

**This document was prepared in conjunction with work accomplished under Contract No. DE-AC09-96SR18500 with the U. S. Department of Energy.**

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## Appendix B - Non-dimensional form of dual-media transport equation set

A non-dimensional form of a common dual-media formulation is derived here for the case of one-dimensional flow and transport and a non-sorbing tracer undergoing first-order decay.

Equations (2.21e) and (2.21f) in MT3D manual:

$$\theta_m R_m \frac{\partial C_m}{\partial t} + \theta_{im} R_{im} \frac{\partial C_{im}}{\partial t} = \frac{\partial}{\partial x_i} \left( \theta_m D_{ij} \frac{\partial C_m}{\partial x_j} \right) - \frac{\partial}{\partial x_i} (\theta_m v_m C_m) + q_s C_s - q_s' C_m \quad (1a)$$

$$- \lambda_{1,m} \theta_m C_m - \lambda_{1,im} \theta_{im} C_{im} - \lambda_{2,m} f \rho_b K_d C_m - \lambda_{2,im} (1-f) \rho_b K_d C_{im}$$

$$\theta_{im} R_{im} \frac{\partial C_{im}}{\partial t} = \zeta (C_m - C_{im}) - \lambda_{1,im} \theta_{im} C_{im} - \lambda_{2,im} (1-f) \rho_b K_d C_{im} \quad (1b)$$

where

$$R_m = 1 + f \rho_b K_d / \theta_m$$

$$R_{im} = 1 + (1-f) \rho_b K_d / \theta_{im}$$

Assuming no sorption,  $K_d = 0$  and the retardation coefficients are one. Suppose also that first-order decay rates are the same for the mobile and immobile regions, and no internal sources exist. Finally, consider only one-dimensional flow and transport. Then equations (1) become

$$\theta_m \frac{\partial C_m}{\partial t} + \theta_{im} \frac{\partial C_{im}}{\partial t} = \frac{\partial}{\partial x} \left( \theta_m D \frac{\partial C_m}{\partial x} \right) - \frac{\partial}{\partial x} (\theta_m v_m C_m) - \lambda (\theta_m C_m + \theta_{im} C_{im}) \quad (2a)$$

$$\theta_{im} \frac{\partial C_{im}}{\partial t} = \zeta (C_m - C_{im}) - \lambda \theta_{im} C_{im} \quad (2b)$$

Let

$$t' = \frac{U}{\theta L} t$$

$$x' = \frac{1}{L} x$$

$$Pe = \frac{L}{D} v_m = \frac{LU}{D\theta_m}$$

$$\zeta' = \frac{L}{U} \zeta$$

$$\lambda' = \frac{\theta L}{U} \lambda$$

$$C_m' = \frac{1}{C_0} C_m$$

$$C_{im}' = \frac{1}{C_0} C_{im}$$

$$\phi = \frac{\theta_m}{\theta}$$

then equation (2a) becomes

$$\begin{aligned} & \left[ \theta_m \frac{\partial C_m'}{\partial t'} + \theta_{im} \frac{\partial C_{im}'}{\partial t'} \right] \frac{U}{\theta L} \\ &= \frac{\theta_m D}{L^2} \frac{\partial^2 C_m'}{\partial x'^2} - \frac{U}{L} \frac{\partial C_m'}{\partial x'} - \lambda' \frac{U}{\theta L} (\theta_m C_m' + \theta_{im} C_{im}') \\ & \left[ \phi \frac{\partial C_m'}{\partial t'} + (1-\phi) \frac{\partial C_{im}'}{\partial t'} \right] \frac{U}{L} \\ &= \frac{\theta_m D}{L^2} \frac{\partial^2 C_m'}{\partial x'^2} - \frac{U}{L} \frac{\partial C_m'}{\partial x'} - \lambda' \frac{U}{L} [\phi C_m' + (1-\phi) C_{im}'] \\ & \left[ \phi \frac{\partial C_m'}{\partial t'} + (1-\phi) \frac{\partial C_{im}'}{\partial t'} \right] = \frac{1}{Pe} \frac{\partial^2 C_m'}{\partial x'^2} - \frac{\partial C_m'}{\partial x'} - \lambda' [\phi C_m' + (1-\phi) C_{im}'] \end{aligned} \quad (3)$$

and equation (2b) becomes

$$\theta_{im} \frac{\partial C_{im}'}{\partial t'} \frac{U}{\theta L} = \zeta' \frac{U}{L} (C_m' - C_{im}') - \lambda' \frac{U}{\theta L} \theta_{im} C_{im}'$$

$$(1-\phi)\frac{\partial C_{im}'}{\partial t'} = \zeta'(C_m' - C_{im}') - \lambda'(1-\phi)C_{im}' \quad (4)$$

Suppose

$$D = \alpha_L v_m \quad (5)$$

then

$$\frac{1}{Pe} = \frac{D}{v_m L} = \frac{\alpha_L v_m}{v_m L} = \frac{\alpha_L}{L} = \alpha_L' \quad (6)$$

In summary

$$\phi \frac{\partial C_m'}{\partial t'} + (1-\phi)\frac{\partial C_{im}'}{\partial t'} = \alpha_L' \frac{\partial^2 C_m'}{\partial x'^2} - \frac{\partial C_m'}{\partial x'} - \lambda'[\phi C_m' + (1-\phi)C_{im}'] \quad (7a)$$

$$(1-\phi)\frac{\partial C_{im}'}{\partial t'} = \zeta'(C_m' - C_{im}') - \lambda'(1-\phi)C_{im}' \quad (7b)$$

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