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DISCLAIMER

Appendix B - Non-dimensional form of dual-media transport equation set

A non-dimensional form of a common dual-media formulation is derived here for the case of one-dimensional flow and transport and a non-sorbing tracer undergoing first-order decay.

Equations (2.21e) and (2.21f) in MT3D manual:

$$\theta_m R_m \frac{\partial C_m}{\partial t} + \theta_{im} R_{im} \frac{\partial C_{im}}{\partial t} = \frac{\partial}{\partial x_i}\left(\theta_m D_{ij} \frac{\partial C_m}{\partial x_j}\right) - \frac{\partial}{\partial x_i}(\theta_m v_m C_m) + q_s C_s - q_s' C_m \tag{1a}$$
$$- \lambda_{1,m}\theta_m C_m - \lambda_{1,im}\theta_{im} C_{im} - \lambda_{2,m} f\rho_b K_d C_m - \lambda_{2,im}(1-f)\rho_b K_d C_{im}$$

$$\theta_{im} R_{im} \frac{\partial C_{im}}{\partial t} = \zeta(C_m - C_{im}) - \lambda_{1,im}\theta_{im} C_{im} - \lambda_{2,im}(1-f)\rho_b K_d C_{im} \tag{1b}$$

where

$$R_m = 1 + f\rho_b K_d / \theta_m$$

$$R_{im} = 1 + (1-f)\rho_b K_d / \theta_{im}$$

Assuming no sorption, $K_d = 0$ and the retardation coefficients are one. Suppose also that first-order decay rates are the same for the mobile and immobile regions, and no internal sources exist. Finally, consider only one-dimensional flow and transport. Then equations (1) become

$$\theta_m \frac{\partial C_m}{\partial t} + \theta_{im} \frac{\partial C_{im}}{\partial t} = \frac{\partial}{\partial x}\left(\theta_m D \frac{\partial C_m}{\partial x}\right) - \frac{\partial}{\partial x}(\theta_m v_m C_m) - \lambda(\theta_m C_m + \theta_{im} C_{im}) \tag{2a}$$

$$\theta_{im} \frac{\partial C_{im}}{\partial t} = \zeta(C_m - C_{im}) - \lambda\theta_{im} C_{im} \tag{2b}$$

Let

$$t' = \frac{U}{\theta L} t$$

$$x' = \frac{1}{L} x$$

$$Pe = \frac{L}{D} v_m = \frac{LU}{D\theta_m}$$

$$\zeta' = \frac{L}{U} \zeta$$

$$\lambda' = \frac{\theta L}{U} \lambda$$

$$C_m{}' = \frac{1}{C_0} C_m$$

$$C_{im}{}' = \frac{1}{C_0} C_{im}$$

$$\phi = \frac{\theta_m}{\theta}$$

then equation (2a) becomes

$$\left[\theta_m \frac{\partial C_m{}'}{\partial t'} + \theta_{im} \frac{\partial C_{im}{}'}{\partial t'}\right] \frac{U}{\theta L} = \frac{\theta_m D}{L^2} \frac{\partial^2 C_m{}'}{\partial x'^2} - \frac{U}{L} \frac{\partial C_m{}'}{\partial x'} - \lambda' \frac{U}{\theta L} \left(\theta_m C_m{}' + \theta_{im} C_{im}{}'\right)$$

$$\left[\phi \frac{\partial C_m{}'}{\partial t'} + (1-\phi) \frac{\partial C_{im}{}'}{\partial t'}\right] \frac{U}{L} = \frac{\theta_m D}{L^2} \frac{\partial^2 C_m{}'}{\partial x'^2} - \frac{U}{L} \frac{\partial C_m{}'}{\partial x'} - \lambda' \frac{U}{L} \left[\phi C_m{}' + (1-\phi) C_{im}{}'\right]$$

$$\left[\phi \frac{\partial C_m{}'}{\partial t'} + (1-\phi) \frac{\partial C_{im}{}'}{\partial t'}\right] = \frac{1}{Pe} \frac{\partial^2 C_m{}'}{\partial x'^2} - \frac{\partial C_m{}'}{\partial x'} - \lambda' \left[\phi C_m{}' + (1-\phi) C_{im}{}'\right] \qquad (3)$$

and equation (2b) becomes

$$\theta_{im} \frac{\partial C_{im}{}'}{\partial t'} \frac{U}{\theta L} = \zeta' \frac{U}{L} \left(C_m{}' - C_{im}{}'\right) - \lambda' \frac{U}{\theta L} \theta_{im} C_{im}{}'$$

$$(1-\phi)\frac{\partial C_{im}{}'}{\partial t'} = \zeta'\left(C_m{}' - C_{im}{}'\right) - \lambda'(1-\phi)C_{im}{}' \tag{4}$$

Suppose

$$D = \alpha_L v_m \tag{5}$$

then

$$\frac{1}{Pe} = \frac{D}{v_m L} = \frac{\alpha_L v_m}{v_m L} = \frac{\alpha_L}{L} = \alpha_L{}' \tag{6}$$

In summary

$$\phi\frac{\partial C_m{}'}{\partial t'} + (1-\phi)\frac{\partial C_{im}{}'}{\partial t'} = \alpha_L{}'\frac{\partial^2 C_m{}'}{\partial x'^2} - \frac{\partial C_m{}'}{\partial x'} - \lambda'\left[\phi C_m{}' + (1-\phi)C_{im}{}'\right] \tag{7a}$$

$$(1-\phi)\frac{\partial C_{im}{}'}{\partial t'} = \zeta'\left(C_m{}' - C_{im}{}'\right) - \lambda'(1-\phi)C_{im}{}' \tag{7b}$$

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