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COMPUTER PROGRAM FOR CORRELATION AND SPECTRAL ANALYSIS OF REACTOR FLOW DATA

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Mathematics and Computers
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COMPUTER PROGRAM FOR CORRELATION AND SPECTRAL ANALYSIS OF REACTOR FLOW DATA

by

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ABSTRACT

The computer program for correlation and spectral analysis of reactor flow data operates on stationary time series data to obtain any or all of the following statistics: mean, variance, standard deviation, probability density function, autocorrelation and crosscorrelation functions, and the frequency spectrum (Fourier transform). The program is limited to two data records, each up to 2000 points in length. These limits could be increased to accommodate longer records. Statistics that have a functional variation are presented graphically with the option provided for detailed listing of functional values. The program can be expanded to include other operations on the data by adding the appropriate subroutine package, the data set being available in common storage.

CONTENTS

INTRODUCTION

Interpretation of continuous data records from reactor flow, temperature, and pressure signals is complicated by random variation in signal level. These variations can be attributed to turbulence, local flow separation, instrument noise, or process variation. For most purposes, the primary interest is in the mean signal level, or in the detection of periodic variation in the signal that may be obscured by the noise. But studies of process variations require analysis of the random noise itself and usually, measurement of the similarity between signals from different parts of the system to establish correlations. Visual comparison for correlation of two or more noisy signals is difficult because the variations are random, and therefore dissimilar.

The computer program described in this report will characterize a noisy signal statistically in both the time and frequency domains. It can be used to detect periodic signals buried in noise, to establish the degree of correlation between random signals, and to aid in establishing the source of process variable fluctuations. Advantages of the program are:

- Modular form, permitting additional subroutines to be easily attached for further analyses
- Separate input subroutine that can be adapted by the user for different types of data records, such as card or tape
- Simple procedures for the user, because the program selects optimal values for lag times in computing correlations, and the number of points for efficient Fourier transform

Appendix A contains detailed input instructions. Appendix B gives a sample problem input; Appendix C, the output.

MATHEMATICAL FORMULATION

This program operates on stationary time series data records in digital form to calculate any or all of the following statistics:

- Mean, variance, standard deviation
- Probability density function
- Autocorrelation function
- Crosscorrelation function (for two simultaneously recorded data sets)
- Frequency spectrum (Fourier transform)

Figure 1 is a block diagram of program options.

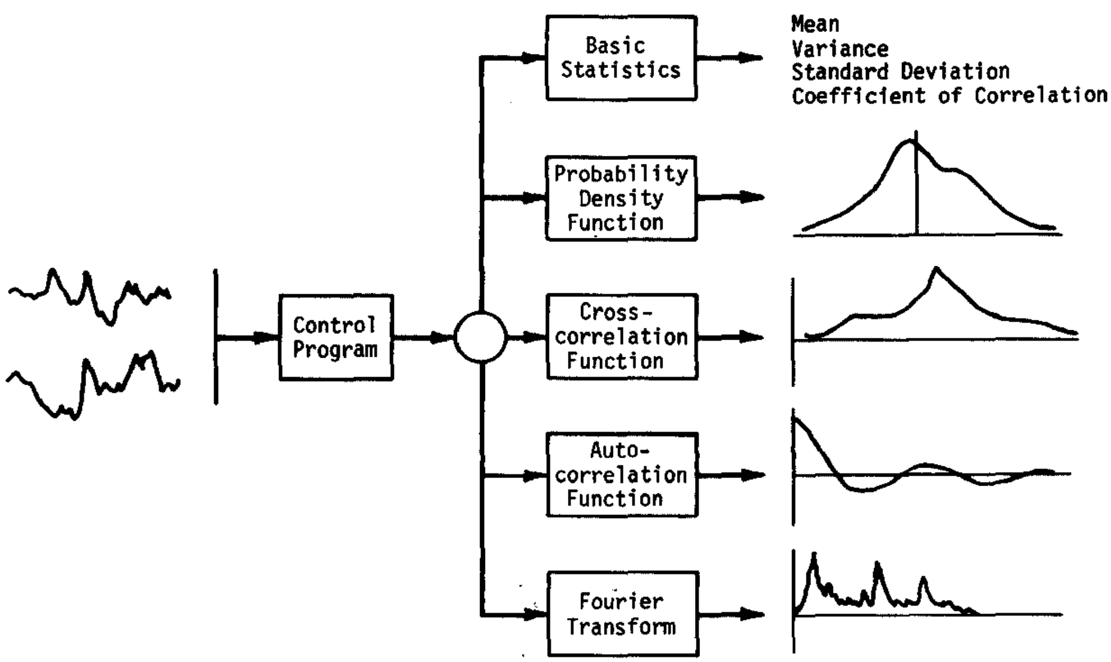


FIG. 1 BLOCK DIAGRAM OF PROGRAM OPTIONS

The mean, variance, and standard deviation of each data set $P_{i,j}$ are computed in the usual manner:

$$\text{Mean} = \overline{P}_j = \frac{1}{N} \sum_{i=1}^{N} P_{i,j} \qquad (1)$$

$$\text{Variance} = s_j^2 = \frac{1}{N-1} \sum_{i=1}^{N} (P_{i,j} - \overline{P}_j)^2 \qquad (2)$$

where N is the number of data points, and subscript i denotes a point in the time series j. The sample standard deviation is the square root of the sample variance. If two data records are being correlated, the covariance is calculated by

$$s_{12}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (P_{i,1} - \overline{P}_1)(P_{i,2} - \overline{P}_2) \qquad (3)$$

and the coefficient of correlation by

$$\rho_{xy} = \frac{s_{12}^2}{s_1 \cdot s_2} \qquad (4)$$

The probability density function is an estimate of the probability of certain amplitudes being assumed by the variable over a period of time. The procedure divides the range of variation about the mean value into 50 intervals, and determines the number of points N_i in the data set that fall into each interval. The probability that the variable will assume a value in a given interval is

$$P\left[x - \frac{\Delta x}{2} < x \leqslant x + \frac{\Delta x}{2}\right] = \frac{N_i}{N} \qquad (5)$$

The range of values is normalized on the largest (absolute) value in the data set and plotted against the probability determined by Equation 5.

Two signals may well be correlated, but not at the same instant in time. In other words a change in one signal precedes a corresponding change in the second signal by a short period of time. A coefficient of correlation of signals recorded simultaneously computed by Equation 4 would be ambiguous. The possibility of a time shift or phase lag can be investigated by developing the crosscorrelation function. One data record is shifted in time relative to the other in discrete steps, and the coefficient of correlation is recomputed after each step using Equations 1, 2, 3, and 4. A peak in the plot of coefficient of correlation versus time lag indicates the time relationship between the variables being correlated. Numerically, the crosscorrelation function is formed from these equations:

$$\Phi_{12}(r) = \frac{1}{N-r} \sum_{i=1}^{N-r} P_{i,1} \cdot P_{i+r,2} \tag{6}$$

$$\Phi_{21}(r) = \frac{1}{N-r} \sum_{i=1}^{N-r} P_{i+r,1} \cdot P_{i,2} \tag{7}$$

where N is the number of data points describing each signal, and $r = 1, 2, \ldots, r_{max}$ is the number of equal time intervals by which one signal is shifted relative to the other.

The autocorrelation function is used to test a signal for similarity to itself at various intervals. Any periodicity will be evidenced by a periodic autocorrelation function. The autocorrelation function for a record of N data values is estimated numerically by

$$\Phi_{jj}(r) = \frac{1}{N-r} \sum_{i=1}^{N-r} P_{i,j} \cdot P_{i+r,j} \qquad r = 1, 2, \ldots, r_{max} \tag{8}$$

where r again is the number of time shifts.

The maximum value of r, for both the autocorrelation and the crosscorrelation calculation, is determined by the recording rate and the bandwidth resolution specified for the Fourier transform:

$$r_{max} = 1/(B_e \cdot \Delta t) = \alpha/B_e \tag{9}$$

where B_e is the specified bandwidth in cycles/second, Δt is the time interval between data points in seconds/point, and α is the recording rate in points/second. If r_{max} computed from Equation 9 is greater than N/2, $r_{max} = N/2$.

Although the mean, variance, and distribution function are sufficient to describe the magnitude of a random variable, they give no indication as to the rapidity of variation with time. For example, two random data signals may both have the same variance, but one could be varying much more rapidly than the other. To describe the time aspect of noisy data, the mean square spectral density, or frequency spectrum, is used. Essentially, the spectrum shows for a random function how the total mean square value is distributed over the frequency range. The spectrum can be computed from the raw data P(t) by numerical integration of

$$F(w) = \int_0^\infty P(t)e^{-jwt}dt \tag{10}$$

where w is the radiant frequency, t is time, and j is $\sqrt{-1}$. The integral is difficult to implement in this form. The code utilizes a fast Fourier transform algorithm[1,2] supplied by IBM to compute the transform. In addition, advantage is taken of the Wiener theorem,[3] which states that the mean square spectral density of a random variable is the Fourier transform of its autocorrelation function. Accordingly, first the autocorrelation function is computed, and then the values are transformed to obtain the frequency spectrum. This procedure is fast and accurate.

The frequency is incremented from 0 Hz to the maximum frequency compatible with the recording rate, $f_{max} = \alpha/r_{max}$, sometimes called the Nyquist frequency.

INPUT FOR THE COMPUTER PROGRAM

The input for this program includes a description of the data: number of points, number of sets (1 or 2), recording rate, and spectral bandwidth resolution (Appendix B). The subroutine for reading in data should include the number of data points. Other information is read as standard input from the main program. The data record length (IMX) is governed only by machine capacity. The program is presently dimensioned to accept two 2000-point records. Longer data records would only require adjustment of the dimensions of arrays P, R, and XXR, which are the data, autocorrelation, and crosscorrelation arrays. The frequency resolution desired for calculation of power spectrum should be no less than $2\alpha/\text{IMX}$, where α is the recording rate in points/second, and IMX is the number of data points. The resolution finally obtained is usually slightly less than specified, because the Fourier transform subroutine pads out the data set with zeros until the number of points is a power of 2.

OUTPUT OF THE COMPUTER PROGRAM

The standard output includes the following:

- Problem title
- List of options specified
- Mean, variance, and standard deviation of each signal
- Single page plots of each statistical function computed
- The Nyquist frequency, number of lag values used to compute correlations, and standard error associated with frequency spectrum

The functions are plotted using the on-line printer, which rounds off values to the nearest line and carriage position. In addition, only 100 points can be plotted, so that information in longer data records may not be shown. Therefore, the option is provided for listing all points computed for each function, including those plotted.

Appendix C includes data for output of sample problem.

REFERENCES

1. *IEEE Audio Transactions.* Special Issue on Fast Fourier Transform (June 1967).
2. *Cooley-Tukey Fast Fourier Transform - FOURI.* IBM Corporation, Contributed Program Library, Hawthorne, New York (September 1968).
3. Y. W. Lee. *Statistical Theory of Communication.* John Wiley & Sons, New York (1964).
4. R. B. Blackman and J. W. Tukey. *The Measurement of Power Spectra.* Dover Publications, New York (1958).
5. J. S. Bendat and A. G. Piersol. *Measurement and Analysis of Random Data.* John Wiley & Sons, New York (1966).

EQN/bch

APPENDIX A

DETAILED INPUT INSTRUCTIONS

Data Record Input. The user should insert an appropriate READ instruction (with FORMAT statement) into subroutine TRASH to read his data into the array P(I,J), where I = 1, 2, ..., IMX; J = 1, 2; and IMX is the number of data points in each record J. If the data are on cards, they should be placed last in the input stream. The sample input described in Appendix B and shown in the listing below provides for reading data punched on cards.

```
      SUBROUTINE  TRASH(NSIG,IMX,ALPHA)

      COMMONP(2000,2),R(1000,2),KUE(5),NO,XXR(1000),V(2)
      DIMENSIONRMKS(8),HEAD(5),A(100,3)
      DATA HEAD/'FUNCTIONS CONSIDERED'/,RMKS/'SAMPLE OF FIRST 100 DATA P
     1OINTS'/,   N,NL,NS,NXS,NYS,XMIN,XMAX,YMIN/  100,0,0,0,1,0.,100.,0.
     2/

         READ NUMBER OF DATA POINTS PER RECORD, IMX

      IMX=201

         LOAD DATA INTO ARRAY P(I,J)
      DO10J=1,NSIG
      READ(5,100)(P(I,J),I=1,IMX)
  100 FORMAT(16F5.0)
   10 CONTINUE

      IF(KUE(1).EQ.0)RETURN
```

IDENTIFICATION CARDS
(Two cards for each run)

Card	Card Column	Identification	Remarks
1	1-14	Number of problems in run	Integer, right-justified
1	5-80	Problem title	Alphameric, up to 156
2	1-80	Problem title (continued)	Character
3	1-5	Options requested	Integer KUE(I) = 0, 1, 2, or 3.
		KUE(1) = 1	Plot of first 100 data points
		KUE(2) = 1	Amplitude probability density function
		= 2	List pf PDF values
		KUE(3) = 1	Autocorrelation
		= 2	Plot of autocorrelation
		= 3	List of autocorrelation function
		KUE(4) = 1	Crosscorrelation of two signals (no plot)
		= 2	Plot of crosscorrelation
		= 3	List of crosscorrelation
		KUE(5) = 1	Fourier transform of the autocorrelation function (mean square spectral density)
		= 2	List of transform coordinates
		KUE(I) = 0 if option not used	
3	6-9	Not used. Reserved for expansion of KUE(I) coding	
3	10	Number of data records in problem	Integer 1 or 2
3	11-15	Recording rate, points/second	Floating point
3	16-20	Frequency resolution desired in calculation of spectrum, cycles/second	Floating point

APPENDIX B

SAMPLE PROBLEM INPUT

As an example, a complete analysis was made of flow rate recordings from two pumps drawing from a common source and discharging into a large header. The pump suction heads were reduced considerably below design value as part of a test of the system behavior under simulated emergency conditions. The pumping rates were unsteady, and it was of interest to correlate the transients in the two systems as well as determine the mean flow rates and characteristics of each.

The data for the input required for the problem were coded on punched cards and read in with the statement shown in subroutine TRASH (page 11). Appendix C shows the output. All options were requested.

COMPUTER INPUT FOR EXAMPLE PROBLEM

TO:	FROM:	OSR 22-157(Rev 10-67)	DATE	PAGE OF
	JOB NUMBER			

1 CORRELATION OF FLOW TRANSIENTS IN PROCESS SYSTEMS 4 AND 6

SIGNAL NO. 1 = SYSTEM 4. SIGNAL NO. 2 = SYSTEM 6. COMPLETE ANALYSIS

12332	2	10	.1												
0.86	0.74	0.67	0.56	0.34	0.10	-0.18	-0.40	-0.56	-0.75	-0.84	-0.86	-0.81	-0.75	-0.79	-0.61
-0.54	-0.44	-0.01	0.37	0.89	0.97	1.11	0.98	0.86	0.71	0.51	0.32	-0.06	-0.30	-0.43	-0.57
-0.70	-0.78	-0.86	-0.78	-0.76	-0.62	-0.52	-0.43	-0.32	-0.26	0.14	0.47	0.97	1.03	0.99	0.73
0.58	0.41	0.28	0.02	-0.15	-0.43	-0.58	-0.77	-0.82	-0.87	-0.79	-0.79	-0.68	-0.62	-0.56	-0.43
-0.21	0.08	0.40	0.90	0.93	1.02	0.90	0.82	0.74	0.48	0.42	0.14	-0.17	-0.34	-0.54	-0.69
-0.81	-0.91	-0.81	-0.97	-0.86	-0.64	-0.60	-0.55	0.0	0.30	0.72	0.94	1.01	0.94	0.88	0.90
0.58	0.31	-0.03	-0.27	-0.40	-0.67	-0.70	-0.86	-0.80	-0.79	-0.82	-0.63	-0.58	-0.42	-0.25	0.09
0.45	0.80	0.80	0.84	0.79	0.66	0.61	0.37	0.17	0.0	-0.27	-0.43	-0.60	-0.70	-0.73	-0.78
-0.73	-0.70	-0.66	-0.53	-0.32	-0.10	0.17	0.58	0.90	0.98	0.93	0.80	0.67	0.53	0.35	0.10
-0.20	-0.26	-0.48	-0.60	-0.73	-0.79	-0.87	-0.88	-0.78	-0.71	-0.43	-0.29	-0.11	0.07	0.39	0.63
0.98	0.97	1.00	0.90	0.73	0.52	0.42	0.12	-0.08	-0.37	-0.57	-0.76	-0.82	-0.90	-0.92	-0.87
-0.82	-0.68	-0.57	-0.34	-0.09	0.13	0.41	0.66	0.97	1.07	1.00	0.85	0.66	0.49	0.30	-0.03
-0.30	-0.44	-0.65	-0.87	-0.90	-0.92	-0.87	-0.86	-0.76							
1.18	1.30	1.44	1.56	1.34	1.43	1.38	1.36	1.45	1.33	1.32	1.39	1.53	1.60	1.49	1.38
1.35	1.23	1.20	1.20	1.22	1.20	1.34	1.62	1.58	1.43	1.50	1.60	1.43	1.34	1.22	1.31
1.36	1.38	1.39	1.49	1.58	1.51	1.44	1.42	1.51	1.25	1.20	1.09	1.11	1.21	1.28	1.28
1.37	1.50	1.62	1.61	1.48	1.42	1.47	1.48	1.54	1.51	1.52	1.30	1.28	1.19	1.13	1.19
1.18	1.27	1.20	1.27	1.46	1.57	1.44	1.42	1.48	1.62	1.48	1.42	1.37	1.30	1.22	1.23
1.38	1.46	1.49	1.39	1.40	1.46	1.33	1.23	1.19	1.30	1.29	1.35	1.41	1.53	1.52	1.50
1.49	1.40	1.42	1.47	1.45	1.33	1.24	1.10	1.29	1.38	1.37	1.43	1.71	1.56	1.46	1.38
1.30	1.17	1.15	1.18	1.37	1.33	1.38	1.44	1.42	1.44	1.46	1.45	1.59	1.56	1.36	1.32
1.32	1.48	1.49	1.42	1.37	1.35	1.34	1.28	1.31	1.31	1.32	1.31	1.87	1.48	1.41	1.83
1.29	1.31	1.41	1.62	1.47	1.52	1.58	1.52	1.38	1.23	1.17	1.27	1.33	1.16	1.12	1.16
1.28	1.47	1.59	1.65	1.59	1.54	1.53	1.29	1.22	1.23	1.27	1.37	1.51	1.50	1.35	1.36
1.27	1.21	1.30	1.50	1.46	1.39	1.49	1.40	1.46	1.39	1.23	1.31	1.59	1.49	1.36	1.34
1.37	1.29	1.20	1.17	1.24	1.30	1.30	1.42	1.43							

APPENDIX C
SAMPLE PROBLEM OUTPUT

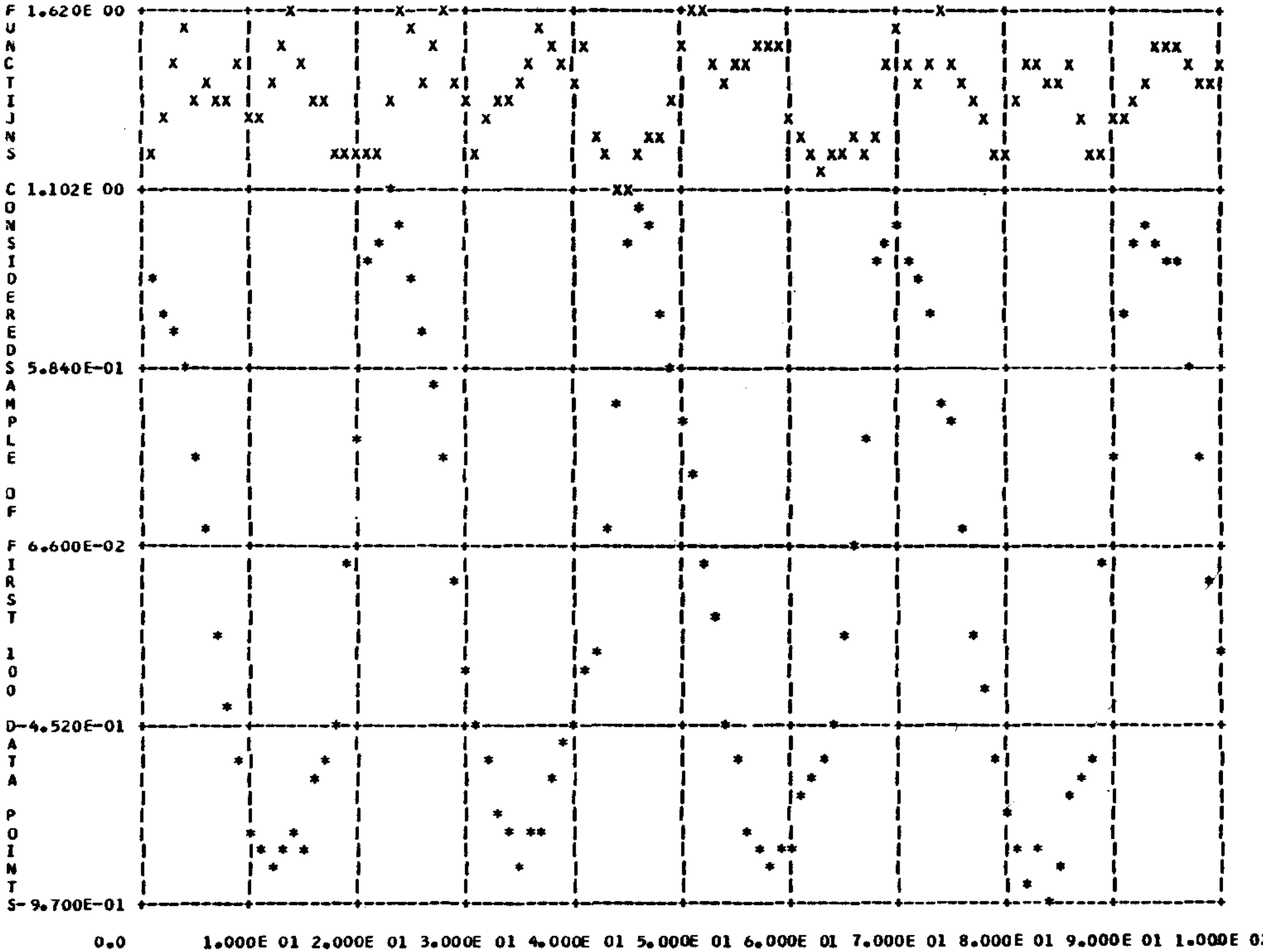
CORRELATION OF FLOW TRANSIENTS IN PROCESS SYSTEMS 4 AND 6

SIGNAL NO.1 = SYSTEM 4. SIGNAL NO.2 = SYSTEM 6. COMPLETE ANALYSIS

THE FOLLOWING OPERATIONS ARE PERFORMED

- BASIC STATISTICS COMPUTED (MEAN,VARIANCE,STANDARD DEVIATION,COEFFICIENT OF CORRELATION)
- FIRST 100 DATA POINTS ARE PLOTTED
- PROBABILITY DENSITY FUNCTION PLOTTED AND ALL COMPUTED VALUES LISTED
- AUTOCORRELATION FUNCTION PLOTTED AND ALL COMPUTED VALUES LISTED
- CROSS CORRELATION FUNCTION PLOTTED AND ALL COMPUTED VALUES LISTED
- MEAN SQUARE SPECTRAL DENSITY COMPUTED FROM THE AUTOCORRELATION FUNCTION AND PLOTTED

CHART 1



SAMPLE OF FIRST 100 DATA POINTS

BASIC STATISTICS ON THESE SIGNALS ARE AS FOLLOW

THE MEAN AMPLITUDES ARE -0.060 AND   1.378

FOR SIGNAL NO. 1 VARIANCE= 0.4322 AND THE STANDARD DEVIATION= 0.6574

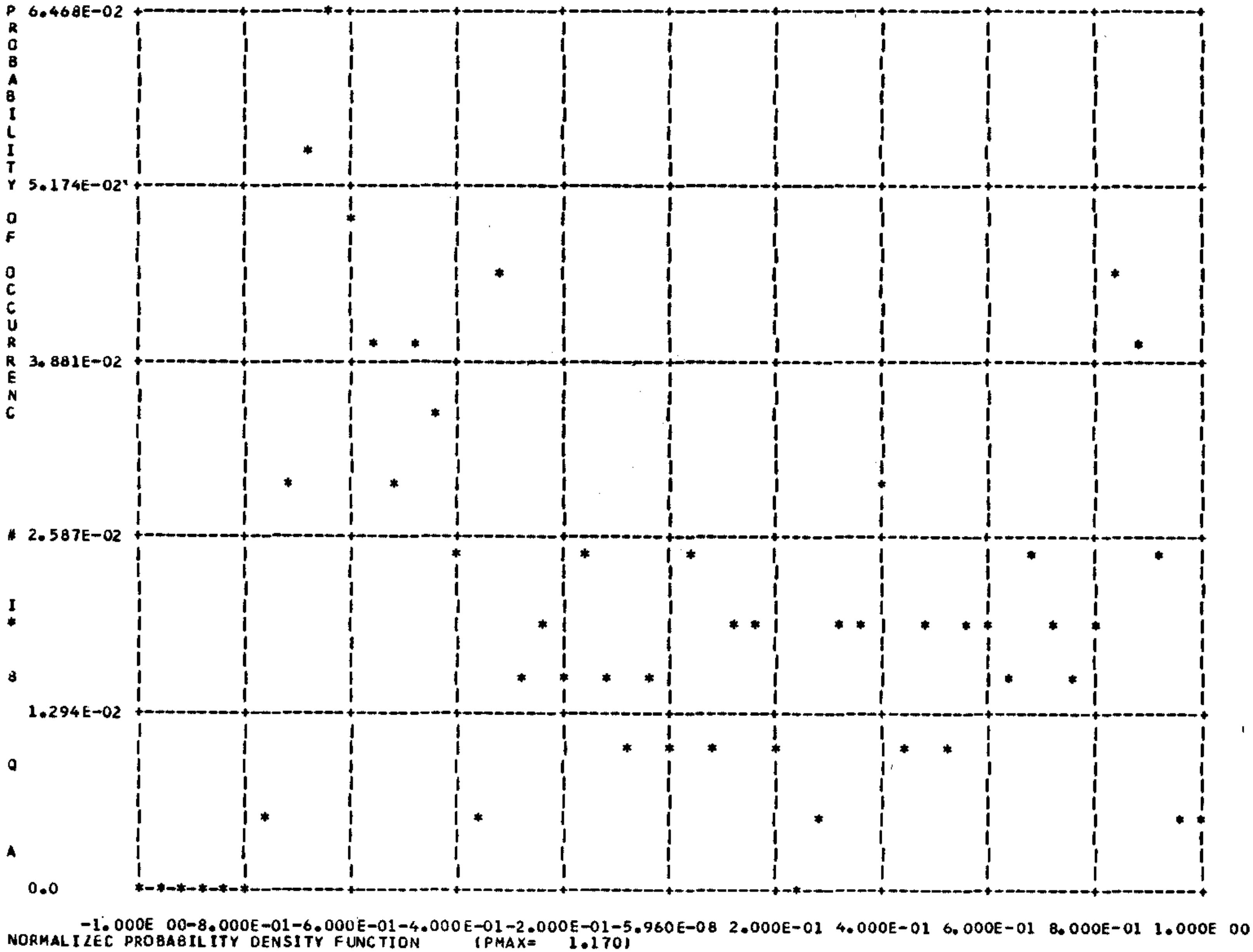
FOR SIGNAL NO. 2 VARIANCE= 0.0165 AND THE STANDARD DEVIATION= 0.1283

FINALLY,THE COVARIANCE IS -0.002 AND THE COEFFICIENT OF CORRELATION IS -0.027

FOR SIGNAL NO. 1 MAXIMUM VALUE= 1.1100 MINIMUM= -0.9700

FOR SIGNAL NO. 2 MAXIMUM VALUE= 1.7100 MINIMUM= 1.0900

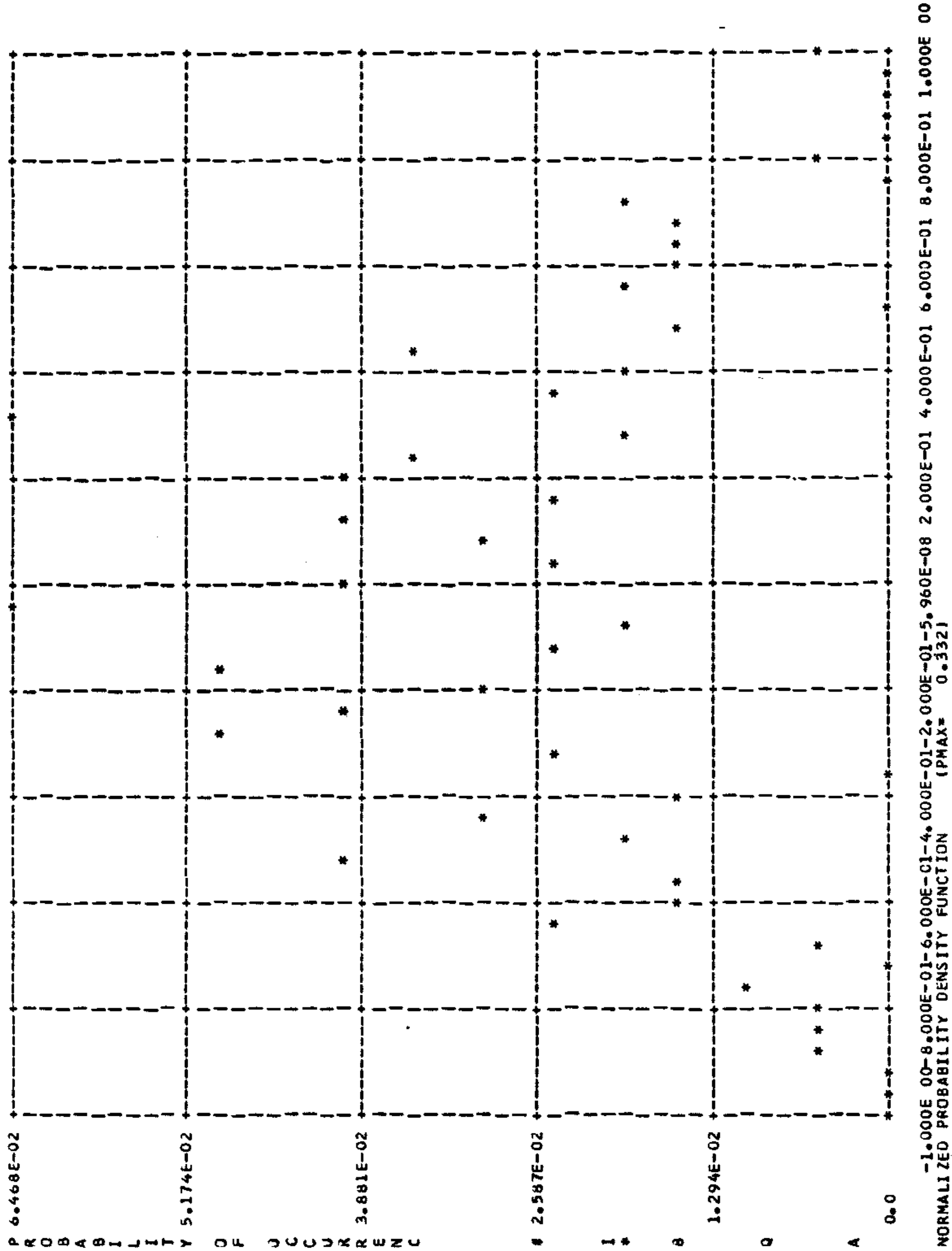
CHART 2


PROBABILITY OF OCCURRENC
I*8 QA
6.468E-02
5.174E-02
3.881E-02
2.587E-02
1.294E-02
0.0
-1.000E 00-8.000E-01-6.000E-01-4.000E-01-2.000E-01-5.960E-08 2.000E-01 4.000E-01 6.000E-01 8.000E-01 1.000E 00
NORMALIZEC PROBABILITY DENSITY FUNCTION (PMAX= 1.170)

NORMALIZED AMPLITUDE (P/PMAX) AND PROBABILITY

```
 P/PMAX=**** PROB=.0     P/PMAX=-.96 PROB=.0     P/PMAX=-.92 PROB=.0     P/PMAX=-.88 PROB=.0     P/PMAX=-.84 PROB=.0
P/PMAX=-.80 PROB=.0     P/PMAX=-.76 PROB=.005   P/PMAX=-.72 PROB=.030   P/PMAX=-.68 PROB=.055   P/PMAX=-.64 PROB=.065
P/PMAX=-.60 PROB=.050   P/PMAX=-.56 PROB=.040   P/PMAX=-.52 PROB=.030   P/PMAX=-.48 PROB=.040   P/PMAX=-.44 PROB=.035
P/PMAX=-.40 PROB=.025   P/PMAX=-.36 PROB=.005   P/PMAX=-.32 PROB=.045   P/PMAX=-.28 PROB=.015   P/PMAX=-.24 PROB=.020
P/PMAX=-.20 PROB=.015   P/PMAX=-.16 PROB=.025   P/PMAX=-.12 PROB=.015   P/PMAX=-.08 PROB=.010   P/PMAX=-.04 PROB=.015
P/PMAX=-.00 PROB=.010   P/PMAX=0.04 PROB=.025   P/PMAX=0.08 PROB=.010   P/PMAX=0.12 PROB=.020   P/PMAX=0.16 PROB=.020
P/PMAX=0.20 PROB=.010   P/PMAX=0.24 PROB=.0     P/PMAX=0.28 PROB=.005   P/PMAX=0.32 PROB=.020   P/PMAX=0.36 PROB=.020
P/PMAX=0.40 PROB=.030   P/PMAX=0.44 PROB=.010   P/PMAX=0.48 PROB=.020   P/PMAX=0.52 PROB=.010   P/PMAX=0.56 PROB=.020
P/PMAX=0.60 PROB=.020   P/PMAX=0.64 PROB=.015   P/PMAX=0.68 PROB=.025   P/PMAX=0.72 PROB=.020   P/PMAX=0.76 PROB=.015
P/PMAX=0.80 PROB=.020   P/PMAX=0.84 PROB=.045   P/PMAX=0.88 PROB=.040   P/PMAX=0.92 PROB=.025   P/PMAX=0.96 PROB=.005
P/PMAX=1.00 PROB=.005   P/PMAX=
```

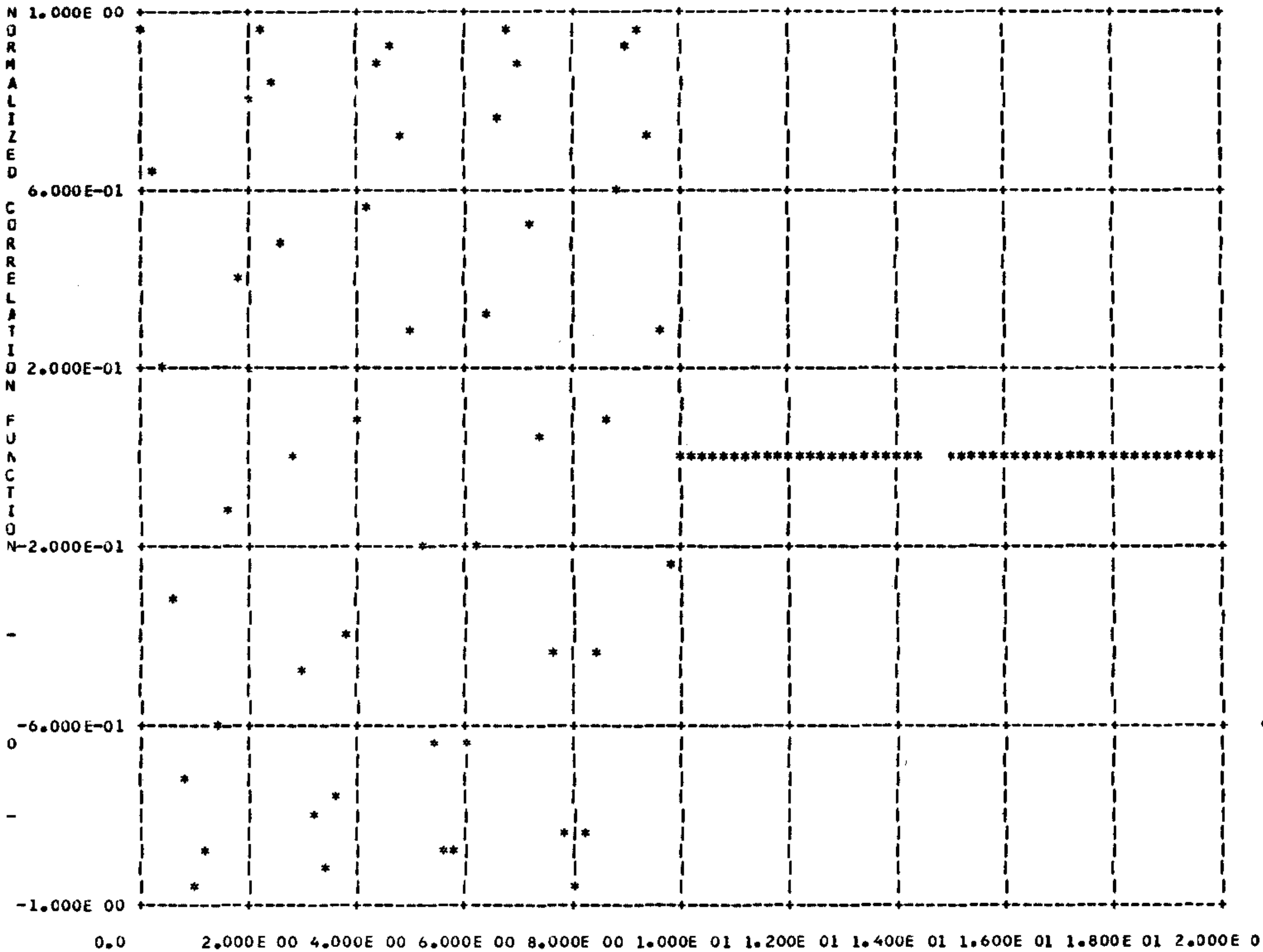
CHART 9


6.468E-02
5.174E-02
3.881E-02
2.587E-02
1.294E-02
0.0
PROBABILITY OF OCCURRENCE
-1.000E 00-8.000E-01-6.000E-01-4.000E-01-2.000E-01-5.960E-08 2.000E-01 4.000E-01 6.000E-01 8.000E-01 1.000E 00
NORMALIZED PROBABILITY DENSITY FUNCTION (PMAX= 0.332)

NORMALIZED AMPLITUDE (P/PMAX) AND PROBABILITY

```
 P/PMAX=****  PROB=.0       P/PMAX=-.96  PROB=.0      P/PMAX=-.92  PROB=.0       P/PMAX=-.88  PROB=.005   P/PMAX=-.84  PROB=.005
P/PMAX=-.80  PROB=.005    P/PMAX=-.76  PROB=.010    P/PMAX=-.72  PROB=.0       P/PMAX=-.68  PROB=.005   P/PMAX=-.64  PROB=.025
P/PMAX=-.60  PROB=.015    P/PMAX=-.56  PROB=.015    P/PMAX=-.52  PROB=.040     P/PMAX=-.48  PROB=.020   P/PMAX=-.44  PROB=.030
P/PMAX=-.40  PROB=.015    P/PMAX=-.36  PROB=.0      P/PMAX=-.32  PROB=.025     P/PMAX=-.28  PROB=.050   P/PMAX=-.24  PROB=.040
P/PMAX=-.20  PROB=.030    P/PMAX=-.16  PROB=.050    P/PMAX=-.12  PROB=.025     P/PMAX=-.08  PROB=.020   P/PMAX=-.04  PROB=.065
P/PMAX=-.00  PROB=.040    P/PMAX=0.04  PROB=.025    P/PMAX=0.08  PROB=.030     P/PMAX=0.12  PROB=.040   P/PMAX=0.16  PROB=.025
P/PMAX=0.20  PROB=.040    P/PMAX=0.24  PROB=.035    P/PMAX=0.28  PROB=.020     P/PMAX=0.32  PROB=.065   P/PMAX=0.36  PROB=.025
P/PMAX=0.40  PROB=.020    P/PMAX=0.44  PROB=.035    P/PMAX=0.48  PROB=.015     P/PMAX=0.52  PROB=.0     P/PMAX=0.56  PROB=.020
P/PMAX=0.60  PROB=.015    P/PMAX=0.64  PROB=.015    P/PMAX=0.68  PROB=.015     P/PMAX=0.72  PROB=.020   P/PMAX=0.76  PROB=.0
P/PMAX=0.80  PROB=.005    P/PMAX=0.84  PROB=.0      P/PMAX=0.88  PROB=.0       P/PMAX=0.92  PROB=.0     P/PMAX=0.96  PROB=.0
P/PMAX=1.00  PROB=.005    P/PMAX=
```

CHART 4

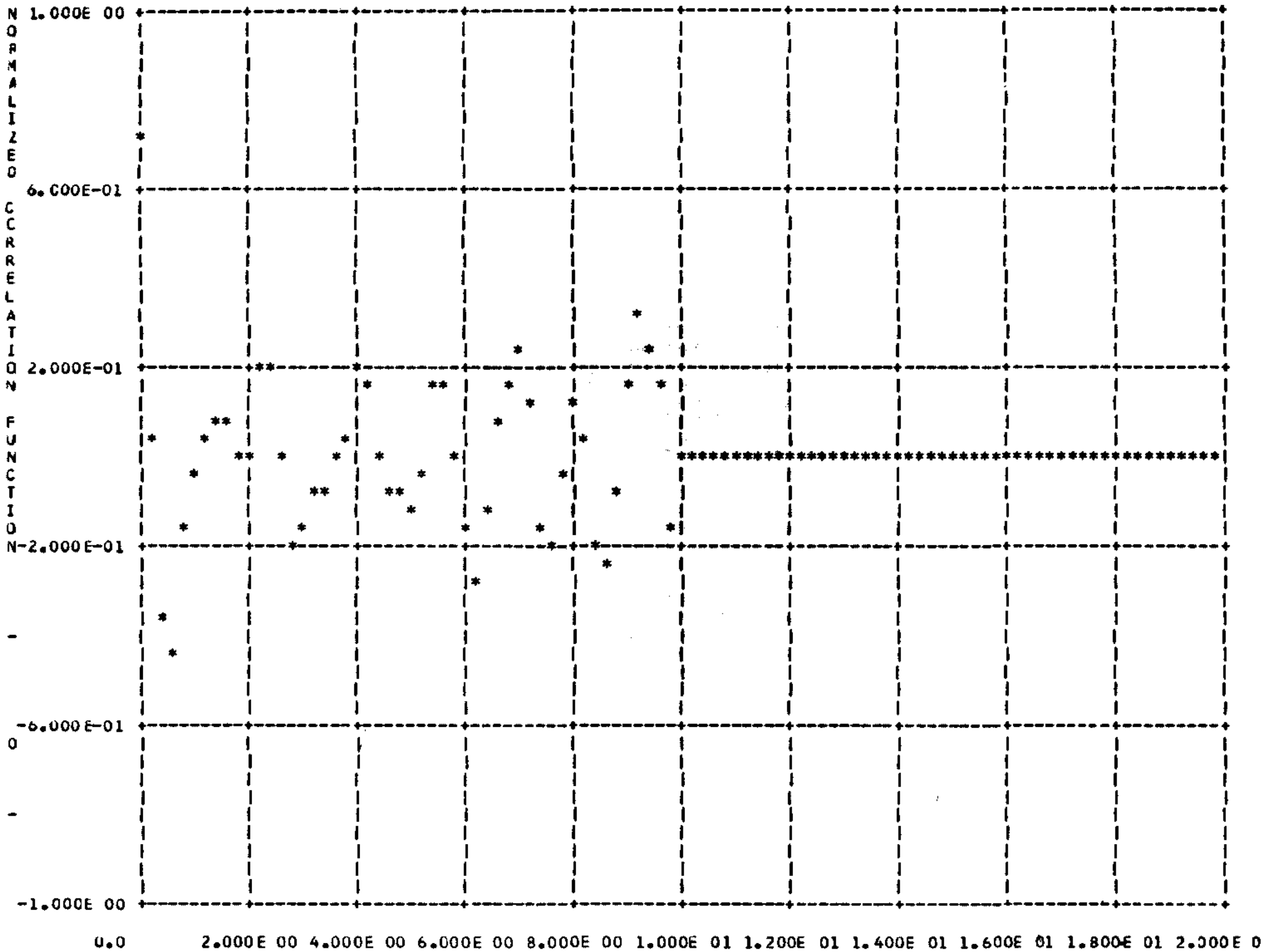


AUTOCORRELATION FUNCTION RR
BANDWIDTH RESOLUTION SPECIFIED TOO SMALL. RESET TO 100

LIST OF AUTOCORRELATION FUNCTION VALUES, NORMALIZED BYR(1,J)= 4.30014E-01. LAG INCREMENT(SEC)= 0.10000

R(1,J)=0.4300	R(2,J)=0.4093	R(3,J)=0.3589	R(4,J)=0.2819	R(5,J)=0.1863
R(6,J)=0.C796	R(7,J)=-.0316	R(8,J)=-.1390	R(9,J)=-.2363	R(10,J)=-.3168
R(11,J)=-.3738	R(12,J)=-.4044	R(13,J)=-.4066	R(14,J)=-.3806	R(15,J)=-.3278
R(16,J)=-.2509	R(17,J)=-.1556	R(18,J)=-.C480	R(19,J)=0.0634	R(20,J)=0.1700
R(21,J)=0.2650	R(22,J)=0.3414	R(23,J)=0.3935	R(24,J)=0.4156	R(25,J)=0.4051
R(26,J)=0.3628	R(27,J)=0.2933	R(28,J)=0.2044	R(29,J)=0.1027	R(30,J)=-.0044
R(31,J)=-.1091	R(32,J)=-.2068	R(33,J)=-.2894	R(34,J)=-.3513	R(35,J)=-.3886
R(36,J)=-.3979	R(37,J)=-.3796	R(38,J)=-.3344	R(39,J)=-.2648	R(40,J)=-.1760
R(41,J)=-.0731	R(42,J)=0.0361	R(43,J)=0.1439	R(44,J)=0.2414	R(45,J)=0.3224
R(46,J)=0.3782	R(47,J)=0.4055	R(48,J)=0.4013	R(49,J)=0.3656	R(50,J)=0.3030
R(51,J)=0.2191	R(52,J)=0.1206	R(53,J)=0.0146	R(54,J)=-.0920	R(55,J)=-.1909
R(56,J)=-.2765	R(57,J)=-.3422	R(58,J)=-.3843	R(59,J)=-.3987	R(60,J)=-.3851
R(61,J)=-.3437	R(62,J)=-.2773	R(63,J)=-.1895	R(64,J)=-.0859	R(65,J)=0.0259
R(66,J)=0.1370	R(67,J)=0.2387	R(68,J)=0.3227	R(69,J)=0.3817	R(70,J)=0.4119
R(71,J)=0.4092	R(72,J)=0.3746	R(73,J)=0.3125	R(74,J)=0.2272	R(75,J)=0.1268
R(76,J)=0.0177	R(77,J)=-.0926	R(78,J)=-.1952	R(79,J)=-.2840	R(80,J)=-.3533
R(81,J)=-.3953	R(82,J)=-.4105	R(83,J)=-.3952	R(84,J)=-.3528	R(85,J)=-.2826
R(86,J)=-.1907	R(87,J)=-.0845	R(88,J)=0.0307	R(89,J)=0.1470	R(90,J)=0.2511
R(91,J)=0.3360	R(92,J)=0.3956	R(93,J)=0.4243	R(94,J)=0.4202	R(95,J)=0.3838
R(96,J)=0.3179	R(97,J)=0.2285	R(98,J)=0.1249	R(99,J)=0.0136	R(100,J)=-.0963

CHART 5

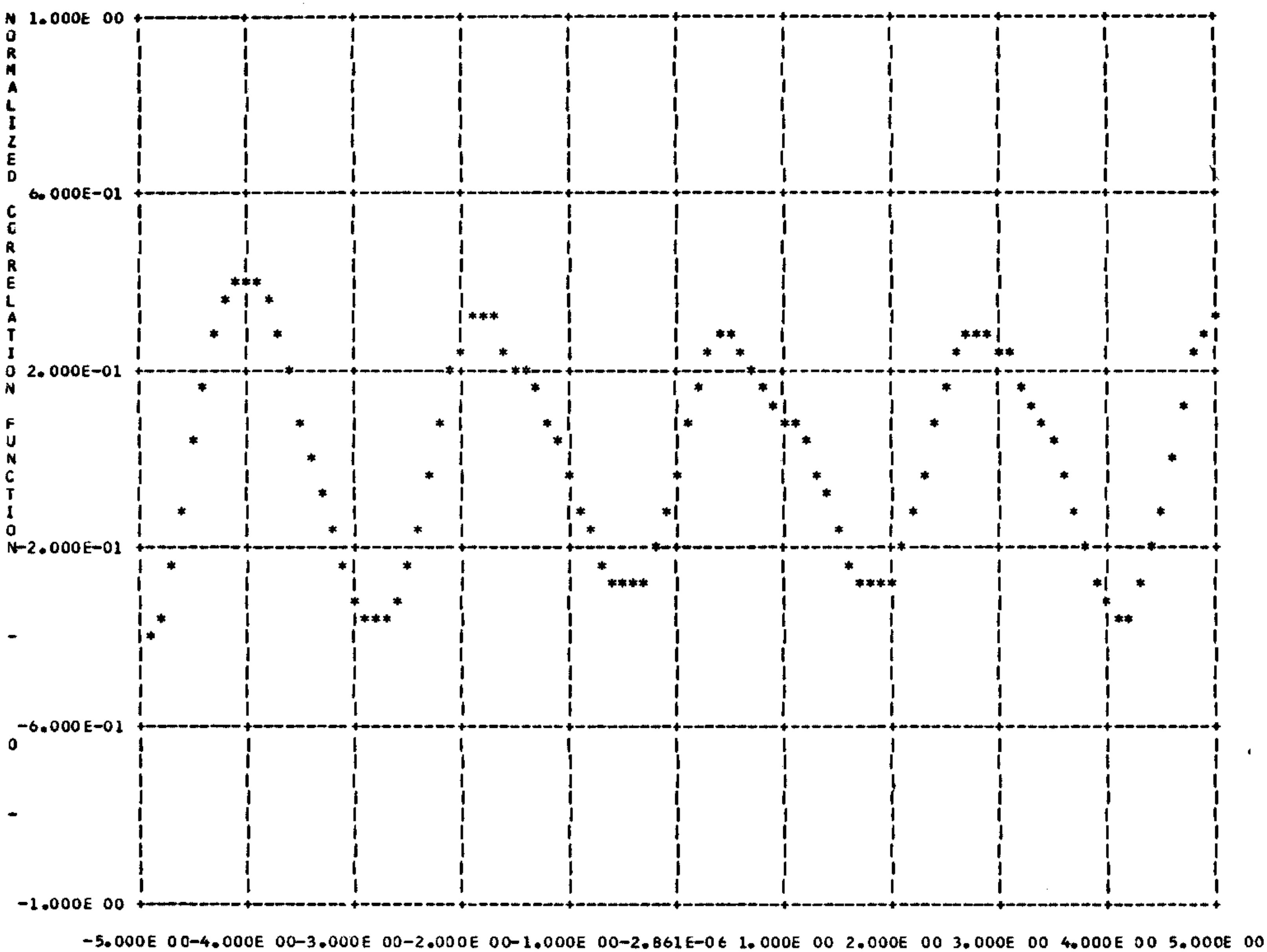


AUTOCORRELATION FUNCTION RR
BANDWIDTH RESOLUTION SPECIFIED TOO SMALL. RESET TO 100

```
LIST OF AUTOCORRELATION FUNCTION VALUES, NORMALIZED BYR(1,J)=    1.63684E-02. LAG INCREMENT(SEC)=    0.10000

 R(   1,J)=0.0164 R(   2,J)=0.0115 R(   3,J)=0.0052 R(   4,J)=0.0008 R(   5,J)=-.0024
R(    6,J)=-.0057 R(   7,J)=-.0076 R(   8,J)=-.0074 R(   9,J)=-.0053 R(  10,J)=-.0029
R(   11,J)=-.0015 R(  12,J)=-.0005 R(  13,J)=0.0000 R(  14,J)=0.0006 R(  15,J)=0.0008
R(   16,J)=0.0012 R(  17,J)=0.0017 R(  18,J)=0.0013 R(  19,J)=0.0003 R(  20,J)=-.0003
R(   21,J)=-.0003 R(  22,J)=0.0001 R(  23,J)=0.0016 R(  24,J)=0.0031 R(  25,J)=0.0039
R(   26,J)=0.0036 R(  27,J)=0.0022 R(  28,J)=0.0001 R(  29,J)=-.0019 R(  30,J)=-.0034
R(   31,J)=-.0040 R(  32,J)=-.0029 R(  33,J)=-.0019 R(  34,J)=-.0014 R(  35,J)=-.0016
R(   36,J)=-.0012 R(  37,J)=-.0006 R(  38,J)=0.0002 R(  39,J)=0.0003 R(  40,J)=0.0008
R(   41,J)=0.0015 R(  42,J)=0.0030 R(  43,J)=0.0035 R(  44,J)=0.0027 R(  45,J)=0.0012
R(   46,J)=0.0002 R(  47,J)=-.0008 R(  48,J)=-.0011 R(  49,J)=-.0013 R(  50,J)=-.0016
R(   51,J)=-.0016 R(  52,J)=-.0018 R(  53,J)=-.0015 R(  54,J)=-.0004 R(  55,J)=0.0011
R(   56,J)=0.0025 R(  57,J)=0.0030 R(  58,J)=0.0027 R(  59,J)=0.0016 R(  60,J)=-.0000
R(   61,J)=-.0012 R(  62,J)=-.0025 R(  63,J)=-.0034 R(  64,J)=-.0043 R(  65,J)=-.0036
R(   66,J)=-.0023 R(  67,J)=-.0003 R(  68,J)=0.0012 R(  69,J)=0.0024 R(  70,J)=0.0029
R(   71,J)=0.0037 R(  72,J)=0.0039 R(  73,J)=0.0033 R(  74,J)=0.0019 R(  75,J)=-.0001
R(   76,J)=-.0024 R(  77,J)=-.0033 R(  78,J)=-.0031 R(  79,J)=-.0022 R(  80,J)=-.0004
R(   81,J)=0.0012 R(  82,J)=0.0020 R(  83,J)=0.0017 R(  84,J)=0.0008 R(  85,J)=-.0008
R(   86,J)=-.0031 R(  87,J)=-.0043 R(  88,J)=-.0041 R(  89,J)=-.0028 R(  90,J)=-.0011
R(   91,J)=0.0007 R(  92,J)=0.0028 R(  93,J)=0.0047 R(  94,J)=0.0053 R(  95,J)=0.0045
R(   96,J)=0.0040 R(  97,J)=0.0036 R(  98,J)=0.0026 R(  99,J)=0.0004 R( 100,J)=-.0023
```

CHART 6



CROSS COVARIANCE FUNCTION OF DATA SIGNAL TIME INTERVAL= 0.1000 SEC/PT

LIST OF ALL VALUES OF CROSS CORRELATION FUNCTION

NO. PTS = 199 DT= 0.1000

-.3707-.4097-.3942-.3624-.2977-.1980-.08240.02100.11720.20090.26690.29970.33090.33320.30750.25250.20830.16010.10450.0421

.0339-.1186-.2089-.3120-.3982-.4307-.4116-.3608-.2925-.1894-.05250.07700.18310.26570.32420.35510.35310.33570.29110.2335

.16940.10430.0428-.0097-.0748-.1676-.2663-.3423-.3945-.4177-.4028-.3405-.2370-.10480.02720.15300.26150.34440.39700.4150

.39660.34750.27710.18900.0936-.0054-.0978-.1736-.2482-.3116-.3568-.3768-.3682-.3307-.2534-.1469-.02550.09380.18810.2542

.30020.30850.30010.25930.21520.18290.14990.09280.0292-.0387-.1072-.1783-.2365-.2752-.2895-.2835-.2617-.2027-.1264-.0266

.08130.17090.23540.26500.27240.24880.21100.17010.12670.09420.06950.0317-.0232-.0935-.1653-.2249-.2761-.2970-.2905-.2607

.2168-.1364-.03490.06100.14630.22180.26950.28440.28190.26000.22390.17670.13520.09130.0289-.0390-.1205-.2070-.2841-.3396

.3699-.3474-.2930-.2136-.10410.00910.12770.22740.29980.33350.32270.28910.24260.18960.12790.0603-.0216-.0944-.1589-.1910

.2151-.2276-.2344-.2201-.1918-.1533-.0988-.03350.03950.10910.17750.22100.23270.19990.16090.11290.07320.04690.02640.0148

.0271-.0810-.1232-.1336-.1455-.1535-.1401-.1194-.0919-.0794-.0531-.01110.03810.08000.10830.13510.14230.12210.0881

FOURIER TRANSFRM OF AUTOCORRELATION FUNCTION FOR POWER DENSITY ANALYSIS

NO. OF PTS TO COMPUTE SPECTRUM=L= 100
RECORDING RATE = ALPHA = 10.0 PTS/SEC

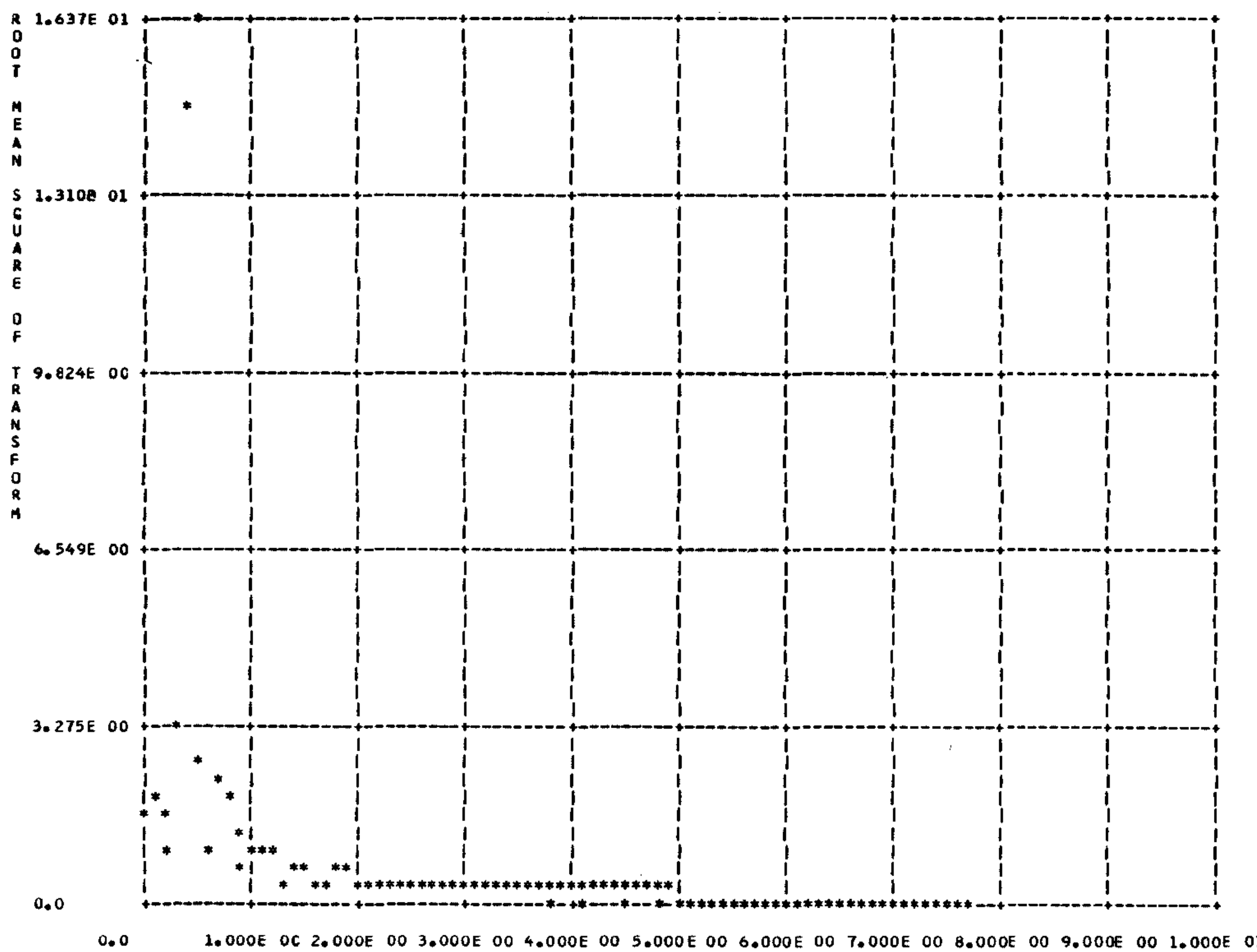
THE NYQUIST FREQUENCY FOR THIS DATA IS 5.00 HZ

THE FRECUENCY INCREMENT IS = 0.100 HZ

REAL TIME STARTING POINT ASSIGNED TO DATA WAS 0.0 SEC,CONTINUING FOR 20.10 SEC.

DATA PADDED OUT TO 128 PTS. COMPUTED RESOLUTICN IS 0.0781 HZ

CHART 7

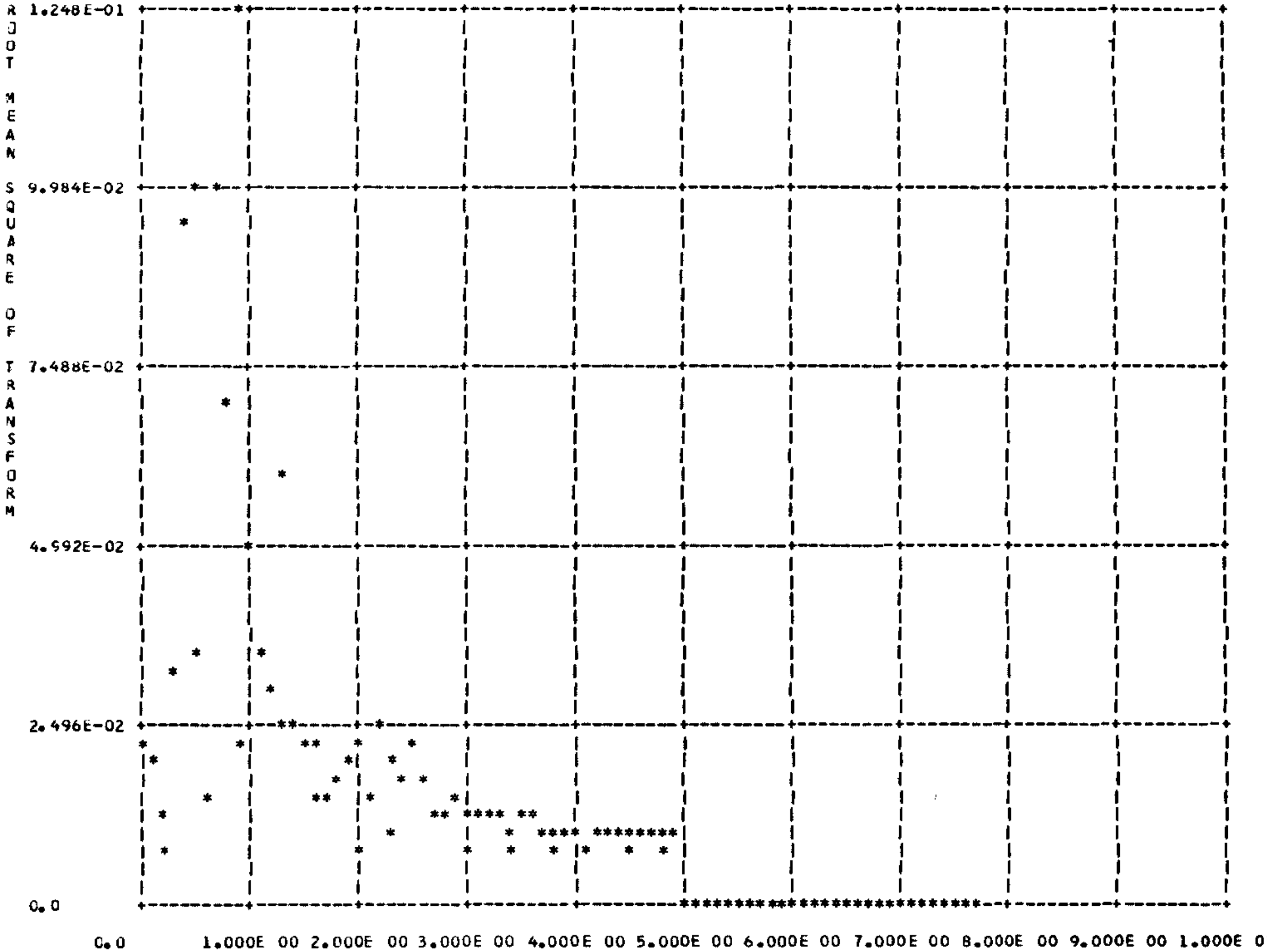


THE NYQUIST FREQUENCY FOR THIS DATA IS 5.00 HZ

THE FREQUENCY INCREMENT IS = 0.078 HZ

0.0 HZ	RMS=	1.629	0.08HZ	RMS=	1.818	0.16HZ	RMS=	1.748	0.23HZ	RMS=	0.874	0.31HZ	RMS=	3.327
0.39HZ	RMS=	14.851	0.47HZ	RMS=	16.373	0.55HZ	RMS=	2.631	0.63HZ	RMS=	1.108	0.70HZ	RMS=	2.163
0.78HZ	RMS=	1.996	0.86HZ	RMS=	1.252	0.94HZ	RMS=	0.515	1.02HZ	RMS=	0.894	1.09HZ	RMS=	1.109
1.17HZ	RMS=	0.871	1.25HZ	RMS=	0.437	1.33HZ	RMS=	0.459	1.41HZ	RMS=	0.723	1.48HZ	RMS=	0.724
1.56HZ	RMS=	0.480	1.64HZ	RMS=	0.274	1.72HZ	RMS=	0.477	1.80HZ	RMS=	0.594	1.87HZ	RMS=	0.494
1.95HZ	RMS=	0.279	2.03HZ	RMS=	0.293	2.11HZ	RMS=	0.449	2.19HZ	RMS=	0.475	2.27HZ	RMS=	0.335
2.34HZ	RMS=	0.209	2.42HZ	RMS=	0.323	2.50HZ	RMS=	0.423	2.58HZ	RMS=	0.371	2.66HZ	RMS=	0.220
2.73HZ	RMS=	0.224	2.81HZ	RMS=	0.344	2.89HZ	RMS=	0.371	2.97HZ	RMS=	0.277	3.05HZ	RMS=	0.170
3.12HZ	RMS=	0.256	3.20HZ	RMS=	0.346	3.28HZ	RMS=	0.313	3.36HZ	RMS=	0.196	3.44HZ	RMS=	0.182
3.52HZ	RMS=	0.289	3.59HZ	RMS=	0.322	3.67HZ	RMS=	0.247	3.75HZ	RMS=	0.156	3.83HZ	RMS=	0.217
3.91HZ	RMS=	0.305	3.98HZ	RMS=	0.288	4.06HZ	RMS=	0.187	4.14HZ	RMS=	0.162	4.22HZ	RMS=	0.255
4.30HZ	RMS=	0.300	4.37HZ	RMS=	0.237	4.45HZ	RMS=	0.147	4.53HZ	RMS=	0.200	4.61HZ	RMS=	0.283
4.69HZ	RMS=	0.275	4.77HZ	RMS=	0.190	4.84HZ	RMS=	0.149	4.92HZ	RMS=	0.240	5.00HZ	RMS=	0.295

DATA PADDED OUT TO 128 PTS. COMPUTED RESOLUTION IS 0.0781 HZ



THE NYQUIST FREQUENCY FOR THIS DATA IS 5.00 HZ

THE FREQUENCY INCREMENT IS = 0.078 HZ

0.0 HZ	RMS=	0.022	0.08HZ	RMS=	0.021	0.16HZ	RMS=	0.013	0.23HZ	RMS=	0.008	0.31HZ	RMS=	0.033
0.39HZ	RMS=	0.096	0.47HZ	RMS=	0.035	0.55HZ	RMS=	0.100	0.63HZ	RMS=	0.016	0.70HZ	RMS=	0.099
0.78HZ	RMS=	0.070	0.86HZ	RMS=	0.125	0.94HZ	RMS=	0.022	1.02HZ	RMS=	0.050	1.09HZ	RMS=	0.035
1.17HZ	RMS=	0.031	1.25HZ	RMS=	0.059	1.33HZ	RMS=	0.026	1.41HZ	RMS=	0.025	1.48HZ	RMS=	0.024
1.56HZ	RMS=	0.014	1.64HZ	RMS=	0.022	1.72HZ	RMS=	0.015	1.80HZ	RMS=	0.017	1.87HZ	RMS=	0.020
1.95HZ	RMS=	0.022	2.03HZ	RMS=	0.009	2.11HZ	RMS=	0.015	2.19HZ	RMS=	0.025	2.27HZ	RMS=	0.019
2.34HZ	RMS=	0.010	2.42HZ	RMS=	0.017	2.50HZ	RMS=	0.022	2.58HZ	RMS=	0.017	2.66HZ	RMS=	0.012
2.73HZ	RMS=	0.012	2.81HZ	RMS=	0.014	2.89HZ	RMS=	0.015	2.97HZ	RMS=	0.012	3.05HZ	RMS=	0.009
3.12HZ	RMS=	0.014	3.20HZ	RMS=	0.013	3.28HZ	RMS=	0.012	3.36HZ	RMS=	0.009	3.44HZ	RMS=	0.009
3.52HZ	RMS=	0.011	3.59HZ	RMS=	0.013	3.67HZ	RMS=	0.010	3.75HZ	RMS=	0.007	3.83HZ	RMS=	0.010
3.91HZ	RMS=	0.011	3.98HZ	RMS=	0.011	4.06HZ	RMS=	0.008	4.14HZ	RMS=	0.007	4.22HZ	RMS=	0.010
4.30HZ	RMS=	0.011	4.37HZ	RMS=	0.009	4.45HZ	RMS=	0.007	4.53HZ	RMS=	0.009	4.61HZ	RMS=	0.011
4.69HZ	RMS=	0.011	4.77HZ	RMS=	0.007	4.84HZ	RMS=	0.009	4.92HZ	RMS=	0.009	5.00HZ	RMS=	0.010

DP-1244
Not included
in Printed Copy

APPENDIX D

LISTING OF CODE (FORTRAN IV)

```
OMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MA
      SUBROUTINE FOUR1(DATA,N,ISIGN)
C     THE COOLEY-TUKEY FAST FOURIER TRANSFORM IN USASI BASIC FORTRAN.
C     NOTE-- IT SHOULD NOT BE NECESSARY TO CHANGE ANY STATEMENT IN THIS
C     PROGRAM SO LONG AS THE FORTRAN COMPILER USED STORES REAL AND
C     IMAGINARY PARTS ADJACENTLY IN STORAGE.
C     TRANSFORM(K) = SUM(DATA(J)*EXP(ISIGN*2*PI*SQRT(-1)*(J-1)*(K-1)
C     /N)), SUMMED OVER ALL J AND K FROM 1 TO N.  DATA IS A ONE-
C     DIMENSIONAL COMPLEX ARRAY (I.E., THE REAL AND IMAGINARY PARTS ARE
C     ADJACENT IN STORAGE, SUCH AS FORTRAN IV PLACES THEM) WHOSE LENGTH
C     N=2**K, K.GE.0 (IF NECESSARY, APPEND ZEROES TO THE DATA).  ISIGN
C     IS +1 OR -1.  IF A -1 TRANSFORM IS FOLLOWED BY A +1 ONE (OR VICE
C     VERSA) THE ORIGINAL DATA REAPPEAR, MULTIPLIED BY N.  TRANSFORM
C     VALUES ARE RETURNED IN ARRAY DATA, REPLACING THE INPUT.  THE TIME
C     IS PROPORTIONAL TO N*LOG2(N), RATHER THAN THE NAIVE N**2.
C     ACCURACY IS ALSO GREATLY IMPROVED, THE RMS RELATIVE ERROR BEING
C     BOUNDED BY 6*SQRT(2)*LOG2(N)*2**(-B), WHERE B IS THE NUMBER OF
C     BITS IN THE FLOATING POINT FRACTION.  WRITTEN BY NORMAN BRENNER OF
C     MIT LINCOLN LABORATORY.  JULY 1967.  THIS IS THE SHORTEST VERSION
C     OF THE FFT KNOWN TO THE AUTHOR.  FASTER PROGRAMS FOUR2 AND FOURT
C     EXIST THAT OPERATE ON ARBITRARILY SIZED MULTIDIMENSIONAL ARRAYS.
C     SEE-- IEEE AUDIO TRANSACTIONS (JUNE 1967), SPECIAL ISSUE ON FFT.
      DIMENSION DATA(1)
      IP0=2
      IP3=IP0*N
      I3REV=1
      DO 50 I3=1,IP3,IP0
      IF(I3-I3REV)10,20,20
10    TEMPR=DATA(I3)
      TEMPI=DATA(I3+1)
      DATA(I3)=DATA(I3REV)
      DATA(I3+1)=DATA(I3REV+1)
      DATA(I3REV)=TEMPR
      DATA(I3REV+1)=TEMPI
20    IP1=IP3/2
30    IF(I3REV-IP1)50,50,40
40    I3REV=I3REV-IP1
      IP1=IP1/2
      IF(IP1-IP0)50,30,30
50    I3REV=I3REV+IP1
      IP1=IP0
60    IF(IP1-IP3)70,100,100
70    IP2=IP1*2
      THETA=6.283185307/FLOAT(ISIGN*IP2/IP0)
      SINTH=SIN(THETA/2.)
      WSTPR=-2.*SINTH*SINTH
      WSTPI=SIN(THETA)
      WR=1.
      WI=0.
      DO 90 I1=1,IP1,IP0
      DO 80 I3=I1,IP3,IP2
      I2A=I3
      I2B=I2A+IP1
      TEMPR=WR*DATA(I2B)-WI*DATA(I2B+1)
      TEMPI=WR*DATA(I2B+1)+WI*DATA(I2B)
```

```
      NPTS=FF/BE1
C        NPTS= NO. OF CALCULATED TRANSFORM PTS LE.NYQUIST FREQ
      IF(NPTS.GT.200)GOTO71
      DO70I=1,100
      A(I,1)=FLOAT(I-1)*BE1
      A(I,2)=0.
      IF(I.LE.NPTS)A(I,2)=SQRT(DATA(1,I)**2+DATA(2,I)**2)
   70 CONTINUE
      M=1
      GOTO73
C       THIS PLOT SCHEME EVENLY SPACES PTS FROM 0 TO NYQUIST FREQ
   71 M=ITMAX/200+1
      DO72I=1,100
      A(I,1)=FLOAT(M*(I-1))*BE1
      RMS=DATA(1,M*I)**2+DATA(2,M*I)**2
   72 A(I,2)=SQRT(RMS)
C       DEFINE UPPER END OF FREQ SCALE
   73 CONTINUE
      X=FLOAT(M)*100.*BE1
      DO74I=1,20
   74 IF(X.LE.5.*FLOAT(I))GOTO77
   77 QWL=5.*FLOAT(I)
      CALLPLOTY(NO,A,100,2,NL,NS,HEAD1,NXS,NYS,QWO,QWL,YMIN,YMAX)
      BEPLOT=FLOAT(M)*BE1
      WRITE(6,103)FF,BEPLOT
C
      IF(KUE(5).NE.2)GOTO80
C       ONLY WRITE VALUES UP TO NYQUIST FREQUENCY
      I2TMAX=NPTS
      DO76I=1,I2TMAX,5
      DO75J=1,5
      HZ(J)=FLOAT(I+J-2)*BE1
   75 RMSS(J)=SQRT(DATA(1,I+J-1)**2+DATA(2,I+J-1)**2)
   76 WRITE(6,111)(HZ(J),RMSS(J),J=1,5)
   80 CONTINUE
  100 FORMAT(18A4)
  102 FORMAT(3F10.0)
  103 FORMAT('0','THE NYQUIST FREQUENCY FOR THIS DATA IS',F6.2,2X'HZ'//
     11X,'THE FREQUENCY INCREMENT IS =' F6.3,' HZ'//)
  104 FORMAT('1',////,1X'FOURIER TRANSFRM OF AUTOCORRELATION FUNCTION FO
     1R POWER DENSITY ANALYSIS')
  106 FORMAT('0','NO. OF PTS TO COMPUTE SPECTRUM=L=',I5/1X'RECORDING RAT
     1E = ALPHA = 'F5.1,' PTS/SEC'/)
  108 FORMAT('0','DATA PADDED OUT TO',I5,' PTS. COMPUTED RESOLUTION IS',
     1F7.4,' HZ')
  109 FORMAT('0','REAL TIME STARTING POINT ASSIGNED TO DATA WAS'F5.2,
     1 1X'SEC,CONTINUING FOR'F10.2, 1X'SEC.'/)
  110 FORMAT(3I1)
  111 FORMAT(1X,5(3XF6.2,'HZ',2X'RMS='F8.3))
      RETURN
      END
```

```
OMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MA
         SUBROUTINESPEC(ITMAX,ALPHA,NSIG,BE)
C         THIS SUBROUTINE ARRANGES THE TRANSFORM CALCULATION AND PLOTS RESULTS
C         ITMAX=NUMBER OF DATA POINTS
C         ALPHA=DATA RECORDING RATE (PTS/SEC)
C         TINIT=REAL TIME ASSIGNED TO BEGINNING OF DATA SIGNAL
C         QWO=INITIAL FREQUENCY, HZ
C         OWL= FINAL FREQUENCY PLOTTED OUT
C         DQWVFREQUENCY INCREMENT, HZ
         COMMONP(2000,2),R(1000,2),KUE(5),NO,XXR(1000),V(2)
         DIMENSIONA(100,2),DATA(2,2000),HEAD1(13),HZ(5),RMSS(5)
         DATAHEAD1/'ROOT MEAN SQUARE OF TRANSFORM'/
         WRITE(6,104)
       1 CONTINUE
         TINIT=0.0
         TIMAX=ITMAX
         L=ALPHA/BE
         IF(L.GE.ITMAX/2)L=ITMAX/2
         BE=ALPHA/FLOAT(L)
         QWO=0.0
         ITMAX=L
         WRITE(6,106)ITMAX,ALPHA
         FF=ALPHA/2.0
C         (FF IS THE FOLDING,OR NYQUIST,FREQUENCY)
         WRITE(6,103)FF,BE
         DT=1.0/ALPHA
         TTOTAL=TIMAX*DT
         WRITE(6,109)TINIT,TTOTAL
C
         DO80K=1,NSIG
C            PREPARATION FOR COOLEY TOOKEY SCHEME
         DO2I=1,ITMAX
         DATA(1,I)=R(I,K)
       2 DATA(2,I)=0.
C         PAD OUT DATA SET TO NEXT POWER OF 2 WITH ZEROS
         ICK=ITMAX+1
         DO4N=4,12
       4 IF(ITMAX.LE.2**N)GOTO5
       5 IND=2**N
C             COMPUTED BANDWIDTH RESOLUTION IS LESS THAN BE, BECAUSE OF PADDING
         BE1=ALPHA/FLOAT(IND)
         WRITE(6,108)IND,BE1
         DO3I=ICK,IND
         DATA(1,I)=0.
       3 DATA(2,I)=0.
         CALL FOUR1(DATA,IND,-1)
C          UPON RETURN, THE TRANSFORM VALUES ARE STRED IN DATA
      12 CONTINUE
C
         NO=NO+1
         NL=0
         NS=0
         NXS=0
         NYS=1
C         PLOTTING SCHEME SEEKS TO PLOT WITH GREATEST POSSIBLE DETAIL
```

```
)MPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MA
          SUBROUTINE CROSS(N,L)
C              SERIES HAS ALREADY BEEN NORMALIZED
C
C
          COMMONP(2000,2),R(1000,2),KUE(5),NO,XXR(1000),V(2)
          DIMENSIONXR(500,2)
          DO130J=1,L
          NJ=N-J+1
          SUMR=0.
          SUMS=0.
          DO120I=1,NJ
          IJ=I+J-1
          SUMR=SUMR+P(I,1)*P(IJ,2)
      120 SUMS=SUMS+P(IJ,1)*P(I,2)
          FNJ=NJ
          XR(J,1)=SUMR/(FNJ*V(1)*V(2))
      130 XR(J,2)=SUMS/(FNJ*V(1)*V(2))
C          THE NEGATIVE SIDE OF A LAGS B (XR(J,1)) IS THE REFLECTION
C           OF B LAGS A THRU THE ORDINATE AXIS.   THUS A CONTINUOUS CROSS COVARIANCE
C           FUNCTION IS FORMED FROM THEM AS FOLLOWS--
          DO140 I=1,L
      140 XXR(I)=XR(L-I+1,2)
          I1=L+1
          IEND=2*L-1
          DO150 I=I1,IEND
      150 XXR(I)=XR(I-L+1,1)
C           THE CROSS-COVARIANCE FUNCTION IS NOW DEFINED BY 2*L-1 PTS.  THE
C           ZERO TIME LAG POINT IS XXR(L)
          RETURN
          END
```

```
)MPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MA
      SUBROUTINEAUTO(K,N,L,INT)
C        THIS SUBROUTINE IS FROM THE SSP-II LIBRARY.  AUTO COMPUTES
C        THE AUTOCORRELATION FUNCTION OF P(I,J) FOR LAG TIMES 0 TO L
C         AND STORES IT IN THE ARRAY R
      COMMONP(2000,2),R(1000,2),KUE(5),NO,XXR(1000),V(2)
      AVER=0.0                                                            AUTO 04
      IF(N-L) 50,50,100                                                   AUTO MO
   50 R(1,K)=0.
      RETURN                                                              AUTO MO
  100 DO 110 I=1,N
  110 AVER=AVER+P(I,K)
      FN=N
      AVER=AVER/FN
C                                                                         AUTO 04
C      CALCULATE AUTOCOVARIANCES                                          AUTO 05
C                                                                         AUTO 051
      DO130J=1,L,INT
      NJ=N-J+1                                                            AUTO 05
      SUM=0.0                                                             AUTO 05
      DO 120 I=1,NJ                                                       AUTO 05
      IJ=I+J-1                                                            AUTO 05
  120 SUM=SUM+(P(I,K)-AVER)*(P(IJ,K)-AVER)
      FNJ=NJ                                                              AUTO 05
  130 R(J,K)=SUM/FNJ
      RETURN                                                              AUTO 06
      END                                                                 AUTO 051
```

```
      CALL PLOTY(NO,A,N,2,NL,NS,HEAD1,NXS,NYS,XMIN,XMAX,YMIN,YMAX)
      WRITE(6,111)(RMKS2(I),I=1,13),DT
      IF(KUE(4).NE.3)GOTO302
      NCCF=2*L-1
      WRITE(6,104)NCCF,DT,(XXR(I),I=1,NCCF)
  302 CONTINUE
  100 FORMAT(20A4)
  101 FORMAT(2I5,3F10.0)
  102 FORMAT(1X,'BANDWIDTH RESOLUTION  SPECIFIED TOO SMALL. RESET TO'I6)
  103 FORMAT('0','************LESS THAN 100 CROSS COVARIANCE POINTS ')
  104 FORMAT('1','LIST OF ALL VALUES OF CROSS CORRELATION FUNCTION'/'0',
     1'NO. PTS ='I5,'   DT=' F8.4/'0',(20F6.4))
  105 FORMAT('1','LIST OF AUTOCORRELATION FUNCTION VALUES, NORMALIZED BY
     1R(1,J)='1PE15.5,'. LAG INCREMENT(SEC)='0PF10.5/'0',( 5(' R('I4,',J
     2)='F6.4)))
  111 FORMAT('0',13A4,3X'TIME INTERVAL='F8.4,' SEC/PT')
      RETURN
      END
```

```
IMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MA
      SUBROUTINEAUCOR(IMX,NSIG,ALPHA,BE)
C        AUCOR SETS UP COMPUTATION OF AUTO- AND CROSS-CORRELATION FUNCTION S
      COMMONP(2000,2),R(1000,2),KUE(5),NO,XXR(1000),V(2)
      DIMENSIONA(100,2),HEAD1(13),RMKS(13),RMKS2(13),RMKS3(13)
C        PLOTTING MATRIX A MUST HAVE EXACT DIMENSIONS
      DATA N,M,NS,NXS,NYS,XMIN,XMAX,YMIN,YMAX/100,2,0,0,0,  0.,100.,-1.,
     11./,RMKS2/'CROSS COVARIANCE FUNCTION OF DATA SIGNAL              '/,
     2RMKS/'AUTOCORRELATION FUNCTION'/,HEAD1/'NORMALIZED CORRELATION FUN
     3CTION'/
      NN=IMX
      NL=0
      L=ALPHA/BE
      IF(L.GT.IMX/2)L=IMX/2
      INT=1
      DT=1./ALPHA
      IF(KUE(3).EQ.0)GOTO250
      IF(KUE(3).EQ.2)GOTO250
      DO250J=1,NSIG
      IF(KUE(3).GE.1)CALL AUTO(J,NN,L,INT)
C        AUTO IS THE SUBROUTINE WHICH COMPUTES THE AUTOCORRELATION FUNCTION
C       NN= NO. OF DATA POINTS
C             L = MAXIMUM LAG TIME TO BE USED
C        THE REMAINDER OF THIS ROUTINE HANDLES THE DETAILS OF
C         PLOTTING THE AUTOCORRELATION FUNCTION(NORMALIZED)
      IF(KUE(3).EQ.1)GOTO250
      III=0
      M=L/100+1
      DO210II=1,100
      A(II,1)=FLOAT(M*(II-1))*DT
      A(II,2)=0.
      IF(M*II.LE.L)A(II,2)=R(M*II,J)/R(1,J)
  210 CONTINUE
      XMAX=FLOAT(100*M)*DT
      NO=NO+1
      XMIN=0.
      CALLPLOTY(NO,A,N,2,NL,NS,HEAD1,NXS,NYS,XMIN,XMAX,YMIN,YMAX)
C        ( OS/360 LIBRARY SUBROUTINE)
      WRITE(6,111)(RMKS(I),I=1,7),DT
      IF(ALPHA/BE.GT.FLOAT(L))WRITE(6,102)L
      IF(KUE(3).NE.3)GOTO250
      WRITE(6,105)R(1,J),DT,(I,R(I,J),I=1,L)
  250 CONTINUE
      IF(KUE(4).EQ.0)GOTO302
      CALLCROSS(IMX,L)
      IF(KUE(4).LT.2)GOTO302
      XMIN=-50.*DT
      XMAX=50.*DT
      IF(2*L-1.LT.100)WRITE(6,103)
      IF(2*L-1.LT.100)GOTO302
C       LOAD PLOT ARRAY
      DO300I=1,100
      A(I,1)=FLOAT(I-50)*DT
  300 A(I,2)=XXR(L+I-50)
      NO=NO+1
```

```
)MPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MA
          SUBROUTINE PDF(IMX,NSIG)
          DIMENSIONAKMX(51),A(51,2),HEAD( 6),RMKS(20)
          COMMONP(2000,2),R(1000,2),KUE(5),NO,XXR(1000),V(2)
          DATA HEAD/'PROBABILITY OF OCCURRENCE'/
          TMAX=IMX
          DO80J=1,NSIG
C           FIND MAXIMUM VALUE OF FUNCTION (ABSOLUTE)
          BIG=0.
          SMALL=0.
          DO10I=1,IMX
          IF(BIG-P(I,J))2,10,4
        2 BIG=P(I,J)
          GOTO10
        4 IF(P(I,J).LT.SMALL)SMALL=P(I,J)
       10 CONTINUE
          IF(BIG+SMALL)11,12,12
       12 PMAX=BIG
          GOTO13
       11 PMAX=-SMALL
       13 CONTINUE
C
          X=-1.
          DX=.02
C          COUNT THE NUMBER OF TIMES THE FUNCTION HAD AN AMPLITUDE INX+-DX
          DO 30M=1,51
          K=0
          DO20I=1,IMX
          PIJ= P(I,J)/PMAX
          IF((X-DX).LT.PIJ.AND.PIJ.LE.(X+DX))K=K+1
       20 CONTINUE
          AKMX(M)=K
          AM=M
          X=-1.0+2.0*AM*DX
       30 CONTINUE
C          BUILD PLOTY MATRIX
C              PLOT X (D'LESS) VS. PROBABILITY OF X
          DO40M=1,51
          AM1=M-1
          A(M,1)=-1.+2.*AM1*DX
       40 A(M,2)=AKMX(M)/TMAX
          NO=NO+1
          NL=0
          NS=0
          NXS=0
          NYS=1
          CALLPLOTY(NO,A,51,2,NL,NS,HEAD,NXS,NYS,-1.,1.,YMIN,YMAX)
          WRITE(6,112)PMAX
          IF(KUE(2).NE.2)GOTO80
          WRITE(6,110)(A(M,1),A(M,2),M=1,51)
       80 CONTINUE
      110 FORMAT('1','NORMALIZED AMPLITUDE (P/PMAX) AND PROBABILITY'/'0',(5(
         1' P/PMAX='F4.2,' PROB='F4.3,1X)))
      112 FORMAT(1X,'NORMALIZED PROBABILITY DENSITY FUNCTION',5X'(PMAX=',
         1 F8.3,')')
```

```
)MPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MA
          SUBROUTINE BASIC(IMX,NSIG,ALPHA)
C           THE BASIC PROGRAM COMPUTES AND WRITES OUT THE BASIC STATISTICS
C             ON THE DATA.
          DIMENSIONAVGP(2),VAR(2)
          COMMONP(2000,2),R(1000,2),KUE(5),NO,XXR(1000),V(2)
          TMX=IMX
C            EACH SIGNAL IS TAKEN IN TURN
          DO50 J=1,NSIG
          SUM=0.0
C            ITS MEAN VALUE (= AVGP) FOUND
          DO20 I=1,IMX
       20 SUM=SUM+P(I,J)
          AVGP(J)=SUM/TMX
          SUM=0.0
C            AND ITS VARIANCE(=VARJ)) & STANDARD DEVIATION(=V(J)) COMPUTED
          DO30 I=1,IMX
       30 SUM=SUM+(P(I,J)-AVGP(J))**2
          VAR(J)=SUM/(TMX-1.0)
       50 V(J)=SQRT(VAR(J))
          IF(NSIG.EQ.1)GOTO65
          SUM=0.0
C            NEXT, IF NSIG=2, THE COVARIANCE & COEFFICIENT OF CORRELATION IS
C              COMPUTED
          DO60 I=1,IMX
       60 SUM=SUM+(P(I,1)-AVGP(1))*(P(I,2)-AVGP(2))
          COVAR=SUM/(TMX-1.)
          COEF=COVAR/(V(1)*V(2))
       65 CONTINUE
C            ALL OF THIS INFORMATION IS WRITTEN OUT
          WRITE(6,100)
          WRITE(6,101)(AVGP(J),J=1,NSIG)
          WRITE(6,102)(J,VAR(J),V(J),J=1,NSIG)
          IF(NSIG.EQ.1)GOTO67
          WRITE(6,103)COVAR,COEF
       67 CONTINUE
C            TO CONTINUE, THE MEAN VALUE OF EACH SIGNAL IS SUBTRACTED
C            FROM THE DATA RECORD, SO THAT THE MEAN IS NOW ZERO.
          DO70J=1,NSIG
          DO70I=1,IMX
       70 P(I,J)=P(I,J)-AVGP(J)
      100 FORMAT('1',10X'BASIC STATISTICS ON THESE SIGNALS ARE AS FOLLOW')
      101 FORMAT('0','THE MEAN AMPLITUDES ARE',F8.3,' AND',F8.3)
      102 FORMAT('0','FOR SIGNAL NO.',I2,' VARIANCE=',F8.4,' AND THE STANDAR
         1D DEVIATION=',F8.4)
      103 FORMAT('0','FINALLY,THE COVARIANCE IS'F8.3,' AND THE COEFFICIENT O
         1F CORRELATION IS',F8.3)
          RETURN
          END
```

```
C         PREPARATION FOR PLOTTING A 100-PT SAMPLE OF THE DATA RECORD
C         (A(I,J)=ARRAY USED BY THE PLOTTING SUBROUTINE WHICH STORES INFO
C         TO BE PLOTTED
      DO20 I=1,IMX
      IF(I.GT.100)GOTO20
      T=I
      A(I,1)=T
      A(I,2)=P(I,1)
      A(I,3)=0.
      IF(NSIG.EQ.2)A(I,3)=P(I,2)
   20 CONTINUE
      NO=1
      M=NSIG+1
      CALL PLOTY(NO,A,N,M,NL,NS,HEAD,NXS,NYS,XMIN,XMAX,YMIN,YMAX)
      WRITE(6,104)(RMKS(I),I=1,8)
  104 FORMAT('0',8A4)
      RETURN
      END
```

```
104 FORMAT('0',40X'FIRST 100 DATA POINTS ARE PLOTTED')
105 FORMAT('0',40X'AMPLITUDE PROBABLITY DISTRIBUTION COMPUTED AND PLOT
   1TED')
106 FORMAT('0',40X'AUTOCORRELATION FUNCTION COMPUTED')
107 FORMAT('0',40X'CROSS CORRELATION FUNCTION COMPUTED')
108 FORMAT('0',40X'MEAN SQUARE SPECTRAL DENSITY COMPUTED FROM THE AUTO
   1CORRELATION FUNCTION AND PLOTTED')
109 FORMAT('0',40X'AUTOCORRELATION FUNCTION PLOTTED')
110 FORMAT('0',40X'AUTOCORRELATION FUNCTION PLOTTED AND ALL COMPUTED V
   1ALUES LISTED')
111 FORMAT('0',40X'CROSS CORRELATION FUNCTION PLOTTED')
112 FORMAT('0',40X'CROSS CORRELATION FUNCTION PLOTTED AND ALL COMPUTED
   1 VALUES LISTED')
113 FORMAT('0',40X'PROBABILITY DENSITY FUNCTION PLOTTED AND ALL COMPUT
   1ED VALUES LISTED')
    STOP
    END
```

```
)MPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=58,SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MA
   C       THIS IS THE CONTROL PROGRAM WHICH DIRECTS THE SEQUENCE OF CALCU-
   C      LATIONS.  IT READS THE BASIC INSTRUCTION CODE AND WRITES A TITLE
   C      PAGE FOR THE PRINT OUT.
          COMMONP(2000,2),R(1000,2),KUE(5),NO,XXR(1000),V(2)
          DIMENSIONRMKS(39)
          READ(5,100)NPROB,RMKS
   C       (RMKS IS AN EXPLANATORY TITLE,TWO CARDS IN LENGTH,WHICH WILL HEAD
   C      UP THE PRINTOUT)
          DO15IPROB=1,NPROB
          READ(5,101)KUE,NSIG,ALPHA,BE
   C       THE KUE(I) CODE DIRECTS THE PROGRAM, AS FOLLOWS
   C                     KUE(1)=1 CALLS FOR PLOT OF FIRST 100 DATA PTS
   C                     KUE(2)=1 CALLS FOR AMPLITUDE PROBABILITY  DENSITY FUNCTION
   C                            =2 CALLS FOR LIST OF PDF VALUES
   C                     KUE(3)=1 CALLS FOR AUTOCORRELATION
   C                            =2 CALLS FOR PLOT OF AUTOCORRELATION
   C                            =3 CALLS FOR LIST OF AUTOCORRELATION FUNCTION
   C                     KUE(4)=1 CALLS FOR CROSSCORRELATION OF TWO SIGNALS(NO PLOT
   C                            =2 CALLS FOR PLOT OF CROSSCORRELATION
   C                            =3 CALLS FOR LIST OF CROSSCORRELATION
   C                     KUE(5)=1 CALLS FOR FOURIER TRANSFORM OF THE AUTOCORRE-
   C                               LATION FUNCTION(MEAN-SQUARE SPECTRAL DENSITY)
   C                            =2 CALLS FOR LIST OF RMS TRANSFORM COORDINATES
   C
   C                    NSIG= NUMBER OF DATA RECORDS IN PROBLEM (1 OR 2)
   C                    ALPHA=RECORDING RATE, PTS/SEC
   C                    BE=CPS RESOLUTION DESIRED FOR POWER SPECTRUM CALCULATION
   C                          (MUST BE LESS THAN ALPHA/0.5*IMX)
          WRITE(6,102)RMKS
          WRITE(6,103)
          IF(KUE(1).EQ.1)WRITE(6,104)
          IF(KUE(2).EQ.1)WRITE(6,105)
          IF(KUE(2).EQ.2)WRITE(6,113)
          IF(KUE(3).EQ.1)WRITE(6,106)
          IF(KUE(3).EQ.2)WRITE(6,109)
          IF(KUE(3).EQ.3)WRITE(6,110)
          IF(KUE(4).EQ.1)WRITE(6,107)
          IF(KUE(4).EQ.2)WRITE(6,111)
          IF(KUE(4).EQ.3)WRITE(6,112)
          IF(KUE(5).NE.0)WRITE(6,108)
          CALL TRASH(NSIG,IMX,ALPHA)
   C       (TRASH READS IN THE BASIC DATA P(I,J). IT ALSO RETURNS THE NO.OF
   C       DATA PTS,IMX)
          CALL BASIC(IMX,NSIG,ALPHA)
   C       (BASIC WILL WRITE OUT ITS FINDINGS BEFORE RETURN)
          IF(KUE(2).NE.0)CALLPDF(IMX,NSIG)
   C
          IF(KUE(3).NE.0)CALL AUCOR(IMX,NSIG,ALPHA,BE)
          IF(KUE(5).NE.0)CALL SPEC(IMX,ALPHA,NSIG,BE)
       15 CONTINUE
      100 FORMAT(I4,19A4/20A4)
      101 FORMAT(5I1,I5,2F5.0)
      102 FORMAT('1',19A4//1X,20A4///)
      103 FORMAT(XX'THE FOLLOWING OPERATIONS ARE PERFORMED'/'0',40X'BASIC ST
         1ATISTICS COMPUTED (MEAN,VARIANCE,STANDARD DEVIATION,COEFFICIENT OF
         2 CORRELATION)')
```

```
      DATA(I2B)=DATA(I2A)-TEMPR
      DATA(I2B+1)=DATA(I2A+1)-TEMPI
      DATA(I2A)=DATA(I2A)+TEMPR
80    DATA(I2A+1)=DATA(I2A+1)+TEMPI
      TEMPR=WR
      WR=WR*WSTPR-WI*WSTPI+WR
90    WI=WI*WSTPR+TEMPR*WSTPI+WI
      IP1=IP2
      GO TO 60
100   RETURN
      END
```