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COMPUTER CODES FOR NUCLEAR CRITICALITY SAFETY CALCULATIONS

H. K. CLARK

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Savannah River Laboratory

Aiken, South Carolina

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by

Hugh K. Clark

Approved by

P. L. Roggenkamp, Research Manager
Theoretical Physics Division

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ABSTRACT

Several computer codes for nuclear criticality safety evaluations are described along with their input requirements. The two basic codes are a code for computing the buckling of fissile material and a two-group diffusion theory code, which uses parameters calculated by the buckling code, for computing critical sizes. The other codes generate cross sections for the buckling code, modify two-group parameters for special cases, compute k_{eff}, and compute the effect of special shapes. Numerous correlations of the codes with data obtained in exponential and critical experiments are given to permit the user to determine the margin that must be allowed to ensure subcriticality for various situations to which the codes may be applied.

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INTRODUCTION

During the past several years, a number of codes have been written at the Savannah River Laboratory for performing calculations required in nuclear safety evaluations. Although general descriptions of some of the computational techniques have been given,[1,2] no detailed descriptions of the codes have been written, with the exception of the code developed for computing the effect of interaction,[3] because they have been under fairly continuous development, partly as the result of changes in computing machinery and in the manner in which it has been operated. Recently, the codes have been converted to FORTRAN IV and have been checked out on an IBM System/360-65. Further changes in the codes due to changes in computing machinery are not anticipated. By now the codes have been used sufficiently long for most (hopefully, all) errors to have been discovered and corrected and for most (but by no means all) changes that are desirable to facilitate calculations to have been made.

The principal codes are a code for computing the buckling of fissile material moderated by water and a two-group diffusion theory code that uses parameters generated by the buckling code. Several approximations are made in the buckling code, but an attempt is made to account for all major effects. The other codes are essentially auxiliary in nature. They are used either to generate cross sections for the buckling code or to operate on the output from the buckling and two-group codes to modify parameters, to take account of special shapes, or to compute k_{eff} for particular situations. For each code the essential features and the theoretical basis are described. This is followed by a detailed listing of the input requirements and the output that is or may be obtained.

With any method of calculation, it is necessary to know the margin that must be allowed to ensure subcriticality. This margin is best determined by correlating the method with the results of critical and exponential experiments. Extensive correlations of the combination of the buckling and two-group codes with such experiments are presented in the Appendix in terms of k_{eff}. If the methods were quite exact, k_{eff} should be approximately unity, and should exhibit no trends. It would presumably be difficult to determine whether deviations of k_{eff} from unity were the result of experimental error or of minor errors in the method of calculation. With the present codes, however, the methods are not exact and appreciable deviations of k_{eff} from unity and appreciable trends are observed. When the codes are applied to a particular situation, reference should therefore be made to similar situations for which correlations have been made to determine the values of k_{eff} that may be considered subcritical.

Source decks for any or all codes may be obtained by writing to the author.

MULTIGROUP BUCKLING CODE
(MGBS1149, L5404)

DESCRIPTION

This code was originally designed to compute the buckling of homogeneous aqueous systems and to provide two-group parameters for subsequent diffusion theory calculations. Modifications were later introduced to permit calculations to be made for heterogeneous cells with cylindrical symmetry. These modifications consisted of incorporating a P_3 subroutine (taken from the IDIOT code[4]) in the thermal group, and Hellstrand's resonance integrals[5] for uranium and uranium oxide and Weitman's resonance integrals[6] for thorium and thorium oxide in the resonance group. Dancoff factor[7] calculations, based on collision probability methods,[8,9] were introduced to give effective surface-to-volume ratios for tubes, as well as rods, to be used in the resonance integral expressions. Otherwise, homogeneity is assumed.

The code may also be used to compute the buckling of homogeneous fast, unmoderated systems. For this case, the thermal and perhaps the resonance group should be omitted. The slow group parameters computed by the code normally correspond to a boundary between fast and slow groups at 0.625 ev, but other group boundaries may be chosen.

The energy range is divided into 12 groups. The first 10 groups correspond to those selected by Loewenstein and Okrent[10] and have a lower bound of 9.1 kev. A resonance group extends from 0.625 ev to 9.1 kev, and the thermal group lies below 0.625 ev. For the 10 fast groups the cross sections of all materials except water, nitric acid, oxygen, beryllium, and carbon are those published by Yiftah, Okrent, and Moldauer,[11] with appropriate averages taken where one of the 10 groups encompassed two groups in this compilation. Cross sections for H_2O and nitric acid solutions up to about 7.5 molar for these 10 groups and for the resonance group are obtained from Lagrange interpolation of calculations made with a zero-dimensional multigroup code (ZDMG), described later, with a lethargy width of 0.1[26]. Since variations with the interpolation parameters are nearly linear, results are presumably good for somewhat higher molarities, but the range of validity has not been determined. This same code was also used to calculate the cross sections for oxygen, beryllium, and carbon.

For homogeneous systems the resonance integrals required in the resonance group were calculated from available data[11-16]. Dresner's[17] treatment of Doppler broadening was used for the

resolved resonances. The NR approximation was made, and interference between potential and resonance scattering was ignored. For ^{240}Pu care was taken not to include contributions lying below 0.625 ev, because these are included in the thermal group. Contributions at energies between 0.625 and the first resonance peak for other nuclides were obtained simply by integrating the cross section curves[12,13] in a 1/E spectrum. The infinite dilution integrals were forced to agree with measurements[16] of resonance integrals adjusted to the range 0.625 ev to 9.1 kev. The adjustment at the upper end was estimated from cross sections for the fast groups. Resonance cross sections (σ_{res}) were obtained by dividing by $\int_{.625}^{9100} \frac{dE}{E}$ and are tabulated in the code for each of 14 species ($^{233}U_f$, $^{235}U_f$, $^{239}Pu_f$, $^{241}Pu_f$, $^{233}U_c$, $^{234}U_c$, $^{235}U_c$, $^{236}U_c$, $^{238}U_c$, $^{239}Pu_c$, $^{240}Pu_c$, $^{241}Pu_c$, $^{242}Pu_c$, $^{232}Th_c$) at 16 equally spaced values of log σ_s from 2.5 through 17.5 where σ_s is the potential scattering cross section in barns per atom of absorber. Cross sections at intermediate values of σ_s are obtained by Lagrange interpolation.

For heterogeneous systems containing slightly enriched uranium or uranium oxide, or thorium or thorium oxide Hellstrand's[5] or Weitman's[6] resonance integrals are used respectively, with 1/v contributions added. Effective ratios of surface to volume are calculated on the assumptions that the metal or oxide is black to resonance neutrons, that the resonance neutrons are uniformly distributed in surrounding moderator in proportion to the slowing down power of the moderator, and that they are emitted isotropically. Transmission through cladding is based on the assumption that currents at interfaces have a cosine distribution[8,9]. The Dancoff factor[7] calculations, required for rods or for the outer surface of tubes, distinguish between square and triangular pitches. (This is the only place in the code where this distinction is made.) Resonance cross sections (σ_{res}) are again obtained by dividing by log $\frac{9100}{.625}$.

By definition, the removal or slowing down cross section for the resonance group is

$$\Sigma_r = \frac{Qp}{\int_{.625}^{9100} \phi(E)dE} \qquad (1)$$

where Q is the number of neutrons entering the group per unit time, p is the probability of escaping resonance capture, and $\phi(E)$ is the energy dependent flux. Again, by definition, the effective absorption cross section for the group is

$$\Sigma_a = \frac{(1 - p)Q}{\int_{.625}^{9100} \phi(E)dE} \qquad (2)$$

Dividing (2) by (1) yields

$$\Sigma_a = \frac{1 - p}{p}\Sigma_r \qquad (3)$$

It is convenient to use removal cross sections calculated for moderator and to assume them to be unaffected by the presence of resonance absorbers. This assumption is made in the code, and Σ_a for the resonance group is calculated from (3) with p calculated as

$$p = e^{-\Sigma_{res}/\Sigma_r} \qquad (4)$$

In the thermal group the cross sections for all nuclides except ^{240}Pu are taken from Amster's compendium[18]; a renormalization of his values for ^{240}Pu was made to take account of data[13] not available to him. In this compendium, the thermal spectrum is assumed to be a function only of the ratios ^{235}U/H, ^{239}Pu/H, and the barns of 1/v absorber per hydrogen atom at 2200 meters per second. Cross sections are tabulated in the code at five values of the first ratio (0, 0.005, 0.010, 0.015, 0.02), at five values of the second ratio (0, 0.0025, 0.005, 0.0075, 0.01), and at three values of the third ratio (0, 2, 4). Three-way Lagrange interpolation is used to obtain cross sections at intermediate values. Scattering by atoms other than H is considered to be scattering by an equivalent number of H atoms, based on the ratio of $\xi\sigma_s$ for the other atoms to $\xi\sigma_s = 20.73$ for H. All absorption cross sections other than those of ^{235}U and ^{239}Pu are assumed to be 1/v for the purpose of determining the shape of the spectrum, but otherwise their true variation with energy is taken into account. The code makes provision for extending the ranges to an H/^{235}U ratio of 2 or to an H/^{239}Pu ratio of 4 with the other fissile nuclide (^{239}Pu or ^{235}U, respectively) treated as a 1/v absorber as regards its effect on the spectrum. The cross sections included for this range are constant values as a function of H/^{235}U and values obtained by extrapolating Amster's results as a function of H/^{239}Pu. The code computes cross sections when the 1/v absorption cross section per H atom is greater than 4 barns, but these are presumably somewhat in error, the error increasing with the extent of the extrapolation. When the 4-barn limit is exceeded, notice to this effect is printed out.

In calculations for cylindrical cells, the thermal flux is first assumed to be flat in computing, by flux and volume weighting, the average ratios on which the spectrum depends. New averages are then calculated with fluxes calculated by the P_3 subroutine from cross sections determined by the first average spectrum, and a new spectrum is obtained. This process is repeated until the flux converges unless more than five iterations are required. Failure to converge can be recognized by the fact that the code prints out the fluxes for the sixth iteration and then those for the fifth. Average cross sections for the group are calculated with the fluxes from the next to the last iteration. In the P_3 calculations, a source of thermal neutrons proportional to the slowing down power is assumed in all regions. The limit of 4 barns per H atom may be exceeded in the first iteration, but not in later iterations. A notice that the limit is exceeded is printed out each time it occurs. Since at least two iterations are required, a single printing of the notice indicates that this limit is exceeded only in the first iteration.

After the average cross sections are calculated for each group, a multigroup calculation is made to obtain the neutron spectrum in the homogenized system. Down-scattering can occur to any group, but because of the large width of the resonance group, significant transfer across this group does not occur. The flux in each group is calculated from the set of equations[4]

$$\frac{B}{\tan^{-1}\frac{B}{\Sigma_i}} - \Sigma_i + \Sigma_{a_i} + \sum_{j=i+1}^{12} \Sigma_{i\to j}\,\phi_i = \chi_i + \sum_{j=1}^{i} \Sigma_{j\to i}\,\phi_j \qquad (5)$$

where B is the square root of the buckling and where, for each group i, Σ_i is the transport cross section, Σ_{a_i} the absorption cross section, $\Sigma_{i\to j}$ the cross section for transfer to group j, and χ_i the fraction of the fission neutrons born in the group.

The leakage term in (5), $\frac{B}{\tan^{-1}\frac{B}{\Sigma_i}} - \Sigma_i$, is derived from transport theory for the case of isotropic scattering. The transport approximation is made; i.e., the total cross section is replaced by the transport cross section and isotropic scattering in the laboratory frame of reference is assumed. This approximation was originally thought to be fairly good for hydrogenous systems[4], due to an error. Correction of the error leads to the results shown in Table I in which leakages calculated for hydrogenous systems in the monoenergetic case are compared for three different approximations: transport, P_1 (linear anisotropy

in the laboratory frame of reference), and diffusion. It is assumed that quadratic terms (through P_2) are sufficient to describe scattering in the laboratory frame of reference accurately. The transport approximation is seen to be superior to the diffusion approximation and to err in the opposite direction.

TABLE I

Comparison of Transport, P_1 (Linear Anisotropy), and Diffusion Approximation[a]

	Relative Leakage					
	Pure Hydrogen			H_2O such that $\sigma_H = \sigma_O$		
B/Σ_T	Transport	P_1	Diffusion	Transport	P_1	Diffusion
0	1.0	1.0	1.0	1.0	1.0	1.0
0.3	0.925	1.022	1.095	0.977	1.009	1.053
0.5	0.875	1.049	1.247	0.955	1.019	1.138
1.0	0.856	1.103	1.831	0.930	1.046	1.471
1.5	0.880			0.933		
2.0	0.903	1.120	3.315	0.942	1.062	2.344

(a) Comparison is in terms of the ratio of the leakage (DB^2) computed by each approximation to that computed by the P_2 approximation.

After the flux in each group is obtained, the total production of neutrons is calculated as $P = \sum_{i=1}^{12} \nu_i \Sigma_{f_i} \phi_i$, the total absorption as $A = \sum_{i=1}^{12} \Sigma_{a_i} \phi_i$, the total leakage as $L = \sum_{i=1}^{12} \left(\frac{B}{\tan^{-1} \frac{B}{\Sigma_i}} - \Sigma_i \right) \phi_i$, the multiplication constant as $k = P/A$, and the migration area as $M^2 = L/AB^2$. If iteration to a converged buckling is desired, a new buckling is calculated as $B'^2 = (k-1)/M^2$, and a new set of fluxes is calculated. The operation is repeated until the change in buckling does not exceed 10^{-6} cm^{-2}.

The number of iterations is limited to 20, but this limit is never approached unless situations occur that cannot converge. Such a situation may, for example, arise when the buckling is negative ($B^2 = -\kappa^2$). Arguments of $\tanh^{-1} \kappa/\Sigma_i$ greater than unity cannot be permitted, and if they occur, the leakage is set equal to zero in all groups.

Average constants are computed for use in a two-group diffusion theory code based on the equations:

$$\frac{\nabla^2\phi_1}{3\Sigma_1} - (1 - c_{11})\Sigma_1\phi_1 + c_{12}\Sigma_2\phi_2 = 0$$
$$\frac{\nabla^2\phi_2}{3\Sigma_2} - (1 - c_{22})\Sigma_2\phi_2 + c_{21}\Sigma_1\phi_1 = 0 \qquad (6)$$

where Σ_i is the transport cross section and c_{ij} is the number of secondary neutrons appearing in group i as the result of an interaction in group j. The cross sections are normally calculated as $\Sigma_1 = \frac{\sum_{i=1}^{11} \phi_i}{\sum_{i=1}^{11} \frac{\phi_i}{\Sigma_i}}$ and $\Sigma_2 = \Sigma_{12}$, but other divisions between the two groups may be selected. In calculating the c_{ij}, the source terms (neutrons per fission, and removal cross section) are increased so that the two-group equations will give the same critical buckling and the same ratio of fast to slow flux corresponding to the eigenvalue as are given by the multigroup calculation. The need for making this adjustment clearly lies in the difference between the leakage terms (transport vs diffusion).

The code is broken up into a number of subroutines which perform various steps in the calculation and contain the cross sections. The atomic densities in each material type are computed in ATDNY. The constants for the 10 fast groups are computed in FAST, the resonance group constants in RES, and the thermal group constants in THRML. The calculation of the effective ratio of surface to volume, required for the resonance group, is performed in EFSTVR, which makes use of CRTVTR, TCYL, TOI, TOO, DCFR, DAN, BEKI3, and BIK. Rational polynomial approximations[19,20] are used for the K_{13} function and for the Bessel functions in the subroutines BEKI3 and BIK, respectively. The P_3 calculation required for the thermal group is performed in P3 and SOL which makes use of BESS and BIK. Lagrange interpolation is performed with the functions G30 and G31 or with G50, G51, and G52. The multigroup calculation of the spectrum, the calculation of energy-

averaged parameters, and the buckling iteration are performed in the MAIN program. The source distribution is contained in SRCE. The cross sections for the 10 fast groups are contained in FC, FF, and FMC; for the resonance group in RCS; and for the thermal group in SO, S2, S4, SU, and SPU. The order in which they appear is stated in comments in the listing of the program.

It has been found that overflows occur in the subroutine SOL in cases where a central moderator region is surrounded by just one additional moderator region. An example of this type of situation would occur if a lattice of MTR elements in water were treated as consisting of cells having a central homogeneous U-Al-H_2O region surrounded by an annulus of water. The overflows apparently do no harm, however. They can be avoided by subdividing the central region into two regions with identical properties, and the results are essentially the same as when the overflows occur.

INPUT

Format	Data
18A4	PN(I), I = 1,5
	PN(I), I = 1,4 is a name of up to 16 characters.
	PN(5) is a number of up to 4 digits.
72I1	IC(I), I = 1,72
	IC(I) is a control vector for input.
	IC(1) and IC(2) denote the number of regions NR (a maximum of 23 are allowed).
	IC(3) denotes the number of types of materials NT (a maximum of 9 are allowed).
	IC(4) denotes geometry: 0 for slab, 1 for cylinder (the slab is homogenized by the code for all groups).
	IC(J+4), J = 1,NR denotes the type of material in region J. The types must be designated in the same order in which they are to be read.
	IC(J+NR+4), J = 1,NR indicates, if non-zero, that the outer radius of region J is to be read. If the radius is not read, it is the same as in the preceding problem. The radius of region 1 for the first problem is preset to 1 cm, but can of course be overridden.
	IC(J+2NR+4), J = 1,NT indicates, if non-zero, that the description of material J is to be read. If it is not read, it is the same as in the preceding problem.

Format	Data
	IC(2NR+NT+5) indicates, if non-zero, that the buckling is to be read. If it is not read, it is the same as in the preceding problem. It is preset to zero by the code.
	IC(2NR+NT+6) denotes the number of low energy groups (exclusive of any that may be omitted) that are included in the slow group. If zero, one is included.
	IC(2NR+NT+7) denotes the number of low energy groups omitted from the 12-group structure.
	IC(2NR+NT+8) indicates, if 0 or 1, a ^{235}U fission source or, if 2, a Pu-Be source.
	IC(2NR+NT+9) denotes type of solution. If 0 or 2, no iteration on buckling; if 1, iteration. If 2, c_{11} is adjusted to agree with input buckling.
	IC(2NR+NT+10) is not used.
	IC(2NR+NT+11) is not used.
	IC(2NR+NT+12) refers to resonance group constants. If 0, code calculates them for a homogeneous system; if 1, they are read in; if 2 or 3, code calculates them from Hellstrand's and Weitman's resonance integrals for square or triangular lattice respectively.
	IC(2NR+NT+13) indicates, if non-zero, that spectrum from preceding problem is to be used and that resonance group absorption cross section is to be set equal to zero.
	IC(2NR+NT+14) denotes number of comment cards (up to 9) to be read.
F10.5	RAD(I)
	Radius (in cm) of region I as called for by control vector. One radius per card.
F10.5	BEST
	Buckling (in cm^{-2}) if called for. An initial guess should be read if iteration is called for. The guess need not be very good, because convergence is rapid.

Format	Data
50I1	ITC(I), I = 1,50

ITC(I) is a control vector for the description of a material type called for by IC.

ITC(1) indicates, if non-zero, that isotopic composition of uranium in material is to be read.

ITC(2) indicates, if non-zero, that isotopic composition of plutonium in material is to be read.

ITC(3) denotes, if 2, that material is aqueous solution or, if 3, metal or oxide.

ITC(K+3), K = 1,47 denotes, if non-zero, that A(K,J) is to be read where J is the material type and A specifies concentrations and densities.

5F10.5 U(I,J), I = 1,5

Weight fractions of ^{233}U, ^{234}U, ^{235}U, ^{236}U, and ^{238}U in material Type J if called for by ITC.

4F10.5 PU(I,J), I = 1,4

Weight fractions of ^{239}Pu, ^{240}Pu, ^{241}Pu, and ^{242}Pu in material Type J if called for by ITC.

F10.5 A(K,J) as called for by ITC, one per card

For aqueous solution, A(1,J) is nitrate ion conc, g/l.

A(2,J) is uranium conc, g/l.

A(3,J) is x in UO_x.

A(4,J) is density of UO_x for natural uranium, g/cm^3.

A(5,J) is plutonium conc, g/l.

A(6,J) is x in PuO_x.

A(7,J) is density of PuO_x for ^{239}Pu, g/cm^3.

etc. for U, Pu, Th, Al, Zr, Fe, Cr, Ni, B, Cd, Gd, Pb, C, Be in this order.

A(44-46,J) is not presently used.

A(47,J) is void fraction.

Format	Data
	For metal or oxide, A(1,J) is density, g/cm^3.
	A(2,J) is weight fraction oxygen.
	A(3,J) is weight fraction uranium.
	A(4,J) is weight fraction plutonium.
	etc. for O, U, Pu, Th, Al, Zr, Fe, Cr, Ni, B, Cd, Gd, Pb, C, Be in this order.
	A(18-47,J) is not used for metal or oxide.
50I1	ITC(I) The cycle of material description cards is read for each type J called for by IC. The number of types may exceed the number of regions. U, PU, and A are all preset to zero and if not read, have the preset values for the first problem or the values in the preceding problem for succeeding problems, except for a metal or oxide region where A(K,J) is preset to zero for each problem. It is necessary to reset A(K,J) to zero, for an aqueous solution by inserting blank cards, if this is desired. This is necessary, and easily overlooked, even when type J changes from metal to solution.
18A4	(COM(I,J), I = 1, 18), J = 1, IC (2NR + NT + 14)
	Comment cards as called for by IC.
5F10.5	TCS(11), TRCS(11), ACS(11), VFCS(11), DCS(66)
	Resonance group constants, if called for by IC(2NR + NT + 12).
72I1	IC(I), I = 1, 72
	Control vector for next problem, etc. Should be blank to terminate a series of problems.

PRINTED OUTPUT

Name and number read as PN(I), I = 1,5

Problem number (problems are numbered consecutively from 1)

Comments

Outer radii of regions

Type of material in each region starting with first region

U, PU, and A for each type of material that is changed

Volume fraction of each region

Thermal disadvantage factor of each region

Thermal absorption cross section (cm^{-1}) in each region

Thermal scattering cross section (cm^{-1}) in each region

Total thermal source and total thermal absorption, meaningless unless P_3 calculation has been made

Number of equivalent atoms of H on basis of slowing down power, numbers of molecules of H_2O, molecules of N_2O_5, extra atoms of O, atoms of ^{233}U, ^{234}U, ^{235}U, ^{236}U, ^{238}U, ^{239}Pu, ^{240}Pu, ^{241}Pu, ^{242}Pu, Th, etc. (in same order as for A(I,J) per cm^3 x 10^{24} for each type of material that is changed

Resonance group constants: Σ_t, Σ_{tr}, Σ_a, $\nu\Sigma_f$, Σ_r, and, for rods or tubes of slightly enriched U or UO_2 or for Th or ThO_2, the actual and effective surface-to-volume ratio

Resonance group constants after being adjusted in accordance with Equation 4

Ratio of fast to slow flux

Total production

Total absorption

Total leakage

Average Σ_{tr}

Average bare extrapolation distance calculated as $\frac{1}{B} \tan^{-1} \frac{2B}{3\Sigma_{tr}}$

B (square root of buckling)

Ratio of fast to slow activations of a 1/v absorber in spectrum

Fraction of source neutrons absorbed and leaking in thermal and resonance groups

Spectrum, i.e., fluxes in all 12 groups

Absorption and fission cross sections of uranium and plutonium isotopes and thorium averaged over spectrum

Fraction of absorptions and fissions occurring in these nuclides in thermal group

Buckling (cm^{-2}), migration area (cm^2), and two-group parameters for diffusion theory calculations

Radius and volume of bare sphere, bare critical mass of uranium or plutonium, and ratio of hydrogen atoms to ^{235}U or ^{239}Pu

PUNCHED OUTPUT

Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22}, M^2, punched with FORMAT (2F10.7, 4F10.8, F10.4, I2). The cards for an entire group of problems are preceded and followed by spacer cards containing PN(5) in columns 75-78, enclosed by slashes in 73-74 and 79-80.

TWO-GROUP ANALYTICAL CODE
(TGAN1149, L5428)

DESCRIPTION

This code solves Equation 6 for plane, cylindrical, or spherical symmetry. Two types of solutions may be chosen: one employing the functions that are solutions of the wave equation $\nabla^2\phi + B^2\phi = 0$ and the other employing transmission and escape probabilities. In the former, finite cylinders and slabs are handled by including a transverse buckling. In the latter, which is appropriate for a thin, heavily absorbing region such as a sheet of cadmium, infinite slabs or cylinders are assumed, and allowance is made for transverse leakage by modifying c_{11} and c_{22}. The code was designed chiefly as a tool for computing critical sizes and extrapolation distances, and considerable care was taken to avoid errors due to round-off. Fast and slow fluxes are calculated after the criticality calculation has been made. The fluxes in some cases may be greatly in error, possibly because of errors due to round-off. These errors in the flux, however, do not indicate that errors are present in the criticality calculation, which has been checked quite thoroughly. Criticality can be searched for by varying either the transverse buckling or the thickness of a region. It is possible also to search for the critical concentration of a solution, which may be an average concentration in the case of nonuniform solutions. For certain special cases, dimensions (and hence extrapolation distances) can be obtained corresponding to particular values of k_{eff}, presumably chosen to be those found to be appropriate for correlating experiment and calculation. In these cases punched output, in the form required as input by the critical dimension code (KEFF), described later, is obtained.

Considerable thought went into choosing the form of the two-group equations and the boundary conditions. As in the Multigroup Buckling Code (MGBS), the transport approximation was made. This approximation is, however, essentially made in any ordinary few-group treatment. If it were not made, source anisotropy would have to be considered, the neutron current in a group would depend not only on the flux in that group but on the fluxes in other groups, and the equations would be much more complicated. The particular form of the equations was chosen to make them symmetrical and to avoid ambiguities that might result due to varying definitions, if parameters such as p, Σ_r, τ, D, L^2, etc., had been used. Provided these latter parameters are clearly defined, the Σ_i and c_{ij} can of course be expressed in terms of them.

Arguments for the choice of boundary conditions and for the leakage expression used in Equation 6 are based on a consideration of the solution of the transport equation in an infinite homogeneous medium with plane symmetry. Similar arguments could perhaps be advanced for a cylinder and sphere but the equations are more complicated. Scattering is assumed to be isotropic in the laboratory frame of reference. (The transport approximation consists of replacing the total cross section by the transport cross section and assuming isotropic scattering.) Two energy groups, characterized by suitably averaged parameters Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, and c_{22}, are assumed. The transport equation in its integral form can then be written quite simply as

$$\phi_1(o) = \int_0^\infty \int_{-1}^{1} \left\{ c_{11}\Sigma_1\phi_1(x) + c_{12}\Sigma_2\phi_2(x) \right\} \frac{e^{-\Sigma_1 r}}{2} \, d\lambda dr$$

$$\phi_2(o) = \int_0^\infty \int_{-1}^{1} \left\{ c_{21}\Sigma_1\phi_1(x) + c_{22}\Sigma_2\phi_2(x) \right\} \frac{e^{-\Sigma_2 r}}{2} \, d\lambda dr \qquad (7)$$

where r is measured from the origin to a source point in the plane at x and $\cos^{-1}\lambda$ is the angle r makes with the x-axis.

Since the origin of the x coordinate may be chosen to be anywhere, there is no loss of generality in choosing to determine ϕ_1 and ϕ_2 at x = 0. In the infinite medium ϕ_1 and ϕ_2 are both solutions of the wave equation $\nabla^2\phi + \mu^2\phi = 0$, and therefore have the form

$$\begin{aligned} \phi_1 &= S(A_1\cos\mu x + A_2\sin\mu x) \\ \phi_2 &= A_1\cos\mu x + A_2\sin\mu x \end{aligned} \qquad (8)$$

Substituting $x = r\lambda$ and performing the integration in (7) give

$$\begin{aligned} SA_1 &= (c_{11}\Sigma_1 S + c_{12}\Sigma_2)\frac{A_1}{\mu}\tan^{-1}\frac{\mu}{\Sigma_1} \\ A_1 &= (c_{21}\Sigma_1 S + c_{22}\Sigma_2)\frac{A_1}{\mu}\tan^{-1}\frac{\mu}{\Sigma_2} \end{aligned} \qquad (9)$$

Eliminating S leads to the following equation that must be satisfied by μ:

$$\left(c_{11} - \frac{\mu/\Sigma_1}{\tan^{-1}\frac{\mu}{\Sigma_1}}\right)\left(c_{22} - \frac{\mu/\Sigma_2}{\tan^{-1}\frac{\mu}{\Sigma_2}}\right) - c_{12}c_{21} = 0 \qquad (10)$$

Two roots may be possible, generally a real root and an imaginary root, although in some cases both may be imaginary. The ratio of the components of ϕ_1 and ϕ_2 corresponding to a root is obtained from (9):

$$S = \frac{\frac{\Sigma_2}{\Sigma_1} c_{12}}{\frac{\mu/\Sigma_1}{\tan^{-1}\frac{\mu}{\Sigma_1}} - c_{11}} \tag{11}$$

The group 1 current crossing the plane at x = 0 in the negative x direction is

$$j_1^-(0) = \int_0^\infty \int_0^1 \left\{ c_{11}\Sigma_1\phi_1(x) + c_{12}\Sigma_2\phi_2(x) \right\} \frac{e^{-\Sigma_1 r}}{2} \lambda d\lambda dr \tag{12}$$

Making use of (8) and carrying out the integration in (12) give

$$j_1^-(0) = \left(c_{11}\Sigma_1 S + c_{12}\Sigma_2\right)\left[A_1\frac{\Sigma_1}{4\mu^2}\log\left(1 + \frac{\mu^2}{\Sigma_1^2}\right) + \frac{A_2}{2}\left(\frac{1}{\mu} - \frac{\Sigma_1}{\mu^2}\tan^{-1}\frac{\mu}{\Sigma_1}\right)\right] \tag{13}$$

Using (9) and noting from (8) that

$$\phi_1(0) = SA_1 \text{ and } \left(\frac{d\phi_1}{dx}\right)_{x=0} = SA_2\mu$$

give

$$j_1^-(0) = \frac{\log\left(1 + \frac{\mu^2}{\Sigma_1^2}\right)}{4\frac{\mu}{\Sigma_1}\tan^{-1}\frac{\mu}{\Sigma_1}}\phi_1(0) + \frac{1 - \frac{\tan^{-1}\frac{\mu}{\Sigma_1}}{\mu/\Sigma_1}}{2\mu\tan^{-1}\frac{\mu}{\Sigma_1}}\left(\frac{d\phi_1}{dx}\right)_{x=0} \tag{14}$$

In general then

$$j_1^{\pm} = \frac{\log\left(1 + \frac{\mu^2}{\Sigma_1^2}\right)}{4\frac{\mu}{\Sigma_1}\tan^{-1}\frac{\mu}{\Sigma_1}}\phi_1 \mp \frac{1 - \frac{\tan^{-1}\frac{\mu}{\Sigma_1}}{\mu/\Sigma_1}}{2\mu\tan^{-1}\frac{\mu}{\Sigma_1}}\frac{d\phi_1}{dx} \tag{15}$$

(It should be noted that this result is valid only for plane symmetry; it does not apply to cylinder or sphere.)

In more simple notation, $$j_i^{\pm} = \frac{\alpha_i(\mu)}{4}\,\phi_i \mp \frac{D_i(\mu)}{2}\,\frac{d\phi_i}{dx} \qquad (16)$$

Thus the current in group i can be expressed entirely in terms of the flux and its derivative in group i.

In a finite medium, far from boundaries, the flux and current are approximated very well by the infinite medium solution. Near a boundary it seems reasonable to expect the current leaving the medium in the positive x direction to be approximated fairly well by j^+ because the current depends on source neutrons deep within the medium as well as near the boundary. If the flux is assumed to satisfy the wave equation, then the current entering the medium must be equated to j^- as evaluated from the properties of the medium. The physical boundary conditions must be continuity of j^+ and j^- for each group. In the finite medium the transient solutions corresponding to the imaginary root $i\nu$ are also present although they are not allowed in the infinite medium because there the ratio of ϕ_1 to ϕ_2 must be the same everywhere. The appropriate boundary conditions thus appear to be to require continuity of the vector

$$\begin{bmatrix} j_1^+ \\ j_1^- \\ j_2^+ \\ j_2^- \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & -\frac{1}{2} & 0 \\ \frac{1}{4} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \alpha_1(\mu)S(\mu)\cos\mu x & \alpha_1(\mu)S(\mu)\sin\mu x & \alpha_1(\nu)S(\nu)\cosh\nu x & \alpha_1(\nu)S(\nu)\sinh\nu x \\ \alpha_2(\mu)\cos\mu x & \alpha_2(\mu)\sin\mu x & \alpha_2(\nu)\cosh\nu x & \alpha_2(\nu)\sinh\nu x \\ -\mu D_1(\mu)S(\mu)\sin\mu x & \mu D_1(\mu)S(\mu)\cos\mu x & \nu D_1(\nu)S(\nu)\sinh\nu x & \nu D_1(\nu)S(\nu)\cosh\nu x \\ -\mu D_2(\mu)\sin\mu x & \mu D_2(\mu)\cos\mu x & \nu D_2(\nu)\sinh\nu x & \nu D_2(\nu)\cosh\nu x \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \qquad (17)$$

or in more simple matrix notation to require continuity of

$$J = CMA \qquad (18)$$

At a boundary x_k between materials i and j then

$$CM_i(x_k)A_i = CM_j(x_k)A_j \qquad (19)$$

$$M_i(x_k)A_i = M_j(x_k)A_j \qquad (20)$$

The net current at a boundary is $j^+ - j^-$ or $-D(\mu)\frac{d\phi(\mu)}{dx} - D(\nu)\frac{d\phi(\nu)}{dx}$. Thus the third and fourth elements of the vectors in (20) are simply the negative of the net currents. The first and second elements, however, are not simply the fluxes; moreover, there are two diffusion coefficients involved in the currents in a group, one a function of μ and the other a function of ν.

$\alpha_2(\mu)$ and $\alpha_2(\nu)$ can be factored out of M and can be included in A so that

$$MA = M'A' = \begin{bmatrix} S'(\mu)\cos\mu x & S'(\mu)\sin\mu x & S'(\nu)\cosh\nu x & S'(\nu)\sinh\nu x \\ \cos\mu x & \sin\mu x & \cosh\nu x & \sinh\nu x \\ -\mu D_1'(\mu)S'(\mu)\sin\mu x & \mu D_1'(\mu)S'(\mu)\cos\mu x & \nu D_1'(\nu)S'(\nu)\sinh\nu x & \nu D_1'(\nu)S'(\nu)\cosh\nu x \\ -\mu D_2'(\mu)\sin\mu x & \mu D_2'(\mu)\cos\mu x & \nu D_2'(\nu)\sinh\nu x & \nu D_2'(\nu)\cosh\nu x \end{bmatrix} \begin{bmatrix} A_1' \\ A_2' \\ A_3' \\ A_4' \end{bmatrix} \tag{21}$$

where

$$S'(\mu) = \frac{\alpha_1(\mu)}{\alpha_2(\mu)}S(\mu) \qquad S'(\nu) = \frac{\alpha_1(\nu)}{\alpha_2(\nu)}S(\nu)$$

$$D_i'(\mu) = \frac{D_i(\mu)}{\alpha_i(\mu)} \qquad D_i'(\nu) = \frac{D_i(\nu)}{\alpha_i(\nu)}$$

$$A_1' = \alpha_2(\mu)A_1 \quad A_2' = \alpha_2(\mu)A_2 \quad A_3' = \alpha_2(\nu)A_3 \quad A_4' = \alpha_2(\nu)A_4$$

The constants A_i or A_i' do not enter into the criticality determination so that (21) may be considered as representing continuity of flux and net current with diffusion constants D'.

$$D_i'(\mu) = \frac{\frac{1}{\Sigma_i}\left(1 - \frac{\tan^{-1}\frac{\mu}{\Sigma_i}}{\mu/\Sigma_i}\right)}{\log\left(1 + \frac{\mu^2}{\Sigma_i^2}\right)} \tag{22}$$

It is instructive to calculate D, D', and α as a function of μ/Σ. Results are given in Table II.

TABLE II

D, D', and α as Functions of μ/Σ

μ/Σ	$3\Sigma D$	$3\Sigma D'$	α	$\alpha/\alpha(0.1\ \mu/\Sigma)$
i	3.0	1.5	2	1.9966
0.8i	1.2741	1.0960	1.1625	1.1612
0.6i	1.1200	1.0437	1.0731	1.0725
0.4i	1.0465	1.0171	1.0289	1.0286
0.2i	1.0107	1.0041	1.0066	1.0065
0	1	1	1	1
0.2	0.9895	0.9961	0.9934	0.9935
0.4	0.9607	0.9852	0.9751	0.9754
0.6	0.9187	0.9688	0.9483	0.9489
0.8	0.8704	0.9497	0.9165	0.9175
1.0	0.8197	0.9288	0.8825	0.8840
1.5	0.7016	0.8776	0.7995	0.8025
2.0	0.6048	0.8321	0.7268	0.7316

Real values of μ/Σ are considered first. A plutonium solution with $H/^{239}Pu = 400$ has a μ/Σ_1 of about 0.7, as does a typical close-packed lattice of U-Al alloy fuel tubes in water. A dense mixture of plutonium and water with $H/^{239}Pu = 5$ has a μ/Σ_1 of about 1.3. In aqueous systems μ/Σ_2 is generally about one-tenth as large as μ/Σ_1. Within the usual range of interest, $1/3\Sigma$ appears to be a reasonably good approximation for D'. It is difficult to say much about the imaginary values of μ/Σ. Since ν/Σ_i must not exceed unity, there will not always be an imaginary root of (10). Moreover, solutions corresponding to the second root are not permitted in the infinite medium. Perhaps about all that can be said is that the trends with increasing ν are in the opposite direction from those with increasing μ.

From the heuristic arguments that have been presented here, including a study of Table II, it appears that the most accurate recipe to follow in performing few-group diffusion theory criticality calculations is:

1. To assume continuity of flux and net current at boundaries with the current given by $-D\nabla\phi$ where $D_i = 1/3\Sigma_i$.

2. To choose two-group constants such that with leakage given by $\mu^2/3\Sigma_i$ the asymptotic solution will be produced in an infinite medium with the proper relative values in all groups. (The Multigroup Buckling Code does this.)

3. The transient solutions will then correspond to ν^2, determined as the other root of (6):

$$\nu^2 = 3\Sigma_1^2(1 - c_{11}) + 3\Sigma_2^2(1 - c_{22}) + \mu^2$$

(There is no requirement here that $\nu \leqslant \Sigma_i$)

When transverse leakage is characterized by a radial buckling, μ^2 in (21) is replaced by $\mu_x^2 = \mu^2 - B_r^2$ and ν^2 by $\nu_x^2 = \nu^2 + B_r^2$. The diffusion constants and coupling coefficients (S) are not affected. The approach taken in the code for the solutions involving transmission and escape probabilities has been described elsewhere[8,9]. A somewhat different formulation of the equations has been employed in the code, however. The current vector in (17) at the right-hand boundary of a medium is related to that at the left-hand boundary by transmission probabilities calculated for the medium and by escape probabilities for source neutrons born within the medium. All secondary neutrons resulting from a collision in the medium are assumed to be uniformly distributed within the medium so that this type of solution should be used only for thin regions. The distribution of neutrons entering a surface of the region is assumed to be proportional to the cosine of the angle made with the normal to the surface. Heuristic arguments, again based on a consideration of the slab case, have been advanced for this approximation[21]. The regions are assumed to have infinite transverse dimensions in calculating transmission and escape probabilities; leakage characterized by a transverse buckling is treated as additional absorption and is accounted for by modifying c_{11} and c_{22}. The two-group parameters are the same as in the diffusion theory case, namely Σ_i and c_{ij}. The coupling between fast and slow groups is contained directly in the equations so that no iteration is involved.

Finally, consider the determination of the critical conditions for a system consisting of several regions. At x_o and at x_n, external boundary conditions must be satisfied. (Here x_o and x_n may be interpreted as radii for cylinder or sphere.) A series of equations of the following form then must be solved:

$$\begin{aligned} J_1(x_o) &= J_L \\ J_2(x_1) &= J_1(x_1) \\ J_3(x_2) &= J_2(x_2) \\ &\cdots \\ J_n(x_{n-1}) &= J_{n-1}(x_{n-1}) \\ J_n(x_n) &= J_R \end{aligned} \qquad (23)$$

where J_L and J_R are vectors giving the external boundary conditions.

If the i^{th} region is characterized by functions that are solutions of the wave equation J_i is given by (18). If, on the other hand, the i^{th} region is characterized by transmission probabilities, $J_{i+1}(x_i) = T_i J_{i-1}(x_{i-1})$, where the matrix T_i contain transmission and escape probabilities.

From (18) and (21)

$$CM_1'(x_o)A_1' = J_L$$

$$CM_2'(x_1)A_2' = CM_1'(x_1)A_1' = CM_1'(x_1)M_1'^{-1}(x_o)C^{-1}J_L$$

$$CM_3'(x_2)A_3' = CM_2'(x_2)A_2' = CM_2'(x_2)M_2'^{-1}(x_1)C^{-1}CM_1'(x_1)M_1'^{-1}(x_o)C^{-1}J_L$$

$$\cdots$$

Finally,
$$J_R = CN_nN_{n-1} \cdots N_2N_1C^{-1}J_L \tag{24}$$

where $N_i = M_i'(x_i)M_i'^{-1}(x_{i-1})$ for a function region

or $N_i = C^{-1}T_iC$ for a transmission region

It is convenient to express the boundary conditions in terms of albedos β, i.e., the ratios j^-/j^+ at the right-hand boundary and j^+/j^- at the left. The product of the vector J_R by the matrix

$$\begin{bmatrix} -\beta_{1R} & 1 & 0 & 0 \\ 0 & 0 & -\beta_{2R} & 1 \end{bmatrix}$$

is thus zero. The vector J_L can be written

$$J_L = \begin{bmatrix} \beta_{1L} & 0 \\ 1 & 0 \\ 0 & \beta_{2L} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} j_{1L}^- \\ j_{2L}^- \end{bmatrix}$$

The critical condition then is that the 2 x 2 determinant

$$\left| QN_nN_{n-1} \cdots N_2N_1P \right| = 0 \tag{25}$$

where

$$Q = \begin{bmatrix} \frac{1 - \beta_{1R}}{4} & 0 & \frac{1 + \beta_{1R}}{2} & 0 \\ 0 & \frac{1 - \beta_{2R}}{4} & 0 & \frac{1 + \beta_{2R}}{2} \end{bmatrix}$$

and

$$P = \begin{vmatrix} 2(1+\beta_{1L}) & 0 \\ 0 & 2(1+\beta_{2L}) \\ 1-\beta_{1L} & 0 \\ 0 & 1-\beta_{2L} \end{vmatrix}$$

The transient solutions in regions characterized by functions present certain problems due to round-off; terms of large magnitude which should cancel, may not do so exactly. When $\beta_L = 1$ or when $\beta_R = \pm 1$, these difficulties can be circumvented in the innermost and the outermost regions (if characterized by functions); and this is done in the code. (An albedo of 1 corresponds to a net current of zero; an albedo of -1 corresponds to a flux of zero.) Otherwise a more general treatment is required. In a single intermediate function region, the matrix N can be broken into two matrices,

$$N = N(\mu) + N(\nu)$$

the former containing only functions of the eigenvalue μ and the latter only functions of ν. The terms in the latter matrix tend to be much larger than those in the former so that a straightforward attempt to solve (25) may be very insensitive to changes in dimension or transverse buckling. The determinant of the product of a 4 x 2 matrix P, by a 4 x 4 matrix N, by a 2 x 4 matrix Q can be shown to be

$$\left|QNP\right| = \sum \begin{vmatrix} q_{1i} & q_{1j} \\ q_{2i} & q_{2j} \end{vmatrix} \times \begin{vmatrix} n_{ik} & n_{im} \\ n_{jk} & n_{jm} \end{vmatrix} \times \begin{vmatrix} p_{k1} & p_{k2} \\ p_{m1} & p_{m2} \end{vmatrix}$$

where $j > i$, $m > k$, and the sum is over all values of i, j, k, and m permitted between 1 and 4. The determinant then can be written

$$\begin{aligned} D &= \left|Q(N(\mu) + N(\nu))P\right| \\ &= \left|QN(\mu)P\right| + \left|QN(\nu)P\right| + \left|QN(\mu)P\begin{bmatrix}1&0\\0&0\end{bmatrix} + QN(\nu)P\begin{bmatrix}0&0\\0&1\end{bmatrix}\right| \\ &\qquad + \left|QN(\mu)P\begin{bmatrix}0&0\\0&1\end{bmatrix} + QN(\nu)P\begin{bmatrix}1&0\\0&0\end{bmatrix}\right| \end{aligned} \qquad (26)$$

Consideration of the products involving $N(\mu)$ and $N(\nu)$ terms in the first two determinants shows that, as a result of terms that are identically zero and of terms that in the slab case are of the form $\cos^2\mu(x_i - x_{i-1}) + \sin^2\mu(x_i - x_{i-1})$ or of the form

$\cosh^2\nu(x_i - x_{i-1}) - \sinh^2\nu(x_i - x_{i-1})$, the only terms present are ones containing the coupling coefficients $S(\mu)$ and $S(\nu)$, the diffusion coefficients, and the ratio of inner to outer radius raised to the power 0, 1, or 2, depending on whether slab, cylinder, or sphere is being treated. These two determinants are usually much smaller than the next two. In the code four matrices P' are formed which are multiplied by Q to produce four determinants. The number of such matrices increases by four for each general function region so that for four such regions, the maximum number permitted by the code, there are 256 P' matrices and 256 determinants that are added to give the required determinant. Even with these precautions, some cases have been found with the IBM System/360-65 where lack of sensitivity was apparent; double precision arithmetic was therefore introduced for the matrix manipulations leading to the evaluation of the determinant.

It is possible for problems not to converge or to converge to the wrong root. The latter case can usually be recognized by the presence of negative fluxes, although negative fluxes do not always indicate trouble. As pointed out above the criticality calculation is considered to be correct, but fluxes may be greatly in error. Particular care is taken in the transverse buckling search to ensure convergence to the right root. The convergence criteria are expressed in terms of the change in the parameter being searched for. When the predicted change for the next iteration becomes sufficiently small, convergence is assumed regardless of the value of the determinant, which in some cases may be very large. The number of iterations is limited to 20, which should generally be more than sufficient if convergence can be achieved with a particular set of initial conditions. Often only 4 or 5 iterations are required. When convergence does not occur or occurs to the wrong root, proper convergence may be achieved by a different choice of initial conditions. To aid in recognizing proper convergence and in choosing other initial conditions, the results of each iteration, including the value of the determinant, are printed.

The input and output options are exercised in the MAIN program. Some of the calculations are performed here also, but most are performed in subroutines. Subroutine ORDER establishes the ordering of function (F) and transmission (T) regions. The bucklings, coupling coefficients, and diffusion constants are calculated by BCKLG. Subroutine PRNT prints data for each region in order. The calculation of the N matrices, the formation of their product, and the evaluation of the determinant are handled by AGAIN with the aid of several subroutines. The elements of N_1 when $\beta_{1L} = \beta_{2L} = 1$ are calculated in MTSYM for an F region and in TRNSYM for a T region. The elements of QN_n when $\beta_{1R} = \beta_{2R} = \pm 1$ are calculated in QSYM for an F region. The elements of a general

F region matrix are calculated in MTXELS. The Bessel functions required for cylinders are calculated in BJY and BIK from polynomial approximations[20,22]. The elements of a general T region matrix are calculated in TRNSMT, which makes use of TOO, TOI, and TCYL in calculating transmission and escape probabilities. These in turn make use of functions calculated in EXIE3 (E_3 function), BIK, and BEKI3 (K_{13} function). A rational approximation[23] for the E_1 function for $x > 1$ is used in EXIE3. Matrix inversion, required in TRNSMT, is performed by MINVS. Matrix multiplication in AGAIN is performed with MLTA and MLTB. The P matrices are calculated in PMAT by means of CSMPLY and MMPLY. Where a T matrix intervenes between F matrices, multiplication of the P matrices by the T matrix to form new P matrices is performed by TPMT. Evaluation of the determinant is performed by DETN. The fluxes and currents at boundaries are calculated in PSI; the necessary functions are carried along in XMF as a side calculation. Transverse buckling iteration is handled by BCON. Calculation of average extrapolation distance for the first slab (starting from the inner boundary) for which it has meaning is performed in SLED. The concentration search is carried out in DIST. Lagrange interpolation of parameters as a function of concentration is performed in GRNG by means of the functions G50, G51, and G52. Subroutine EFTM (40) handles machine overflows and underflows. When an underflow occurs, zero is substituted; when an overflow occurs, the result is set equal to the largest possible positive number; and when division by zero occurs, zero is substituted. In the last two cases, the location of the interrupt is printed.

INPUT

Format	Data
5A4	PN(I),I = 1,5
	PN(I),I = 1,4 is a name of up to 16 characters.
	PN(5) is a number of up to 4 digits.
30I1, 12I3	(ICON(I),I = 1,30), (JD(I),I = 1,12)
	ICON(I) is a control vector.
	ICON(1) is number of T regions ($\leqslant 6$).
	ICON(2) is number of F regions. In general case the limit is 4. If the first region is an F region and if $\beta_1 = \beta_2 = 1$, the limit is increased by 1. If the last region is an F region and if $\beta_1 = \beta_2 = \pm 1$, the limit is increased by 1.

Format	Data

ICON(I+2), I = 1,6 indicates: if zero, that the data for the I^{th} T region are not to be read or are not to be selected and are the same as in the preceding problem; if unity, that a data card for the I^{th} T regions is to be read; or if greater than unity, that the data are to be selected from a set read initially. The order is that in which T regions appear, without regard to the location of F regions.

ICON(I+8), I = 1,6 indicates: if zero, that the data for the I^{th} F region are not to be read or are not to be selected and are the same as in the preceding problem; if unity, that a data card for the I^{th} F region is to be read; or if greater than unity that the data are to be selected from an initial set. The order is that in which F regions appear, without regard to the location of T regions. If the number of T regions changes, data are required for all F regions even though there is no change.

ICON(I+14), I = 1,12 indicates if non-zero, that the thickness (cm) of region either I or I-1 is to be read depending on whether or not $\beta_1 = \beta_2 = 1$ at the inner or left-hand boundary. In the latter case if ICON(15) $\neq$ 0 the inner radius of region 1 is to be read. If dimensions are not read, they are the same as they were initially in the preceding problem.

ICON(27) denotes the type of problem. If ICON(27) = 0,1 or 4, the type is standard; if 1 or 4, a general data card is to be read; if 0, the general data are the same as in the preceding problem. If ICON(27) = 4, data cards for all types of T regions and then for all types of F regions are read before the general data card is read. Numbering is considered to start with unity for each type of data. If ICON(27) = 2, a concentration search is called for. If ICON(27) = 3, abbreviated input for a series of cores surrounded by a reflector of fixed composition and thickness is called for, and punched output suitable for input to the KEFF code is obtained.

ICON(28) indicates, if non-zero, that the transverse buckling is to be read. If the buckling is not read, it is the same as initially in the preceding problem. The buckling is preset to 0. If the transverse buckling is to be used as an initial guess, it must be read for each problem even though its value is not changed.

Format	Data
	ICON(29) indicates the number of comment cards (≤ 9).
	ICON(30) denotes the problem series. The series starts with 100 × ICON(30) + 1 and is numbered consecutively.
	JD(I), I = 1,6 indicates the T data to be selected for the I^{th} T region.
	JD(I), I = 7,12 indicates the F data to be selected for the $(I-6)^{th}$ F region. Unless ICON(I), I = 3,14 is greater than unity, JD should be left blank.
18A4	(CT(I,J), I = 1,18), J = 1, ICON(29)
	Comment cards as called for.
2I5	NKG, NKC
	NKG is the number of T data cards to be read initially and NKC the number of F data cards. NKC must be greater than zero. Unless ICON(27) = 4, NKG and NKC are zero, and this card is omitted.
2F10.7, 4F10.8	Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22} for NKG T regions.
2F10.7, 4F10.8	Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22} for NKC F regions.
4F5.2, 15I1, 3F5.2, 6I2	BETFI,BETSI,BETFO,BETSO,(IGEN(I), I = 1,15), FTR,SFR,SDR, (IFP(I), I = 1,6)
	General data, if called for: Fast and slow inner and outer albedos. Order in which T and F regions are interlaced: 0 for T and 1 for F region through total number. IGEN(13) denotes slab, cylinder, or sphere by 0, 1, or 2. IGEN(14) denotes negative exponent of 10 for convergence criterion; cannot exceed 8 for buckling (10^{-8} cm^{-2}) or 4 for a dimension (10^{-4} cm). IGEN(15) denotes type of search: 0 denotes transverse buckling and I denotes thickness of F region I. (All other region thicknesses remain unaltered.) FTR is a factor to be applied to an initial guess either of transverse buckling or of extrapolation distance for a central core, depending on whether SDR is or is not zero. SFR is a speed-up factor to be applied to the linearly predicted change in buckling or dimension; without prior experience, use unity, never zero. SDR is the initial guess either of the increment in thickness to be subtracted in a critical dimension search or of the extrapolation distance, depending on

Format	Data
	whether FTR is or is not zero. IFP(1) denotes the number of internal points in region 1 at which the flux may be calculated. IFP(I), I = 2,6 is not used.
2F10.7, 4F10.8	DATA(I,J), I = 1,6 Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22} for the J^{th} T region, if called for.
2F10.7, 4F10.8	DATA(I,J+NT), I = 1,6 Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22} for the J^{th} F region, if called for. NT is number of T regions.
F10.4	PAD(I) Radius or thickness (cm) as called for, one per card. In a dimension search for the radius of a symmetrical F region core, a guess of the radius and its change will be calculated from a guess of the extrapolation distance and the factor FTR, and PAD(1) should not be read.
F10.4	BEST Transverse buckling (cm^{-2}) if called for. In a buckling search if an initial guess is not read or is not computed by the code from an initial guess of extrapolation distance, it is taken to be 1% greater than the maximum material buckling in any F region. Convergence to a greater buckling than that initially selected by any means is not permitted by the code; hence if the initial value is too small, convergence to the proper root will not occur.
30I1	(ICON(I), I = 1,30), (JD(I), I = 1,12) Control vector for next problem. If ICON(2) = 0, series is terminated.

Abbreviated Type ICON(27) = 3

Intended only for obtaining for a particular value of k_{eff}, the sizes of symmetrical F region cores that are either bare or are surrounded by a reflector of fixed composition and thickness.

Format	Abbreviated Type ICON(27) = 3
2I5	NKG, NKC NKG = 0, 1, 2, 3, or 4 denotes respectively IGEN(13) as it is read, slab and cylinder, slab and sphere, cylinder and sphere, or all 3. IGEN(13) should denote the first one of the group. If NKC = 0, a k_{eff} and concentration are required for each core. If NKC = 1, the first k_{eff} and concentration apply to all cores.
4F5.2, 15I1	General data card as in standard type.
3F5.2, 6I2	IGEN(13) should be as indicated above. IGEN(15) must be 1. Neither FTR nor SDR may be zero.
2F10.7, 4F10.8	DATA(I,J), I = 1,6 Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22} for the J^{th} T region, if called for.
7F10.4	DATA(I,NT+1), I = 1,6, XMS Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22}, and M^2 for core. If card is blank, next card is interpreted as ICON.
2F10.7, 4F10.8	DATA(I,J+NT+1), I = 1,6 Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22} for J^{th} F region beyond the core as called for ICON(9+J).
F10.4	PAD(I+1) Thickness of I^{th} region beyond the core as called for by ICON(15+I). ICON(15) should be zero.
F10.4	BEST Transverse buckling if called for. Read only for first core. Intention is that it be zero.
2F10.4	EK, GPL EK is k_{eff} to which sizes are to correspond. GPL is concentration in core, g/l. The code makes no use of GPL except to punch it out for input to the KEFF code. These are read for the first core, but not thereafter if NKC = 1.
7F10.4	Data for second core.
2F10.4	EK, GPL for second core if called for by NKC = 0, etc.

Concentration Search ICON(27) = 2

No T regions are permitted here. This section is intended chiefly for finding the average critical concentration for nonuniform distributions.

Format	Concentration Search ICON(27) = 2
I5	NCONC NCONC ($\leqslant$50) is the number of concentrations in the table of two-group constants to be read in. If NCONC = 0, the next card read is interpreted as ICON.
14F5.0	CONC(I), I = 1, NCONC Concentrations in g/l for which group constants are to be read. As many as four cards may be required. Concentrations must be equally spaced within each of a maximum of three groups. The spacing of concentrations in a succeeding group must be an even multiple of that in the preceding group; e.g., concentrations of 0, 5, 10, 15, 20, 30, 40, 50, 60, 80, 100, 120 are acceptable.
2F10.7, 4F10.8	(D(I,J), I = 1,6), J = 1, NCONC Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22} for each of NCONC concentrations.
14I5	NR,NST,KR,NVC,KGY,NFP,NFR,NSU,NS,NFC,NBFI,NBSI,NBFO,NBSO NR is number of regions, including reflectors. NST is IGEN(15) and denotes type of search. KR denotes number of identical reflectors: 0, 1, or 2. NVC is IGEN(14), the convergence exponent. KGY is IGEN(13), the geometry index. NFP is IFP(1), the flux points in region 1. NFR is 10 × FTR, the buckling or extrapolation distance factor. NSU is 10 × SFR, the speed-up factor. NS is 10 × SDR, the extrapolation distance or the radius increment. NFC is percent by which concentration is changed if a concentration search is desired. NBFI, NBSI, NBFO, and NBSO are the fast and slow inner and outer albedos multiplied by 10. If card is blank, next card is interpreted as NCONC.
2F10.7, 4F10.8	DATA(I,NR), I = 1,6 Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22} for reflector.

Format	Concentration Search ICON(27) = 2
7F10.4	RAD(I), I = 1, NR, BUCK

Radii (cm) of all regions and transverse buckling (cm^2).

If card is blank, next card is interpreted as NR, NST, KR, etc.

7F10.4 CNC(I), I = 1, (NR-KR)

Concentration in each region, excepting reflectors.

If card is blank, next card is interpreted as RAD(I).

PRINTED OUTPUT

Name and number read as PN(I), I = 1,5

Problem number and geometry

Boundary conditions

Region number, type (0 for T, 1 for F), and two-group constants

Bucklings for F regions

Transverse buckling, value of determinant, extrapolation distance for core or, in slab case, average extrapolation distance for first region where it is meaningful, and radii, for each iteration

Fast and slow fluxes and currents at boundaries and interfaces and, if called for, fluxes at intermediate points in first region

For abbreviated type (ICON(27) = 3): k_{eff}, M^2, and critical volume/cm^2, volume/cm, or volume for slab, cylinder or sphere

For concentration search (ICON(27) = 2): critical mass/cm^2 and radial buckling, critical mass/cm and axial buckling, or critical mass and excess buckling for slab, cylinder, or sphere, respectively; concentrations in each region.

PUNCHED OUTPUT

For abbreviated type only: concentration (g/l), B_m^2, M^2, three dimensions, three extrapolation distances, two bare extrapolation distances, index for card reading, index for type of solution. Format is F7.2, F8.6, F7.3, 3F6.1, 5F5.2, 2II. This output is in the form required as input for KEFF. The concentration and M^2 are those read as input by TGAN. The material buckling B_m^2 is that (B^2) calculated from the two-group parameters and then adjusted to k_{eff}, i.e.,

$$B_m^2 = \frac{\frac{1 - k_{eff}}{M^2} + B^2}{k_{eff}}$$

The dimensions are either 0 or ∞ (9999 in.). The extrapolation distance on the finite dimension to be calculated by KEFF is that calculated by TGAN. The bare extrapolation distances are arbitrarily 3.0 cm. The card reading index is 1 and the solution type index is 2, 3, or 1 for infinite slab, infinite cylinder, or sphere. A third index on at least the first card is required for running problems with KEFF. (See description of KEFF for significance of these indices.) Spacer cards containing PN(5) in columns 75-78 precede and follow these cards.

CRITICAL DIMENSIONS CODE
(KEFF1149, L5403)

DESCRIPTION

This code computes critical dimensions and from them volumes and masses corresponding to a number ($\leqslant 9$) of selected values of k_{eff}, where

$$k_{eff} = \frac{1 + M^2 B_m^2}{1 + M^2 B_g^2} \tag{27}$$

and where for a slab

$$B_g^2 = \left(\frac{\pi}{W + 2\bar{S}_W}\right)^2 + \left(\frac{\pi}{L + 2\bar{S}_L}\right)^2 + \left(\frac{\pi}{H + 2\bar{S}_H}\right)^2$$

for a cylinder

$$B_g^2 = \left(\frac{2.4048}{R + S_R}\right)^2 + \left(\frac{\pi}{H + 2\bar{S}_H}\right)^2$$

for a sphere

$$B_g^2 = \left(\frac{\pi}{R + S}\right)^2$$

Alternatively, given all the dimensions and extrapolation distances and of course B_m^2 and M^2, the code computes k_{eff}; if the extrapolation distances are not given, the code computes an effective extrapolation distance. The code also computes one-group albedos at outer surfaces from the following expressions:

$$\beta = \frac{\sin B_W(S - S_o)}{\sin B_W(S + S_o)} \qquad \text{for a slab}$$

$$\beta = \frac{\dfrac{J_o(2.4048 - B_r S)}{J_1(2.4048 - B_r S)} - \dfrac{J_o(2.4048 - B_r S_o)}{J_1(2.4048 - B_r S_o)}}{\dfrac{J_o(2.4048 - B_r S)}{J_1(2.4048 - B_r S)} + \dfrac{J_o(2.4048 - B_r S_o)}{J_1(2.4048 - B_r S_o)}} \qquad \text{for a cylinder}$$

$$\beta = \frac{\dfrac{\pi - BS}{1 + (\pi - BS)\cot BS} - \dfrac{\pi - BS_o}{1 + (\pi - BS_o)\cot BS_o}}{\dfrac{\pi - BS}{1 + (\pi - BS)\cot BS} + \dfrac{\pi - BS_o}{1 + (\pi - BS_o)\cot BS_o}} \qquad \text{for a sphere} \tag{28}$$

where S is the extrapolation distance at the surface, S_o is the bare extrapolation distance there, and B_W and B_r are the components

of the buckling in the direction normal to the surface. These albedos are useful in performing calculations concerned with the interaction of units.

To reduce the amount of card punching required, there are a number of options that indicate which positions on a card are meaningful. The others are ignored, and data for which these positions are allocated are taken to be the same as in the preceding problem.

INPUT

Format	Data
F7.2,F8.6,F7.2,	Group 1 concentration (g/l), B_m^2 (cm^{-2}), M^2 (cm^2)
3F6.2,	Group 2 three dimensions, inches
3F5.2,	Group 3 three extrapolation distances, cm
2F5.2,	Group 4 two bare extrapolation distances, cm
3I1	Indices I, J, K
	In group 2 the dimensions are: width, height, and length for a slab, diameter and height for a cylinder, and diameter for a sphere. The unused dimensions should be left blank including dimensions to be calculated by the code.
	In group 3 the extrapolation distances are in the same order as the dimensions, and again those not used should be left blank.
	In group 4 the extrapolation distances are in the same order as in group 3. No bare extrapolation distance for slab length is read.
	Index I indicates which groups of data are ignored.
	If I = 1, none. First card should have I = 1.
	If I = 2, group 4.
	If I = 3, groups 2-4.
	If I = 4, group 2.
	If I = 5, groups 2 and 4.
	If I = 6, groups 1 and 4.
	If I = 7, groups 1, 3, and 4.
	If I = 8, groups 1, 2, and 4.

Format	Data
	Index J indicates what is computed. If J = 1, sphere diameter. If J = 2, height of cylinder with fixed diameter. If J = 3, diameter of cylinder with fixed height. If J = 4, width of slab with fixed height and length. If J = 5, k_{eff}. If J = 6, effective extrapolation distance.
	Index K denotes number ($\leqslant 9$) of values of k_{eff} to be read. Must not be zero for J = 1 through 4 for first problem. Should be zero for J = 5, 6.
9F5.3	Values of k_{eff} called for by index K. If K = 0, this card is omitted.
F7.2,F8.6,F7.2 3F6.2,5F5.2,3I1	Data for next problem. A blank card terminates the series.

OUTPUT

The output consists of a label specifying shape, the concentration, B^2, M^2, the extrapolation distances, the dimensions, the mass, albedos, and k_{eff}.

TAPERED SLAB CODE
(TSLA1149, L5406)

DESCRIPTION

This code computes, corresponding to various choices of k_{eff}, the height of a slab that may be tapered or may be reflected, in effect, by a tapered reflector. The taper is assumed to be linear, the slab thickness or width increasing linearly with height. The calculation is one-dimensional and one-group. The height is divided into a number of segments. Within a segment

$$B_H^2 = B_m^2 - B_W^2 - B_L^2$$

where

$$B_L^2 = \left(\frac{\pi}{L + 2\overline{S}_L}\right)^2$$

$$B_W^2 = \left(\frac{\pi}{W_O + MH + 2\overline{S}_W(H)}\right)^2$$

and where L is the length of the slab, $\overline{S}_L$ is the average extrapolation distance for the length, W_O is the width of the slab at zero height, H is the height of the upper boundary of the segment, M is the linear rate of increase of width with height, and $2\overline{S}_W(H)$ is the average extrapolation distance for width as a function of height as determined from Lagrange interpolation of values at three heights.

External boundary conditions at top and bottom are expressed in terms of extrapolation distances. At interfaces between segments, the boundary condition is continuity of the vector

$$\begin{bmatrix} \phi(X) \\ \frac{d\phi(X)}{dx} \end{bmatrix} = \begin{bmatrix} \sin B_H X & \cos B_H X \\ B_H \cos B_H X & -B_H \sin B_H X \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

where B_H differs in adjacent segments and may be imaginary toward the bottom of the slab, in which case the functions are the hyperbolic sine and cosine.

Starting from a guess of initial height, which is such that the volume contained in the tapered slab equals that contained in a nontapered slab with height corresponding to k_{eff} and with width equal to that at the top of the tapered slab, the code iterates on the height until the boundary and interface conditions are

satisfied adequately. The convergence criterion is that the height predicted for the next iteration should differ by less than 0.01 cm from that just calculated. The number of iterations is limited to 20.

INPUT

Format	Data
4I5	N, M, NN, ND N is number of segments into which height is subdivided. If N = 0, number is same as in previous problem. If N < 0, series is terminated. M is number ($\leqslant 9$) of k_{eff} values to be read. If M = 0, values are same as in previous problem. NN is number ($\leqslant 9$) of comment cards to be read. ND indicates, if non-zero, that length, width at zero height, and slope are to be read; otherwise they are not read and are the same as in previous problem.
18A4	Comment cards as called for by NN.
7F10.5	k_{eff} values as called for by M.
3F10.5	Width (in.) at zero height, length (in.), linear rate of increase of width with height if called for by ND $\neq$ 0.
6F10.5	Concentration within slab (g/l), material buckling (cm^{-2}), migration area (cm^2), extrapolation distances (cm) from bottom and from top, and SL. If SL $\geqslant$ 1, it is interpreted as the average extrapolation distance (cm) for the length. If SL < 1, it is interpreted as the geometric buckling (cm^{-2}) transverse to the thickness in a critical slab with thickness equal to the length of the tapered slab.
6F10.5	G(I), I = 1,3; SW(I), I = 1,3 G(I) are three equally spaced heights (in.) at which SW(I) is read. G(1) must be zero. G(3) should be greater than the largest height of the tapered slab that is of interest. If SW(I) $\geqslant$ 1, it is interpreted as the average extrapolation distance (cm) for the width at height G(I). If SW(I) < 1, it is interpreted as the buckling (cm^{-2}) transverse to the thickness in a critical slab having a thickness equal to the width of the tapered slab at height G(I).
4I5	N, M, NN, ND for next problem.

OUTPUT

The output consists of the comments read; the length (in.), width at zero height (in.), the slope, and the number of segments into which the height is divided; the concentration, material buckling, migration area, and all the extrapolation distances; the height (cm) of the tapered slab for each iteration; the value of k_{eff}, the initial height (in.) from which iteration started, the converged height (in.), the volume (liters) at the converged height, and the mass (kg). If the initial height is greater than G(3), a statement that the height of the tapered slab is greater than G(3) is printed out, and no iteration is carried out.

CODE FOR CYLINDER WITH DISHED BOTTOM
(DISH1149, L5402)

DESCRIPTION

This code computes, corresponding to various choices of k_{eff}, the maximum height (i.e., the height along the axis, at the lowest part of the dish) in a cylinder with a dished bottom. The calculation is one-dimensional and one-group. The cylinder is divided into a number of annuli. Within an annulus

$$B_R^2 = B_m^2 - B_H^2$$

where

$$B_H^2 = \left(\frac{\pi}{H + 2\bar{S}_M}\right)^2$$

and where H is the height above the curved bottom at the inner radius of the annulus and $\bar{S}_H$ is the average extrapolation distance for the height. The radial boundary condition is expressed in terms of a radial extrapolation distance. At interfaces between annuli, the boundary condition is continuity of the vector.

$$\begin{bmatrix} \phi(r) \\ \frac{d\phi(r)}{dr} \end{bmatrix} = \begin{bmatrix} J_o(B_R R) & Y_o(B_R R) \\ -B_R J_1(B_R R) & -B_R Y_1(B_R R) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

If, as is possible, B_R is imaginary the Bessel functions I_o, I_1, K_o, and K_1 are used.

Starting from a guess of maximum height, which is such that the volume within the dished cylinder equals that in a non-dished cylinder with the same diameter at the same k_{eff}, the code iterates on height until it converges within 0.0001 cm. If no finite initial height exists, the code proceeds to the next problem.

The code has been applied to the dished cylinder formed by dividing a sphere into a number ($\geq$10) of cylindrical annuli and then displacing these annuli and deforming them so that their tops would be flat and all lie in a plane. When the maximum height of such a cylinder is calculated to equal its diameter, k_{eff} is slightly greater than for a sphere with the same diameter; hence the method is conservative.

INPUT

Format	Data
3I5	N, L, LP N is number of radial boundaries. If zero, series is terminated. L indicates, if non-zero, that the next problem will start by reading N, L, LP, and if zero, by reading another concentration, etc. LP indicates, if > 0, that intermediate steps in the iteration will be printed.
7F10.5	RAD(I), I = 1, N; DH(I), I = 1, N RAD(I) is the radius (inches) at which an annular boundary is located. DH(I) is the difference between the maximum height (inches) and the height (inches) at this radius. As many cards are read as are required.
6F10.5	Concentration (g/l), material buckling (cm^{-2}), migration area (cm^2), average extrapolation distance (cm) for height, radial extrapolation distance (cm), number of values of k_{eff} to be read. If concentration is zero, the next card is interpreted as N, L, LP. If number of k_{eff}'s is zero, values are same as in previous problem.
7F10.5	k_{eff}'s as called for by preceding card. Limit is 7.
3I5 or 6F10.5	N, L, LP or concentration, etc. for next problem, depending on whether L is or is not zero.

OUTPUT

The output consists of concentration (g/l), material buckling (cm^{-2}), migration area (cm^2), axial and radial extrapolation distances (cm), k_{eff}, and the corresponding maximum height (inches), volume (liters), and mass (g). The volume and mass are calculated on the assumption that the height of an annulus varies linearly from inner to outer boundary. If LP > 0, intermediate heights (cm) and values of a determinant that are calculated in the course of an iteration are printed.

PARAMETER ADJUSTMENT CODE
(PADJ1149, L5405)

DESCRIPTION

This code adjusts two-group parameters for use in subsequent two-group calculations. Two types of adjustment are possible. In the first, c_{22} is modified to account for absorption. For example, a two-group calculation may have been run for an infinite system of slabs of fissionable material alternating with thin sheets of cadmium. The critical transverse buckling is then in effect the material buckling of a homogenized material containing fissionable material and cadmium. Presumably only c_{22} should be significantly different for the homogenized material from its value for the fissionable material without cadmium. The code accordingly adjusts c_{22} to conform with the material buckling and the other two-group parameters. Adjustments in the other parameters can of course be made, if indicated, prior to reading them into the code.

The other type of adjustment consists of adjusting c_{11} and c_{22} to give effective values that account for a transverse buckling and/or for a change in buckling to conform to a k_{eff} differing from unity. The two-group code requires the same transverse buckling in all regions. For some cases, however, different transverse bucklings and k_{eff}'s correlating calculation and experiment may be desirable in different regions. The adjustment in c_{11} and c_{22} permits calculations of this type to be carried out with the two-group code.

INPUT

Format	Data
4A4,I4	PN(I), I = 1,4,NB PN is a name of up to 16 characters, NB is a number of up to 4 digits.
I10, 5F10.6	L, GUCK, HUCK, TA, TB, EFFL If L = 1, adjustment is made in c_{22} to conform to GUCK (material buckling (cm^{-2})) and to other two-group parameters; rest of card should be blank. If L = 2, adjustment is made in c_{11} and c_{22} to give effective values that account for transverse leakage as characterized by GUCK and HUCK (transverse bucklings (cm^{-2})). TA and TB should be left blank. EFFL is k_{eff} to which material buckling is adjusted. Parameters

Format	Data
	left blank will have the values in the preceding problem or preset values for the first problem. The preset values are 0 for GUCK, HUCK, TA, and TB, and 1 for EFFL. If L = 3, same adjustment is made as when L = 2, but GUCK and HUCK are interpreted as transverse dimensions (cm) and TA and TB as the respective average extrapolation distances (cm). This option applies only to a parallelepiped. If L = 0, series is terminated.
2F10.7, 4F10.8, F10.4	Two-group parameters Σ_1, Σ_2, c_{11}, c_{21}, c_{12}, c_{22}, and M^2. Migration area not required unless $k_{eff} \neq 1$.
I10, 5F10.6	L etc. for next problem.

PRINTED OUTPUT

Identifying name and number followed by two-group parameters, material buckling (for c_{22} adjustment), or effective buckling adjusted for transverse leakage and k_{eff}, migration area, and k_{eff}.

PUNCHED OUTPUT

Two-group parameters and migration area. Spacer cards containing the identifying number in columns 75-78 precede and follow these cards.

TWO-GROUP INPUT CODE
(TGIN1149)

DESCRIPTION

If many problems are to be run, it is somewhat tedious to prepare input for the two-group code when searches for the average concentration in nonuniform distributions are to be made; this code supplies the required punched cards. In addition it contains the arbitrary set of concentration profiles used in a recent study[24] of the effect of nonuniform distributions, and it contains the relative radii for subdivision of a slab, cylinder, or sphere into five regions with equal volumes or with equal ratios of surface to volume.

INPUT

Format	Data
5A4	PN(I), I = 1,5 PN(I), I = 1,4 is an identifying name. PN(5) is an identifying number.
4F10.5, 3I10	Outer radius (cm) of core, outer radius (cm) of reflector, average concentration (g/l) in core, transverse buckling (cm^{-2}), IG, ID, and ITR. IG = 0, 1, or 2 denotes slab, cylinder, or sphere. ID indicates, if 0, subdivision into five regions with equal ratios of surface to volume, or, if 1, into five regions with equal volume. ITR indicates, if 0, that series of problems is terminated; if 1, that average relative concentrations in each subdivision are to be calculated for each of the five concentration profiles; or if 2, that these concentrations are taken to be the same as in the previous problem.
4F10.5, 3I10	Next core radius, etc.

PRINTED OUTPUT

Identifying name and number. Then for each of the five profiles: the radii (cm) of the subdivisions and the reflector and the transverse buckling (cm^{-2}), the concentration (g/l) in each region, and zero.

PUNCHED OUTPUT

A card giving radii and buckling.

A card giving concentrations.

A card with zero.

Three such cards are obtained for each of the five profiles. The entire batch of cards is preceded and followed by spacer cards containing the identifying number PN(5) in columns 75-78.

ZERO-DIMENSIONAL MULTIGROUP CODE (ZDMG1149)

DESCRIPTION

This code computes the spectrum in an infinite homogeneous medium in which there is a uniformly distributed fission source, in which there is no leakage, and in which the only events are elastic scattering and absorption. This spectrum is useful for computing constants for larger groups than employed here, where leakage is considered, as in the Multigroup Buckling Code. Although leakage of course affects the spectrum, cross sections averaged over a group width are not apt to be affected significantly by whether or not the spectrum used for calculating them takes account of leakage, provided the group is not so wide that the number of neutrons lost by leakage is no longer small compared with the numbers lost by absorption and by transfer to other groups. Elastic scattering may be anisotropic in the center of mass frame of reference. No provision is made for inelastic scattering or for resonance absorption expressed in terms of resonance parameters. All groups must have equal lethargy width. Only down scattering is considered.

The neutron balance equation for a group k may be written

$$\sum_{i=1}^{N} N_i \sigma_{ik} \phi_k = \chi_k + \sum_{i=1}^{N} \sum_{j=1}^{k} N_i c_{ikj} \sigma_{ij} \phi_j \qquad (29)$$

where N_i is the atomic density of material i; σ_{ik} is its total cross section in group k; χ_k is the fraction of the fission spectrum in group k; c_{ikj} is the number of secondary neutrons transferred, per elastic scattering collision with material i, from group j to group k; and ϕ_j is the flux integrated over group j. The code starts with the group of highest energy and works downward, calculating the flux integrals as it goes. In the code the absorption and scattering are treated as coming from different species and different types of absorption may be kept separate. Thus oxygen is considered to consist of a scattering species and an absorption species which may in turn be considered to consist of an (n,α) species and an (n,p) species.

From the mechanics of elastic collisions, the probability density p for transfer from E' to E is

$$p(E,E') = \frac{\sum_{n=0}^{\infty} (2n+1)b_n(E')P_n\left(\frac{2E - E'(1+\alpha)}{E'(1-\alpha)}\right)}{E'(1-\alpha)} \qquad (30)$$

where $\alpha = \left(\frac{A-1}{A+1}\right)^2$, A is the mass of the scatterer, the P_n's are Legendre polynomials, and the $b_n(E')$'s are the coefficients in the expansion of σ_s in terms of λ, the cosine of the angle of scattering in the center of mass frame of reference. In the code, the number of terms in the expansion is limited to six, i.e.,

$$\sigma_s = \frac{\sigma_{s_o}}{4\pi}\sum_{n=0}^{6}(2n+1)b_nP_n(\lambda)$$

It is convenient to use lethargy u rather than energy E as a variable where $u = \log E_o/E$ and hence $E = E_oe^{-u}$. Then

$$C(u,u') = p(E',E)\frac{dE}{du}$$

$$= -\frac{e^{-(u-u')}}{1-\alpha}\sum_{n=0}^{6}(2n+1)B_n(u')P_n\left(\frac{2e^{-(u-u')} - (1+\alpha)}{1-\alpha}\right) \qquad (31)$$

where, of course, $B_n(u') = b_n(E')$. Except for the dependence of B_n on u', $C(u,u')$ depends only on u-u'.

Considerable simplification results if all neutrons within a group are treated as having the lethargy of the midpoint of the group. The transfer coefficient c_{kj} then is

$$c_{kj} = \frac{1}{2}\sum_{n=0}^{6}(2n+1)B_n\int_a^b P_n(\lambda)d\lambda \qquad (32)$$

where

$$a = \frac{2e^{-(k-j+\frac{1}{2})\Delta u} - (1+\alpha)}{1-\alpha} \quad \text{or } -1, \text{ whichever is greater}$$

and

$$b = \frac{2e^{-(k-j-\frac{1}{2})\Delta u} - (1+\alpha)}{1-\alpha} \quad \text{or } 1, \text{ whichever is less.}$$

The accuracy of this approximation depends on the width of the groups. For H_2O, results obtained with widths of 0.1 and 0.05 differ only slightly.

Among the quantities calculated by the code is the transport cross section. This requires a calculation of the average value of $\bar{\mu}$, the cosine of the angle of scattering in the laboratory frame of reference. From the mechanics of elastic collisions,

$$\bar{\mu} = \int_{-1}^{1} \frac{1 + A\lambda}{\sqrt{1 + A^2 + 2A\lambda}} \sum_{n=0}^{6} \frac{(2n+1)B_n}{2} P_n(\lambda)d\lambda$$

$$= \sum_{n=0}^{6} (-1)^{n+1} B_n \left(\frac{n}{(2n-1)A^{n-1}} - \frac{n+2}{(2n+3)A^{n+1}} \right) \qquad (33)$$

The reading of input data and the writing of output are controlled by the MAIN program; most of the calculation is performed in subroutines. The flux integral for a group is calculated by FLUX from source neutrons furnished by fission and by transfer from groups with higher energy as calculated by SRCE. The fission spectrum is calculated by FSPM, with the spectrum expressed as $N(E) = a\sqrt{E}\, e^{-E/T}$. The coefficients for transfer are calculated by STR or STRL for scattering that is, respectively, isotropic or anisotropic in the center-of-mass frame of reference. The average cosine of the angle of scattering in the laboratory frame of reference is computed by COSAL. Average group constants for groups of greater width than employed in the code are computed in GRCST which uses WEAV to compute the flux-weighted averages. All cross sections except the transport cross section are weighted directly; its reciprocal is weighted by the flux in calculating average values.

INPUT

Format	Data
5A4	PN(I), I = 1,5
	PN(I), I = 1,4 is a name of up to 16 characters
	PN(5) is a number of up to 4 digits.
13I5, I3,I2	(IC(I), I = 1,3), K, L, KI, KJ, KK, KL, KM, KN, KO, KP, IP, IPP
	IC(1) is number ($\leqslant$5) of anisotropic scatterers.

Format	Data

IC(2) is number of isotropic scatterers.

IC(3) is number of absorbers. IC(2)+(C(3)) $\leqslant$15.

K is number of groups ($\leqslant$400, except that number for which Legendre coefficients are >0 for n >0 must be the smaller of 160 and 5.0001/D + L where D is group width and L is number of groups above 10 Mev).

L is number of groups above 10 Mev, which must be a group boundary.

KI is number of groups within first coarse group for which averages are desired.

KJ is the number of groups within each of the next KK coarse groups.

KL is the number of groups within each of the next KM coarse groups.

KN, KO, and KP are the number of absorbers of types 1, 2, and 3. The number of type 4 is IC(3) - KN - KO - KP. A type might, for example, be (n,α).

IP indicates, if non-zero, that fluxes from previous problem are to be used in computing group constants.

IPP indicates, if non-zero, that output is to be punched.

2F10.5 Group width D in lethargy units. Temperature of fission spectrum T, Mev.

2A5 SPN(J,I), J = 1,2

A ten-letter label for species I. (Scattering, (n,α) absorption, etc., from the same nuclide are treated as separate species. To provide spaces on printout, only the middle eight positions should be used.)

7F10.5 SG(J,I), J = 1,K

Microscopic cross section (barns) of species I in group J. As many cards are used as are required for a species. The series of cards SPN, SG is read for each I from 1 through IC(1) + IC(2) + IC(3) before proceeding to read CL. The order must be the same as in IC, i.e., anisotropic scatterers, isotropic scatterers, and absorbers.

7F10.5 CL(J,N,I), J = 1,M

Legendre coefficients B_N for N $\geqslant$ 1. The number (M) read is the smaller of the number (K) of groups and 5.0001/D + L. As many cards are used as are required to hold all the B_N for a species. A new card is

Format	Data
	started each time N advances. The same number of cards is used for each N even though at low energy and high N many B_N are zero. The B_N for N = 1,6 are read for a species before advancing to B_1 for the next species.
7F10.5	Atomic mass of each of IC(1) + IC(2) scatterers in the order in which labels and cross sections are read.
7F10.5	Atomic density × 10^{24} of each species of scatterer and absorber in the order read.
BI5, I3, I2	IC,K,etc. for next problem. If K = 0, series is terminated.

PRINTED OUTPUT

The identifying name and number, read as PN.

The labels SPN, read for each species and, directly beneath each label, the atomic density.

The group width, the number (NG) of coarse groups, and the lethargies of the coarse group boundaries.

The temperature of the fission spectrum.

The flux integral in each group.

If there are exactly two coarse groups, the fission and transfer sources for the second group are then printed. These can be used as sources in a subsequent calculation, for example in a thermalization calculation where up-scattering occurs.

The flux integral, the fission source, Σ_t, Σ_{tr}, Σ_s, Σ_{a_1}, Σ_{a_2}, Σ_{a_3}, and Σ_{a_4} for each of the coarse groups.

The transfer cross sections Σ_{kj} for the coarse groups listed as Σ_{11}, Σ_{21}, Σ_{22}, Σ_{31}, ..., $\Sigma_{NG+1,NG}$.

PUNCHED OUTPUT

The following data are punched, consecutively, if called for by IPP≠0: Σ_t for each coarse group, Σ_{tr}, Σ_a, 2 × NG zeros, Σ_{kj} (in same order as above), and the relative molecular weight of the mixture of nuclides. These data are preceded and followed by spacer cards with the identifying number PN(5) in columns 75-78. (This format is that required as input by the code HEETR[16].)

PARAMETER AVERAGES CODE
(PAVE1149)

DESCRIPTION

This code takes point cross sections, weights them by the fission spectrum, and computes average cross sections for groups with equal lethargy width. Similarly, Legendre coefficients are weighted by σ_{el} and by the spectrum, and averages are calculated. The code is used chiefly for preparing input for ZDMG. The integration required to obtain averages is based on the trapezoidal rule. All computation except the integration is performed in the main program.

INPUT

Format	Data
5A4	PN(I),I = 1,5 Identifying name and number as in ZDMG.
14I5	(N1(I),I = 1,6), (N2(I),I = 1,6), KF, KN N1(I) is the number of values to be read for each of at most six cross sections. If N1(1) = 0, the series of problems is terminated. N2(I) is the number of values to be read for each of at most six Legendre coefficients. KF denotes read format. KN denotes number of types of cross sections to be punched (e.g., elastic scattering, (n,α), etc.).
3F10.4	Group width in lethargy units, the initial lethargy (i.e., the lethargy corresponding to the upper bound of the group of highest energy), the temperature of the fission spectrum, Mev.

The manner in which the remainder of the data are read depends on the value of KF.

KF	Format	Data
0	7F10.4	Energies (Mev) to which neutron cross sections correspond starting with highest energy.
	7F10.4	(A(I,J), I = 1, N1(J)), J = 1,6 A is cross section of type J at I^{th} energy. Type 1 must be the elastic scattering cross section. A new card is started for each new type.
	18A4	Six 12-character labels for the six cross section types.
	18A4	Six 12-character labels for the Legendre coefficients B_n in the expansion of $$\sigma_s(1) = \frac{\sigma_{s_o}}{4\pi} \sum_{n=0}^{6} (2n+1)B_n P_n(\lambda)$$ If there are no coefficients to be read, this card should be omitted.
	7F10.4	B(I,J), I = 1, N2(J) B is J^{th} Legendre coefficient at I^{th} energy. A new card is started for each J.
1	4F10.4	Energy (Mev), σ_3, σ_1, σ_2, starting with lowest energy. As above, $\sigma_1 = \sigma_{el}$.
	18A4	Six 12-character labels for cross sections. Six 12-character labels for Legendre coefficients.
	7F10.4	Energy (Mev), B_1, B_2, B_3, B_4, B_5, B_6, starting with lowest energy.
2,3,4		Same as with KF = 1 except that respective orders for reading cross sections are: σ_5, σ_2, σ_3, σ_4, σ_1; σ_6, σ_2, σ_3, σ_4, σ_5, σ_1; σ_4, σ_2, σ_3, σ_1. The formats for KF > 0 are designed to handle the data in the form reported in Reference 25.

Data for the next problem are read, starting with the card containing N1 etc.

PRINTED OUTPUT

The lethargy and energy corresponding to group boundaries, the average cross sections, and the average Legendre coefficients are printed. These are followed by the input data consisting of the cross sections, Legendre coefficients, and energies.

PUNCHED OUTPUT

The first KN types of average cross sections are punched by FORMAT 7F10.5. A new card is started for each new type. The average Legendre coefficients, if any, are then punched by the same format, and again a new card is started for each new coefficient. Both the cross sections and coefficients are arranged in order of decreasing energy. The output cards are preceded and followed by spacer cards with the identifying number PN(5) in columns 75-78.

—— APPENDIX ——

CORRELATIONS WITH EXPERIMENT

The combination of TGAN and MGBS has been correlated with a large number of exponential and critical experiments. All correlations are expressed as the value of k_{eff} calculated for configurations found experimentally to be critical. (An exponential experiment is considered to be critical with a buckling that is less than the geometric buckling calculated from the dimensions.) Many of these correlations have been reported in internal memoranda; others have been given in external reports. All the correlations that have been reported internally together with many that have not previously been reported are included in this appendix. In addition the early correlations[26] with critical experiments performed with aqueous solutions of plutonium have been repeated. Correlations with data for lattices of rods of slightly enriched uranium and uranium oxide in water have been reported recently[1] and are not included here.

No attempt is made here to express the correlations as functions of variables to which they appear sensitive as was done for the rod lattices[1]. This is left as an exercise for the user of the codes described in this report. When one wishes to use these codes for a particular situation, he should examine correlations for similar situations and should choose k_{eff} such that he has reasonable assurance that the system will be subcritical. In general this k_{eff} will not be a single value but will be a function of variables such as $H/^{235}U$. Where the correlations show the codes to overestimate reactivity appreciably, as with U-Al alloy fuel tubes, the subcritical k_{eff} may perhaps be chosen to be slightly greater than unity. Where correlations show the codes to err in the opposite direction, as with lattices of slightly enriched uranium rods at low ratios of water to uranium, the subcritical k_{eff} must necessarily be chosen to be appreciably less than unity. It is easy to choose k_{eff} so low that subcriticality is obviously assured, but there are generally economic reasons for not picking values too low. There is therefore generally an incentive to study the correlations so as to choose an adequate but not excessive margin of safety in terms of k_{eff}. This margin is necessarily in addition to operational margins of safety that allow for errors in measuring concentrations, double batching, and other contingencies, the consideration of which is outside the scope of this report.

For correlations to have meaning, it is necessary to describe how the calculations were performed. Most of the systems were reflected by water. Two-group parameters for water were chosen

to be those characteristic of an infinite sea containing a uniformly distributed fission source, and are those calculated by MGBS with zero buckling and no iteration. In some experiments thin walls of aluminum, stainless steel, or cadmium separated the core and the reflector or separated various parts of the core. Two-group parameters for these materials were calculated by MGBS in the spectrum calculated for water. (Thin aluminum walls have little effect, and in some cases were ignored.) Two-group parameters, the material buckling, and the migration area were calculated for fissionable material by MGBS, with iteration. For heterogeneous systems cylindrical cells with the same area as the actual square or hexagonal cells were assumed.

Most experiments were not performed with shapes that can be considered one-dimensional. In these cases the flux was assumed to be separable within the core. Thus for a finite cylinder a calculation was made in the radial direction by TGAN, and the critical transverse (axial) buckling (and hence the radial extrapolation distance) was found corresponding to the experimental diameter. A calculation in the axial direction then gave the critical radial buckling (and hence the axial extrapolation distance) corresponding to the critical height. These extrapolation distances together with the experimentally determined critical dimensions permitted a geometric buckling (B_g^2) to be calculated and hence a k_{eff}, calculated as $k_{eff} = \frac{1 + M^2 B_m^2}{1 + M^2 B_g^2}$, correlating this buckling with the material buckling (B_m^2) calculated by MGBS. For exponential experiments in which the axial buckling was measured by a flux traverse, the radial buckling was calculated in the foregoing manner, and the geometric buckling was taken to be the sum of the calculated radial buckling and the experimentally determined axial buckling.

^{235}U

Extensive correlations of experimental data for aqueous solutions of highly enriched uranium have been made with a very simple method of calculation in terms of an effective extrapolation distance[27]. In a recent study of the effect of nonuniform distributions[24], critical radii of water-reflected spheres of solutions of UO_2F_2 were derived from this correlation and were corrected to critical radii of spheres of homogeneous dispersions of pure ^{235}U, with the density of the metal, in water. These radii have been correlated with the present methods of calculation in terms of k_{eff}, with the results given in Table III.

TABLE III

k_{eff} Calculated for Critical Water-Reflected Spheres of Homogeneous Mixtures of H_2O and ^{235}U

$H/^{235}U$	^{235}U, g/l	Critical Radius, cm	k_{eff}
1735	15	48.87	1.0000
1301	20	28.99	0.9999
1041	25	23.09	1.0018
867	30	20.19	1.0056
743	35	18.48	1.0108
650	40	17.30	1.0151
578	45	16.44	1.0187
520	50	15.77	1.0216
433	60	14.80	1.0261
371	70	14.13	1.0293
324	80	13.63	1.0313
259	100	12.93	1.0342
216	120	12.46	1.0354
143	180	11.71	1.0388
98.9	260	11.27	1.0423
63.7	400	10.97	1.0456

Critical experiments have been performed[28] showing the effect of lateral reflectors of various thicknesses of stainless steel on the critical heights of cylinders of solutions of uranyl nitrate where the uranium contained 93.3% ^{235}U. The comparisons of calculation and experiment in terms of k_{eff} calculated for the experiments are presented in Table IV.

A survey of data[29,30] obtained with aqueous solutions of uranium enriched to about 30% ^{235}U has been made. Some data[31] for an enrichment of 44.6% were included. In all cases the solutions were of UO_2F_2, which was taken to be UO_4 in the calculations because MGBS does not have cross sections for F. The reported solution concentrations were used in the calculations, but the apparent densities of UO_2F_2 were obtained from data at 25°C analyzed by Johnson and Kraus[32] rather than from the reported ratios of $H/^{235}U$. Ratios calculated by MGBS from these densities (adjusted to UO_4) differed only slightly from those reported. Most of the experiments were performed in stainless steel cylinders with and without thick radial water reflectors. For the experiments at 30.3% enrichment the cylinder walls were all 0.064 inch thick. At the 44.6% enrichment the wall thickness was stated[31]

TABLE IV

k_{eff} Calculated for Experimentally Critical Cylinders of Solutions of $UO_2(NO_3)_2$ in Water

Uranium = 93.3% ^{235}U, 1.1% ^{234}U, 5.6% ^{238}U

Water-Reflected Cylinders

U Conc, g/l	NO_3^- Conc, g/l	H/^{235}U[a] Calc	H/^{235}U[a] Reported	k_{eff} for Cylinder with Diameter of 8 in.	9 in.	10 in.	12 in.	15 in.
384.8	271	62.4	61.8	1.0440	1.0462	1.0494		1.0267
111.7	78.7	240.1	240	1.0350	1.0343	1.0325	1.0296	1.0339
77.5	54.6	350.4	352		1.0296	1.0309	1.0357	1.0281
56.6	39.9	483.5	493			1.0215	1.0272	1.0250
35.3	24.9	781.2	733				0.9909	0.9937

Bare Cylinders

U Conc, g/l	k_{eff} for Cylinder with Diameter of 10 in.	12 in.	15 in.
111.7	1.0348	1.0288	1.0197
77.5		1.0258	1.0197
56.6		1.0075	1.0108
35.3			0.9831

Lateral Stainless Steel Reflector[b]

Thickness, in.	k_{eff} 384.8 g/l 8 in.	384.8 g/l 10 in.	10 in. 111.7 g/l	10 in. 77.5 g/l	10 in. 56.6 g/l
0			1.0348		
0.25			1.0321		
0.50			1.0317	1.0240	
0.75				1.0252	
1.00		1.0426	1.0310		
1.50	1.0497			1.0252	
2.00	1.0495				
2.50		1.0035[c]	1.0272	1.0286	1.0302
3.00	1.0461				
3.50	1.0435				

TABLE IV (Continued)

Lateral Water Reflector

Thickness, in.	k_{eff}				
	10 in.			12 in.	
	384.8 g/l	111.7 g/l	77.5 g/l	77.9 g/l	56.6 g/l
0		1.0348		1.0258	1.0075
0.88	1.0368	1.0278	1.0198	1.0229	1.0102
1.75	1.0325	1.0248	1.0212	1.0197	
2.63	1.0311	1.0247			
3.50	1.0307	1.0267	1.0250		
4.50				1.0193	1.0153

Lateral Stainless Steel Reflector Surrounded by Large Thickness of Water

Thickness, in.	k_{eff}				
	384.8 g/l		10 in.		
	8 in.	10 in.	111.7 g/l	77.5 g/l	56.6 g/l
0	1.0440	1.0462	1.0325	1.0309	1.0215
0.12	1.0392				
0.25	1.0366		1.0293		
0.50	1.0331	1.0398	1.0277	1.0211	
0.75	1.0334			1.0229	
1.00	1.0376	1.0406	1.0312		
1.50	1.0396			1.0302	
2.00	1.0438				
2.50		1.0477	1.0372	1.0366	1.0343
3.50	1.0487				

(a) Concentrations were calculated from reported densities. Calculated $H/^{235}U$ ratios were obtained from these concentrations.
(b) Type 347, 7.9 g/cm^3. Composition assumed to be 18% Cr, 11% Ni, 0.006% B, 70.994% Fe. Fictitious B is to simulate 1/v absorbers other than Fe, Cr, and Ni.
(c) This point appears in a table in Reference 33 but is ignored in graphs; it was probably considered in error by the experimenters.

to be 1/8 inch, but the same tank was stated[30] to have been used for both enrichments; hence the wall thickness was taken to be 0.064 inch. In no cases were there top or bottom reflectors. The bottoms were somewhat complex and assumptions had to be made as to their effective thickness; the thicknesses assumed for the 16, 12, and 8-inch diameter tanks were, respectively, 1.7, 1.0, and 2.5 cm. The sphere experiments were performed in vessels with 1/8-inch-thick aluminum walls[33]. Values of k_{eff} calculated for these experiments are listed in Table V.

Aqueous solutions and homogeneous mixtures of uranium with lower enrichments have been included in a recent study of both heterogeneous and homogeneous systems of low enrichment. Average values of k_{eff} correlating the calculations and the experiments within, for the most part, ±0.01 are given in Appendix A of Reference 1. Minor errors in Reference 1 are:

1. In Appendix A the expression for k_{eff} for oxide rod and solution data should have been stated to be valid for the range $1.5 < Z < 5.0$. For $Z < 1.5$, the expression for metal rods is valid for oxide rods and solutions.

2. The heading on p 29 of Appendix B should have stated that the oxide has a density of 10.9 g/cm^3.

3. The enrichment of the uranium in Table III (according to a private communication from E. B. Johnson) is 4.98% for the 890 g/l solution; the others are all 4.89%, as given. (In Reference 7 of Reference 1 Volume 7 Number 2 of the Transactions should be cited rather than Number 1.) Taking the correct enrichment into account reduces the first four deviations from the average k_{eff} from 0.79, 0.64, 0.64, 0.52 to 0.27, 0.12, 0.14, 0.00 respectively. This same error exists in Table III of Reference 2.

4. In Table II the headings for the second and third columns should be: V_{H_2O}/V_U[c] and Lattice Type and Dim.[d], cm.

The foregoing study considered only metal and oxide rods and homogeneous mixtures or solutions. At the time the study was made, however, correlations were also made with data for tubular metal and oxide fuel, with data obtained at SRL with 2 and 3-inch diameter metal rods enriched to 3.0% ^{235}U, and with some data for ThO_2 rods enriched with ^{235}U. These correlations were not included in the reports of the study. Some data have appeared since the reports were written (including data for three lattices of oxide

TABLE V

k_{eff} Calculated by MGBS and TGAN for Solutions of Moderately Enriched Uranium as UO_2F_2

Cylinders: 0.35% ^{234}U, 30.30% ^{235}U, 0.07% ^{236}U, 69.28% ^{238}U

Diameter, in.	$H/^{235}U$	k_{eff} Lateral Reflector	k_{eff} Bare
16	75.6	1.0188	1.0186
	108.7	1.0243	1.0248
	170.3	1.0270	1.0289
	265.3	1.0253	1.0221
	434.0	1.0176	1.0164
	650.3	1.0124	1.0107
	807.9	1.0067	1.0031
	933.9	1.0042	0.9981
12	81.2	1.0205	
	81.7	1.0202	1.0245
	105.2	1.0227	1.0279
	133.9	1.0248	1.0299
	165.3	1.0254	1.0305
	254.1	1.0244	1.0289
	374.0	1.0206	1.0239
	436.4	1.0191	1.0210
	526.1	1.0162	
	616.5	1.0153	
	645.2	1.0146	
8	81.2	1.0307	
	92.8	1.0321	
	103.7	1.0332	
	113.5	1.0337	
	139.2	1.0346	
	158.5	1.0349	
	193.5	1.0350	

Spheres: 30.45% ^{235}U (assumed: 0.35% ^{234}U, 69.2% ^{238}U)

Diameter, in.	Water Reflected $H/^{235}U$	Water Reflected k_{eff}	Bare $H/^{235}U$	Bare k_{eff}
12.01	352.3	1.0276		
13.74	574.4	1.0122	76.7	1.0297
			218.3	1.0360
15.96	784.2	1.0070	534.4	1.0144
22.00	1196.0	0.9975	1040.3	0.9997

Cylinders: 44.6% ^{235}U (assumed: 0.5% ^{234}U, 54.9% ^{238}U)

Diameter, in.	$H/^{235}U$	k_{eff} Lateral Reflector	k_{eff} Bare
12	259.4	1.0264	0.9936
	494.8	1.0231	
	679.0	1.0209	

rods enriched to 5.74% ^{235}U), and correlations have been made with these data. These various correlations are presented for metal tubes and rods in Table VI, for uranium oxide tubes and rods in Table VII, and for thorium oxide rods in Table VIII.

The form of the tables is identical with that used previously, except that k_{eff} is tabulated rather than the deviation of k_{eff} from an average value (note that for the three oxide rod lattices at the 5.74% enrichment the deviations from the appropriate average k_{eff} in Appendix A of Reference 1 are -0.0087, -0.0100, and -0.0168 for the order in which the data are given in Table VII), that references are listed for the data rather than just the laboratory where the experiments were performed, and that the atomic ratio of H/Th is tabulated rather than ratio of the volumes of water and thorium. The cell radii and the fuel and cladding compositions are those actually used in MGBS. In some cases a certain amount of guesswork was involved, because references are not sufficiently explicit. The ratios of the volumes of water and uranium are those calculated by MGBS and may occasionally differ slightly from the ratios reported in the references. In no case did stainless steel cladding contain boron, but boron was assumed to be present to mock up strong absorbers, chiefly manganese. For ThO_2 it was necessary to assume the uranium to be 100% ^{235}U rather than 93% because of the way MGBS treats resonance absorption in rods.

The SRP tubular fuel elements studied at SRL were slugs 8 inches long with 0.2-inch end caps on each end. The axial bucklings given in Table VI have been corrected for the end caps by adding to the experimental axial bucklings the material bucklings calculated by MGBS for lattices of tubes without end caps and then subtracting the radial bucklings calculated by TGAN for an infinite series of 8-inch slabs of fuel tubes alternating with 0.4-inch-thick slabs having properties calculated by MGBS, with zero input buckling and no iteration, for lattices of aluminum tubes (i.e., end caps). The correction made the axial buckling more negative at small pitches and more positive at large pitches and had a maximum absolute value of 3.75 m^{-2}.

Little effort has been made to fit the correlations to polynomials in the ratio of water to uranium, but there appears to be the same general trend as was observed for rods[1]. The values of k_{eff} for tubes tend to be somewhat larger than for rods with diameters such that the ratios of surface to volume are the same. For the SRP tubular elements the difference averages about 0.018.

Many fuel elements are fabricated from an alloy of highly enriched uranium and aluminum. These include both MTR-type elements and SRP tubular elements, and critical and exponential experiments have been performed with both types.

TABLE VI

Lattices of Uranium Metal Tubes and Rods in Water

Materials[a]	Cell Radii[b], cm				V_{H_2O}/V_U	Lattice Type and Dimensions[c], cm			B_H^2, m^{-2} or (H, cm)	k_{eff}	Reference
WAW	0.616	1.734	2.374		1.15	T	115.32		(121.92)	0.9610	34
							123.20		(106.68)	0.9613	
			2.667		1.71		87.38		(121.92)	0.9745	
							89.92		(106.68)	0.9736	
							95.50		(91.44)	0.9737	
							105.16		(76.20)	0.9730	
WPWQ-BQW	1.281	1.429	1.481	1.557	0.98	S	40.11	40.11	-58.52	0.9592	35
		2.431	2.527	2.829							
				3.008	1.28		42.65	42.65	-45.01	0.9632	
				3.187	1.60		45.19	45.19	-38.73	0.9822	
				3.366	1.93		47.73	47.73	-31.06	0.9827	
				3.545	2.29		50.27	50.27	-30.48	0.9926	
				3.725	2.66		52.81	52.81	-32.58	0.9999	
				3.904	3.06		55.35	55.35	-33.83	0.9963	
				4.262	3.89		60.43	60.43	-47.48	1.0057	
WPWQ-BQW-QBQ-W	1.281	1.429	1.481	1.557	1.06	S	62.50	62.50	-27.52	0.9634	35
		2.431	2.527	3.048							
		3.124	3.831	3.907							
		4.408									
		4.587			1.25		56.91	65.04	-24.46	0.9711	
							65.04	65.04	-19.61	0.9679	
		4.766			1.45		59.14	67.58	-20.01	0.9796	
							67.58	67.58	-16.35	0.9796	
		4.945			1.65		70.12	70.12	-14.95	0.9886	
		5.125			1.87		72.66	72.66	-14.24	0.9923	
		5.304			2.09		75.20	75.20	-16.72	0.9969	
WPVC-VPW	2.205	2.256	2.278	2.968	1.62	T	38.94		-54.55	0.9517	36
		2.991	3.042	3.201			47.48		-36.29	0.9596	
				3.734	2.64		45.42		-40.01	0.9857	
							55.38		-22.81	0.9874	
				4.134	3.51		50.30		-39.51	0.9965	
							59.04		-27.61	0.9997	
				4.534	4.47		55.16		-48.23	1.0038	
WPVC-VPW-PVC-VPW	0.554	0.605	0.627	1.500	0.67	T	47.48		-61.81	0.9054	36
		1.523	1.574	2.205							
		2.256	2.278	2.968							
		2.991	3.042	3.201							
				3.734	1.35		45.42		-41.62	0.9613	
							55.38		-25.40	0.9633	
				4.134	1.92		50.30		-31.18	0.9788	
							59.04		-19.27	0.9813	
				4.534	2.56		55.16		-34.32	0.9878	
WPVD-VPW	1.095	1.176	1.194	2.108	1.28	T	50.40		-32.25	0.9654	37
		2.126	2.197	2.739							
				2.940	1.66		54.10		-19.70	0.9743	
				3.274	2.35		60.24		-14.66	0.9883	
				3.608	3.11		66.40		-21.06	0.9952	
VPVD-VPW	1.095	1.176	1.194	2.108	0.89	T	50.40		-37.36	0.9336	37
		2.126	2.197	2.739							
				2.940	1.26		54.10		-24.19	0.9575	
				3.274	1.95		60.24		-16.36	0.9804	
				3.608	2.71		66.40		-19.97	0.9889	

(TABLE CONTINUED)

TABLE VI (Continued)

Materials[a]	Cell Radii[b], cm				V_{H_2O}/V_U	Lattice Type and Dimensions[c], cm		B_H^2, m^{-2} or (H, cm)	k_{eff}	Reference
WPVE-VPW	0.574	0.625	0.635	1.697	1.31	T	36.60	-39.51	0.9581	37
		1.709	1.781	2.467				-39.56	0.9583	
				2.667	1.73		39.56	-26.96	0.9732	
				2.801	2.02		41.54	-22.83	0.9803	
								-23.43	0.9818	
				2.934	2.33		43.52	-22.57	0.9886	
								-21.37	0.9854	
				3.201	2.99		47.48	-25.71	0.9940	
								-24.75	0.9914	
VEVP-W	0.635	1.697	1.709	1.781	1.18	T	36.60	-40.57	0.9486	37
		2.467								
		2.667			1.59		39.56	-28.72	0.9693	
		2.801			1.89		41.54	-24.41	0.9777	
		2.934			2.20		43.52	-22.74	0.9840	
		3.201			2.86		47.48	-25.20	0.9899	
WRER-W	2.240	2.289	3.007	3.056	1.57	T	39.04	-31.04	0.9485	36
		3.209								
		3.734			2.53		32.56	-53.59	0.9785	
							45.42	-29.22	1.0252	
		4.134			3.36		36.04	-50.66	0.9991	
							50.30	-16.06	1.0085	
		4.534			4.27		39.52	-57.05	1.0122	
							55.16	-25.69	1.0198	
WFWP-W	0.610	1.740	1.781	1.905	1.51	T	37.34	-24.31	0.9689	37
		2.667								
		2.801			1.78		39.20	-19.37	0.9820	
		2.934			2.07		39.36	-20.33	0.9873	
		3.201			2.69		44.80	-15.39	0.9961	
		3.467			3.36		48.54	-23.63	1.0034	
VFVP-W	0.610	1.740	1.781	1.905	1.31	T	37.34	-31.70	0.9681	37
		2.667								
		2.801			1.59		39.20	-22.48	0.9746	
		2.934			1.88		39.36	-23.14	0.9839	
		3.201			2.49		41.98	-23.09	0.9933	
		3.467			3.16		48.54	-22.48	1.0012	
WGWP-W	0.589	1.770	1.781	1.905	1.39	T	29.70	-56.79	0.9669	37
		2.667					34.98	-33.11	0.9669	
							37.34	-24.29	0.9661	
							39.56	-17.88	0.9678	
		2.801			1.65		41.54	- 9.69	0.9757	
		2.934			1.92		32.68	-42.61	0.9830	
							38.48	-20.02	0.9831	
							43.52	- 4.86	0.9816	
		3.201			2.51		41.98	-17.91	0.9921	
							44.80	-11.65	0.9948	
							47.48	- 3.48	0.9895	
		3.467			3.15		38.62	-42.61	0.9964	
							45.48	-23.89	0.9979	
							51.42	-10.20	0.9936	

TABLE VI (Continued)

Materials[a]	Cell Radii[b], cm				V_{H_2O}/V_U	Lattice Type and Dimensions[c], cm			B_H^2, m^{-2} or (H, cm)	k_{eff}	Reference
WHVP-W	0.589	1.770	1.781	1.905	1.38	T	34.98		-26.33	0.9663	37
		2.667									
		2.801			1.64		36.72		-18.08	0.9767	
		2.934			1.91		38.48		-13.11	0.9849	
							45.10		(101.60)	0.9815	
							45.48		(101.60)	0.9794	
		3.201			2.50		41.98		- 9.95	0.9932	
							48.50		(101.60)	0.9880	
							48.80		(101.60)	0.9894	
		3.467			3.14		45.48		-15.56	0.9990	
VHVP-W	0.589	1.770	1.781	1.905	1.25	T	34.98		-31.18	0.9635	37
		2.667									
		2.801			1.51		36.72		-21.98	0.9734	
		2.934			1.79		32.68		-38.06	0.9812	
							38.48		-16.54	0.9825	
							43.52		- 2.80	0.9837	
							47.02		(101.60)	0.9847	
							48.42		(101.60)	0.9772	
		3.201			2.37		41.98		-12.83	0.9923	
							50.52		(101.60)	0.9885	
							51.56		(101.60)	0.9935	
		3.467			3.01		45.48		-17.36	0.9977	
WRIR-W	1.720	1.796	2.889	2.966	1.58	T	41.56		(69.34)	0.9765	36
		3.734									
		4.134			2.20		42.54		(69.34)	0.9974	
		4.534			2.87		47.88		(69.34)	1.0038	
JW	2.540	2.866			0.27	S	20.32	20.32	-97.64	0.9065	38
							20.32	25.40	-88.22	0.9155	
							20.32	30.48	-71.15	0.8946	
							20.32	35.56	-68.73	0.9068	
							20.32	40.64	-63.18	0.9063	
		3.224			0.61		28.57	28.57	-18.42	0.9239	
							28.57	34.29	-10.71	0.9318	
							28.57	40.00	- 4.40	0.9358	
		3.583			0.99		25.40	25.40	-26.76	0.9650	
							25.40	31.75	- 7.99	0.9622	
							25.40	38.10	- 3.32	0.9767	
		3.941			1.41		20.95	41.91	-12.50	0.9801	
							20.95	48.89	- 5.01	0.9800	
							20.95	55.88	- 0.69	0.9739	
							20.95	62.86	2.64	0.9749	
							27.94	27.94	-11.11	0.9769	
							27.94	34.92	6.43	0.9754	
		4.299			1.87		22.86	30.48	-38.05	0.9862	
							30.48	30.48	- 9.88	0.9821	
		4.657			2.36		24.76	33.02	-43.23	0.9824	
							33.02	33.02	-18.71	0.9823	
		5.016			2.90		26.67	35.56	-54.77	0.9769	
							35.56	35.56	-29.72	0.9704	
		5.374			3.48		28.57	38.10	-70.42	1.0146	

(TABLE CONTINUED)

TABLE VI (Continued)

Materials[a]	Cell Radii[b], cm	V_{H_2O}/V_U	Lattice Type and Dimensions[c], cm	B_H^2, m^{-2} or (H, cm)	k_{eff}	Reference
JW	3.810 4.299	0.27	S 15.24 22.86	-117.66	0.9376	38
			22.86 22.86	-82.47	0.9122	
			22.86 45.72	-50.26	0.9143	
			30.48 30.48	-43.56	0.8931	
			30.48 38.10	-47.11	0.9302	
	4.657	0.49	33.02 33.02	-16.71	0.9288	
			33.02 41.27	- 4.61	0.9281	
	5.016	0.73	35.56 35.56	- 3.47	0.9585	
	5.374	0.99	28.57 28.57	-28.52	0.9761	
			28.57 38.10	- 6.25	0.9718	
	5.732	1.26	30.48 40.64	- 7.68	0.9830	
			30.48 50.80	4.15	0.9809	
	6.449	1.87	45.72 45.72	- 7.20	0.9769	
			45.72 57.15	2.34	0.9764	

(a) Materials used in the calculations are designated as follows:
A - J are uranium; P - R are cladding; V is void; and W is water. The various weight % ^{235}U and metal densities are:
A - 0.94 , 18.78; B - 0.95, 18.9;
C - 1.002, 18.9; D - 1.007, 18.9;
E - 1.25, 18.9; F - 1.44, 18.9;
G - 1.466, 18.9; H - 1.60, 18.9;
I - 1.95, 18.9; J - 3.00, 18.9.
The various cladding materials, percentage compositions, and densities are:
P - Al, 100 Al, 2.7; Q - Al, 96.78 Al, 3.22 Ni, 2.76;
R - Zr, 100 Zr, 6.4.

(b) Radii are outer radii of regions, ending with equivalent cylindrical cell radius.

(c) Lattice type is either square (S) or triangular (T). Dimensions are length and width of a rectangular array if two are given or equivalent diameter of a circular array if only one is given.

TABLE VII

Lattices of Uranium Oxide Tubes and Rods in Water

Materials[a]	Cell Radii[b], cm				V_{H_2O}/V_U[c]	Lattice Type and Dimensions[d], cm			B_H^2, m^{-2} or (H, cm)	k_{eff}	Reference
WQAQ- WQA- QW	0.301	0.342 1.184 1.905	0.907 1.270 2.501	0.953 1.824	3.08	T	69.48 73.66 78.60 83.32		(84.73) (73.41) (63.80) (59.64)	0.9928 0.9939 0.9931 0.9937	39
WQAQ- WQA- QW	0.301	0.342 1.184 1.905	0.907 1.270 2.790	0.953 1.824	4.54	T	65.78 68.56 71.24 72.54 72.54 76.30		(94.23) (81.05) (73.46) (71.09) (71.40) (63.03)	1.0046 1.0040 1.0041 1.0047 1.0051 1.0027	39
VQAQ- VQA- QW	0.301	0.342 1.184 1.905	0.907 1.270 2.501	0.953 1.824	2.52	T	82.32		(89.18)	0.9960	39
VQAQ- VQA- QW	0.301	0.342 1.184 1.905	0.907 1.270 2.790	0.953 1.824	3.98	T	71.24 72.54 72.54 76.30		(99.59) (94.87) (94.59) (80.90)	1.0093 1.0102 1.0100 1.0092	39
WQAQ- WQA- QWQ- W	0.301	0.342 1.184 1.905 2.790	0.907 1.270 1.956	0.953 1.824 2.007	4.35	T	85.54 91.86 96.80 101.52		(103.48) (85.14) (77.50) (71.55)	1.0040 1.0030 1.0029 1.0020	39
VQAQ- VQA- QWA- W	0.301	0.342 1.184 1.905 2.790	0.907 1.270 1.956	0.953 1.824 2.007	3.79	T	101.52		(89.13)	1.0082	39
WBW	0.410	0.896	1.334 1.600 1.867		3.71 6.25 9.25	T	20.84 25.00 29.16		-80.19 -50.72 -43.20	1.0115 1.0154 1.0159	40
VBW	0.410	0.896	1.334 1.600 1.867		3.17 5.71 8.71	T	20.84 25.00 29.16		-79.89 -52.82 -44.00	0.9660 0.9945 1.0036	40
VRCR- VRW	0.723	0.794 1.842	1.516 1.905	1.588 2.580	3.98	S	59.44	60.12	(121.92)	1.0144	41
VRCR- PRW	0.723	0.794 1.842	1.516 1.905	1.588 2.580	3.98	S	59.44	60.24	(121.92)	1.0136	41
WRCR- PRW	0.723	0.794 1.842	1.516 1.905	1.588 2.580	4.67	S	59.44	46.40	(121.92)	1.0113	41
WRCR- VRW	0.723	0.794 1.842	1.516 1.905	1.588 2.580	4.67	S	59.44	46.08	(121.92)	1.0123	41
VRCR- WRW	0.723	0.794 1.842	1.516 1.905	1.588 2.580	5.13	S	54.86	54.86	(121.92)	1.0183	41
WRCR- WRW	0.723	0.794 1.842	1.516 1.905	1.588 2.580	5.82	S	59.44	41.52	(121.92)	1.0152	41
VRCR- PRW	0.723	0.794 1.842	1.516 1.905	1.588 2.723	4.98	S	67.56 67.56	53.42 53.76	(121.92) (121.92)	1.0201 1.0210	41

(TABLE CONTINUED)

TABLE VII (Continued)

Materials[a]	Cell Radii[b], cm				V_{H_2O}/V_U [c]	Lattice Type and Dimensions[d], cm			B_H^2, m^{-2} or (H, cm)	k_{eff}	Reference
WRCR-VRW	0.723	0.794	1.516	1.588	5.67	S	48.26	57.92	(121.92)	1.0188	41
		1.842	1.905	2.723							
VRCR-WRW	0.723	0.794	1.516	1.588	6.13	S	59.12	57.92	(121.92)	1.0220	41
		1.842	1.905	2.723							
WRCR-WRW	0.723	0.794	0.516	1.588	6.82	S	48.26	57.92	(121.92)	1.0191	41
		1.842	1.905	2.723							
VRCR-PRW	0.723	0.794	1.516	1.588	6.04	S	60.96	60.96	(121.92)	1.0208	41
		1.842	1.905	2.866							
WRCR-WRW	0.723	0.794	1.516	1.588	7.87	S	57.16	60.96	(121.92)	1.0227	41
		1.842	1.905	2.866			50.08	71.12	(121.92)	1.0208	
DVSW	0.453	0.458	0.497	0.745	3.16	S	31.51		(92.96)	0.9916	42
				0.803	4.07		29.55		(92.96)	1.0002	
				1.135	10.66		29.86		(92.96)	1.0259	

(a) Materials used in the calculations are designated as follows:
A - D are uranium oxide; P - S are cladding; V is void; and W is water. The various weight % ^{235}U, wt % oxygen, and oxide densities are:
A - 2.55, 12.00, 9.265; B - 2.60, 11.85, 10.43;
C - 3.39, 11.86, 9.754; D - 5.742, 11.87, 10.193.
The various cladding materials, percentage compositions, and densities are:
P - Al 100 Al, 2.7; Q - SS 71.747 Fe, 18.5 Cr, 9.75 Ni, 0.003 B, 7.9; R - SS 70.9 Fe, 18.5 Cr, 10.6 Ni, 0.002 B, 7.9;
S - SS 72.096 Fe, 18.4 Cr, 9.5 Ni, 0.004 B.

(b) See note (b) of Table VI.

(c) V_{H_2O}/V_U in the ratio of the volume of water to the volume of uranium having a density of 18.9 g/cm^3.

(d) See note (c) of Table VI.

TABLE VIII

Lattices of Thorium Oxide Rods in Water

Materials[a]	Cell Radii[b], cm				H/Th	Lattice Type and Dimensions[c], cm		B_H^2, m^{-2} or (Ht, cm)	k_{eff}	Reference
AVQW	0.330	0.348	0.396	0.545	4.70	S	88.12	(121.92)	0.9730	43
				0.577	5.89		79.72	(121.92)	0.9843	
BVQW	0.330	0.348	0.396	0.545	4.81	S	48.04	(121.92)	0.9855	43
				0.577	6.04		43.98	(121.92)	0.9934	
				0.629	8.20		40.06	(121.92)	0.9999	
				0.771	15.01		38.36	(121.92)	1.0139	
				1.091	35.42		55.86	(121.92)	1.0316	
CVPW	0.297	0.306	0.392	0.689	13.07	S	47.28	(140.50)	1.0044	44
				0.816	20.83		55.24	(135.27)	1.0146	
BVQW	0.330	0.357	0.392	0.552	5.16	S	36.66	(127.21)	0.9837	43
				0.577	6.15		34.24	(133.44)	0.9866	
				0.689	11.02		31.26	(135.05)	1.0119	

(a) Materials used in calculations are designated as follows:
A - C are thorium oxide enriched with ^{235}U; P - Q are cladding;
V is void; and W is water.
The various wt % ^{235}U, wt % oxygen, and oxide densities are:
A - 3.31, 12.32, 8.35; B - 5.53, 12.16, 8.35; C - 3.37, 11.96, 8.45.
The various cladding materials, percentage compositions, and densities are:
P - Al, 100 Al, 2.7; Q - SS, 71.5 Fe, 19 Cr, 9.5 Ni, 0.004 B, 7.9.

(b) See (b) of Table VI.

(c) See (c) of Table VI.

Critical experiments have been performed at ORNL[45] with ORR elements, which are of the MTR type. Three concentrations of uranium were used in the alloy giving masses of ^{235}U per element of 140, 168, and 200 g. The enrichment of the uranium was 93.2%. The equivalent cell radius was calculated to be 4.548 cm, and within this cell the respective concentrations of uranium were calculated to be 38.53, 46.26, and 55.08 g/l within the active length. The aluminum concentrations were calculated to be respectively 1039, 1038, and 1036 g/l with effective densities of 2.695, 2.694, and 2.692 g/cm^3. The uranium density was taken to be 18.8 g/cm^3 in all cases. The respective $H/^{235}U$ ratios were 445, 370, and 311. The average uranium concentrations were 3.57, 4.27, and 5.05%.

In calculations of the buckling by the Multigroup Buckling Code for the ORR elements, the uranium and aluminum within the boundaries of the elements were assumed to be homogeneously dispersed. The larger cells resulting from moving the elements apart were then treated as consisting of two regions: the central homogeneous mixture of aluminum, uranium, and water and an outer annulus of water. The interaction of lines of elements in water and the reflector savings of surrounding water reflectors were calculated by the Two-Group Analytical Code. The arrays of ORR elements were all treated as though they were rectangular because this was approximately the experimental arrangement. Results obtained for these elements are given in Table IX. There is a trend toward increased conservatism in the calculations as the spacing between elements increases.

In addition to the experiments with the ORR elements, ORNL has recently performed experiments with a heavier MTR-type fuel designated as SPERT-D[46]. Each element contained 306.46 g ^{235}U, on the average, in 22 fuel plates. The plates were 0.020 inch thick and were clad with 0.020 inch of aluminum. The uranium enrichment was 93.17% ^{235}U.

Critical experiments were performed with several types of arrays. The plates within an element were removable so that criticality could be achieved by adjusting the number of plates within an outer row of elements. In one group of experiments nearly square arrays were made critical at various spacings between elements. (In one experiment an approximately cylindrical array was formed.) In another group of experiments the nearly square arrays were three elements long rather than one element long as in the first group. Lattices of slabs formed by placing several elements in a row were made critical at various spacings depending on the number of rows. In two cases cadmium sheets were inserted between rows. A lattice of elements was immersed in a dilute solution of $UO_2(NO_3)_2$ containing 3.99 g $^{235}U/l$ and various con-

TABLE IX

Comparison of Calculations with Critical Experiments with ORR Fuel

Fuel Elements	Spacing, in.	Approximate Critical Number		k_{eff}
140 g	0	18.75		1.018
168 g	0	15.5		1.024
	1/8	15		1.027
	1/4	15		1.029
	3/8	15.5		1.033
	1/2	16.5		1.035
	3/4	20.5		1.036
200 g	0	13		1.023
140 and 168 g	1	28	168-g elements	1.035
		3	140-g elements on outside edge	
140, 168, and 200 g	1-1/4	6	200-g elements in center	1.024
		28	168-g elements	
		27	140-g elements on outside edge	

centrations of boron, and the critical number of elements was determined as a function of the boron concentration. The uranium in solution contained 92.6% ^{235}U.

Calculations were made for all these arrays. Two approaches were followed with MGBS. In the first, the entire element was homogenized as was done for the ORR elements. In the second, the element was treated as though it consisted of two homogeneous regions: a central 2.454 in. x 2.703 in. U-Al-H_2O region and an outer Al-H_2O annulus. The concentrations and densities within the regions were:

- R = 4.2934 cm
  93.16 g U/l (1% ^{234}U, 93.17% ^{235}U, 5.83% ^{238}U); 13.71 g/cm^3
  1429 g Al/l; 2.7 g/cm^3

- R_1 = 3.6907 cm
  126.08 g U/l; 13.71 g/cm^3
  1294 g Al/l; 2.7 g/cm^3

  R_2 = 4.2934 cm
  1811 g Al/l

Elements separated by various distances (measured in the experiments from the outer surfaces) were treated as having the central homogeneous region or regions surrounded by an annulus of H_2O. (In the solution experiments, solution replaced the H_2O.) The solution experiments were performed in a 37.75-inch ID tank. The thicknesses of the top and bottom reflectors were respectively 5 and 4 inches. The thicknesses of the side reflectors decreased as the size of the lattice increased with boron concentration. The calculations took account of these finite thicknesses. When three elements were placed end to end, a 1-1/8 in. thick Al-H_2O region separated fuel-containing regions in adjacent elements. Two-group parameters for this thin (homogenized) region were provided by the buckling code. Results of the calculations are presented in Tables X-XIII. The trends are similar to those observed with ORR fuel[45]. The method of treating the fuel region makes quite a difference. Better results are clearly obtained when it is subdivided into two regions.

TABLE X

Approximately Square Arrays of SPERT-D Elements in Water

Separation, in.	L, in.	W, in.	Critical Number	k_{eff} No. of Regions 1	k_{eff} No. of Regions 2
0.00	11.98	11.29	15.09	1.0236	1.0183
0.25	12.98	10.26	12.63	1.0435	1.0255
0.50	13.98	10.80	12.36	1.0648	1.0341
0.50	cyl,D =	13.29	11.35	1.0538	1.0242
0.75	14.98	11.84	12.63	1.0673	1.0266
1.00	15.98	14.79	14.82	1.0745	1.0243
1.25	21.23	17.07	20.11	1.0762	1.0183
1.50	26.98	24.00	32.04	1.0710	1.0071
1.60	32.17	30.70	46.77	1.0749	1.0088

TABLE XI

Approximately Square Arrays of SPERT-D Elements, Three High, in Water

Separation, in.	L, in.	W, in.	Critical Number(a)	k_{eff} No. of Regions 1	k_{eff} No. of Regions 2
0.00	11.98	9.59	12.81	1.0307	1.0250
0.50	11.75	10.49	10.09	1.0603	1.0298
1.25	16.98	16.98	16.00	1.0793	1.0211

(a) Refers to a single tier.

TABLE XII

Arrays of Slabs of SPERT-D Elements in Water

Surface Separation, in.		No. of	Row Length,	k_{eff} No. of Regions	
Within Row	Between Rows	Rows	in.(a)	1	2
0.00	0.00	2.32	47.94	1.0387	1.0330
0.50	2.19	3	55.94	1.1320	1.0923
0.50	2.56	4	55.94	1.1387	1.0988
0.50	6.37	2	55.94	1.0809	1.0501
0.00	0.75	4(b)	15.10	1.0511	1.0431
0.00	0.75	4(c)	21.09	1.0571	1.0490

(a) Length includes thickness of water associated with each element.
(b) One 0.025-inch sheet of cadmium between rows 2 and 3.
(c) One sheet of 0.025-inch cadmium between rows 1 and 2 and another between 3 and 4.

TABLE XIII

Approximately Square Arrays of SPERT-D Elements at 0.5-inch Spacing in Solution Containing 3.99 g $^{235}U/l$

Boron, g/l	L, in.	W, in.	Critical Number	k_{eff} No. of Regions 1	2
0	10.49	10.80	9.33	1.0476	1.0224
0	cyl,D =	11.84	9.01	1.0501	1.0246
0.389	13.98	14.68	16.80	1.0519	1.0205
0.579	17.48	15.42	22.06	1.0565	1.0229
0.773	20.98	17.34	29.04	1.0638	1.0279
0.871	20.98	19.40	33.12	1.0626	1.0255
1.118	cyl,D =	26.28	44.38	1.0614	1.0225

Exponential experiments have been performed[47] with SRP tubular uranium-aluminum alloy fuel elements in H_2O. These experiments together with some more recent ones have been summarized by F. E. Kinard[48], and the elements are described in his Table I. The designations for these elements that will be used here are Mark VI, VIA, VIJ, VIB inner, and VIB; they refer, respectively, to tubes 1, 4 (a nesting of 2 and 3), 5, 6, and 8 (a nesting of 6 and 7) in this table. Very recently experiments have been completed with fuel elements consisting of nested sets of three fuel tubes, designated as Mark XIIA.[49] Except for the Mark VI tubes, where a triangular pitch was used in some experiments, all pitches were rectangular. The arrays having triangular pitches were approximated by cylinders having cross-sectional areas equal to the products of the areas of the hexagonal cells and the number of cells. The boundaries of the rectangular arrays were taken to coincide with cell boundaries. Where the rectangular cells were not square, square cells having edges equal to the shorter edge of the rectangle were assumed in computing the buckling, and two-group calculations were used to calculate the interaction between lines of such cells through the thickness of water corresponding to the difference between the longer and shorter sides. Correlations of calculation and experiment are given in Table XIV.

Table XV gives correlations with experiments in which sheets of cadmium separated rows of tubular elements in water.

TABLE XV

Rows of Tubular Elements Separated by Cadmium Sheets

Element	No. of Rows	No. per Row	Pitch, cm		k_{eff}	Reference
			Within Row	Between Rows		
Mark VI	6	8	7.54	8.81	1.312	47,50
Mark VIA	6	6	7.54	8.81	1.081	47,50
Mark VIA	3	7	7.54	8.81	1.101	47,50
Mark VIJ	5	10	7.54	12.70	1.225	48,50
Mark VIB	2	6	7.79	9.06	1.068	48
Mark VIB	2	6	7.79	10.33	1.066	48
Mark VIB	2	6	7.79	12.87	1.068	48
Mark VIB	2	7	7.79	25.4	1.122	48
Mark VIB	3	7	7.79	14.14	1.067	48
Mark VIB	3	7	7.79	16.68	1.069	48
Mark XIIA	4	6	9.04	12.70	1.083	49

TABLE XIV

k_{eff} Correlating Calculations and Experiments with U-Al Alloy Tubes

Fuel	Triangular Pitch, cm	keff for an Array of 19	keff for an Array of 31	Square: Pitch Within Row, cm	Square: Pitch of Rows, cm	Square: No. per Row	Square: No. of Rows	keff
Mark VI	7.57	1.034						
	8.81	1.040						
	10.08	1.043	1.047					
	11.35	1.049	1.046					
				7.54	7.54	6	4	1.047
						5	4	1.045
					8.81	5	5	1.028
						4	6	1.027
					10.08	8	4	1.016
						9	4	1.017
					12.62	5	6	1.016
						10	6	1.010
Mark VIA				7.54	7.54	3	3	1.062
					8.81	3	4	1.073
					10.08	4	3	1.072
						3	5	1.075
					12.62	7	3	1.075
						5	4	1.075
					15.16	7	3	1.068
Mark VIJ				7.54	7.54	3	3	1.046
						4	3	1.049
						10	2	1.043
					13.97	10	5	1.022
					15.24	10	5	1.048
					27.86	20(a)	2	1.045
					30.40	20(a)	2	1.045
				8.81	8.81	4	3	1.064
Mark VIB				7.79	7.79	4	2	1.057
						3	2	1.057
					8.42	4	2	1.059
						3	2	1.059
					9.06	3	2	1.055
					10.26	3	2	1.051
					12.87	4	2	1.048
						3	2	1.051
					17.95	5	2	1.065
				8.42	8.42	3	2	1.073
					12.87	4	2	1.082
Mark VIB, inner tubes				6.23	6.55	3	3	1.041
						4	3	1.042
				6.55	8.77	4	3	1.042
					11.31	4	3	1.034
				6.86	6.86	4	3	1.059
Mark XIIA				9.04	9.04	6	2	1.049
						3	3	1.048
					10.16	6	2	1.050
					12.70	6	3	1.048
					15.24	6	4	1.054
				10.16	10.16	6	2	1.053
						3	3	1.048
					15.24	6	4	1.077
				11.43	11.43	5	4	1.044

(a) Double rows, 2 tubes wide, 10 tubes long.

^{239}Pu

Correlations with experiments performed with aqueous solutions of plutonium have been presented previously[26]. These have been repeated because there has been a change in the manner in which MGBS computes the resonance group constants and because previously the data were adjusted empirically to take account of the vessel walls. In the present calculations the vessel walls are included in the two-group calculations. The results are given in Table XVI. The new values of k_{eff} are about 0.005 lower than the previous ones for water-reflected spheres and about 0.007 lower for water-reflected cylinders.

TABLE XVI

Aqueous Solutions of Plutonium

Water-Reflected Spheres

Pu Conc, g/l	^{240}Pu, %	NO_3^-, g/l	Diameter, in.	k_{eff}
26.51	0.54	77.8	14.02	0.9980
26.50		107		0.9925
27.65		137.5		0.9969
28.57		187.5		0.9953
27.94	1.76	109.5	14.02	0.9957
33.62		86.7	13.02	1.0011
35.65		145		1.0018
25.06	3.12	116	15.01	0.9924
25.83		147		0.9940
27.05		212		0.9929
28.78		87.4	14.02	0.9966
30.33		143		0.9978
31.79		208		0.9960
35.68		309.5		0.9998
39.62		408		0.9998
38.05		130	13.02	1.0062
40.90		205		1.0050
44.38		269.5		1.0063
36.27		93.1	12.81(a)	1.0059
37.11		125		1.0043
50.32		138.5	12.10	1.0085
56.48		207		1.0111
63.75		270		1.0133
77.22		359		1.0146
29.88	4.05	87.5	14.02	0.9981
31.72		146.8		0.9999
33.94		210.7		1.0016
36.38		272.4		1.0033
41.12		384.9		1.0035
32.41	4.40	158		0.9996

(a) Aluminum walls, others are stainless steel.

TABLE XVI (Continued)

Bare Spheres

35.51	4.15	106.9	15.84	1.0094
43.20		281.8		1.0078
22.35	4.20	76.1	17.87(b)	1.0001
23.85		148.4		1.0024
25.18		199.2		1.0057
27.48		285.8		1.0090

Water-Reflected Cylinder

Pu Conc, g/l(c)	NO_3^-, g/l	Diameter, in.	Height, in.	k_{eff}
54.53	119.8	9.00	17.55	1.0090
61.49	134		15.78	1.0104
73.92	125.6		13.94	1.0159
85.14	151		12.73	1.0118
99.09	136.5		12.09	1.0173
109.16	166		11.76	1.0141
39.10	138	10.00	16.44	0.9955
49.26	142		12.85	1.0007
62.47	146		10.73	1.0023
77.17	152		9.84	1.0069
33.54	137	11.00	15.57	0.9993
36.90	300		16.93	0.9975
39.10	138		12.28	0.9963
41.73	215		12.85	1.0002
47.21	117		10.66	1.0071
48.75	116.3		10.18	1.0033
48.98	139		10.22	1.0008
63.99	121.1		8.98	1.0119
26.45	134	12.00	17.50	0.9940
31.14	114		13.16	0.9990
36.52	107.1		11.21	1.0062
42.29	126.6		9.94	1.0065
48.75	116.3		8.80	1.0056
109.16	166		6.82	1.0160

(b) Covered with 0.02 inch of cadmium.
(c) ^{240}Pu ranged from 2.83 to 2.90%.

Some experiments[42,51] have been performed with lattices of rods of uranium oxide enriched with plutonium oxide. Correlations with these experiments are given in Table XVII. Presumably caution should be used in extrapolating the results to other concentrations of ^{240}Pu, because MGBS treats ^{240}Pu as though it were homogeneously distributed in computing the resonance group parameters.

In addition to the exponential experiments with U-Al alloy tubular fuel, Kinard[48] performed some experiments with Pu-Al alloy rods clad with aluminum. Correlations with these experiments are given in Table XVIII.

TABLE XVII

Lattices of Rods of UO_2-PuO_2 in Water

Materials[a]	Cell Radii[b], cm				V_{H_2O}/V_U[c]	Lattice Type and Dimensions[d], cm		B_H^2, m^{-2} or (H, cm)	k_{eff}	Reference
APW	0.472	0.541	0.734		2.50	T	56.56	(123.20)	0.9552	51
			0.800		3.55		46.08	(123.20)	0.9629	
			0.947		6.17		41.66	(123.20)	0.9778	
			1.067		8.63		43.72	(123.20)	0.9824	
			1.200		11.72		51.04	(123.20)	0.9875	
			1.240		12.72		54.80	(123.20)	0.9900	
BVPW	0.429	0.438	0.497	0.745	3.74	S	32.38	(92.96)	0.9499	42
				0.803	4.81		29.46	(92.96)	0.9562	
				1.053	10.44		25.89	(92.96)	0.9790	
				1.135	12.61		26.18	(92.96)	0.9843	
				1.490	23.90		32.24	(92.96)	1.0027	

(a) Materials used in calculations are designated as follows:
A - B are UO_2-PuO_2, P is cladding, W is water, U is void. The densities and compositions of the mixed oxide are:
A - 9.569 g/cm^3 12.00% O, 86.68% U, 1.32% Pu; 0.22% ^{235}U, 99.78% ^{238}U; 91.41% ^{239}Pu, 7.83% ^{240}Pu, 0.73% ^{241}Pu, 0.03% ^{242}Pu.
B - 10.333 g/cm^3 11.865% O, 82.313% U, 5.821% Pu; Nat U; 90.49% ^{239}Pu, 8.57% ^{240}Pu, 0.89% ^{241}Pu, 0.04% ^{242}Pu.
P - Zr 6.4 g/cm^3, 100% Zr.
(b) See note (b) of Table VI.
(c) V_{H_2O}/V_U is the ratio of the volume of water to the volume of uranium having a density of 18.9 g/cm^3.
(d) See note (c) of Table VI.

TABLE XVIII

Lattices of Pu-Al Rods in H_2O

Square Pitch, cm	H/Pu	Array	keff
3.18	254	8 x 8	1.000
		7 x 8	1.000
3.81	456	7 x 8	1.013
		7 x 7	1.006
4.44	694	8 x 8	1.008
		7 x 8	1.003
5.08	969	8 x 8	0.988
		7 x 7	0.982
5.72	1280	8 x 8	0.960
		7 x 7	0.952
6.35	1630	8 x 8	0.913

^{233}U

Because ^{233}U is treated as a 1/v absorber, solutions with concentrations much above 100 g ^{233}U per liter cannot be treated without exceeding the limit of 4 barns per H atom in MGBS. Correlations with experiment, moreover, show that the code tends to overestimate the bucklings of these solutions by rather large amounts. This code has, therefore, not been correlated with critical experiments performed with solutions of ^{233}U.

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35. Experiments performed at SRL in 1963 by W. B. Rogers.

Direct experimental results were as follows:

Fuel Tube	Square Pitch, in.	Configuration	V_{H_2O}/V_U	κ_z^2, m^{-2}
Inner	1.974	8 x 8	0.982	57.12
	2.099	8 x 8	1.282	44.54
	2.224	8 x 8	1.627	39.21
	2.349	8 x 8	1.937	32.35
	2.474	8 x 8	2.292	32.45
	2.599	8 x 8	2.666	35.13
	2.724	8 x 8	3.058	36.85
	2.974	8 x 8	3.897	51.23
Outer	3.076	8 x 8	1.057	26.00
	3.201	8 x 8	1.249	18.86
		7 x 8	1.249	23.71
	3.326	8 x 8	1.449	16.33
		7 x 8	1.449	19.99
	3.451	8 x 8	1.656	15.59
	3.576	8 x 8	1.871	15.48
	3.701	8 x 8	2.093	18.47

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Direct experimental results were as follows:

Rows	Fuel Assemblies per Row	Center-to-Center Spacing, inches: Within Row	Center-to-Center Spacing, inches: Between Rows	κ_z^2, m^{-2}
2	6	3.56	3.56	6.15 ±0.10
3	3	3.56	3.56	4.85 ±0.10
2	6	3.56	4.00	7.55 ±0.10
3	6	3.56	5.00	-6.00 ±0.30
4(a)	6	3.56	5.00	45.00 ±2.00
4	6	3.56	6.00	14.75 ±0.20
2	6	4.00	4.00	16.30 ±0.30
3	3	4.00	4.00	12.25 ±0.20
4	6	4.00	6.00	24.60 ±0.60
4	5	4.50	4.50	-4.00 ±0.30

(a) A 0.030-inch sheet of cadmium was centered between each pair of rows.

50. H. K. Clark. Handbook of Nuclear Safety - Supplement, Uranium-Aluminum Alloy. USAEC Report DP-532 (Suppl.), E. I. du Pont de Nemours & Co., Savannah River Laboratory, Aiken, S. C. (1961)(Confidential).

51. L. C. Schmid, et al. "Critical Masses and Bucklings of PuO_2-UO_2-H_2O Systems." Trans. Am. Nucl. Soc. 7, 216 (1964). (HW-83187)