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AEC RESEARCH AND DEVELOPMENT REPORT

CUDS: A COMPUTER PROGRAM FOR PROCESSING CUMULATIVE DATA STATISTICS

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CUDS: A COMPUTER PROGRAM FOR PROCESSING CUMULATIVE DATA STATISTICS

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ABSTRACT

The Cumulative Data Statistics (CUDS) program was written in FORTRAN IV for an IBM System/360-195 to statistically analyze volume calibrations for the accountability tanks at the Savannah River Plant. The management of nuclear materials requires accurate measurement data and a knowledge of the uncertainty associated with the measurements (weighing, sampling, analyzing, and converting instrument readings to volume). The program offers optimum flexibility in processing and listing discrete data, thus providing a valuable tool for calculating liquid tank inventories.

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INTRODUCTION

The management of nuclear materials requires accurate measurement data and a knowledge of the uncertainty associated with the measurements, including random and systematic errors. It is assumed that knowledge of an existing bias, its direction, and magnitude is applied to the data before the measurements are reported. Measurements include weighing, sampling, analyzing, and converting instrument readings to volume in accordance with a calibration equation.

Many of the materials accounted for in the Nuclear Materials Management System at the Savannah River Plant are measured in solution. The accurate measurement of the volume of solution and the assessment of the uncertainty associated with that volume measurement are important parts of the total measurement.

Calibration data are obtained for various transfer and storage tanks used for accounting of material. Because many of the tanks are not of single vertical cross-sectional dimensions [e.g., coils are installed in and displace liquid only in the lower portion of the tank (Figure 1)], volume calibrations are more appropriate, and each such tank calibration is subdivided into appropriate sections of constant vertical cross section. Each section is treated statistically as if it were a separate tank, and the equations developed are applied only to the appropriate section. If a transition region exists at the junction of the sections of the tank, a gap is left in the data to cover the transition--the equations are not solved for their intersection.

The Cumulative Data Statistics (CUDS) computer program was written to convert the raw data associated with a tank calibration and to analyze the data using the cumulative data statistics method. The output includes the best estimate of the calibration equation, the estimates of the random error and the three separate systematic errors, and details that are useful in identifying maverick data points. The application of the output from the CUDS computer program is described on page 26.

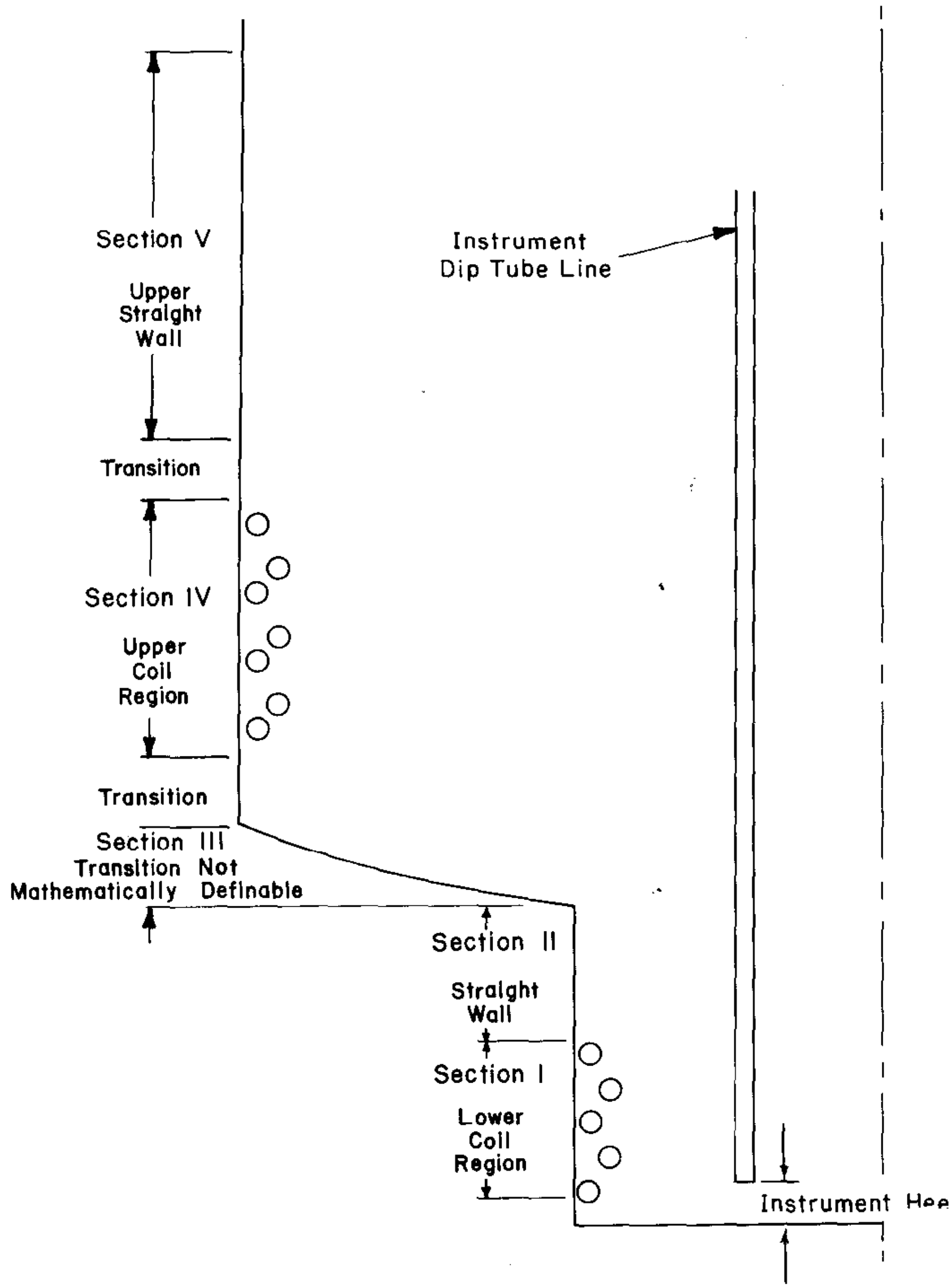


FIGURE 1 A Typical Tank with Nonuniform Vertical Cross Sections

SALIENT POINTS

The computer code for Savannah River Plant purposes is programmed to process a maximum of 9 sets of calibration data. Each set may contain a maximum of 250 data points comprising a maximum of 5 subsets. The input cards within a set need not be sequential because the computer code orders the data correctly prior to processing. The ability to define subsets and the use of variable formatting for decimal precision permit optimum flexibility in obtaining desired output.

If desired, an instrument factor constant may be used to convert the data before analysis to provide results for use with a replacement instrument of different type, make, or range for routine operation other than the instrument used for the calibration. This presumes the uncertainty in the calibration correlation between the instrument used for the tank calibration and the substitute instrument to be small (or negligible) compared to the uncertainty in the tank calibration. This is usually the case, because the cross-calibration between instruments may be treated by normal independent data techniques, which give lower uncertainties than the cumulative data method required of the tank calibration data.

The data for each subset of each calibration run are treated according to the cumulative data statistics methods of J. Mandel[1] as restated by R. J. Jones.[2] The systematic and random error variances are computed from the equations derived by C. G. Hough.[3]

The raw data are cumulated and each data point is converted to volume and standard instrument reading before being treated statistically. Data points that make unusually large contributions to the total variance are flagged as either "suspect" or "maverick" depending on the magnitude of the contribution. If either end point for a subset exceeds the maverick test value, that point is deleted and the calculation is redone without it. This is repeated as often as necessary until neither end-point contribution exceeds the maverick criterion. Interior or imbedded mavericks are not deleted; they are merely flagged.

Bartlett's test is performed on the residual variance data to detect unusually good or poor runs before combining the data from the different runs to obtain the best representation of the calibration. Fisher's test is performed on the combined data to detect significant differences in slopes of the individual calibration curves.

Considerable effort is expended in preparing to calibrate a tank and collecting the data, especially for the remote (nonaccessible) tanks. In addition, plant operations may have to be interrupted for a few days to collect the data; therefore, overall effort is reduced if a greater number of data points are obtained than may prove necessary as opposed to having to reschedule the work. Therefore, provision is made to treat the data using the cumulative data points incrementing by 1, 2, 3, or 4. Incrementing by 1 treats each data point, incrementing by 2 treats every second data point, etc. All possibilities are investigated, e.g., incrementing by 3 treats data points 1, 4, 7, etc., points 2, 5, 8, etc., and points 3, 6, 9, etc. This permits an after-the-fact choice of the optimum number and location of data points to establish a lower residual variance and a sufficient level of confidence.

EQUATIONS USED IN THE PROGRAM

The equations used in the CUDS program are described below and are written using the acronyms illustrated in the Appendix.

Conversion of Raw Input Data

Equations 1-3 are used to convert the raw input data to standard values. The raw data are usually in calibration readings for X (normally a manometer) and weight units for Y. X values are corrected for density of the manometer fluid and the calibration fluid at their respective temperatures, and Y values are converted to cumulative volume:

$$Y_i = \left(\sum_{j=1}^{i} YI_j \right) SV/DT_i \qquad 1 \leqslant i \leqslant N \tag{1}$$

where

Y_i = corrected cumulative volume of Y for the ith point

YI_j = measured incremental addition of weight or volume of Y between the (j - 1)th and the jth point

SV = standard value (conversion constant between YI and Y at standard conditions, e.g., liters/pound at 4°C)

DT_i = correction to standard value of calibrating fluid at fluid temperature at time of measurement

N = total number of data points in a set

$$X_i = XI_i(DM_i/DT_i) \qquad 1 \leqslant i \leqslant N \tag{2}$$

where

X_i = corrected value of X (calibration instrument reading) at the ith point.

XI_i = measured value of X for the ith point

DM_i = correction to measuring instrument (density of manometer fluid, etc.) at temperature of the instrument

DT_i = correction to standard value of calibrating fluid at fluid temperature at time of measurement

N = total number of data points in a set

$$R_i = (X_i/IF) \qquad 1 \leqslant i \leqslant N \tag{3}$$

where

R_i = corrected reading expected from a substitute or replacement instrument

X_i = corrected value of X (calibration instrument reading) at the ith point

IF = conversion factor (substitute instrument to calibration instrument at standard conditions)

N = total number of data points in a set

Calculation of Cumulative Statistics

Equations 4-10 are used to calculate the slope and intercept of the straight line representing the data, the residual random variance, the variance of the slope, the variance of the intercept, and the covariance of the slope and intercept based on cumulative data.

$$A = \frac{(X_{nn})(Y_L) - (X_L)(Y_{nn})}{X_{nn} - X_L} \tag{4}$$

$$B = \frac{Y_{nn} - Y_L}{X_{nn} - X_L} \tag{5}$$

where

A = alpha parameter for a subset

B = beta parameter for a subset

X_L = first X value of a subset

X_{nn} = last X value of a subset

Y_L = first Y value of a subset

Y_{nn} = last Y value of a subset

$$RV = \sum_{i=L+num}^{nn} \left(\frac{\left(B(X_i - X_{i-num}) - (Y_i - Y_{i-num})\right)^2}{(X_i - X_{i-num})} \right) \left(\frac{1}{JCNT - 1} \right) \tag{6}$$

RV = random residual variance

B = beta parameter (Equation 5)

L = subscript of first X and Y values of a subset

nn = subscript of last X and Y values of a subset

num = incremental value defined to a subset, e.g., L = 1, nn = 7, num = 2. This would permit processing the 1st, 3rd, 5th, and 7th X and Y variables of a subset

JCNT = number of pairs of X,Y data points processed from a subset

$$BV = \frac{RV}{X_{nn} - X_L} \tag{7}$$

where

BV = beta variance for a subset

RV = random residual variance (Equation 6)

X_L = first X value of a subset

X_{nn} = last X value of a subset

$$ABC = \frac{(-RV)(X_L)}{X_{nn} - X_L} \tag{8}$$

where

ABC = alpha-beta covariance of a subset

RV = random residual variance (Equation 6)

X_L = first X value of a subset

X_{nn} = last X value of a subset

$$AV = \frac{(RV)(X_L)(X_{nn})}{X_{nn} - X_L} \tag{9}$$

where

AV = alpha variance of a subset

RV = random residual variance (Equation 6)

X_L = first X value of a subset

X_{nn} = last X value of a subset

$$F = JCNT - 1 \tag{10}$$

where

F = degrees of freedom of a subset

JCNT = number of X or Y values processed from a subset

$$ICV_i = \frac{[B(X_i - X_{i-num}) - (Y_i - Y_{i-num})]^2}{(X_i - X_{i-num})} \qquad 1 < i \leqslant nn \qquad (11)$$

where

ICV_i = individual contribution

B = beta parameter for a subset (Equation 5)

X_i = individual X value of a subset

Y_i = individual Y value of a subset

nn = subscript of last X and Y value of a subset

num = incremental value defined to a subset, e.g., L = 1, nn = 7, num = 2. This would permit processing the 1st, 3rd, 5th, and 7th X and Y variables of a subset

Equations 12-23 are used if and only if a nonzero value is entered in Column 72 of the subset and decision card (Figure 5), and the number of calibration runs is greater than one. Equation 12 calculates Bartlett's test for homogeneity of the residual random variances for each subset from multiple calibration runs.

$$CCS = \frac{\left(\sum_{i=1}^{kilo} LDF_i\right) \ln VA - \left(\sum_{i=1}^{kilo} LDF_i \ \ln ZULU_i\right)}{\left(1 + \frac{1}{3(kilo - 1)}\right)\left[\left(\sum_{i=1}^{kilo} \frac{1}{LDF_i}\right) - \left(\frac{1}{\sum_{i=1}^{kilo} LDF_i}\right)\right]} \qquad (12)$$

where

CCS = corrected chi square

LDF = degrees of freedom for individual residual variance of the subsets

ln = natural log

VA = average residual variance (Equation 13)

ZULU = individual residual variances of the subsets

kilo = number of calibration runs processed

Equation 13 calculates the statistically weighted average residual variance for each subset.

$$VA = \frac{\sum_{i=1}^{kilo} LDF_i \; ZULU_i}{\sum_{i=1}^{kilo} LDF_i} \tag{13}$$

where

VA = average residual variance

LDF = degrees of freedom for individual residual variance of the subsets

ZULU = individual residual variance of the subsets

kilo = number of calibration runs processed

Equations 14-17 calculate the average value for X1, XN, Y1, and YN for each subset, and assume that the X1, Y1 values do not differ significantly among runs.

$$X1B = \left(\sum_{i=1}^{kilo} X1_i \right) / kilo \tag{14}$$

$$XNB = \left(\sum_{i=1}^{kilo} XN_i \right) / kilo \tag{15}$$

$$Y1B = \left(\sum_{i=1}^{kilo} Y1_i \right) / kilo \qquad (16)$$

$$YNB = \left(\sum_{i=1}^{kilo} YN_i \right) / kilo \qquad (17)$$

where

XIB = average value of all X1's

X1 = first X value of each subset

XNB = average value of all Xn's

XN = last X value of each subset

Y1B = average value for all Y1's

Y1 = first Y value of each subset

YNB = average Y value for all Yn's

YN = last Y value of each subset

kilo = number of calibration runs to be processed

Equation 18 calculates the F ratio of between slope to within slope variance for each subset.

$$FR = \left(kilo \left(\sum_{i=1}^{kilo} BVX_i^{\,2} \right) - \left(\sum_{i=1}^{kilo} BVX_i \right)^2 \right) \frac{(XNB - XIB)}{VA(kilo - 1)} \qquad (18)$$

where

FR = F test ratio

BVX = beta value of each subset (Equation 5)

X1B = average value for all X1's (Equation 14)

XNB = average value for all XN's (Equation 15)

kilo = number of calibration runs processed

VA = average residual variance

If the F test ratio for the subset is smaller than the value of P = 0.10 for the values of $\sum_{i=1}^{kilo} LDF_i$ and (kilo - 1), average statistics are calculated by Equations 19-23.

$$ALP = \frac{(XNB)(Y1B) - (X1B)(YNB)}{XNB - X1B} \qquad (19)$$

where

ALP = alpha average

$$BET = \frac{YNB - Y1B}{XNB - X1B} \qquad (20)$$

where

BET = beta average

$$BVA = \frac{VA}{(XNB - X1B)kilo} \qquad (21)$$

where

BVA = beta variance average

VA = average residual variance (Equation 13)

$$ABCA = \frac{(-VA)(X1B)}{(XNB - X1B)kilo} \qquad (22)$$

where

ABCA = alpha-beta covariance average

$$AVA = \frac{(VA)(X1B)(XNB)}{(XNB - X1B)kilo} \qquad (23)$$

where

AVA = alpha variance average

DESCRIPTION OF THE PROGRAM

CUDS was written in FORTRAN IV for an IBM System/360-195 using card input. The flowchart is shown in Figure 2, and the FORTRAN listing is given in the Appendix.

The CUDS program permits considerable user flexibility in obtaining desired data processing and output. This is accomplished by allowing the user to define to the computer code the conversion constants, data card sequence numbers, subset ranges, a subset sample selection code, required decimal precision, units of measure, and an equation default code.

Program Input

Program input includes two cards containing chi square and F test probability points (Figure 3). These values are used with Equations 12 and 18 to accept or reject the hypothesis that the individual slope variances are homogeneous, and between slope to within slope variances are also homogeneous.

A title card (Figure 4) is used to enter the title of each set of calibration data and the date the data were obtained. A conversion constant (Columns 41-47) may be entered if needed to correct the raw data to corrected data. If this field is left blank or zero, the program sets the conversion constant equal to '1' to prevent multiplication by zero. The units of measure (pounds, inches, percent, etc) and the desired number of decimal places for each unit output are required input by the user. No default options are available for these fields. An instrument factor constant may be entered if readings are desired from a replacement instrument (perhaps used for routine operation) of different type, make, or range other than the instrument used for the calibration. If such a constant is entered, the program prints statistics based on the calibration instrument and also for the converted (replacement) instrument values. If such a constant is not entered, the program prints statistics based only on the calibration instrument readings.

A subset and decision card (Figure 5) is used to divide each set of calibration data into subsets, to define the sample to be processed from each subset, to specify whether multiple runs are to be averaged, and to enter the decision value for determining which variables are to be discarded as maverick data points.

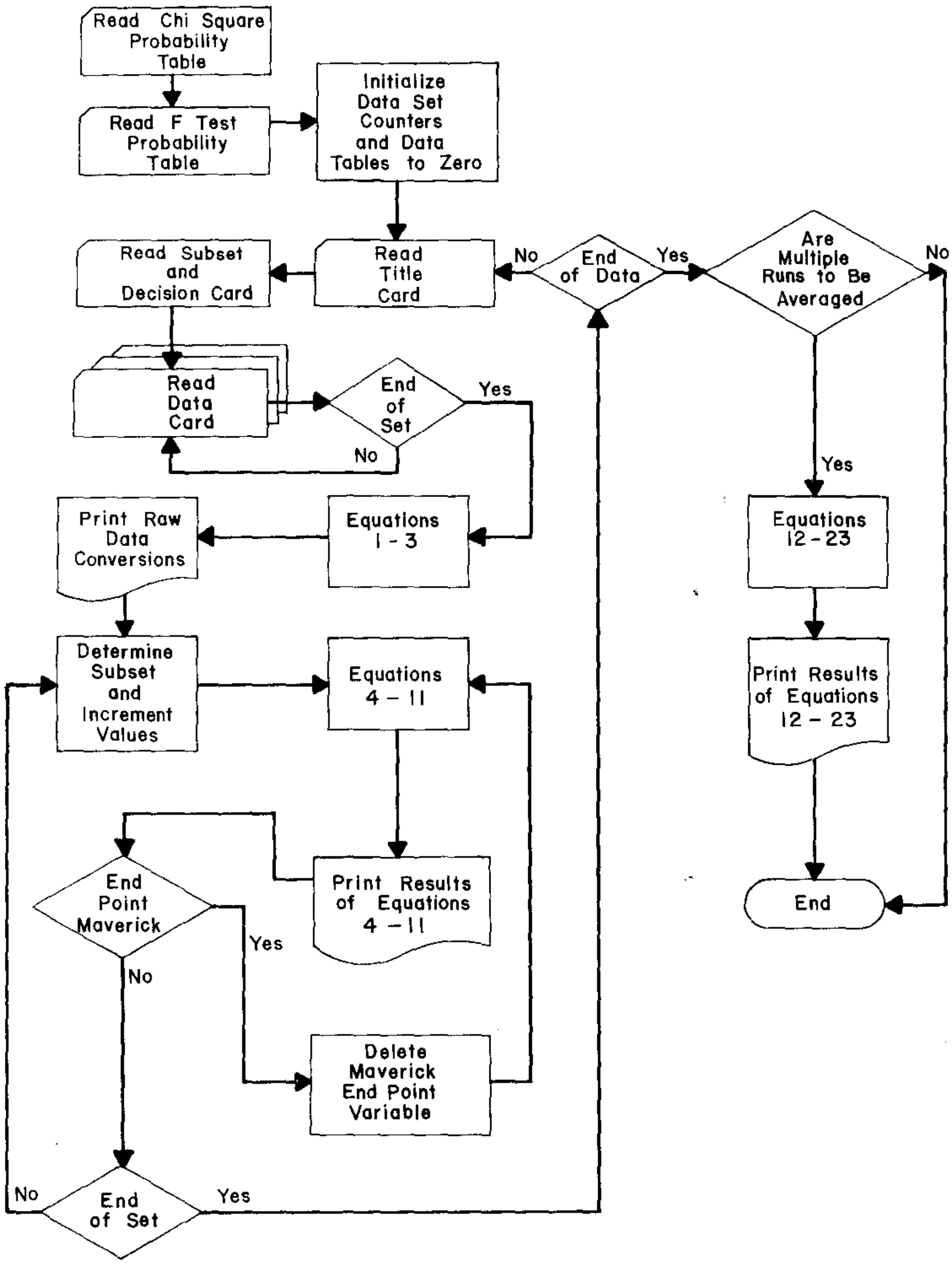
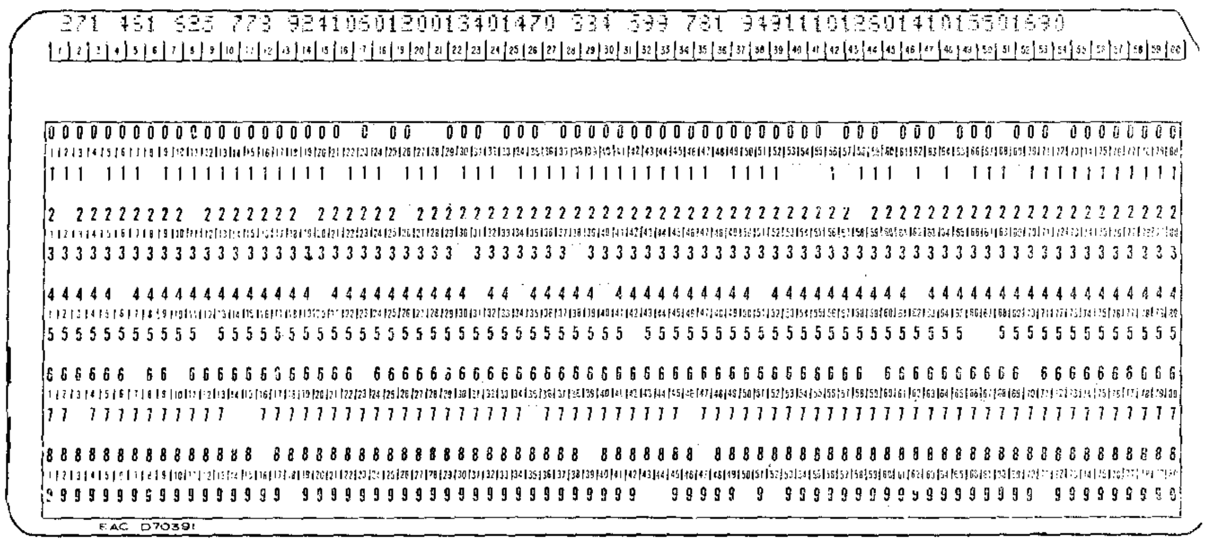


FIGURE 2 Flowchart for CUDS Program



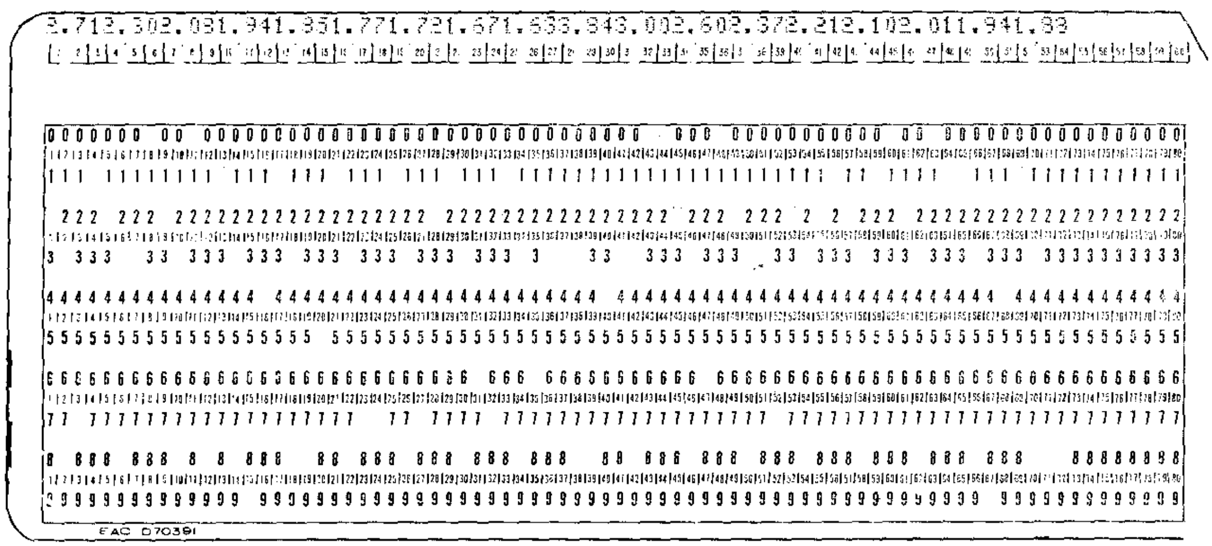


FIGURE 3 Chi Square and F Test Probability Points

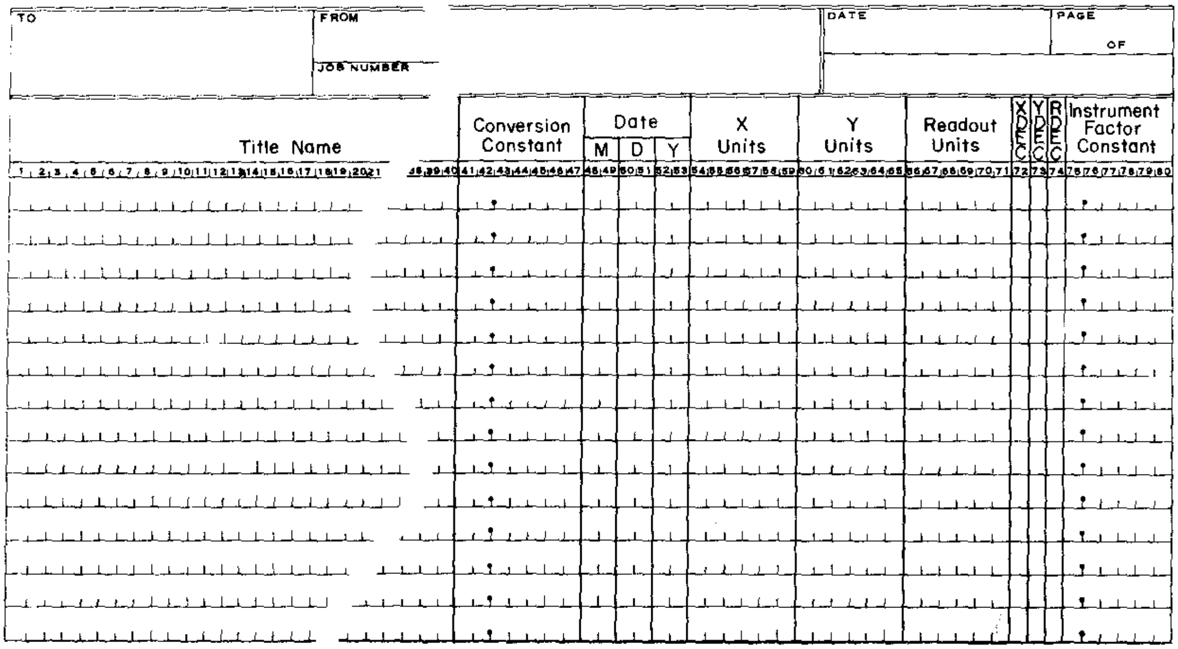


FIGURE 4 Title Card

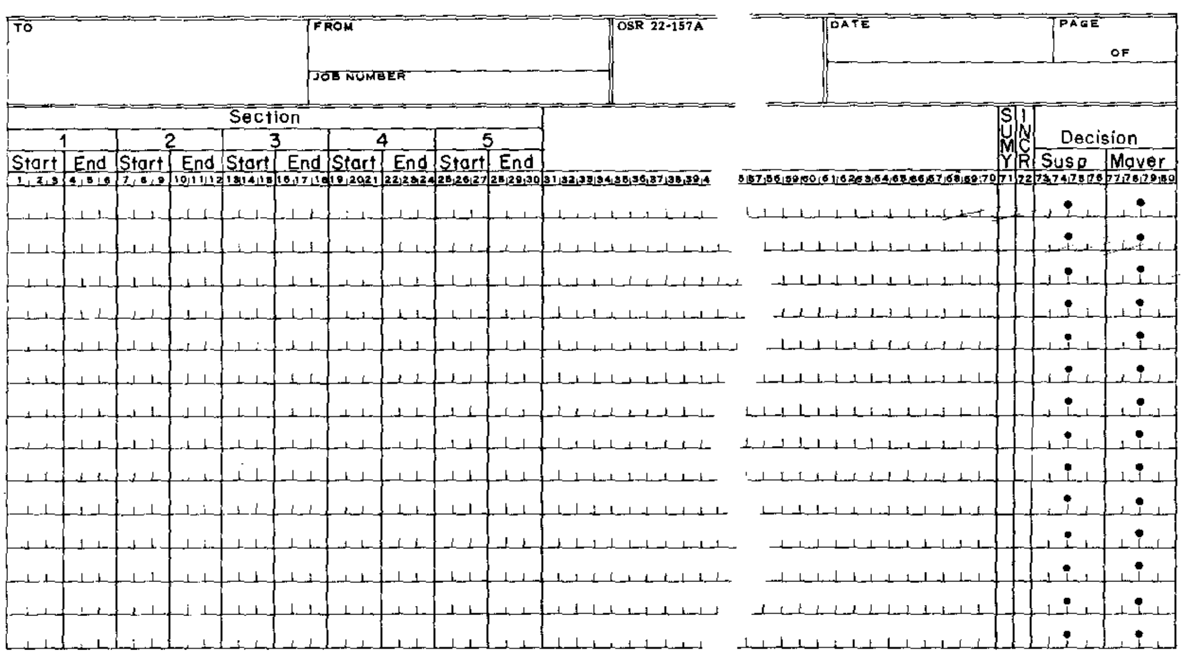


FIGURE 5 Subset and Decision Card

The option code (Column 71) is used to define the sample to be processed from each subset and permits using every data point, or every second, third, or fourth data point starting with the first, second, third, or fourth data point for a maximum of 10 combinations within a subset. The option codes are 0, 1, 2, 3, or 4 (Figure 6). A numeric default code greater than zero entered in Column 72 causes the multiple sets for a calibration to be averaged.

Individual data cards (Figure 7) follow the subset and decision card. Each data card must have its sequence number entered in Columns 51-53 because the program uses this number to order the data. As a result, the data cards may be submitted for processing in any order as long as they remain in the same set and the last data card is followed by a card with a '1' in Column 80.

Program Output

The following output listings are examples obtained from typical inventory tanks.

- Raw input data converted to corrected data in the standard units desired (Figure 8).
- Statistics based on the calibration instrument readings (Figure 9).
- Statistics based on a replacement instrument (Figure 10). This list will not be printed if an instrument factor conversion constant has not been entered on the title card.
- The weighted average variance for all calibration runs on a tank and the average values for the calibration equation and the average variances (Figure 11). These statistics will not be printed if the calibration sets are less than two or the default option code (Column 72, Figure 5) is zero or blank.

The correct deck setup for processing the CUDS program is shown in Figure 12.

Option Code 0									
1	1	2	1	2	3	1	2	3	4
2	3	4	4	5	6	5	6	7	8
3	5	6	7	8	9	9	10	11	12
4	7	8	10	11	12	13	14	15	16

Option Code 1
1
2
3
4

Option Code 2		Option Code 3			Option Code 4			
1	2	1	2	3	1	2	3	4
3	4	4	5	6	5	6	7	8
5	6	7	8	9	9	10	11	12
7	8	10	11	12	13	14	15	16

The first row of each matrix denotes the starting variable and each column denotes the incremental value in each subset.

FIGURE 6 Subset Sample Definition

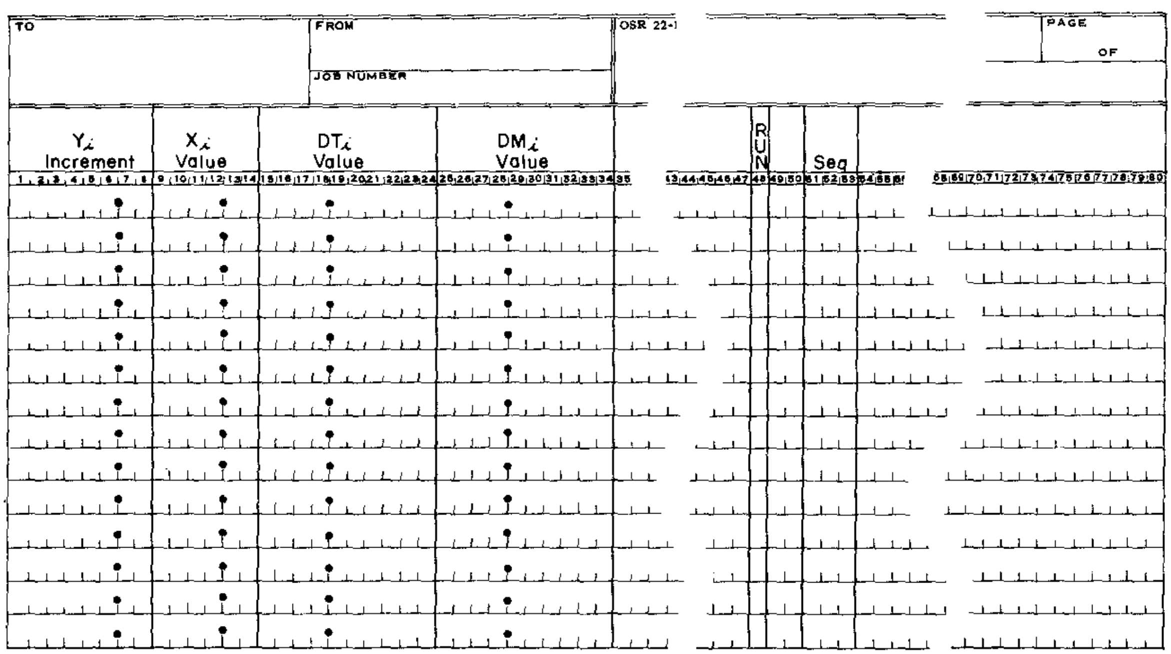


FIGURE 7 Data Card

8.3-H TANK CALIBRATION RUN #1 9/11/71

COMPUTER RUN ON 03/16/73 AT 08 HRS 13 MIN

DATA CONVERSION

	R-VALUE	X-VALUE	Y-VALUE
1	0.	1.33	458.08
2	0.	2.75	685.55
3	0.	4.14	913.01
4	0.	5.53	1141.39
5	0.	6.89	1368.86
6	0.	8.28	1596.33
7	0.	9.68	1823.80
8	0.	11.01	2051.26
9	0.	12.37	2278.27
10	0.	13.61	2485.69
11	0.	14.94	2712.24
12	0.	16.27	2939.71
13	0.	17.75	3167.63
14	0.	19.08	3395.10
15	0.	20.42	3622.57
16	0.	21.84	3850.49
17	0.	23.20	4078.41
18	0.	24.56	4306.34
19	0.	25.86	4534.26
20	0.	27.28	4761.73
21	0.	28.70	4989.19
22	0.	30.00	5216.66
23	0.	31.39	5443.67
24	0.	32.84	5671.60
25	0.	34.17	5898.61

FIGURE 8 Data Conversion

CUMULATIVE STATISTICS

COMPUTER RUN ON 03/16/73 AT 08 HRS 13 MIN

8.3-H TANK CALIBRATION RUN #1 9/11/71

CALCULATION # 8, 1ST SECTION, STARTING WITH THE 2ND VARIABLE AND INCREMENTING BY 4

FIRST X 2.75 INCHES FIRST Y 685.55 LITERS

LAST X 68.29 INCHES LAST Y 11588.03 LITERS

ALPHA	228.0917	LITERS
BETA	166.3485	LITERS / INCHES
RESIDUAL VARIANCE	41.7850	
BETA VARIANCE	0.6375	
ALPHA-BETA COVARIANCE	-1.7533	
ALPHA VARIANCE	119.7302	

DEGREES OF FREEDOM 12

	X-VALUE	Y-VALUE	NUMERATOR SIGN	INDIVIDUAL CONTRIBUTION	ICV/RV	FLAGGED VARIABLES	FACTOR
6	8.28	1596.33		15.0642	0.361		
10	13.61	2485.69	NEGATIVE	1.3907	0.033		
14	19.08	3395.10	NEGATIVE	0.0487	0.001		
18	24.56	4306.34	NEGATIVE	0.0223	0.001		
22	30.00	5216.66	NEGATIVE	5.3290	0.128		
26	35.48	6126.53	NEGATIVE	0.5397	0.013		
30	40.68	7036.40	NEGATIVE	386.9668	9.261	MAVERICK	3.50
34	46.19	7946.27	NEGATIVE	8.1718	0.196		
38	51.75	8855.68		43.1413	1.032		
42	57.25	9766.92	NEGATIVE	2.4578	0.059		
46	62.73	10677.70	NEGATIVE	0.1196	0.003		
50	68.29	11588.03		38.1682	0.913		

FIGURE 9 Statistics Based on Actual Converted Instrument Reading

CUMULATIVE STATISTICS

COMPUTER RUN ON 03/16/73 AT 08 HRS 13 MIN

8.3-H TANK CALIBRATION RUN #5 9/20/71

CALCULATION # 1, 2ND SECTION, STARTING WITH THE 1ST VARIABLE AND INCREMENTING BY 1

FIRST R 94. PERCEN FIRST Y 12908.32 LITERS

LAST R 150. PERCEN LAST Y 20867.91 LITERS

ALPHA -452.4204 LITERS

BETA 142.1355 LITERS / PERCEN

RESIDUAL VARIANCE 1384.3064

BETA VARIANCE 24.7198

ALPHA-BETA COVARIANCE -2323.6572

ALPHA VARIANCE 348548.5759

DEGREES OF FREEDOM 25

	R-VALUE	Y-VALUE	INDIVIDUAL CONTRIBUTION	NUMERATOR SIGN	ICV/RV	FLAGGED VARIABLES	FACTOR
41	96.	13226.30	568.1459	NEGATIVE	0.410		
42	99.	13546.11	3787.6122		2.736	SUSPECT	2.30
43	101.	13864.09	568.1459	NEGATIVE	0.410		
44	103.	14182.53	583.7578	NEGATIVE	0.422		
45	105.	14500.97	583.7578	NEGATIVE	0.422		
46	108.	14818.49	3952.0977		2.855	SUSPECT	2.30
47	110.	15137.39	599.5813	NEGATIVE	0.433		
48	112.	15455.37	568.1459	NEGATIVE	0.410		
49	114.	15773.35	568.1459	NEGATIVE	0.410		
50	116.	16094.53	681.1345	NEGATIVE	0.492		
51	119.	16412.05	3952.0977		2.855	SUSPECT	2.30
52	121.	16730.03	568.1459	NEGATIVE	0.410		
53	123.	17048.93	599.5813	NEGATIVE	0.433		
54	125.	17367.37	583.7578	NEGATIVE	0.422		
55	128.	17685.81	3885.5961		2.807	SUSPECT	2.30
56	130.	18004.24	583.4162	NEGATIVE	0.421		
57	132.	18321.77	553.0782	NEGATIVE	0.400		
58	134.	18640.66	599.2351	NEGATIVE	0.433		
59	136.	18958.19	553.0782	NEGATIVE	0.400		
60	139.	19275.72	3951.3719		2.854	SUSPECT	2.30
61	141.	19595.07	615.2656	NEGATIVE	0.444		
62	143.	19913.05	568.1459	NEGATIVE	0.410		
63	145.	20231.94	599.2351	NEGATIVE	0.433		
64	148.	20549.47	3951.3719		2.854	SUSPECT	2.30
65	150.	20867.91	583.7578	NEGATIVE	0.422		

FIGURE 10 Statistics Based on Replacement Instrument Reading

```
BARTLETTS TEST USING 'X' VALUES

COMPUTER RUN ON 03/16/73    AT  08 HRS  13 MIN

** SECTION 1 **    CORRECTED CHI SQUARE PROBABILITY GREATER THAN P = 0.10

                   F-TEST BETWEEN TO WITHIN SLOPES =      2.7357

                   CORRECTED CHI SQUARE =      1.0681

                   DEGREES OF FREEDOM =    50

** BETWEEN SLOPES VARIANCE GREATER THAN WITHIN SLOPES VARIANCE WITH
     PROBABILITY LESS THAN P = 0.10 **

** SECTION 2 **    CORRECTED CHI SQUARE PROBABILITY LESS THAN OR EQUAL TO P = 0.05

                   F-TEST BETWEEN TO WITHIN SLOPES =      2.4719

                   CORRECTED CHI SQUARE =     28.3203

                   DEGREES OF FREEDOM =    31

** BETWEEN SLOPES VARIANCE GREATER THAN WITHIN SLOPES VARIANCE WITH
     PROBABILITY LESS THAN P = 0.10 **
```

FIGURE 11   Bartlett's Test Using Instrument Readings

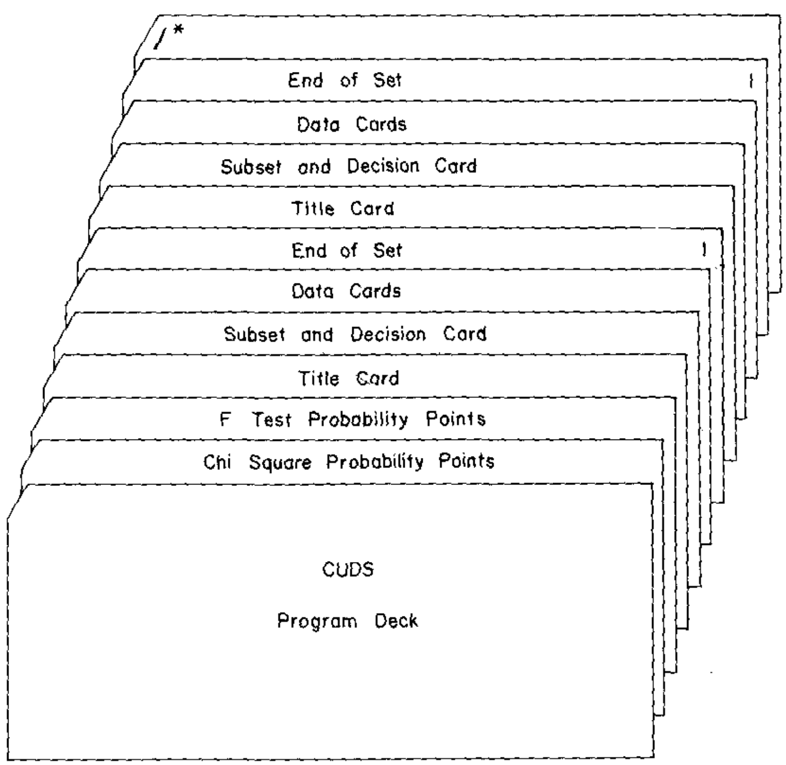


FIGURE 12   Program Deck

USING THE PROGRAM OUTPUT

Calibration equations are simply a restatement of the values of alpha and beta for each section of the tank:

$$\text{Volume} = \alpha + \beta X \Big]_{L}^{\mu} \qquad (24)$$

where L is the lowest accepted value of X for that section, and μ is the highest accepted value of X for that section. Equation 24 should not be used outside the range of data used in the correlation. Although the intersections of the calibration equations for the adjacent sections of the tank can be calculated from the simultaneous solution of the equations, the value should be used only to confirm the location of the junction between sections indicated by the blueprint. The transition from one section of tank to another is seldom sharp; therefore, operation with liquid levels in the transition region should be avoided because of the inability to assess the volume and the uncertainty associated with an estimate of the volume in that region.

The uncertainty associated with a volume calculated from the calibration equation for a section of tank is composed of two components: systematic and random errors.

The variance associated with the systematic and the random errors for the section of tank containing the liquid surface must be combined before extracting the square root to determine the uncertainty of the volume. The systematic variance is given by

$$V(\ell)_S = (\text{alpha variance}) + (2)(X)(\text{alpha-beta covariance}) + (X)^2(\text{beta variance}) \qquad (25)$$

The random variance is given by

$$V(\ell)_R = (X)(\text{random variance}) \qquad (26)$$

and the uncertainty of the volume measurement is

$$\text{Uncertainty} = \pm t \sqrt{V(\ell)_S + V(\ell)_R} \qquad (27)$$

where t is the value of Student's t at the desired confidence level.

REFERENCES

1. John Mandel. "Fitting a Straight Line to Certain Types of Cumulative Data." *American Statistical Assoc. J. 52,* 552-66 (1957).

2. R. J. Jones. *Selected Measurement Methods for Plutonium and Uranium in the Nuclear Fuel Cycle.* USAEC Report TID-7029, USAEC Division of Nuclear Materials Management (1963).

3. C. G. Hough. *Statistical Analysis - Accuracy of Volume Measurements in a Large Process Vessel.* USAEC Report HW-62177, Hanford Atomic Products Operation, Richland, Wash. (October 1959).

APPENDIX – FORTRAN Listing

```
COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SIZE=0000K,
                  SOURCE,EBCDIC,LIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
      C     THE CUMULATIVE DATA STATISTICS (CUDS) PROGRAM WAS WRITTEN TO
      C     STATISTICALLY ANALYZE CALIBRATION DATA OBTAINED FROM SAVANNAH
      C     RIVER PLANT INVENTORY TANKS.  THE FOLLOWING LIST INCLUDES A
      C     DEFINITION OF EACH ACRONYM DEFINED TO THE PROGRAM.
      C
      C
      C     DM AND DT ARE VARIABLES USED TO CONVERT XI AND YI VALUES TO
      C     CORRECTED VALUES.
      C     XI AND YI ARE RAW INPUT DATA POINTS.
      C     'IF' IS AN INSTRUMENT FACTOR CONSTANT USED TO CALCULATE 'R' VALUES
      C     R VALUES ARE OBTAINED BY DIVIDING X(I) BY 'IF'.
      C     X AND Y ARE CORRECTED XI AND YI VALUES.
      C     D IS A CUMULATIVE TOTAL OF YI'S.
      C     SV IS A CONVERSION CONSTANT USED IN CALCULATING Y(J'S).
      C     A - CALCULATED ALPHA PARAMETER.
      C     B - CALCULATED BETA PARAMETER.
      C     RV - CALCULATED RANDOM VARIANCE PARAMETER.
      C     ICV - CALCULATED INDIVIDUAL CONTRIBUTION PARAMETER.
      C     ABC - CALCULATED ALPHA-BETA COVARIANCE PARAMETER.
      C     CLASS - DATA STATEMENT USED TO DENOTE ARITHMETIC SIGN OF ICV
      C     NUMERATOR BEFORE SQUARING.
      C     BB - NUMERATOR OF ICV EQUATION.
      C     ICRV - INDIVIDUAL CONTRIBUTION DIVIDED BY RANDOM VARIANCE.
      C     BV - CALCULATED BETA VARIANCE PARAMETER.
      C     AV - CALCULATED ALPHA VARIANCE PARAMETER.
      C     GRADE - LEVEL OF ICRV SIGNIFICANCE.
      C     SIGN - CLASS(I),I=1,2
      C     CODE - GRADE(I),I=1,3
      C     DA - DM VALUE READ FROM INPUT CARD.
      C     DB - DT VALUE READ FROM INPUT CARD.
      C     YA - YI VALUE READ FROM INPUT CARD.
      C     XB - XI VALUE READ FROM INPUT CARD.
      C     ZULU - THE INDIVIDUAL SUBSET RANDOM VARIANCES FOR X VALUES.
      C     VA - AVERAGE RANDOM VARIANCE.
      C     CCS - CORRECTED CHI SQUARE.
      C     TBIRD - SUMMATION OF (DEGREES OF FREEDOM FOR SUBSET VARIANCES
      C     MULTIPLIED BY NATURAL LOG OF SUBSET VARIANCES).
      C     JBIRD - SUMMATION OF (1 DIVIDED BY INDIVIDUAL SUBSET DEGREES OF
      C     FREEDOM).
      C     BULU - THE INDIVIDUAL SUBSET RANDOM VARIANCES FOR 'R' VALUES.
      C     LOG - DENOMINATOR OF BARTLETTS' TEST EQUATION.
      C     X1 - THE FIRST 'X' VALUE OF EACH SUBSET.
      C     XN - THE LAST 'X' VALUE OF EACH SUBSET.
      C     Y1 - THE FIRST 'Y' VALUE OF EACH SUBSET USING 'X' VALUES.
      C     YN - THE LAST 'Y' VALUE OF EACH SUBSET USING 'X' VALUES.
      C     R1 - THE FIRST 'R' VALUE OF EACH SUBSET.
      C     RN - THE LAST 'R' VALUE OF EACH SUBSET.
      C     Y2 - THE FIRST 'Y' VALUE OF EACH SUBSET USING 'R' VALUES.
      C     YM - THE LAST 'Y' VALUE OF EACH SUBSET USING 'R' VALUES.
      C     KILO - TOTAL NUMBER OF SETS TO BE PROCESSED.
      C     NK - TOTAL NUMBER OF SUBSETS PER SET.
      C     MIKE - TOTAL NUMBER OF SETS USING AN 'IF' CONVERSION CONSTANT.
      C     ISUM - ANY ONE DIGIT NON ZERO NUMBER DENOTING MULTIPLE RUNS ARE TO
      C     BE AVERAGED.
```

```
      C     LTD - SET EQUAL TO '1' IF ISUM GREATER THAN ZERO AND PERMITS
      C     MULTIPLE RUNS TO BE AVERAGED.
      C     ALP - ALPHA AVERAGE
      C     BET - BETA AVERAGE.
      C     BVA - BETA VARIANCE AVERAGE.
      C     ABCA - ALPHA BETA COVARIANCE AVERAGE.
      C     AVA - ALPHA VARIANCE AVERAGE.
      C
      C
  2          REAL *8 X(250),XI(250),DT(250),Y(250),YI(250),DM(250),R(250),IF,D,
            1SV,A,B,RV,ICV(250),ABC,CLASS(2)/'        NEGATIVE'/,BB(250),ICRV(2
            2250),BV,AV,GRADE(3)/'           SUSPECTMAVERICK'/,CODE,SIGN,YA,XB,DA,
            3DB,ZULU(9,5),VA,CCS,TBIRD,JBIRD,LOG,BULU(9,5),X1(9,5),XN(9,5),Y1(9
            4,5),YN(9,5),R1(9,5),RN(9,5),Y2(9,5),YM(9,5)
  3          REAL *8 X1B,XNB,Y1B,YNB,R1B,RNB,Y2B,YMB,BVX(9,5),BVR(9,5),BV2,FR,A
            1LP,BET,BVA,ABCA,AVA
      C
  4          REAL *4 ABLE(5)/' 1ST 2ND 3RD 4TH 5TH'/,FOX,BOX,BEE/'    '/,PROB(9
            1,2)
      C
  5          REAL *4 TROB(9,2)
  6          INTEGER *2 F,LDF(9,5),IDF(9,5)
  7          INTEGER *4 LX,LR,LY,RX
  8          DIMENSION XW(11),XX(41),XY(57),XZ(18)
  9          EQUIVALENCE (XW(1),FT(1)), (XX(1),Z1(1)), (XY(1),Z2(1)), (XZ(1),FS
            1(1))
      C
      C
 10          LOGICAL *1 XU(6),YU(6),RU(6),TITLE(40),IFMT(5),IFMR(7),FT(41),Z1(1
            163),Z2(225),FS(72),CAT(8),DD(9),TT(8),BLK(4)/'    '/
      C
 11          DATA FS/'(1H0,T2,I4,T8,F9.0,T26,F9.0,T45,F12.0,T71,A8,T89,F6.3,T10
            18,A8,T121,F4.2)'/,CAT/'  A4F4.2'/
      C
      C
 12          DATA FT/'(1H0,T2,I3,T23,F10.0,T57,F10.0,T91,F10.0)'/,IFMT/'01234'/
      C
      C
 13          DATA Z1/'(1H0,T27,7HFIRST X,T35,F10.3,T48,6A1,T79,7HFIRST Y,T87,F1
            10.3,T99,6A1//T27,6HLAST X,T35,F10.3,T48,6A1,T79,6HLAST Y,T87,F10.3
            2,T99,6A1//T27,5HALPHA,T50,F15.4,T68,6A1)'/,IFMR/'02468RX'/
      C
      C
 14          DATA Z2/'(1H0,T27,4HBETA,T50,F15.4,T68,6A1,T75,1H/,T77,6A1//T27,17
            1HRESIDUAL VARIANCE,T50,F15.4//T27,13HBETA VARIANCE,T50,F15.4//T27,
            221HALPHA-BETA COVARIANCE,T50,F15.4//T27,14HALPHA VARIANCE,T50,F15.
            34//T27,18HDEGREES OF FREEDOM,T46,I3)'/
      C
      C     READ CHI SQUARE PROBABILITY TABLE.
      C
 15          READ (5,94) ((PROB(I,J),I=1,9),J=1,2)
      C
      C     READ F TEST PROBABILITY TABLE.
      C
 16          READ (5,94) ((TROB(I,J),I=1,9),J=1,2)
      C
```

```
C       INITIALIZE DATA SET COUNTERS AND DATA TABLES TO ZERO.
C
        LTD=0
        MIKE=0
        KILO=0
        DO 2 I=1,9
        DO 1 J=1,5
        X1(I,J)=0.
        XN(I,J)=0.
        Y1(I,J)=0.
        YN(I,J)=0.
        R1(I,J)=0.
        RN(I,J)=0.
        Y2(I,J)=0.
        YM(I,J)=0.
1       CONTINUE
2       CONTINUE
C
C       READ TITLE CARD AND INCREMENT DATA SET COUNTERS BY '1'.
C
3       READ (5,95,END=84) TITLE,SV,M,MD,MY,XU,YU,RU,IX,IY,IR,IF
        KILO=KILO+1
        IF (IF.NE.0.) MIKE=MIKE+1
C
C       READ SUBSET AND DECISION CARD.
C
        READ (5,96) N1,M1,N2,M2,N3,M3,N4,M4,N5,M5,S1,S2,LF,ISUM
        IF (ISUM.EQ.0) LTD=1
C
        D=0.
        NN=0
        L1=0
        NK=1
        NX=IX+1
        NY=IY+1
        NR=IR+1
        JX=NX
        IF (IX.LT.IY) JX=NY
        RX=NR
        IF (IR.LT.IY) RX=NY
C
C
        CALL CDATE(DD,TT)
        DD(7)=BLK(1)
        DD(8)=BLK(2)
        DD(9)=BLK(3)
        TT(5)=BLK(1)
        TT(6)=BLK(2)
        TT(7)=BLK(3)
        TT(8)=BLK(4)
C
C       READ DATA CARDS.
C
        DO 4 I=1,250
        READ (5,97,ERR=4,END=5) YA,XB,DA,DB,N,LC
        IF (LC.EQ.1) GO TO 5
```

```
      YI(N)=YA
      XI(N)=XB
      DT(N)=DA
      DM(N)=DB
      NN=NN+1
    4 CONTINUE
C
    5 IF (S1.GE.S2) GO TO 6
      GO TO 7
    6 WRITE (6,98) TITLE,M,MD,MY,DD,TT
      GO TO 3
    7 CONTINUE
C
      WRITE (6,99) TITLE,M,MD,MY,DD,TT
      WRITE (6,100)
C
C     EQUATIONS 1 THRU 3.
C
      DO 10 J=1,NN
      X(J)=XI(J)*(DM(J)/DT(J))
      R(J)=0.
      IF (IF.EQ.0) GO TO 8
      R(J)=X(J)/IF
    8 D=D+YI(J)
      IF (SV.EQ.0.D0) SV=1.D0
      Y(J)=(SV*D)/DT(J)
      LX=X(J)*(10**IX)+.5D0
      X(J)=LX/(10.D0**IX)
      LR=R(J)*(10**IR)+.5D0
      R(J)=LR/(10.D0**IR)
      LY=Y(J)*(10**IY)+.5D0
      Y(J)=LY/(10.D0**IY)
      L1=L1+1
      IF (L1.NE.26) GO TO 9
      WRITE (6,99) TITLE,M,MD,MY,DD,TT
      L1=0
    9 CONTINUE
      FT(20)=IFMT(NR)
      FT(30)=IFMT(NX)
      FT(40)=IFMT(NY)
C
C     PRINT RAW DATA CONVERSIONS.
C
      WRITE (6,FT) J,R(J),X(J),Y(J)
   10 CONTINUE
C
C
      NUZ=1
      NUMB=0
      JJ=N1
      NN=M1
      NUM=1
C
C
   11 KEY=1
C
```

```
      C      DETERMINE SUBSET AND INCREMENT VALUES.
      C
111          IF (LF.EQ.0) GO TO 12
113          IF (LF.EQ.2) GO TO 21
115          IF (LF.EQ.3) GO TO 23
117          IF (LF.EQ.4) GO TO 26
119          IF (LF.EQ.1.AND.NUZ.EQ.1) GO TO 30
121          GO TO 40
122   12     IF (LF.EQ.0.AND.NUZ.EQ.1) NUM=1
124          IF (LF.EQ.0.AND.NUZ.EQ.1) GO TO 30
126          IF (LF.EQ.0.AND.NUZ.NE.2) GO TO 13
128          NUM=2
129          GO TO 30
130   13     IF (LF.EQ.0.AND.NUZ.NE.3) GO TO 14
132          KEY=2
133          JJ=JJ+1
134          GO TO 30
135   14     IF (LF.EQ.0.AND.NUZ.NE.4) GO TO 15
137          NUM=3
138          GO TO 30
139   15     IF (LF.EQ.0.AND.NUZ.NE.5) GO TO 16
141          KEY=2
142          JJ=JJ+1
143          GO TO 30
144   16     IF (LF.EQ.0.AND.NUZ.NE.6) GO TO 17
146          KEY=3
147          JJ=JJ+2
148          GO TO 30
149   17     IF (LF.EQ.0.AND.NUZ.NE.7) GO TO 18
151          NUM=4
152          GO TO 30
153   18     IF (LF.EQ.0.AND.NUZ.NE.8) GO TO 19
155          KEY=2
156          JJ=JJ+1
157          GO TO 30
158   19     IF (LF.EQ.0.AND.NUZ.NE.9) GO TO 20
160          KEY=3
161          JJ=JJ+2
162          GO TO 30
163   20     IF (LF.EQ.0.AND.NUZ.NE.10) GO TO 40
165          KEY=4
166          JJ=JJ+3
167          GO TO 30
168   21     IF (LF.EQ.2.AND.NUZ.NE.1) GO TO 22
170          NUM=2
171          GO TO 30
172   22     IF (LF.EQ.2.AND.NUZ.NE.2) GO TO 40
174          KEY=2
175          JJ=JJ+1
176          GO TO 30
177   23     IF (LF.EQ.3.AND.NUZ.NE.1) GO TO 24
179          NUM=3
180          GO TO 30
181   24     IF (LF.EQ.3.AND.NUZ.NE.2) GO TO 25
183          KEY=2
184          JJ=JJ+1
```

```
185          GO TO 30
186    25    IF (LF.EQ.3.AND.NUZ.NE.3) GO TO 40
188          KEY=3
189          JJ=JJ+2
190          GO TO 30
191    26    IF (LF.EQ.4.AND.NUZ.NE.1) GO TO 27
193          NUM=4
194          GO TO 30
195    27    IF (LF.EQ.4.AND.NUZ.NE.2) GO TO 28
197          KEY=2
198          JJ=JJ+1
199          GO TO 30
200    28    IF (LF.EQ.4.AND.NUZ.NE.3) GO TO 29
202          KEY=3
203          JJ=JJ+2
204          GO TO 30
205    29    IF (LF.EQ.4.AND.NUZ.NE.4) GO TO 40
207          KEY=4
208          JJ=JJ+3
209    30    L=JJ
210          DO 31 I=L,NN,NUM
       C
       C     EQUATIONS 4 THRU 11.
       C
211    31    CONTINUE
212          NN=I-NUM
213          A=((X(NN)*Y(L))-(X(L)*Y(NN)))/(X(NN)-X(L))
214          B=(Y(NN)-Y(L))/(X(NN)-X(L))
215          BVX(KILO,NK)=B
216          JCNT=0
217          K=JJ+NUM
218          RV=0.
219          DO 32 I=K,NN,NUM
220          JCNT=JCNT+1
221          IF ((X(I)-X(I-NUM)).EQ.0.) GO TO 32
223          ICV(I)=(B*(X(I)-X(I-NUM))-(Y(I)-Y(I-NUM)))**2/(X(I)-X(I-NUM))
224          RV=ICV(I)+RV
225    32    CONTINUE
226          RV=RV/JCNT
227          ZULU(KILO,NK)=RV
228          BV=RV/(X(NN)-X(L))
229          ABC=-RV*X(L)/(X(NN)-X(L))
230          AV=RV*X(L)*X(NN)/(X(NN)-X(L))
231          F=JCNT
232          LDF(KILO,NK)=F
233          NUMB=NUMB+1
234          BOX=ABLE(KEY)
235          FOX=ABLE(NK)
236          WRITE (6,101) DD,TT,TITLE,M,MD,MY,NUMB,FOX,BOX,NUM
237          Z1(18)=IFMR(7)
238          Z1(28)=IFMT(NX)
239          Z1(60)=IFMT(NY)
240          Z1(82)=IFMR(7)
241          Z1(92)=IFMT(NX)
242          Z1(123)=IFMT(NY)
243          Z1(154)=IFMR(JX)
```

```
C
C     PRINT RESULT OF EQUATION 4.
C
      WRITE (6,Z1) X(JJ),XU,Y(JJ),YU,X(NN),XU,Y(NN),YU,A,YU
      Z2(25)=IFMR(JX)
      Z2(85)=IFMR(JX)
      Z2(117)=IFMR(JX)
      Z2(157)=IFMR(JX)
      Z2(190)=IFMR(JX)
C
C     PRINT RESULTS OF EQUATIONS 5 THRU 10.
C
      WRITE (6,Z2) B,YU,XU,RV,BV,ABC,AV,F
      WRITE (6,102)
C
      L2=0
      LT=15
      DO 38 I=K,NN,NUM
      BB(I)=B*(X(I)-X(I-1))-(Y(I)-Y(I-1))
      SIGN=CLASS(1)
      IF (BB(I).LT.0.) SIGN=CLASS(2)
      IF ((X(I)-X(I-1)).EQ.0.) GO TO 38
      ICRV(I)=ICV(I)/RV
      IF (ICRV(I).LT.S2) GO TO 33
      CODE=GRADE(3)
      GO TO 34
   33 IF (ICRV(I).GE.S1) CODE=GRADE(2)
      IF (ICRV(I).LT.S1) CODE=GRADE(1)
      IF (ICRV(I).LT.S1) CODE=GRADE(1)
   34 CONTINUE
      L2=L2+1
      LT=LT-1
      IF (LT.EQ.0) GO TO 35
      IF (L2.NE.26) GO TO 36
   35 WRITE (6,103) DD,TT
      L2=0
   36 CONTINUE
      KB=5
      BIRD=S2
      IF (ICRV(I).LT.S2) BIRD=S1
      IF (ICRV(I).GE.S1) GO TO 37
      BIRD=BEE
      KB=1
   37 FS(18)=IFMT(NX)
      FS(27)=IFMT(NY)
      FS(37)=IFMR(JX)
      FS(68)=CAT(KB)
      FS(69)=CAT(KB+1)
      FS(70)=CAT(KB+2)
      FS(71)=CAT(KB+3)
C
C     PRINT RESULT OF EQUATION 11.
C
      WRITE (6,FS) I,X(I),Y(I),ICV(I),SIGN,ICRV(I),CODE,BIRD
   38 CONTINUE
      NUZ=NUZ+1
```

```
300           JK=K
        C
        C     DELETE MAVERICK END POINT VARIABLE.
        C
301           IF (ICRV(JK).LT.S2.AND.ICRV(NN).LT.S2) GO TO 41
303           IF (ICRV(JK).GE.S2.AND.ICRV(NN).GE.S2) GO TO 39
305           IF (ICRV(JK).GE.S2) JJ=JJ+NUM
307           IF (ICRV(NN).GE.S2) NN=NN-NUM
309           NUZ=NUZ-1
310           GO TO 30
311     39    IF (ICRV(JK).GT.ICRV(NN)) JJ=JJ+NUM
313           IF (ICRV(JK).LE.ICRV(NN)) NN=NN-NUM
315           NUZ=NUZ-1
316           GO TO 30
317     40    X1(KILO,NK)=X(JJ)
318           XN(KILO,NK)=X(NN)
319           Y1(KILO,NK)=Y(JJ)
320           YN(KILO,NK)=Y(NN)
321           NK=NK+1
322           NUMB=0
323           NUZ=1
324           IF (NK.EQ.6) GO TO 47
326     41    GO TO (42,43,44,45,46),NK
327     42    JJ=N1
328           NN=M1
329           GO TO 11
330     43    IF (N2.LT.1) GO TO 47
332           JJ=N2
333           NN=M2
334           GO TO 11
335     44    IF (N3.LT.1) GO TO 47
337           JJ=N3
338           NN=M3
339           GO TO 11
340     45    IF (N4.LT.1) GO TO 47
342           JJ=N4
343           NN=M4
344           GO TO 11
345     46    IF (N5.LT.1) GO TO 47
347           JJ=N5
348           NN=M5
349           GO TO 11
350     47    IF (IF.EQ.0.) GO TO 3
352           NK=1
353           NUMB=0
354           NUZ=1
355           JJ=N1
356           NN=M1
357           NUM=1
358     48    KEY=1
        C
        C     DETERMINE SUBSET AND INCREMENT VALUES.
        C
359           IF (LF.EQ.0) GO TO 49
361           IF (LF.EQ.2) GO TO 58
363           IF (LF.EQ.3) GO TO 60
```

```
      IF (LF.EQ.4) GO TO 63
      IF (LF.EQ.1.AND.NUZ.EQ.1) GO TO 67
      GO TO 77
49    IF (LF.EQ.0.AND.NUZ.EQ.1) NUM=1
      IF (LF.EQ.0.AND.NUZ.EQ.1) GO TO 67
      IF (LF.EQ.0.AND.NUZ.NE.2) GO TO 50
      NUM=2
      GO TO 67
50    IF (LF.EQ.0.AND.NUZ.NE.3) GO TO 51
      KEY=2
      JJ=JJ+1
      GO TO 67
51    IF (LF.EQ.0.AND.NUZ.NE.4) GO TO 52
      NUM=3
      GO TO 67
52    IF (LF.EQ.0.AND.NUZ.NE.5) GO TO 53
      KEY=2
      JJ=JJ+1
      GO TO 67
53    IF (LF.EQ.0.AND.NUZ.NE.6) GO TO 54
      KEY=3
      JJ=JJ+2
      GO TO 67
54    IF (LF.EQ.0.AND.NUZ.NE.7) GO TO 55
      NUM=4
      GO TO 67
55    IF (LF.EQ.0.AND.NUZ.NE.8) GO TO 56
      KEY=2
      JJ=JJ+1
      GO TO 67
56    IF (LF.EQ.0.AND.NUZ.NE.9) GO TO 57
      KEY=3
      JJ=JJ+2
      GO TO 67
57    IF (NUZ.NE.10) GO TO 77
      KEY=4
      JJ=JJ+3
      GO TO 67
58    IF (LF.EQ.2.AND.NUZ.NE.1) GO TO 59
      JJ=JJ+2
      NUM=2
      GO TO 67
59    IF (NUZ.NE.2) GO TO 77
      KEY=2
      JJ=JJ+1
      GO TO 67
60    IF (LF.EQ.3.AND.NUZ.NE.1) GO TO 61
      NUM=3
      GO TO 67
61    IF (LF.EQ.3.AND.NUZ.NE.2) GO TO 62
      KEY=2
      JJ=JJ+1
      GO TO 67
62    IF (NUZ.NE.3) GO TO 77
      KEY=3
      JJ=JJ+2
```

```
      GO TO 67
   63 IF (LF.EQ.4.AND.NUZ.NE.1) GO TO 64
      NUM=4
      GO TO 67
   64 IF (LF.EQ.4.AND.NUZ.NE.2) GO TO 65
      KEY=2
      JJ=JJ+1
      GO TO 67
   65 IF (LF.EQ.4.AND.NUZ.NE.3) GO TO 66
      KEY=3
      JJ=JJ+2
      GO TO 67
   66 IF (LF.EQ.4.AND.NUZ.NE.4) GO TO 77
      KEY=4
      JJ=JJ+3
   67 L=JJ
      DO 68 I=L,NN,NUM
   68 CONTINUE
      NN=I-NUM
C
C     EQUATIONS 4 THRU 11.
C
      A=((R(NN)*Y(L))-(R(L)*Y(NN)))/(R(NN)-R(L))
      B=(Y(NN)-Y(L))/(R(NN)-R(L))
      BVR(MIKE,NK)=B
      JCNT=0
      RV=0.
      K=JJ+NUM
      DO 69 I=K,NN,NUM
      JCNT=JCNT+1
      IF ((R(I)-R(I-NUM)).EQ.0.) GO TO 69
      ICV(I)=(B*(R(I)-R(I-NUM))-(Y(I)-Y(I-NUM)))**2/(R(I)-R(I-NUM))
      RV=ICV(I)+RV
   69 CONTINUE
      RV=RV/JCNT
      BULU(MIKE,NK)=RV
      BV=RV/(R(NN)-R(L))
      ABC=-RV*R(L)/(R(NN)-R(L))
      AV=RV*R(L)*R(NN)/(R(NN)-R(L))
      F=JCNT
      IDF(MIKE,NK)=F
      NUMB=NUMB+1
      BOX=ABLE(KEY)
      FOX=ABLE(NK)
      WRITE (6,101) DD,TT,TITLE,M,MD,MY,NUMB,FOX,BOX,NUM
      Z1(18)=IFMR(6)
      Z1(28)=IFMT(NR)
      Z1(60)=IFMT(NY)
      Z1(82)=IFMR(6)
      Z1(92)=IFMT(NR)
      Z1(123)=IFMT(NY)
      Z1(154)=IFMR(RX)
C
C     PRINT RESULT OF EQUATION 4.
C
      WRITE (6,Z1) R(JJ),RU,Y(JJ),YU,R(NN),RU,Y(NN),YU,A,YU
```

```
494          Z2(25)=IFMR(RX)
495          Z2(85)=IFMR(RX)
496          Z2(117)=IFMR(RX)
497          Z2(157)=IFMR(RX)
498          Z2(190)=IFMR(RX)
       C
       C     PRINT RESULTS OF EQUATIONS 5 THRU 10.
       C
499          WRITE (6,Z2) B,YU,RU,RV,BV,ABC,AV,F
500          WRITE (6,104)
501          L2=0
502          LT=15
503          DO 75 I=K,NN,NUM
504          BB(I)=B*(R(I)-R(I-1))-(Y(I)-Y(I-1))
505          SIGN=CLASS(1)
506          IF (BB(I).LT.0.) SIGN=CLASS(2)
508          IF ((R(I)-R(I-1)).EQ.0.) GO TO 75
510          ICRV(I)=ICV(I)/RV
511          IF (ICRV(I).LT.S2) GO TO 70
513          CODE=GRADE(3)
514          GO TO 71
515    70    IF (ICRV(I).GE.S1) CODE=GRADE(2)
517          IF (ICRV(I).LT.S1) CODE=GRADE(1)
519    71    CONTINUE
520          L2=L2+1
521          LT=LT-1
522          IF (LT.EQ.0) GO TO 72
524          IF (L2.NE.26) GO TO 73
526    72    WRITE (6,105) DD,TT
527          L2=0
528    73    CONTINUE
529          KB=5
530          BIRD=S2
531          IF (ICRV(I).LT.S2) BIRD=S1
533          IF (ICRV(I).GE.S1) GO TO 74
535          KB=1
536          BIRD=BEE
537    74    FS(18)=IFMT(NR)
538          FS(27)=IFMT(NY)
539          FS(37)=IFMR(RX)
540          FS(68)=CAT(KB)
541          FS(69)=CAT(KB+1)
542          FS(70)=CAT(KB+2)
543          FS(71)=CAT(KB+3)
       C
       C     PRINT RESULT OF EQUATION 11.
       C
544          WRITE (6,FS) I,R(I),Y(I),ICV(I),SIGN,ICRV(I),CODE,BIRD
545    75    CONTINUE
546          NUZ=NUZ+1
547          JK=K
       C
       C     DELETE MAVERICK END POINT VARIABLE.
       C
548          IF (ICRV(JK).LT.S2.AND.ICRV(NN).LT.S2) GO TO 78
550          IF (ICRV(JK).GE.S2.AND.ICRV(NN).GE.S2) GO TO 76
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```
552          IF (ICRV(JK).GE.S2) JJ=JJ+NUM
554          IF (ICRV(NN).GE.S2) NN=NN-NUM
556          NUZ=NUZ-1
557          GO TO 67
558     76   IF (ICRV(JK).GT.ICRV(NN)) JJ=JJ+NUM
560          IF (ICRV(JK).LE.ICRV(NN)) NN=NN-NUM
562          NUZ=NUZ-1
563          GO TO 67
564     77   R1(MIKE,NK)=R(JJ)
565          RN(MIKE,NK)=R(NN)
566          Y2(MIKE,NK)=Y(JJ)
567          YM(MIKE,NK)=Y(NN)
568          NK=NK+1
569          NUMB=0
570          NUZ=1
571          IF (NK.EQ.6) GO TO 3
573     78   GO TO (79,80,81,82,83),NK
574     79   JJ=N1
575          NN=M1
576          GO TO 48
577     80   IF (N2.LT.1) GO TO 3
579          JJ=N2
580          NN=M2
581          GO TO 48
582     81   IF (N3.LT.1) GO TO 3
584          JJ=N3
585          NN=M3
586          GO TO 48
587     82   IF (N4.LT.1) GO TO 3
589          JJ=N4
590          NN=M4
591          GO TO 48
592     83   IF (N5.LT.1) GO TO 3
594          JJ=N5
595          NN=M5
596          GO TO 48
        C
        C    IF MULTIPLE RUNS ARE TO BE AVERAGED (EQUATIONS 12 THRU 23).
        C
597     84   IF (KILO.LE.1.OR.LTD.EQ.1) GO TO 93
599          NK=NK-1
600          WRITE (6,106) DD,TT
601          DO 88 J=1,NK
602          VA=0.
603          IPH=0
604          TBIRD=0.
605          JBIRD=0.
606          BV=0.
607          BV2=0.
608          DO 85 I=1,KILO
609          VA=VA+(LDF(I,J)*ZULU(I,J))
610          IPH=IPH+LDF(I,J)
611          TBIRD=TBIRD+(LDF(I,J)*DLOG(ZULU(I,J)))
612          JBIRD=JBIRD+(1.D0/LDF(I,J))
613          BV=BVX(I,J)**2+BV
614          BV2=BV2+BVX(I,J)
```

```
615    85    CONTINUE
616          VA=VA/IPH
617          X1B=0.
618          XNB=0.
619          Y1B=0.
620          YNB=0.
621          DO 86 I=1,KILO
622          X1B=X1B+X1(I,J)
623          XNB=XNB+XN(I,J)
624          Y1B=Y1B+Y1(I,J)
625          YNB=YNB+YN(I,J)
626    86    CONTINUE
627          X1B=X1B/KILO
628          XNB=XNB/KILO
629          Y1B=Y1B/KILO
630          YNB=YNB/KILO
631          ALP=((XNB*Y1B)-(X1B*YNB))/(XNB-X1B)
632          BET=(YNB-Y1B)/(XNB-X1B)
633          BVA=VA/((XNB-X1B)*KILO)
634          ABCA=(-VA*X1B)/((XNB-X1B)*KILO)
635          AVA=(VA*X1B*XNB)/((XNB-X1B)*KILO)
636          BV2=BV2**2
637          FR=(((KILO*BV)-BV2)/(KILO*(KILO-1)))*(((XNB-X1B)*KILO)/VA)
638          LOG=1.D0+((1.D0/(3.D0*(KILO-1)))*(JBIRD-(1.D0/IPH)))
639          CCS=((IPH*DLOG(VA))-TBIRD)/LOG
       C
       C     PRINT CALCULATED 'F' TEST AND CORRECTED CHI SQUARE VALUES.
       C
640          IF (CCS.LT.PROB(KILO-1,1)) WRITE (6,109) J,FR,CCS,IPH
642          IF (CCS.GE.PROB(KILO-1,2)) WRITE (6,108) J,FR,CCS,IPH
644          IF (CCS.GE.PROB(KILO-1,1).AND.CCS.LT.PROB(KILO-1,2)) WRITE (6,107)
            1 J,FR,CCS,IPH
646          IF (FR.LE.TROB(KILO-1,2)) GO TO 87
648          WRITE (6,110)
649          GO TO 88
       C
       C     PRINT RESULTS OF EQUATIONS 14, 15, 19, 13, 22, 16, 17, 20, 21,
       C     AND 23.
       C
650    87    WRITE (6,111) X1B,Y1B,ALP,VA,ABCA,XNB,YNB,BET,BVA,AVA
651    88    CONTINUE
       C
       C     IF MULTIPLE RUNS ARE TO BE AVERAGED FOR 'R' VARIABLES (EQUATIONS
       C     12 THRU 24).
       C
652          IF (MIKE.LE.1.OR.IF.EQ.0.) GO TO 93
654          WRITE (6,112) DD,TT
655          DO 92 J=1,NK
656          VA=0.
657          IPH=0
658          TBIRD=0.
659          JBIRD=0.
660          BV=0.
661          BV2=0.
662          DO 89 I=1,MIKE
663          VA=VA+(IDF(I,J)*BULU(I,J))
```

```
      IPH=IPH+IDF(I,J)
      TBIRD=TBIRD+(IDF(I,J)*DLOG(BULU(I,J)))
      JBIRD=JBIRD+(1.DO/IDF(I,J))
      BV=BVR(I,J)**2+BV
      BV2=BV2+BVR(I,J)
   89 CONTINUE
      VA=VA/IPH
      R1B=0.
      RNB=0.
      Y2B=0.
      YMB=0.
      DO 90 I=1,MIKE
      R1B=R1B+R1(I,J)
      RNB=RNB+RN(I,J)
      Y2B=Y2B+Y2(I,J)
      YMB=YMB+YM(I,J)
   90 CONTINUE
      R1B=R1B/MIKE
      RNB=RNB/MIKE
      Y2B=Y2B/MIKE
      YMB=YMB/MIKE
      ALP=((RNB*Y2B)-(R1B*YMB))/(RNB-R1B)
      BET=(YMB-Y2B)/(RNB-R1B)
      BVA=VA/((RNB-R1B)*MIKE)
      ABCA=(-VA*R1B)/((RNB-R1B)*MIKE)
      AVA=(VA*R1B*RNB)/((RNB-R1B)*MIKE)
      BV2=BV2**2
      FR=(((MIKE*BV)-BV2)/(MIKE*(MIKE-1)))*(((RNB-R1B)*MIKE)/VA)
      LOG=1.DO+((1.DO/(3.DO*(MIKE-1)))*(JBIRD-(1.DO/IPH)))
      CCS=((IPH*DLOG(VA))-TBIRD)/LOG
C
C     PRINT CALCULATED 'F' TEST AND CORRECTED CHI SQUARE VALUES.
C
      IF (CCS.LT.PROB(MIKE-1,1)) WRITE (6,109) J,FR,CCS,IPH
      IF (CCS.GE.PROB(MIKE-1,2)) WRITE (6,108) J,FR,CCS,IPH
      IF (CCS.GE.PRO3(MIKE-1,1).AND.CCS.LT.PROB(MIKE-1,2)) WRITE (6,107)
     1 J,FR,CCS,IPH
      IF (FR.LE.TROB(MIKE-1,2)) GO TO 91
      WRITE (6,110)
      GO TO 92
C
C     PRINT RESULTS OF EQUATIONS 14, 15, 19, 13, 22, 16, 17, 20, 21,
C     AND 23.
C
   91 WRITE (6,113) R1B,Y2B,ALP,VA,ABCA,RNB,YMB,BET,BVA,AVA
   92 CONTINUE
   93 CONTINUE
      STOP
C
      STOP
C
   94 FORMAT (18F4.2)
   95 FORMAT (40A1,F7.5,3I2,6A1,6A1,6A1,3I1,F6.5)
   96 FORMAT (10I3,T73,2F4.2,T71,2I1)
   97 FORMAT (F8.2,F6.2,2F10.6,T51,I3,T80,I1)
   98 FORMAT (1H1,T38,40A1,T88,I2,'/',I2,'/',I2//T47,'COMPUTER RUN ON',1
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             1X,2A1,'/',2A1,'/',5A1,'AT  ',2A1,' HRS',2X,2A1,' MIN'////////////4
             2A1,T32,'SUSPECT TEST VALUE IS GREATER THAN OR EQUAL TO MAVERICK TE
             3ST VALUE')
714     99    FORMAT (1H1,T38,40A1,T88,I2,'/',I2,'/',I2//T47,'COMPUTER RUN ON',1
             11X,2A1,'/',2A1,'/',5A1,'AT  ',2A1,' HRS',2X,2A1,' MIN',4A1)
715    100    FORMAT (1H0,T59,'DATA CONVERSION'//T27,'R-VALUE',T61,'X-VALUE',T95
             1,'Y-VALUE')
716    101    FORMAT (1H1,T56,'CUMULATIVE STATISTICS'//T47,'COMPUTER RUN ON',1X,
             12A1,'/',2A1,'/',5A1,'AT  ',2A1,' HRS  ',2A1,' MIN',4A1//T38,40A1,T
             288,I2,'/',I2,'/',I2//T26,'CALCULATION #',I3,',',A4,' SECTION, STAR
             3TING WITH THE',A4,' VARIABLE AND INCREMENTING BY ',I2)
717    102    FORMAT (1H0,T48,'INDIVIDUAL',T70,'NUMERATOR',T109,'FLAGGED'/T11,'X
             1-VALUE',T29,'Y-VALUE',T47,'CONTRIBUTION',T73,'SIGN',T90,'ICV/RV',T
             2107,'VARIABLES',T120,'FACTOR')
718    103    FORMAT (1H1,T47,'COMPUTER RUN ON',1X,2A1,'/',2A1,'/',5A1,'AT  ',2A
             11,' HRS  ',2A1,' MIN',4A1//T48,'INDIVIDUAL',T70,'NUMERATOR',T109,'
             2FLAGGED'/T11,'X-VALUE',T29,'Y-VALUE',T47,'CONTRIBUTION',T73,'SIGN'
             3,T90,'ICV/RV',T107,'VARIABLES',T120,'FACTOR')
719    104    FORMAT (1H0,T48,'INDIVIDUAL',T70,'NUMERATOR',T109,'FLAGGED'/T11,'R
             1-VALUE',T29,'Y-VALUE',T47,'CONTRIBUTION',T73,'SIGN',T90,'ICV/RV',T
             2107,'VARIABLES',T120,'FACTOR')
720    105    FORMAT (1H1,T47,'COMPUTER RUN ON',1X,2A1,'/',2A1,'/',5A1,'AT  ',2A
             11,' HRS  ',2A1,' MIN',4A1//T48,'INDIVIDUAL',T70,'NUMERATOR',T109,'
             2FLAGGED'/T11,'R-VALUE',T29,'Y-VALUE',T47,'CONTRIBUTION',T73,'SIGN'
             3,T90,'ICV/RV',T107,'VARIABLES',T120,'FACTOR')
721    106    FORMAT (1H1,T50,'BARTLETTS TEST USING ''X'' VALUES'//T43,'COMPUTER
             1 RUN ON',1X,2A1,'/',2A1,'/',5A1,'AT  ',2A1,' HRS',2X,2A1,' MIN',4A
             21/)
722    107    FORMAT (1H0,T35,'** SECTION ',I1,' **',4X,'CORRECTED CHI SQUARE PR
             1OBABILITY LESS THAN OR EQUAL TO P = 0.10'//T11,'F-TEST BETWEEN TO
             2WITHIN SLOPES = ',F10.4,5X,'CORRECTED CHI SQUARE = ',F10.4,5X,'DEG
             3REES OF FREEDOM = ',I4/)
723    108    FORMAT (1H0,T35,'** SECTION ',I1,' **',4X,'CORRECTED CHI SQUARE PR
             1OBABILITY LESS THAN OR EQUAL TO P = 0.05'//T11,'F-TEST BETWEEN TO
             2WITHIN SLOPES = ',F10.4,5X,'CORRECTED CHI SQUARE = ',F10.4,5X,'DEG
             3REES OF FREEDOM = ',I4/)
724    109    FORMAT (1H0,T35,'** SECTION ',I1,' **',4X,'CORRECTED CHI SQUARE PR
             1OBABILITY GREATER THAN P = 0.10'//T14,'F-TEST BETWEEN TO WITHIN SL
             2OPES = ',F10.4,5X,'CORRECTED CHI SQUARE = ',F10.4,5X,'DEGREES OF F
             3REEDOM = ',I4/)
725    110    FORMAT (1H0,T16,'** BETWEEN SLOPES VARIANCE GREATER THAN WITHIN SL
             1OPES VARIANCE WITH PROBABILITY LESS THAN P = 0.10 **')
726    111    FORMAT (1H0,T2,'X1-BAR ',F10.4,2X,'Y1-BAR ',F10.4,T40,'ALPHA AVERA
             1GE ',F10.4,T66,'RANDOM VARIANCE AVERAGE',F9.4,T100,'A-B COVARIANCE
             2 AVERAGE ',F10.4//T2,'XN-BAR ',F10.4,T21,'YN-BAR ',F10.4,T41,'BETA
             3 AVERAGE ',F10.4,T66,'BETA VARIANCE AVERAGE ',F10.4,T100,'ALPHA VA
             4RIANCE AVERAGE ',F10.4//)
727    112    FORMAT (1H1,T50,'BARTLETTS TEST USING ''R'' VALUES'//T43,'COMPUTER
             1 RUN ON',1X,2A1,'/',2A1,'/',5A1,'AT  ',2A1,' HRS',2X,2A1,' MIN',4A
             21/)
728    113    FORMAT (1H0,T2,'R1-BAR ',F10.4,'Y1-BAR ',F10.4,T40,'ALPHA AVERAGE
             1',F10.4,T66,'RANDOM VARIANCE AVERAGE',F9.4,T100,'A-B COVARIANCE AV
             2ERAGE ',F10.4//T2,'RN-BAR ',F10.4,T21,'YN-BAR ',F10.4,T41,'BETA AV
             3ERAGE ',F10.4,T66,'BETA VARIANCE AVERAGE ',F10.4,T100,'ALPHA VARIA
             4NCE AVERAGE ',F10.4//)
729           END
```