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Appendix D - Estimation of mobile and immobile volume fractions from conductivity distribution using mean/effective conductivity cutoff

A concept for estimating mobile and immobile volume fractions is developed in this appendix. The idea is to equate the immobile volume fraction with the probability of a conductivity value being below the effective conductivity of the K field. A log-normal distribution of conductivity is assumed below to derive a quantitative estimate. In preparation for this analysis, key statistical properties of normal and log-normal distributions are first presented.

1 Normal distribution

The normal distribution has the probability density function (Walpole and Myers, 1978, p.110)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(x-\mu)/\sigma]^2} \equiv n(x;\mu,\sigma) \tag{1.1}$$

where μ and σ are the mean and standard deviation. The latter can be verified as follows. The mean of a distribution is computed from the probability distribution function as

$$\begin{aligned} E(X) &= \int_{-\infty}^{+\infty} x f(x) dx \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} x e^{-(1/2)[(x-\mu)/\sigma]^2} dx \end{aligned} \tag{1.2}$$

Letting

$$z = (x-\mu)/\sigma \tag{1.3}$$

we obtain

$$\begin{aligned} E(X) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (\mu + \sigma z) e^{-z^2/2} dz \\ &= \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-z^2/2} dz + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} z e^{-z^2/2} dz \end{aligned} \tag{1.4}$$

The first integral is evaluated as (Beyer, 1984, definite integral #604, p. 285)

$$\begin{aligned}\int_{-\infty}^{+\infty} e^{-z^2/2} dz &= 2\int_{0}^{+\infty} e^{-z^2/2} dz \\ &= 2\sqrt{2}\int_{0}^{+\infty} e^{-z^2/2} d(z/\sqrt{2}) \\ &= 2\sqrt{2}\int_{0}^{+\infty} e^{-t^2} dt \\ &= \sqrt{2\pi}\end{aligned} \tag{1.5}$$

By direct integration, the second integral is

$$\begin{aligned}\int_{-\infty}^{+\infty} z e^{-z^2/2} dz &= -\int_{-\infty}^{+\infty} e^{-z^2/2}(-z dz) \\ &= \left[-e^{-z^2/2}\right]_{-\infty}^{+\infty} \\ &= 0\end{aligned} \tag{1.6}$$

Equation (1.4) becomes

$$E(X) = \frac{\mu}{\sqrt{2\pi}}\sqrt{2\pi} + \frac{\sigma}{\sqrt{2\pi}} 0 = \mu \tag{1.7}$$

The variance is computed as

$$\begin{aligned}E\left[(X-\mu)^2\right] &= \int_{-\infty}^{+\infty} (x-\mu)^2 f(x) dx \\ &= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty} (x-\mu)^2 e^{-(1/2)[(x-\mu)/\sigma]^2} dx\end{aligned} \tag{1.8}$$

Again using equation (1.3)

$$\begin{aligned} E\left[(X-\mu)^2\right] &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} (\sigma z)^2 e^{-z^2/2} d(\sigma z) \\ &= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} z^2 e^{-z^2/2} dz \end{aligned} \tag{1.9}$$

Integrating by parts with

$$u = z \tag{1.10}$$

$$dv = ze^{-z^2/2} dz \tag{1.11}$$

yields

$$v = -e^{-z^2/2} \tag{1.12}$$

$$\begin{aligned} \int_{-\infty}^{+\infty} z^2 e^{-z^2/2} dz &= \int_{-\infty}^{+\infty} u dv \\ &= [uv]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} v du \\ &= \left[-ze^{-z^2/2}\right]_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} e^{-z^2/2} dz \\ &= 0 + \sqrt{2\pi} \end{aligned} \tag{1.13}$$

where the last integral was evaluated using equation (1.6). Returning to equation (1.9)

$$E\left[(X-\mu)^2\right] = \frac{\sigma^2}{\sqrt{2\pi}} \sqrt{2\pi} = \sigma^2 \tag{1.14}$$

The median and mode of the normal distribution are also equal to μ.

2 Transformations

If the one-to-one transformation from one variable to a second variable is known, the probability distribution function of the second variable can be computed from knowledge of the first. Let

$$X, f(x) \tag{2.1}$$

denote a random variable and its distribution, and

$$Y, g(y) \tag{2.2}$$

denote the same quantities for the second variable. Let the transformations between X and Y be defined by

$$y = F(x) \tag{2.3}$$

$$x = G(y) \tag{2.4}$$

The probability distribution of the Y is defined by (Walpole and Myers, 1978, Theorem 5.3)

$$g(y) = f\left[G(y)\right] \cdot G'(y) \tag{2.5}$$

Equation (2.5) provides a method for deriving the probability distribution functions of log normal distributions, as shown below.

3 Natural log normal distribution

If the natural logarithm ("ln") of a variable has a normal distribution, the variable is said to have a log normal distribution. The probability distribution function of the log normal distribution can be derived through application of equations (2.1) through (2.5). If

$$x = \ln(y) = G(y) \tag{3.1}$$

then (Aitchison and Brown, 1957, equation (2.5))

$$\begin{aligned} g(y) &= f\left[G(y)\right] \cdot G'(y) \\ &= \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(\ln y - \mu)/\sigma]^2} \cdot \frac{1}{y} \end{aligned} \tag{3.2}$$

where

$$\mu \equiv \mu_{\ln y} = E(\ln y) \tag{3.3}$$

$$\begin{aligned} \sigma^2 &\equiv \sigma^2_{\ln y} \\ &= E\left[(\ln y - E(\ln y))^2\right] \\ &= E\left[(\ln y - \mu)^2\right] \end{aligned} \tag{3.4}$$

The mean of Y is

$$\begin{aligned} E(Y) &= \int_{-\infty}^{+\infty} y g(y) dy \\ &= \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(\ln y - \mu)/\sigma]^2} dy \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-(1/2)[(\ln y - \mu)/\sigma]^2} dy \end{aligned} \tag{3.5}$$

Letting

$$z = (\ln y - \mu)/\sigma \tag{3.6}$$

$$\begin{aligned} dz &= dy/\sigma y \\ dy &= e^{\mu + \sigma z} \sigma dz \end{aligned} \tag{3.7}$$

produces

$$\begin{aligned} E(Y) &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-z^2/2} e^{\mu + \sigma z} \sigma dz \\ &= \frac{e^{\mu}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-z^2/2} e^{\sigma z} dz \\ &= \frac{e^{\mu}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{\sigma z - z^2/2} dz \end{aligned} \tag{3.8}$$

Note that

$$\begin{aligned} (z-\sigma)^2 &= z^2 - 2\sigma z + \sigma^2 \\ &= 2(z^2/2 - \sigma z) + \sigma^2 \end{aligned} \tag{3.9}$$

or

$$\begin{aligned} z^2/2 - \sigma z &= \left[(z-\sigma)^2 - \sigma^2\right]/2 \\ \sigma z - z^2/2 &= \left[\sigma^2 - (z-\sigma)^2\right]/2 \end{aligned} \tag{3.10}$$

Substituting equation (3.10) into (3.8) and using equation (1.5) yields

$$
\begin{aligned}
E(Y) &= \frac{e^{\mu}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{[\sigma^2-(z-\sigma)^2]/2} dz \\
&= \frac{e^{\mu}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{\sigma^2/2} e^{-(z-\sigma)^2/2} dz \\
&= \frac{e^{\mu+\sigma^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-(z-\sigma)^2/2} d(z-\sigma) \\
&= \frac{e^{\mu+\sigma^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-u^2/2} du \\
&= \frac{e^{\mu+\sigma^2/2}}{\sqrt{2\pi}} \sqrt{2\pi} \\
&= e^{\mu+\sigma^2/2}
\end{aligned}
\tag{3.11}
$$

The median of Y is simply

$$
\begin{aligned}
G(y_{\text{median}}) = \ln(y_{\text{median}}) = \mu \\
y_{\text{median}} = e^{\mu}
\end{aligned}
\tag{3.12}
$$

The mode of Y is computed by setting the first derivative of the p.d.f. to zero

$$
\begin{aligned}
g'(y) &= 0 \\
\frac{d}{dy}\left\{ \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(\ln y-\mu)/\sigma]^2} \cdot \frac{1}{y} \right\} &= 0 \\
e^{-(1/2)[(\ln y-\mu)/\sigma]^2} \frac{d}{dy}\left\{\frac{1}{y}\right\} + \frac{d}{dy}\left\{ e^{-(1/2)[(\ln y-\mu)/\sigma]^2} \right\} \cdot \frac{1}{y} &= 0 \\
e^{-(1/2)[(\ln y-\mu)/\sigma]^2} \left\{-\frac{1}{y^2}\right\} + \left\{ e^{-(1/2)[(\ln y-\mu)/\sigma]^2} \cdot -\frac{(\ln y-\mu)/\sigma}{\sigma y} \right\} \cdot \frac{1}{y} &= 0 \\
1 + \left\{ \frac{\ln y - \mu}{\sigma^2} \right\} &= 0 \\
\ln y &= \mu - \sigma^2
\end{aligned}
\tag{3.13}
$$

Therefore

$$
y_{\text{mode}} = e^{\mu-\sigma^2} \tag{3.14}
$$

In summary (Aitchison and Brown, 1957, p.9; Rivoirard, 1994, p 51-52)

$$y_{\text{mean}} = e^{\mu+\sigma^2/2} \tag{3.15}$$

$$y_{\text{median}} = e^{\mu} \tag{3.16}$$

$$y_{\text{mode}} = e^{\mu-\sigma^2} \tag{3.17}$$

4 Base 10 log normal distribution

The base 10 logarithm ("log") is often more convenient than the natural logarithm ("ln") for data plotting and analysis. Useful relationships between the two logarithms are (Beyer, 1994, p. 157)

$$\log x = \log e \cdot \ln x \tag{4.1}$$

$$\ln x = \ln 10 \cdot \log x \tag{4.2}$$

Note from (4.1) and (4.2) that

$$\ln 10 \cdot \log e = 1 \tag{4.3}$$

Also the derivative of the base 10 logarithm is

$$\frac{d}{dx}\log x = \frac{\log e}{x} = \frac{1}{x\ln 10} \tag{4.4}$$

Suppose

$$x = \log(z) = H(z) \tag{4.5}$$

then

$$\begin{aligned} h(z) &= f\left[H(z)\right] \cdot H(z) \\ &= \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(\log z-\mu)/\sigma]^2} \cdot \frac{\log e}{z} \\ &= \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\log e \ln z-\mu)^2}{2\sigma^2}\right] \cdot \frac{\log e}{z} \\ &= \frac{\log e}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln z-\mu/\log e)^2}{2(\sigma/\log e)^2}\right] \cdot \frac{1}{z} \end{aligned} \tag{4.6}$$

where

$$\mu \equiv \mu_{\log y} = E(\log y) \tag{4.7}$$

$$\begin{aligned} \sigma^2 &\equiv \sigma^2_{\log y} \\ &= E\left[(\log y - E(\log y))^2\right] \\ &= E\left[(\log y - \mu)^2\right] \end{aligned} \tag{4.8}$$

Defining

$$\mu' = \mu / \log e = \mu \ln 10 \tag{4.9}$$

$$\sigma' = \sigma / \log e = \sigma \ln 10 \tag{4.10}$$

yields

$$h(z) = \frac{1}{\sigma' \sqrt{2\pi}} \exp\left[-\frac{(\ln z - \mu')^2}{2\sigma'^2}\right] \cdot \frac{1}{z} \tag{4.11}$$

which is the same form as equation (3.2). Therefore

$$\begin{aligned} z_{\text{mean}} &= e^{\mu' + \sigma'^2 / 2} \\ &= e^{\mu \ln 10 + (\sigma \ln 10)^2 / 2} \\ &= e^{\ln 10 [\mu + \sigma^2 \ln 10 / 2]} \\ &= 10^{\mu + \sigma^2 \ln 10 / 2} \\ &= 10^{\mu + \sigma^2 / 2 \log e} \end{aligned} \tag{4.12}$$

$$\begin{aligned} z_{\text{median}} &= e^{\mu'} \\ &= e^{\mu \ln 10} \\ &= 10^{\mu} \end{aligned} \tag{4.13}$$

$$
\begin{aligned}
z_{\text{mode}} &= e^{\mu' - \sigma'^2} \\
&= e^{\mu \ln 10 - (\sigma \ln 10)^2} \\
&= e^{\ln 10[\mu - \sigma^2 \ln 10]} \\
&= 10^{\mu - \sigma^2 \ln 10} \\
&= 10^{\mu - \sigma^2 / \log e}
\end{aligned}
\tag{4.14}
$$

5 Summary of means, medians and modes

In summary, mean, median and mode for the normal, “ln normal” and “log normal” distributions are

$$x_{\text{mean}} = \mu \tag{5.1}$$

$$x_{\text{median}} = \mu \tag{5.2}$$

$$x_{\text{mode}} = \mu \tag{5.3}$$

$$y_{\text{mean}} = e^{\mu + \sigma^2 / 2} \tag{5.4}$$

$$y_{\text{median}} = e^{\mu} \tag{5.5}$$

$$y_{\text{mode}} = e^{\mu - \sigma^2} \tag{5.6}$$

$$z_{\text{mean}} = 10^{\mu + \sigma^2 \ln 10 / 2} = 10^{\mu + \sigma^2 / 2 \log e} \tag{5.7}$$

$$z_{\text{median}} = e^{\mu \ln 10} = 10^{\mu} \tag{5.8}$$

$$z_{\text{mode}} = 10^{\mu - \sigma^2 \ln 10} = 10^{\mu - \sigma^2 / \log e} \tag{5.9}$$

The two log normal representations are actually equivalent, so that $y_{\text{mean}} = z_{\text{mean}}$, $y_{\text{median}} = z_{\text{median}}$, $y_{\text{mode}} = z_{\text{mode}}$. The latter can be demonstrated by first noting that μ and σ are defined differently for the two distributions. Because $\log(\cdot)$ is merely a rescaling of $\ln(\cdot)$

$$\log x = \log e \cdot \ln x \tag{5.10}$$

equations (4.7) and (4.8) can be written as a rescaling of equations (3.3) and (3.4) (Walpole and Myers, 1978, Theorem 5.11)

$$\mu_{\log x} = \log e \cdot \mu_{\ln x} \tag{5.11}$$

$$\sigma^2_{\log x} = (\log e)^2 \cdot \sigma^2_{\ln x} \tag{5.12}$$

Substituting (5.11) and (5.12) into (5.7) through (5.9) yields

$$\begin{aligned} z_{\text{mean}} &= 10^{\mu_{\log z} + \sigma^2_{\log z} / 2 \log e} \\ &= 10^{\log e \cdot \mu_{\ln z} + (\log e \cdot \sigma_{\ln z})^2 / 2 \log e} \\ &= 10^{\log e \left(\mu_{\ln z} + \sigma^2_{\ln z} / 2 \right)} \\ &= e^{\mu_{\ln z} + \sigma^2_{\ln z} / 2} \end{aligned} \tag{5.13}$$

$$\begin{aligned} z_{\text{median}} &= 10^{\mu_{\log z}} \\ &= 10^{\log e \cdot \mu_{\ln z}} \\ &= e^{\mu_{\ln y}} \end{aligned} \tag{5.14}$$

$$\begin{aligned} z_{\text{mode}} &= 10^{\mu_{\log z} - \sigma^2_{\log z} / \log e} \\ &= 10^{\log e \cdot \mu_{\ln z} - \log e \cdot \sigma^2_{\ln z}} \\ &= e^{\mu_{\ln z} - \sigma^2_{\ln z}} \end{aligned} \tag{5.15}$$

The exact equivalence of the two distributions can be shown by observing from (4.6), (5.11) and (5.12) that

$$\begin{aligned} h(z) &= \frac{1}{\sigma_{\log z} \sqrt{2\pi}} \exp\left[- \frac{(\log z - \mu_{\log z})^2}{2 \sigma_{\log z}{}^2} \right] \cdot \frac{\log e}{z} \\ &= \frac{\log e}{\sigma_{\log z} \sqrt{2\pi}} \exp\left[- \frac{(\ln z - \mu_{\log z} / \log e)^2}{2 (\sigma_{\log z} / \log e)^2} \right] \cdot \frac{1}{z} \\ &= \frac{1}{\sigma_{\ln z} \sqrt{2\pi}} \exp\left[- \frac{(\ln z - \mu_{\ln z})^2}{2 \sigma_{\ln z}{}^2} \right] \cdot \frac{1}{z} \end{aligned} \tag{5.16}$$

6 Application to dual media transport

Suppose that the (relatively) mobile and immobile regions are defined according to whether the conductivity is above or below the average (mean), and that conductivities are log-normally distributed. Then the volume fraction occupied by each region can be computed from the cumulative probability of the conductivity distribution at the mean value of conductivity. The volume fraction of the immobile region is

$$\phi_{im} \equiv \frac{V_{im}}{V} = P(K < \mu_K) \tag{6.1}$$

where

$$\mu_K \equiv E(K) \tag{6.2}$$

The volume fraction of the mobile region becomes

$$\phi_m \equiv \frac{V_m}{V} = 1 - \phi_{im} = 1 - P(K < \mu_K) \tag{6.3}$$

As a refinement, the effective conductivity value for the media could be used in place of the arithmetic mean. For example, the effective conductivity of a three-dimensional, statistically homogeneous, but anisotropic, random ln K field subjected to a uniform mean flow was derived analytically by Gelhar and Axness (1983). The three-dimensional anisotropy of the heterogeneous medium is defined in terms an exponential autocovariance function with distinct correlation scales for each coordinate direction, λ_1, λ_2 and λ_3. When the mean flow is aligned with the bedding ($\lambda_1 = \lambda_2 > \lambda_3$), the non-zero components of the conductivity tensor are

$$\overline{K}_{11} = \overline{K}_{22} = \overline{K}_h = K_g\left[1 + \sigma^2\left(\frac{1}{2} - g_{11}\right)\right] \tag{6.4}$$

$$\overline{K}_{33} = \overline{K}_v = K_g\left[1 + \sigma^2\left(\frac{1}{2} - g_{33}\right)\right] \tag{6.5}$$

where

$\overline{K}_h \equiv$ effective horizontal conductivity

$\overline{K}_v \equiv$ effective vertical conductivity

$K_g \equiv \exp[E(\ln K)] = e^{\mu}$; geometric mean of point conductivity field

$\mu \equiv E(\ln K)$; mean of the natural logarithm of point conductivities

$\sigma^2 \equiv$ variance of the natural logarithm of point conductivities

and g_{11} and g_{33} are functions of the correlation scales. For case being considered here, they are defined in terms of the ratio of horizontal to vertical correlation, $\rho = \lambda_h/\lambda_v > 1$, as follows:

$$g_{11} = \frac{1}{2}\frac{1}{\rho^2 - 1}\left[\frac{\rho^2}{(\rho^2 - 1)^{1/2}}\tan^{-1}(\rho^2 - 1)^{1/2} - 1\right] \tag{6.6}$$

$$g_{33} = \frac{\rho^2}{\rho^2 - 1}\left[1 - \frac{1}{(\rho^2 - 1)^{1/2}}\tan^{-1}(\rho^2 - 1)^{1/2}\right] \tag{6.7}$$

The above analytical results are based on a first-order perturbation analysis, and strictly speaking, only exact in the limit as the variance approaches zero. Accurate results can be expected for small variances. For large variances, the predictions may become increasingly inaccurate, or even nonphysical. For example, $\overline{K}_v$ is negative when $\lambda_h/\lambda_v \rightarrow \infty$ and the variance of lnK exceeds 2. To remedy such nonphysical results and hopefully extend the range of applicability of effective conductivity predictions, Gelhar and Axness (1983) proposed the following generalization of equations (6.4) and (6.5)

$$\overline{K}_h = K_g \exp\left[\sigma^2\left(\frac{1}{2} - g_{11}\right)\right] \tag{6.8}$$

$$\overline{K}_v = K_g \exp\left[\sigma^2\left(\frac{1}{2} - g_{33}\right)\right] \tag{6.9}$$

The generalization is motivated by the observation that a Taylor series expansion of equations (6.8) and (6.9) contains equations (6.4) and (6.5), respectively, as the first two terms. Subsequent comparison of equation (6.8) to numerical simulations indicates that the exponential generalization is accurate for isotropic systems and variances up to 7, but overpredicts effective conductivity for anisotropic systems (Gelhar, 1997, p. 161).

By defining the constants

$$p_h = 1 - 2g_{11} \tag{6.10}$$

$$p_v = 1 - 2g_{33} \tag{6.11}$$

and incorporating the definition of K_g, equations (6.8) and (6.9) can be rewritten as

$$\overline{K}_h = \exp\left(\mu + \frac{p_h\sigma^2}{2}\right) = e^{\mu + p_h\sigma^2/2} \tag{6.12}$$

$$\overline{K}_v = \exp\left(\mu + \frac{p_v\sigma^2}{2}\right) = e^{\mu + p_v\sigma^2/2} \tag{6.13}$$

Assuming dual-media transport occurs under horizontal flow conditions,

$$K_{eff} = e^{\mu + p_h\sigma^2/2} \tag{6.14}$$

For an anisotropic media with $\lambda_h = 10\lambda_v$, $p_h = 0.86$ (and $p_v = -0.72$) and (6.14) becomes

$$K_{eff} = e^{\mu + 0.86\sigma^2/2} \tag{6.15}$$

In comparison, the mean value of K used in equation (6.3) is

$$K_{mean} = \mu_K = e^{\mu_{\ln K} + (\sigma_{\ln K})^2/2} = e^{\mu + \sigma^2/2} \tag{6.16}$$

from equation (5.4). Criteria (6.2) and (6.3) can be generalized as

$$\phi_{im} = P(K < K_{eff}) \tag{6.17}$$

$$\phi_m = 1 - \phi_{im} = 1 - P(K < K_{eff}) \tag{6.18}$$

where K_{eff} denote an effective conductivity value, such as from equation (6.14). Equation (6.14) can be rewritten in terms of log base 10 using equations (5.11) and (5.12)

$$
\begin{aligned}
K_{eff} &= e^{\mu_{\ln K} + p_h (\sigma_{\ln K})^2 / 2} \\
&= e^{\mu_{\log K / \log e} + p_h (\sigma_{\log K / \log e})^2 / 2} \\
&= 10^{\log e \left[\mu_{\log K / \log e} + p_h (\sigma_{\log K / \log e})^2 / 2\right]} \\
&= 10^{\mu_{\log K} + p_h \sigma_{\log K}{}^2 / (2 \log e)} \\
&= 10^{\mu + p_h \sigma^2 / 2 \log e}
\end{aligned}
\tag{6.19}
$$

where μ and σ^2 are based on log K.

Other estimates of effective conductivity could be used. Through numerical modeling Desbarats (1992) generated an equivalent conductivity estimate of p_h = 0.59 for an anisotropic media with $\lambda_h = 10\lambda_v$ and finite block dimensions of $L_h/l_h = L_v/l_v = 3$.

References

Aitchison, J. and J. A. C. Brown, 1957, The Lognormal Distribution, Cambridge University Press.

Beyer, W. H., ed., 1984, CRC Standard Mathematical Tables, 27th edition, CRC Press, Boca Raton, Florida.

Desbarats, A. J., 1992, Spatial averaging of hydraulic conductivity in three-dimensional heterogeneous porous media, Mathematical Geology, v24, n3, 249-267.

Gelhar, L. W., 1997, Perspectives on field-scale application of stochastic subsurface hydrology, in G. Dagan and S. P. Neuman, eds., Subsurface flow and transport: a stochastic approach, Cambridge University Press.

Gelhar, L. W. and C. L. Axness, 1983, Three-dimensional stochastic analysis of macrodispersion in aquifers, Water Resources Research, v19, n1, 161-180.

Rivoirard, J., 1994, Introduction to Disjunctive Kriging and Non-Linear Geostatistics, Clarendon Press, Oxford.

Walpole, R. E. and R. H. Myers, 1978, Probability and Statistics for Engineers and Scientists, 2nd Edition, MacMillan, New York.