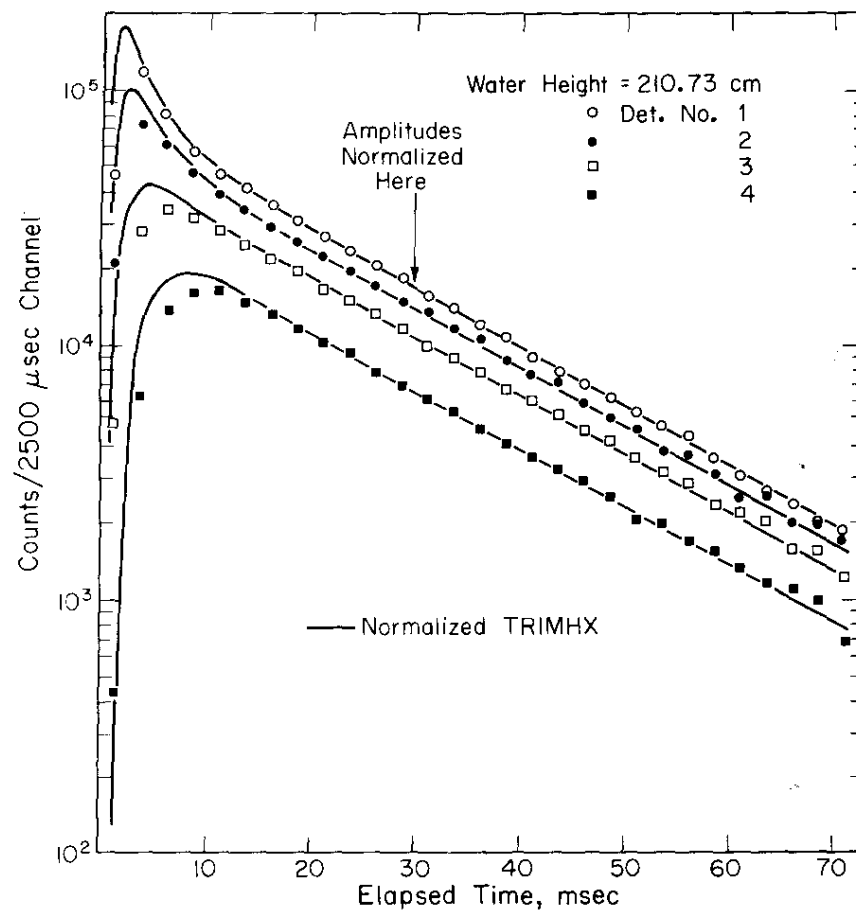


FIGURE 4 Measured and Calculated Prompt Neutron Responses in the Reflected Lattice at 210.73-cm Water Height - Normalized Diffusion Parameters

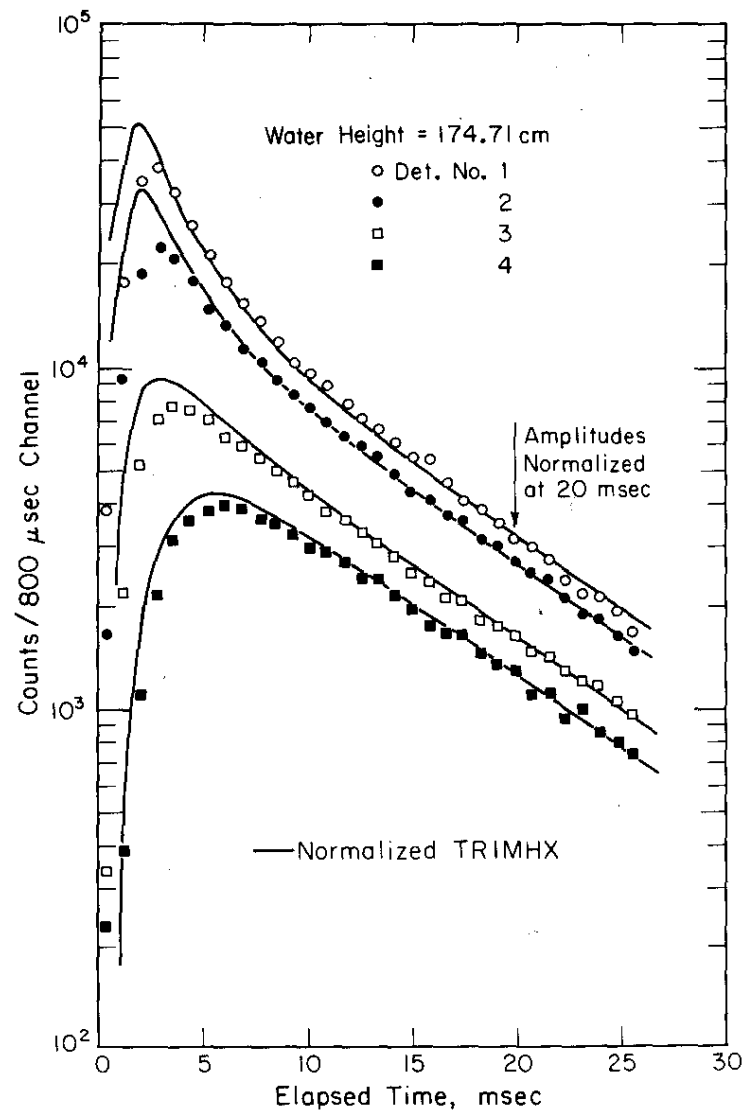


FIGURE 5 Measured and Calculated Prompt Neutron Responses in the Reflected Lattice at 174.71-cm Water Height - Normalized Diffusion Parameters

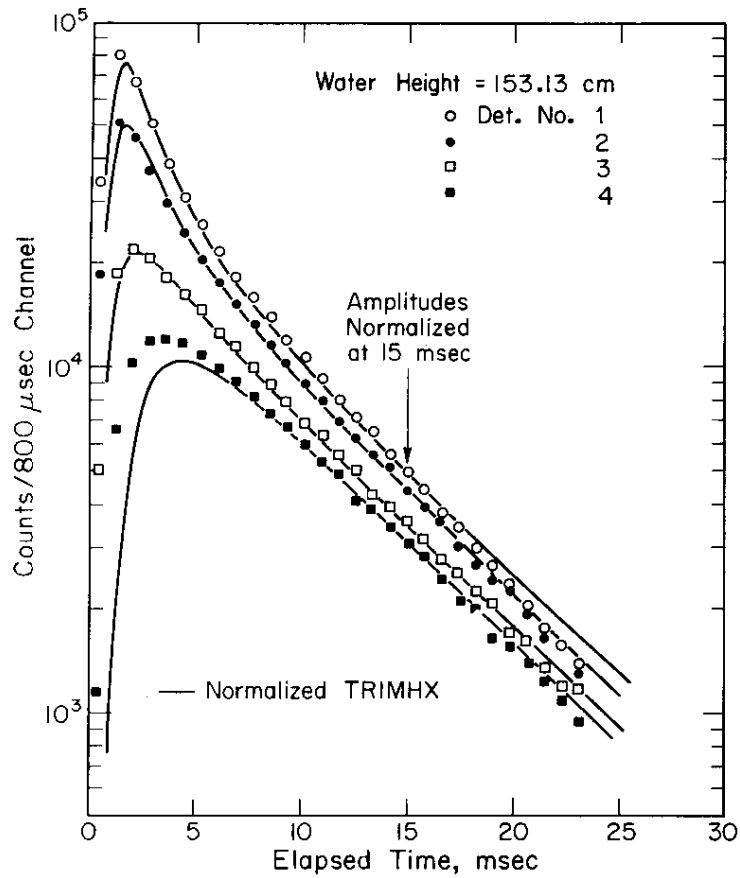


FIGURE 6 Measured and Calculated Prompt Neutron Responses in the Reflected Lattice at 153.13-cm Water Height - Normalized Diffusion Parameters

The other methods for analyzing pulsed neutron experiments reviewed in this report involve more assumptions than does the Space-Time method. Thus, the Space-Time method, where applicable, should be the most general and provide the best possible accuracy for the reactivity. However, the Space-Time method may not always be applicable. Obviously, detailed knowledge of the interior of the reactor is necessary for the calculations. The method is not suited to those field applications where little is known of the subcritical assembly composition. In principle, the entire prompt response, or any part of it, would be suitable for normalizing the calculation. However, the limitations of the source treatment reported here prevent the use of data far from the fundamental mode decay.

INHOUR METHOD

Preskitt et al.² have derived the Inhour method of pulsed neutron analysis for subcritical reactivity that is expressed in the equation

$$\rho_s = \alpha_o^p \bar{\Lambda}_o + \beta_{T_o} \quad (1)$$

where

$$\rho_s = \text{static reactivity from delayed critical} = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$$

$$\alpha_o^p = \text{measured prompt neutron decay constant, fundamental mode}$$

$$\bar{\Lambda}_o = \text{calculated prompt neutron generation time, fundamental mode}$$

$$\beta_{T_o} = \text{calculated effective delayed neutron fraction, fundamental mode}$$

$\bar{\Lambda}_o$ and β_{T_o} are calculated with static reactor codes, and α_o^p is a purely measured quantity. Since a time-dependent calculation is not necessary, the Inhour method affords a potentially large savings in computer time over the Space-Time method. The calculational model does need to be just as accurate as that used in a Space-Time analysis. Moreover, the implied additional assumption of Equation 1 is that the fundamental mode decay constant exists and is measurable. Thus, in principle, the Inhour method is not as general as the Space-Time method. The validity of the Inhour method can be judged better after reviewing Preskitt's derivation of Equation 1.

Summary of Preskitt's Derivation

The space-time and energy-dependent equations that govern the behavior of the neutron flux and precursors under the influence of a time-varying external source are:

$$\frac{1}{v(E)} \frac{\partial \Phi(\vec{r}, E, \vec{\Omega}, t)}{\partial t} = [(1 - \beta_T) \chi_p(E) \bar{P} - \bar{D}] \Phi(\vec{r}, E, \vec{\Omega}, t) + \sum_{i=1}^M \chi_i(E) \lambda_i C_i(\vec{r}, t) + S(\vec{r}, E, \vec{\Omega}, t) \quad (2)$$

and

$$\frac{\partial C_i(\vec{r}, t)}{\partial t} = \beta_i \bar{P} \Phi(\vec{r}, E, \vec{\Omega}, t) - \lambda_i C_i(\vec{r}, t) \quad (3)$$

where

$\Phi(\vec{r}, E, \vec{\Omega}, t)$ = total neutron flux (prompt plus delayed).

$C_i(\vec{r}, t)$ = concentration of the i^{th} delayed neutron precursor

\bar{P} = fission production operator

\bar{D} = destruction operator for the net scattering, absorption, and leakage

λ_i = decay constant of the i^{th} delayed neutron precursor (M total delayed groups)

β_i = fractional yield of the i^{th} delayed neutron precursor

β_T = the total delayed neutron fraction, $\sum_{i=1}^M \beta_i$

χ_p = linear operator describing the fission spectrum of prompt neutrons

χ_i = linear operator describing the energy spectrum of neutrons from the i^{th} precursor

$S(\vec{r}, E, \vec{\Omega}, t)$ = extraneous neutron source

A harmonic expansion for the total neutron flux and for the precursors (assuming space and time separation) can be made as follows:

$$\begin{aligned}\Phi(\vec{r}, \vec{\Omega}, E, t) &= \sum_{n=1}^{\infty} \sum_{i=1}^M \Phi_{ni}(\vec{r}, \vec{\Omega}, E) e^{\alpha_{ni} t} \\ C(\vec{r}, t) &= \sum_{n=1}^{\infty} \sum_{i=1}^M C_{ni}(\vec{r}) e^{\alpha_{ni} t}\end{aligned}\quad (4)$$

By substituting Equations 4 into Equations 2 and 3, and by citing well known properties of the pulsed neutron experiment, Preskitt was able to show that the static reactivity could be very closely approximated by

$$\rho_s = \alpha_o^p \frac{\langle \Phi_s^+, v^{-1} \phi_o^p \rangle}{\langle \Phi_s^+, \chi_T \bar{p} \phi_o^p \rangle} + \frac{\sum_{i=1}^M \langle \Phi_s^+, \chi_i \beta_i \bar{p} \phi_o^p \rangle}{\langle \Phi_s^+, \chi_T \bar{p} \phi_o^p \rangle} \quad (5)$$

where ϕ_o^p is the forward eigenfunction of the prompt neutron flux decaying in the fundamental mode with decay constant α_o^p ; Φ_s^+ is the adjoint of the static total flux eigenfunction, and χ_T is the total neutron energy spectrum operator. The bra-ket notation indicates multivariable integration over space and energy. The delayed neutron fraction in the prompt fundamental mode is defined as

$$\beta_{T_o} \equiv \frac{\sum_{i=1}^M \langle \Phi_s^+, \chi_i \beta_i \bar{p} \phi_o^p \rangle}{\langle \Phi_s^+, \chi_T \bar{p} \phi_o^p \rangle} \quad (6)$$

The definition of the fundamental prompt neutron generation time is

$$\bar{\Lambda}_o \equiv \frac{\langle \Phi_s^+, v^{-1} \phi_o^p \rangle}{\langle \Phi_s^+, \chi_T \bar{p} \phi_o^p \rangle} \quad (7)$$

Thus Equation 5 simplifies to Equation 1.

A more detailed derivation of Equation 5 is given in the Appendix.

Normalization

In Preskitt's original paper, the prompt neutron generation time was viewed as a purely calculated quantity. Thus, the accuracy of the generation time calculation effectively limited the accuracy of the Inhour method. However, this limitation can be avoided. The generation time calculation can be normalized by a procedure analogous to the Space-Time method normalization.

Equation 7 involves integrals over energy and reactor volume in both numerator and denominator. The adjoint of the normal static eigenfunction, Φ_S^+ , and the direct prompt eigenfunction, Φ_O^P , are found with a static diffusion theory code such as GRIMHX, the static counterpart of TRIMHX.¹¹ The static adjoint solution is straightforward. The prompt solution is obtained when the input macroscopic capture cross section of each region, by group, is replaced by $\Sigma_c + \alpha_O^P/v$, where α_O^P is the measured fundamental decay constant, and v is the calculated group average velocity. The total neutron fission spectrum is also replaced by $(1-\beta_{TO})\chi_p$. However, all other cross sections and diffusion coefficients remain the same as in the normal static solution. These substitutions convert the normal static eigenequation into the prompt neutron eigenequation.

If the input cross sections were correct, the eigenvalue of the prompt neutron solution, k_{eff}^P , would be unity. Most probably, this result will not be obtained in the first prompt calculation. However, the cross sections can be normalized to force the prompt eigenvalue to unity simply by dividing all the regional $v\Sigma_f$ values by k_{eff}^P from the first calculation. Equation 7 can then be used to compute the normalized generation time using the normalized cross sections and eigenfunctions. Then Equation 1 can be used to compute the subcritical reactivity.

An alternative method avoiding Equations 7 and 1 is simply to compute the normal static eigenvalue directly in a k_{eff} search with the normalized cross sections as derived in the above paragraph. The resulting reactivity values by either method are equivalent.

Application to a Reflected Lattice

The reflected lattice experiments described in the discussion of the Space-Time method can also serve as a test of the Inhour method. It seems reasonably clear that a fundamental mode was established in the experimental data for the cases at water heights of 210.73 and 174.71 cm. If the TRIMHX calculation for the 153.13-cm case is examined well beyond the end of the measured prompt data, it is found that the decay constant is continuing to decrease. Thus, the Inhour method can be applied only to the 210.73- and 174.71-cm cases with any rigor.

GRIMHX was used to provide the static calculations of ϕ_s^+ and ϕ_o^p . The cross sections normalized in the Space-Time analysis were used unaltered in the GRIMHX calculations. The resulting generation times and reactivities are listed in Table 1 and compared to the Space-Time method reactivities. The calculated prompt k_{eff}^p is also listed to provide a check on the adequacy of the normalization.

The Space-Time and Inhour method reactivities agree very closely, as indeed they must if the calculational model is valid. The change in generation time with subcriticality simply reflects the increasing neutron importance of the reflector with its more thermal spectrum. The small deviation of the prompt eigenvalue, $k_{eff}^p(\text{calc})$, from unity demonstrates the essential correctness of the normalized diffusion parameters generated in the Space-Time normalization. Almost the same cross sections would have been generated in an Inhour normalization.

TABLE 1

Derived Reactivities for the Reflected Lattice
by Space-Time and Inhour Methods

| Water Height, cm | Reactivity | | $\bar{\Lambda}_o$, μsec | Avg α_o^p , sec ⁻¹ | $k_{eff}^p(\text{calc})$ |
|------------------------|------------|---------------------|-----------------------------|---|--------------------------|
| | Space-Time | Inhour ^a | | | |
| 210.73 | -0.01128 | -0.01132 | 357.6 | - 52.76 | 0.9998 |
| 174.71 | -0.03139 | -0.03268 | 399.1 | -100.8 | 1.0015 |

a. (Inhour) $\rho_s = \alpha_o^p \bar{\Lambda}_o + \beta_{T_o}$, where $\beta_{T_o} = 0.007576$.

SIMMONS-KING METHOD

The Simmons-King method³ also uses Equation 1 to relate the reactivity and the fundamental mode decay constant. Historically, it was developed before the Inhour method but is, in reality, simply an attempt to avoid using a calculated generation time. Equation 1 is rewritten as

$$\alpha_o^p = \frac{\rho_s - \beta_{T_o}}{\bar{\Lambda}_o}$$

If the reactor is pulsed at delayed critical, $\rho_s = 0$, and

$$\alpha_c^p = \frac{-\beta_{T_o}}{\bar{\Lambda}_c}$$

Simmons and King then assume that $\bar{\Lambda}_c \approx \bar{\Lambda}_o$. Thus, the following approximation is obtained:

$$\bar{\Lambda}_o \approx - \frac{\beta_{T_o}}{\alpha_c^p}$$

and

$$\rho_s \approx \beta_{T_o} \left(- \frac{\alpha_o}{\alpha_c} + 1 \right)$$

Several difficulties are associated with the Simmons-King method. First, there is the practical difficulty of measuring the decay constant at delayed critical, for with every source burst the flux level increases. Second, the assumption that $\bar{\Lambda}_c = \bar{\Lambda}_o$ is often a poor approximation. Table 1 demonstrates that $\bar{\Lambda}_o$ is varying with the degree of subcriticality in the reflected lattice. Finally, if the reactor can be made critical, it isn't clear that pulsing is even necessary. Reactivity worths could probably be obtained from critical measurements alone. For these reasons, the Simmons-King method has not been used at the Savannah River Laboratory. However, it has been used by others.

METHODS YIELDING SPATIALLY DEPENDENT RESULTS

Historically, the methods that yield spatially dependent results are among the earliest methods devised for the deduction of subcritical reactivity from pulsed neutron data. However, they are all affected in varying degrees by harmonics and kinetic distortion. It will be seen that this is a direct consequence of the assumption of space and time separability and the use of both prompt and delayed neutron data from the experiment.

The spatially dependent methods of Sjöstrand,⁴ Gozani,⁵ and Garelis-Russell⁶ all involve some form of integrals over time of both the prompt and delayed neutron responses to the source bursts. Thus, they are sometimes referred to as area methods. Becker and Quisenberry⁷ have devised correction factors for kinetic distortion in reflected reactors to be applied to the reported reactivities from the area methods. Their treatment assumes that only the fundamental mode of decay exists for both the prompt and the delayed neutrons. Thus, harmonic distortion is not accounted for and can cause significant errors in B-Q corrected area method reactivities, as will be shown later in specific examples.

Preskitt et al.² have given a more complete derivation than that of Becker and Quisenberry⁷ that includes the kinetic distortion correction factors in the presence of harmonic distortion. However, to apply these factors, it is necessary to calculate harmonic flux distributions, a difficult if not impossible task. Preskitt's results go over to the B-Q results in the limit of the fundamental mode only.

The description of kinetic distortion given by Becker and Quisenberry is the starting point of this section. Their basic assumption that harmonic distortions do not exist is seldom met, but nevertheless their treatment allows an understanding of the origin of kinetic distortion without the extraneous complications of harmonic distortions. Moreover, their derived expression for the subcritical reactivity leads very simply to the Sjöstrand and Gozani expressions if it is assumed that kinetic distortion is absent. When kinetic distortion is present, the conventional methods have a simple correction factor applied.

SUMMARY OF THE BECKER-QUISENBERY DERIVATION

The mathematical concept of kinetic distortion independent of prompt and delayed distortion is developed in this derivation. The total neutron flux is divided into prompt and delayed neutron fluxes.

$$\Phi(\vec{r}, \vec{\Omega}, E, t) = \phi_p(\vec{r}, \vec{\Omega}, E, t) + \phi_d(\vec{r}, \vec{\Omega}, E, t) \quad (8)$$

When Equation 8 is substituted into Equations 2 and 3, three new equations result (dropping the space, energy, and time notation).

$$\frac{1}{v} \frac{\partial \phi_p}{\partial t} = \left[(1 - \beta_T) \chi_p \bar{P} - \bar{D} \right] \phi_p + S \quad (9)$$

$$\frac{1}{v} \frac{\partial \phi_d}{\partial t} = \left[(1 - \beta_T) \chi_p \bar{P} - \bar{D} \right] \phi_d + \sum_{i=1}^M \lambda_i \lambda_i C_i \quad (10)$$

$$\frac{\partial C_i}{\partial t} = \beta_i \bar{P} \phi_p + \beta_i \bar{P} \phi_d - \lambda_i C_i \quad (11)$$

At this point a few assumptions are made that are usually met in the pulsing experiments. First, assume that the reactor is pulsed with period T between pulses, and integrate Equations 9-11 over a single period. Second, assume that the pulse period T is very long compared to the time required for essentially complete decay of the prompt neutrons caused by the source burst. Thus,

$$\phi_p(0) \approx \phi_p(T)$$

Third, assume that pulsing has been going on for some time before the pulse at time = 0+, in fact long enough for delayed neutron equilibrium to be reached.

$$C_i(0) = C_i(T)$$

Implementation of these three assumptions and elimination of $\lambda_i \int_0^T C_i dt$ reduce Equations 9-11 to two equations involving only ϕ_p , ϕ_d , and S.

$$0 = \left[(1 - \beta_T) \chi_p \bar{P} - \bar{D} \right] \int_0^T \phi_p dt + \int_0^T S dt \quad (12)$$

$$0 = \left[\chi_T \bar{P} - \bar{D} \right] \int_0^T \phi_d dt + \sum_{i=1}^M \chi_i \beta_i \bar{P} \int_0^T \phi_p dt \quad (13)$$

At this point assume that the prompt and delayed fluxes are each separable into functions of space and time. Furthermore, assume that the fundamental mode dominates (no harmonics).

$$\phi_p(\vec{r}, \vec{\Omega}, E, t) = \psi_p(\vec{r}, \vec{\Omega}, E) T_p(t) \quad (14)$$

and

$$\phi_d(\vec{r}, \vec{\Omega}, E, t) = \psi_d(\vec{r}, \vec{\Omega}, E) T_d(t) \quad (15)$$

Substituting Equations 14 and 15 into 13 yields

$$\frac{\int_0^T T_p(t) dt}{\int_0^T T_d(t) dt} = - \frac{\left[\chi_T \bar{P} - \bar{D} \right] \psi_d(\vec{r}, \vec{\Omega}, E)}{\sum_{i=1}^M \beta_i \chi_i \bar{P} \psi_p(\vec{r}, \vec{\Omega}, E)}$$

Arbitrarily weighting the numerator and denominator of the right side of Equation 15 by the same weighting function $W(\vec{r}, \vec{\Omega}, E)$ and then integrating both numerator and denominator over all space and energy yield

$$\frac{\int_0^T T_p(t) dt}{\int_0^T T_d(t) dt} = - \frac{\left\langle W(\vec{r}, \vec{\Omega}, E), \left[\chi_T \bar{P} - \bar{D} \right] \psi_d(\vec{r}, \vec{\Omega}, E) \right\rangle}{\sum_{i=1}^M \left\langle W(\vec{r}, \vec{\Omega}, E), \beta_i \chi_i \bar{P} \psi_p(\vec{r}, \vec{\Omega}, E) \right\rangle} \quad (16)$$

Again, the bra-ket notation indicates multivariable integration.

A convenient weighting function is the adjoint of the delayed neutron distribution, ψ_d^+ . With $\psi_d^+ = W$ in Equation 16 and the definition for reactivity measured from a base of zero at delayed critical,

$$\rho_s \equiv \frac{\langle \psi_d^+, [\chi_T \bar{P} - \bar{D}] \psi_d \rangle}{\langle \psi_d^+, \chi_T \bar{P} \psi_d \rangle} \quad (17)$$

then Equation 16 transforms into

$$\frac{\int_0^T T_p(t) dt}{\int_0^T T_d(t) dt} = -\rho_s \left[\frac{\langle \psi_d^+(\vec{r}, \vec{\Omega}, E), \chi_T \bar{P} \psi_d(\vec{r}, \vec{\Omega}, E) \rangle}{\sum_{i=1}^M \langle \psi_d^+(\vec{r}, \vec{\Omega}, E), \beta_i \chi_i \bar{P} \psi_p(\vec{r}, \vec{\Omega}, E) \rangle} \right] \quad (18)$$

The term in the bracket on the right-hand side of Equation 18 is similar to the definition of the inverse effective delayed neutron fraction $\bar{\beta}_{T_o}^{-1}$, where

$$\bar{\beta}_{T_o} \equiv \frac{\sum_{i=1}^M \langle \psi_d^+(\vec{r}, \vec{\Omega}, E), \beta_i \chi_i \bar{P} \psi_p(\vec{r}, \vec{\Omega}, E) \rangle}{\langle \psi_d^+(\vec{r}, \vec{\Omega}, E), \chi_T \bar{P} \psi_d(\vec{r}, \vec{\Omega}, E) \rangle}$$

The appearance of $\psi_d(\vec{r}, \vec{\Omega}, E)$ prevents exact equivalence. However, the integral involving $\psi_d(r, \Omega, E)$ can be normalized by the following condition:

$$\langle \psi_d^+(\vec{r}, \vec{\Omega}, E), \chi_T \bar{P} \psi_d(\vec{r}, \vec{\Omega}, E) \rangle N_c = \langle \psi_d^+(\vec{r}, \vec{\Omega}, E), \chi_T \bar{P} \psi_p(\vec{r}, \vec{\Omega}, E) \rangle \quad (19)$$

With this condition Equation 18 becomes

$$\frac{\int_0^T T_p(t) dt}{\int_0^T T_d(t) dt} = -\frac{\rho_s}{N_c \bar{\beta}_{T_o}} \quad (20)$$

In terms of ratios of flux integrals

$$\frac{\int_0^T \phi_p(\vec{r}, \vec{\Omega}, E, t) dt}{\int_0^T \phi_d(\vec{r}, \vec{\Omega}, E, t) dt} = - \frac{\rho_s}{N_c \beta_{T_o}} \left[\frac{\psi_p(\vec{r}, \vec{\Omega}, E)}{\psi_d(\vec{r}, \vec{\Omega}, E)} \right] \quad (21)$$

In the following sections, approximations developed by Sjöstrand, Gozani, and Garelis-Russell are given. The Sjöstrand method follows directly from Equation 21 if kinetic distortion is ignored. The Gozani method extends the Sjöstrand method to account for prompt harmonics but still ignores kinetic distortion. Finally, the Garelis-Russell method is shown to be similar to the Simmons-King treatment except that the integral quantities in Equation 21 are used.

SJÖSTRAND APPROXIMATION

Sjöstrand has derived the following expression for the subcritical reactivity in terms of the prompt and delayed flux integrals.⁴

$$- \frac{\rho_s}{\beta_{T_o}} = \frac{\int_0^T \phi_p dt}{\int_0^T \phi_d dt} \quad (22)$$

The integrals $\int_0^T \phi_p dt$ and $\int_0^T \phi_d dt$ are evaluated from the measured data alone; i.e., the prompt and delayed counts from the detector are separated, and the area under each curve is measured separately. No calculated flux responses are used.

For Equation 22 to be valid, three conditions must be met. No kinetic distortion may exist; i.e. $N_c \psi_d = \psi_p$, and Equations 21 and 22 are identical. No prompt neutron harmonics may exist. Finally, no delayed neutron harmonics may exist. The first condition is often met as long as no reflectors are present. The last two conditions arise from the assumption of fundamental mode dominance in the derivation of Equation 21; only rarely are these two conditions met.

Considerable caution should be used in applying the Sjöstrand method because the limitations are so rarely satisfied. However, Kosály has shown that, in reflected reactors, the Sjöstrand inaccuracies are often smaller than might be anticipated.¹²

GOZANI METHOD

Like Sjöstrand's, Gozani's derivation⁵ does not allow for kinetic distortion; i.e., $\psi_p = \psi_d N_c$. Gozani also assumes that delayed harmonics can be neglected. However, Gozani's method does specifically deal with the problem of prompt neutron harmonic distortion by the following procedure. First, the fundamental mode decay constant α is measured by considering only data from time t_α , after all higher order harmonics have disappeared. The fundamental is then extrapolated back to time $t = 0$, or

$$\int_0^T \phi_p \text{ (fundamental) } dt = e^{-\alpha t_\alpha} \int_{t_\alpha}^T \phi_p dt$$

Gozani's form of Equation 22 is thus

$$-\frac{\rho_s}{\beta_{T_0}} = \frac{e^{-\alpha t_\alpha} \int_{t_\alpha}^T \phi_p dt}{\int_0^T \phi_d dt}$$

Further refinements and corrections that Gozani has made take into account the finite width of the source burst, d ; the decay of the delayed neutrons over the pulse period, T , involving $\bar{\lambda}$, the average delayed neutron decay constant; and the missed portion of the prompt integral by cutting off the integration at T . These corrections yield Gozani's equation.

$$-\frac{\rho_s}{\beta_{T_0}} = \left[\frac{C_d(t_d)}{1 + \frac{2\bar{\lambda}}{|\alpha|}} \right] \left[\frac{1 - e^{|\alpha|T}}{e^{|\alpha|d} - 1} \right] \left[\frac{d}{T} \frac{N_p(0)}{N_d(t_d)} \right] \quad (23)$$

where

$N_p(0)$ = measured prompt fundamental amplitude by extrapolation backwards to $t = 0$

$N_d(t_d)$ = measured delayed amplitude at $t = t_d$ well within the delayed tail

$$C_d(t_d) \approx 1 + \left(\frac{T}{2} - t_d \right) \frac{\bar{\lambda} |\rho_s|}{1 + |\rho_s|} \text{ (delayed correction factor)}$$

$C_d(t_d)$ and ρ_s are solved for iteratively; one iteration is usually sufficient.

Delayed harmonic distortion is not treated in Gozani's derivation; and moreover, the method will be seen to break down for reflected lattices when both kinetic and delayed harmonic distortions are present.

GARELIS-RUSSELL METHOD

The Garelis-Russell method⁶ is somewhat akin to the Simmons-King method.

$$\frac{\rho_s}{\beta_{T_0}} = - \frac{\alpha}{\alpha_c} + 1 \quad (24)$$

where

$$\alpha_c = - \frac{\beta_{T_0}}{\Lambda_c}$$

was supposed to be measured by pulsing a critical reactor. Garelis and Russell have devised another technique, using only the pulsed subcritical data for obtaining α_c , more like an area analysis treatment. The end result is

$$\int_0^T \left(e^{\alpha_c t} - 1 \right) \phi_p dt = \int_0^T \phi_d dt$$

where ϕ_p and ϕ_d are the measured prompt and delayed detector responses. The decay constant α_c is to be searched for iteratively. Once α_c is found, Equation 24 is used to find the reactivity.

Gozani has given a more accurate form of the Garelis-Russell equation that corrects for the effects of finite burst width and delayed neutron decay.⁵

$$\frac{\alpha_c d}{\alpha_c d - 1} \int_0^T (e^{\alpha_c t} - 1) \phi_p dt = \frac{TN_d(t_d)}{C_d(t_d)} \quad (25)$$

The delayed neutron correction factor $C_d(t_d)$ is defined just as it was in the Gozani treatment.

The Garelis-Russell derivation has ignored the problem of kinetic distortion. The results will also be affected by both prompt and delayed neutron harmonics. Like the Gozani method, the Garelis-Russell method is found to break down for reflected lattices with significant kinetic as well as harmonic distortions.

APPLICATION TO AN UNREFLECTED LATTICE

A pulsing experiment on an unreflected lattice has been chosen as one example to show the use of the area methods of analyses because there is no kinetic distortion.¹ The only distortions are due to prompt and delayed neutron harmonics.

The unreflected lattice (Figure 7) was formed of slightly enriched fuel assemblies distributed uniformly in the Subcritical Experiment (SE) in D₂O moderator and was pulsed from the top by an external ³H(d,n)⁴He source of 14-MeV neutrons. Figure 8 shows the recorded prompt neutron responses (obtained from the raw data after deadtime correction and subtraction of average noise and delayed neutron tail). Thus, the plotted values are the prompt neutron responses of the two detectors. The smooth curve is a normalized DISCOTHEQUE Space-Time calculation.

Both Gozani and Garelis-Russell methods are valid only when a fundamental mode has been established in the experimentally observed prompt neutron decay. The data in Figure 8 clearly show that this has occurred by 10 msec after the burst initiation. The measured decay constants used in the Gozani and Garelis-Russell treatments are taken from data accumulated after 10 msec.

The results of the three conventional area methods are shown in Table 2 and are compared to the Space-Time result obtained with the DISCOTHEQUE-DISCO combination of codes. Note the smaller space dependence (on detector location) of the Gozani and Garelis-Russell values when compared to the Sjöstrand value. Note also that the direction of discrepancy of the Gozani method from the mean of the two Gozani values is always opposite that of the Garelis-Russell method compared to its mean for each detector.

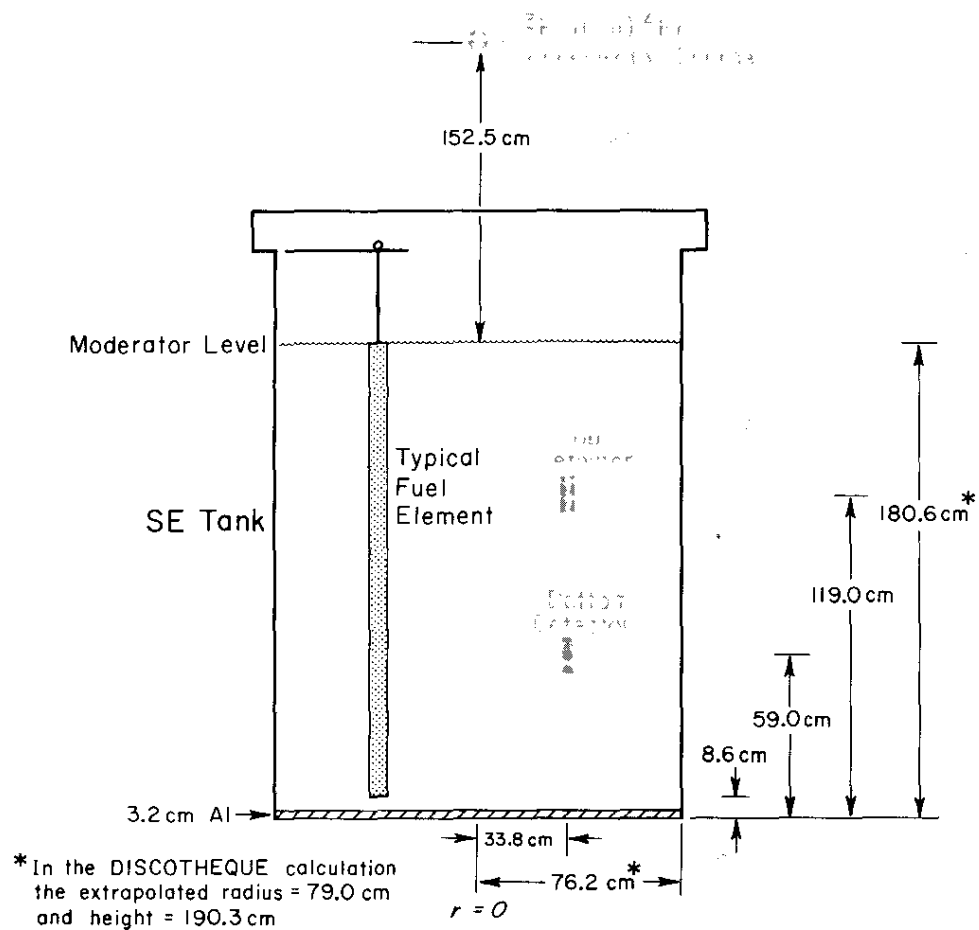


FIGURE 7 Experimental Arrangement for the Unreflected Lattice in the SE

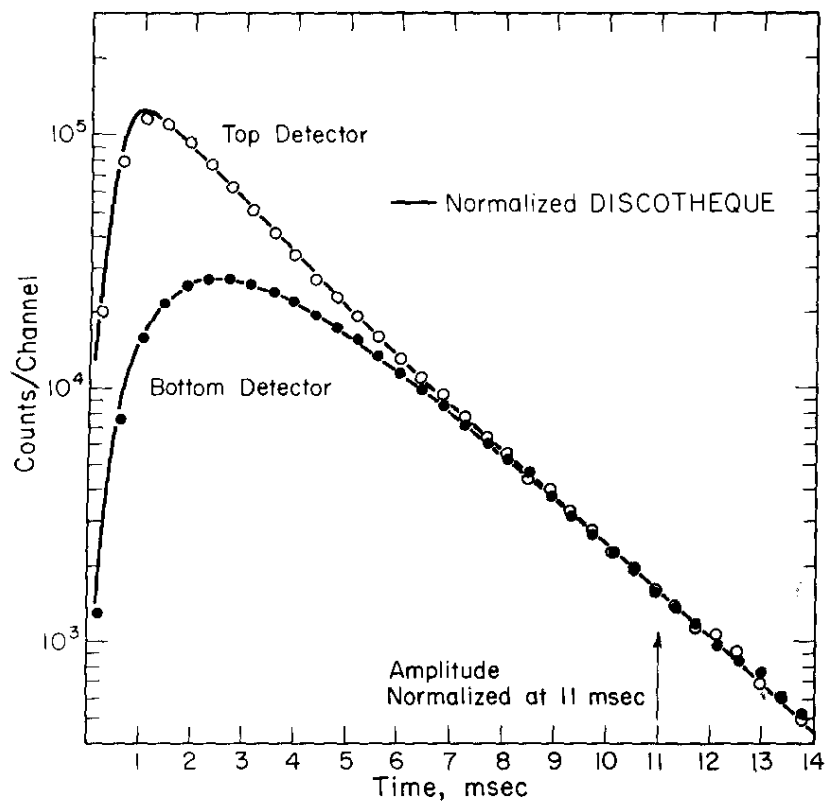


FIGURE 8 Measured and Calculated Prompt Neutron Responses in the Unreflected Lattice - Normalized Diffusion Parameters

TABLE 2

Area and Space-Time Method Reactivities for the Unreflected Lattice

| <u>Space-Time</u> | <u>Detector</u> | <u>Reactivity^a</u> | | |
|-------------------|-----------------|-------------------------------|------------------------|------------------|
| | | <u>Gozani</u> | <u>Garelis-Russell</u> | <u>Sjöstrand</u> |
| -0.0865 ± 0.0020 | Top | -0.0759 | -0.0856 | -0.0870 |
| | Bottom | -0.0980 | -0.0758 | -0.0481 |
| | Mean | -0.0870 | -0.0807 | -0.0676 |

a. (Reactivity) $\rho = \beta \beta$, where $\beta = 0.007982$.

Gozani has pointed out that in deeply subcritical states of unreflected reactors, higher order harmonics do affect the derived reactivities in both the Gozani and the Garelis-Russell methods.¹³ In the Gozani version, the distortion is caused by delayed neutron harmonics. In the Garelis-Russell version, both delayed neutron harmonics and prompt neutron harmonics affect the derivation of α_c (Equation 25).

In a complicated series of arguments, Gozani shows that where positive harmonics predominate (near the source)

$$|\rho(\text{Go.})| < |\rho| < |\rho(\text{GR})|$$

where ρ is the true reactivity. However, where negative harmonics dominate (far from the source)

$$|\rho(\text{Go.})| > |\rho| > |\rho(\text{GR})|$$

Thus, a better approximation for the actual reactivity of an unreflected system can be found by the "bracket procedure" where

$$\rho(\text{bracket}) = 1/2[\rho(\text{Go.}) + \rho(\text{GR})] \quad (26)$$

The bracket procedure applied to the data of Table 2 yields

$$\rho(\text{top-bracket}) = -0.0807$$

and

$$\rho(\text{bottom-bracket}) = -0.0869$$

These values compare well with the Space-Time value of -0.0865 (presumably the most accurate).

CORRECTIONS FOR KINETIC DISTORTION

Becker and Quisenberry have given expressions for each of the area methods of analyses corrected for kinetic distortion.⁷

The corrected Sjöstrand equation is

$$\left. \frac{\rho_s}{\beta_{T_o}} \right|_{\text{corr. Sjö.}} = \left. \frac{\rho_s}{\beta_{T_o}} \right|_{\text{Sjö.}} \left(\frac{\psi_d(\vec{r}, \vec{\Omega}, E)}{\psi_p(\vec{r}, \vec{\Omega}, E)} \right)^{N_c} \quad (27)$$

Similarly, the corrected Gozani equation is

$$\left. \frac{\rho_s}{\beta_T} \right|_{\text{corr. Go.}} = \left. \frac{\rho_s}{\beta_T} \right|_{\text{Go.}} \left(\frac{\psi_d(\vec{r}, \vec{\Omega}, E)}{\psi_p(\vec{r}, \vec{\Omega}, E)} \right)^{N_c} \quad (28)$$

Finally, the Garelis-Russell equations are corrected as

$$\left. \frac{\rho_s}{\beta_T} \right|_{\text{corr. G-R}} = - \frac{\alpha}{\alpha_c} + 1$$

where

$$\left(\frac{\alpha_c^d}{\alpha_c^d - 1} \right) \int_0^T (e^{\alpha_c t} - 1) \phi_p dt = \left(\frac{T N_d(t_d)}{C_d(t_d)} \right) \left(\frac{\psi_p(\vec{r}, \vec{\Omega}, E)}{\psi_d(\vec{r}, \vec{\Omega}, E) N_c} \right) \quad (29)$$

It has been stated earlier that kinetic distortion does not exist for the unreflected uniform reactor but does exist for the reflected reactor. In the unreflected homogeneous reactor, the shapes of ψ_p and ψ_d are the same. The amplitudes may differ, but even if they do, that problem is taken care of in the normalization. Thus for the unreflected, homogeneous reactor,

$$N_c \psi_d = \psi_p$$

and

$$\frac{\int_0^T \phi_p(\vec{r}, \vec{\Omega}, E, t) dt}{\int_0^T \phi_d(\vec{r}, \vec{\Omega}, E, t) dt} = - \frac{\rho_s}{\beta_T}$$

This equation holds well even for inhomogeneous unreflected lattices as long as there are no gross cross section changes from one region to the next and as long as there are no prompt or delayed harmonic distortions.

Consider now the reflected lattice. To make the argument simple, assume a slab reactor in the one-group diffusion theory approximation. The greatest difference between ψ_p and ψ_d occurs in the reflector.⁷ In the reflector, the prompt neutron eigenequation is

$$\nabla^2 \psi_p^r - \left(\frac{\Sigma_a}{D} \right)^r \left[\frac{\alpha}{(v\Sigma_a)^r} + 1 \right] \psi_p^r = 0$$

whereas the delayed neutron eigenequation is

$$\nabla^2 \psi_d^r - \left(\frac{\Sigma_a}{D} \right)^r \psi_d^r = 0$$

Because Σ_a^r is usually small in reflectors, the correction term multiplying $\left(\frac{\Sigma_a}{D} \right)^r$ in the prompt equation is not 1 and in fact may reverse sign. Thus ψ_p will have a larger peak in the reflector than does ψ_d . For D₂O and graphite reflectors, the difference can be very large.

APPLICATION TO A REFLECTED LATTICE

The reflected lattice experiments detailed in the description of the Space-Time method are influenced by both harmonic distortion and kinetic distortion. Table 3 lists the reactivities found at the different water heights by the conventional area analyses without any correction being applied for the distortions. The Gozani and Garelis-Russell results for the 153.13-cm case are not listed because of the failure to establish the fundamental decay.

The procedures outlined on page 19 have been followed to compute the ratio $N_c \psi_d / \psi_p$. The results of correcting the Sjöstrand and Gozani methods for kinetic distortion by the Becker-Quisenberry approach are listed in Table 4. The Space-Time and Inhour results are listed for comparison.

As shown in Table 4, that the Becker-Quisenberry correction for kinetic distortion is generally insufficient to bring the area results into agreement with the Space-Time and Inhour results or even to self-consistency for the four detectors. It is interesting to observe that the Sjöstrand results for Detector 1 closest to the source and center of the core are in good agreement with the spatially independent results, but this agreement may be accidental. The progressively poorer agreement of the Sjöstrand result with increasing radius is due most probably to the unaccounted presence of prompt and delayed harmonics. The strong prompt harmonic distortion is visible in Figures 3-6. The error of the Gozani result is due most probably to the unaccounted presence of the delayed harmonics only.

TABLE 3

Area Method Reactivities for the
Reflected Lattice

| <u>Water Height, cm</u> | <u>Detector</u> | <u>Reactivity^a</u> | | |
|---------------------------------|-----------------|-------------------------------|-----------------------------|------------------|
| | | <u>Gozani</u> | <u>Garelis- Russell</u> | <u>Sjöstrand</u> |
| 210.73 | 1 | -0.00918 | -0.00855 | -0.01038 |
| | 2 | -0.00952 | -0.00939 | -0.00995 |
| | 3 | -0.01043 | -0.01049 | -0.00939 |
| | 4 | -0.01157 | -0.01068 | -0.00914 |
| 174.71 | 1 | -0.02278 | -0.02269 | -0.02805 |
| | 2 | -0.02430 | -0.02344 | -0.02669 |
| | 3 | -0.03122 | -0.02967 | -0.02971 |
| | 4 | -0.03818 | -0.03515 | -0.02972 |
| 153.13 | 1 | | | -0.04857 |
| | 2 | | | -0.04553 |
| | 3 | | | -0.04412 |
| | 4 | | | -0.05427 |

a. (Reactivity) $\rho = \beta_{\text{eff}}$, where $\beta_{\text{eff}} = 0.007576$.

TABLE 4

Space-Time, Inhour, and Kinetic Distortion Corrected
Area Method Reactivities for the Reflected Lattice

| <u>Water Height, cm</u> | <u>Reactivity</u> | | <u>Detector</u> | $N_c \frac{\psi_d}{\psi_p}$ | <u>Sjöstrand Corrected</u> | <u>Gozani Corrected</u> |
|---------------------------------|-------------------|---------------|-----------------|-----------------------------|--------------------------------|-----------------------------|
| | <u>Space-Time</u> | <u>Inhour</u> | | | | |
| 210.73 | -0.01128 | -0.01132 | 1 | 1.056 | -0.01096 | -0.00969 |
| | | | 2 | 1.034 | -0.01029 | -0.00984 |
| | | | 3 | 0.9509 | -0.00893 | -0.00992 |
| | | | 4 | 0.8328 | -0.00761 | -0.00964 |
| 174.71 | -0.03139 | -0.03268 | 1 | 1.117 | -0.03133 | -0.02545 |
| | | | 2 | 1.071 | -0.02858 | -0.02603 |
| | | | 3 | 0.9040 | -0.02686 | -0.02822 |
| | | | 4 | 0.6999 | -0.02080 | -0.02672 |

CONCLUSIONS

A wide variety of methods for analysis of pulsed neutron data for subcritical reactivity have been reviewed in this paper. In most cases, the methods have been applied to experimental data.

There is no single method which, a priori, will define an absolute measure of the reactivity with guaranteed freedom from error. Thus, in judging the relative accuracy of the methods, we conclude that the method that is the most sophisticated and the most general (involving the fewest assumptions) should be the criterion against which other less general methods are judged. The Space-Time method, where applicable, is that criterion.

Accuracy is not the sole criterion that determines the choice of method for analysis of pulsed data. Appropriateness to a given situation must also be considered. The decision-making flow diagram (Figure 9) shows which method should be used to extract the reactivity for various situations that may be encountered. The first question asked is whether sufficient detail of the subcritical assembly construction is known to allow a calculational model of the assembly. Obviously, methods such as the Space-Time or Inhour are not appropriate to "black box" applications, or to situations with irregular geometries.

Of the methods using calculational models, only the Space-Time method can be used for the case where the fundamental mode is not clearly measurable. In principle, comparisons of experiment and calculation anywhere in the prompt harmonic region would suffice to establish the reactivity. With the source treatment given in this paper, however, the best accuracy can be derived only with comparisons of experiment and calculation near the fundamental mode region; i.e. when the instantaneous decay constant is not changing too rapidly with time.

When the fundamental mode can be accurately measured, the Inhour method can be used as a substitute for the Space-Time method with little or no attendant loss of accuracy. Moreover, the Inhour method is much cheaper in terms of computing costs because only static calculations are involved.

In principle, the Inhour method can be used for both reflected and unreflected lattices with equally accurate results. However, for unreflected lattices, Gozani's bracket procedure should give reasonably accurate results without having to employ

any calculational modeling. For many applications, the bracket procedure may be sufficiently accurate.

The kinetic distortion corrections of Becker and Quisenberry assume that calculational modeling is possible. However, an example has been given that shows that this correction fails to remove the space dependence of conventional area-method reactivities when the reactor is large enough to have significant harmonic distortions. Moreover, the Inhour method is a much more accurate use of the same modeling. Therefore, the use of the Becker-Quisenberry correction is not recommended.

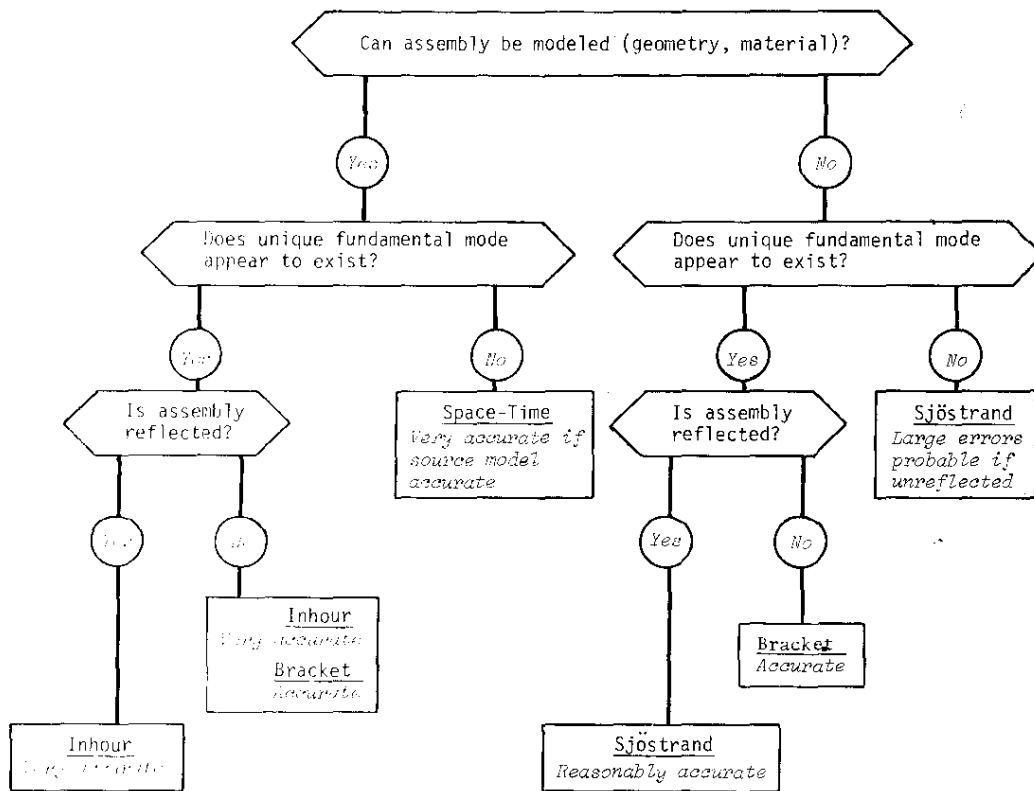


FIGURE 9 Preferred Methods of Analysis for Pulsing Situation

Recently, Valkó and Pór¹⁴ have suggested that complicated cores are too difficult to calculate with the precision needed for application of the Space-Time method (and by implication the Inhour method). Preskitt introduced the Inhour method in 1967 and analyzed quite successfully the startup pulsed neutron experiments on the Peach Bottom HTGR. The author has used both the Space-Time and Inhour methods for the analysis of numerous pulsed reactors at Savannah River. Accurate modeling appears to be determined, in the main, by having computer facilities of sufficient size to handle the large computer core requirements entailed in the description of real reactors. It is admitted that initial calculated results (decay constant, eigenvalues) will probably be in error to some degree. However, it is the function of the normalization procedures in the Space-Time method and the Inhour method to provide the correction which is necessary to make the calculational model nearly exact.

The "black box" application, where insufficient detail is known about the interior of the subcritical assembly, precludes calculational modeling. Too small a computer facility, likewise, may preclude modeling. The more conventional area methods of pulsed neutron analysis may still be used but with increased risk of analysis error. The next question is whether a fundamental mode appears to exist and whether it is clearly measurable. If the fundamental mode is not clearly measurable, only the Sjöstrand method is applicable. All the other methods (Gozani, Garelis-Russell) use the measured decay constant. However, in large unreflected lattices with significant prompt and delayed neutron harmonics, large errors will be probable. For small reactors, where the harmonics will be small, the Sjöstrand method will achieve reasonably accurate results.

If the subcritical assembly is known to be reasonably homogeneous (specifically precluding low absorbing, low leakage regions such as reflectors), Gozani's bracket procedure is recommended. None of the area methods will produce very accurate reactivities for large reflected lattices. However, the Sjöstrand method does appear somewhat more accurate than either the Gozani or the Garelis-Russell methods.¹²

In those applications involving use of the conventional area analyses, the flux response should be sampled at many places in the assembly to determine the space dependence of the reported reactivity. This procedure will help to determine the limits of confidence in the results of the measurements.

APPENDIX

Preskitt's Detailed Derivation of the Inhour Method

Substitute Equations 4 into Equations 2 and 3 of the main text; restrict attention to time after completion of the source burst; and eliminate C_{ni} . The result is

$$0 = \left[\left(\frac{-\alpha_{ni}}{v} - \bar{D} \right) + (1-\beta_T)\chi_p \bar{P} + \sum_{i=1}^M \chi_i \lambda_i \frac{\beta_i \bar{P}}{(\lambda_i + \alpha_{ni} \bar{I})} \right] \phi_{ni} \quad (A1)$$

where \bar{I} is the identity operator.

The quantity χ_k is defined

$$\chi_k \equiv (1-\beta_T)\chi_p + \sum_{i=1}^M \chi_i \lambda_i \frac{\beta_i}{(\lambda_i + \alpha_{ni} \bar{I})} \quad (A2)$$

as the kinetic spectrum operator.

Equations 2 and 3 have evolved to the much simpler form

$$0 = \left[\left(\frac{-\alpha_{ni}}{v} - \bar{D} \right) + \chi_k \bar{P} \right] \phi_{ni} \quad (A3)$$

The pulsed neutron experiment has two very distinct time domains. Just after the source burst, the prompt neutrons are far more numerous than the delayed neutrons. Thus, $\phi \approx \phi^p$ in most of the prompt decay region. All the prompt mode decay constants are very much larger than any of the λ_i values, the precursor decay constants. Thus, in the prompt region

$$\chi_k \approx (1-\beta_T)\chi_p$$

and the eigenequation for the prompt neutrons is approximated by

$$0 = \left[\left(\frac{-\alpha_{ni}}{v} - \bar{D} \right) + (1-\beta_T)\chi_p \bar{P} \right] \phi_{ni}^p \quad (A4)$$

The other time domain of importance is the delayed neutron region where $\phi \approx \phi^d$. In this region, the flux is decaying very slowly, in fact at rates similar to the precursor decay constants λ_i . Thus, the delayed modes must have decay constants that cluster about the λ_i values. But because $\lambda_i \ll \nu$, and the diffusion coefficients are on the order of unity, then in the delayed region $\frac{\alpha_{ni}}{\nu} \ll \bar{D}$. The delayed eigenequation is given closely by

$$0 = \left(-\bar{D} + \chi_k \bar{P} \right) \phi_{ni}^d$$

But in the limit that α_{ni} is very small, the kinetic spectrum operator (Equation A2) approaches the total fission spectrum operator defined by

$$\chi_T \equiv (1 - \beta_T) \chi_p + \sum_{i=1}^M \chi_i \beta_i \quad (A5)$$

Thus,

$$0 = \left(-\bar{D} + \chi_T \bar{P} \right) \phi_{ni}^d \quad (A6)$$

Equations A4 and A6 are simply static equations. In principle, static codes can be used to solve for all ϕ_{ni}^p and ϕ_{ni}^d . However, in practice, harmonic solutions for any modes other than the fundamental are possible only in one-dimensional codes. It turns out that this is not restrictive, as shown in the following derivation of Equation 5 of the main text.

The static reactivity ρ_s is defined as the algebraically largest eigenvalue of the eigenequation.

$$0 = \left\{ -\bar{D} + (1 - \rho_s) \left[(1 - \beta_T) \chi_p \bar{P} + \sum_{i=1}^M \chi_i \beta_i \bar{P} \right] \right\} \phi_s \quad (A7)$$

where ϕ_s is the static eigenfunction.

Preskitt's expression for the static reactivity,

$$\rho_s = \alpha_{ni} \frac{\langle \phi_s^+, v^{-1} \phi_{ni}^p \rangle}{\langle \phi_s^+, \chi_T \bar{p} \phi_{ni}^p \rangle} - \frac{\langle \phi_s^+, \left[\sum_{i=1}^M \chi_i \alpha_{ni} \frac{\beta_i}{(\lambda_i + \alpha_{ni} \bar{I})} \right] \bar{p} \phi_{ni}^p \rangle}{\langle \phi_s^+, \chi_T \bar{p} \phi_{ni}^p \rangle} \quad (A8)$$

is obtained by the following procedure.

- 1) Multiply Equation A7 from the right by ϕ_{ni}^p and integrate over space and energy. The bra-ket notation simply indicates the multivariable integration.
- 2) Multiply Equation A1 from the left by ϕ_s^+ , the adjoint of ϕ_s , and integrate over all space and energy.
- 3) Use the definition for adjoint operators and then subtract the results of Step 1) from the results of Step 2) to eliminate terms involving \bar{D} . The result is Equation A8.

Preskitt has thus arrived at an expression for the reactivity ρ_s that is the same for any given n, i pair of indices, because a unique reactivity ρ_s exists independent of any algebra leading to Equation A8. Thus attention can be confined to the prompt fundamental mode. Now $|\alpha_0^p| \gg \lambda_i$. To a very close approximation Equation A8 goes over to

$$\rho_s = \alpha_0^p \frac{\langle \phi_s^+, v^{-1} \phi_0^p \rangle}{\langle \phi_s^+, \chi_T \bar{p} \phi_0^p \rangle} + \frac{\sum_{i=1}^M \langle \phi_s^+, \chi_i \beta_i \bar{p} \phi_0^p \rangle}{\langle \phi_s^+, \chi_T \bar{p} \phi_0^p \rangle} \quad (A9)$$

which is identical to Equation 5 in the main text.

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