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ENRICHMENT MONITOR FOR <sup>235</sup>U FUEL TUBES

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## ENRICHMENT MONITOR FOR $^{235}\text{U}$ FUEL TUBES

### I. Introduction

In 1976 measurements were needed to appraise  $^{235}\text{U}$ -enrichment uniformity of fuel tubes produced in the powder metallurgy program. Because the design enrichment ( $\sim 60\text{e}\%$ ) is attained by blending high-enrichment ( $\sim 75\text{e}\%$ ) and low-enrichment ( $\sim 40\text{e}\%$ ) oxides, inadequate blending can cause enrichment non-uniformities. Enrichment measurements for  $1/4$ "-diameter fuel tube areas are required to appraise uniformity.<sup>1</sup> The Fuel Distribution Analyzer<sup>2</sup> and the Nuclear Test Gauge<sup>3,4</sup> were available to measure U-uniformity and  $^{235}\text{U}$ -uniformity, to yield enrichment uniformity; however, the fuel tube areas tested were much too large. Thus, alternative methods were sought.

By early 1977, mass spectroscopy had appraised enrichment uniformity in representative tubes.<sup>1</sup> Sample disks of  $1/4$ "-diameter were cut from tubes and transferred to Analytical Chemistry Division for analysis. The enrichments of these samples agreed well within the desired  $\pm 0.5\text{e}\%$ . Unfortunately, this inherently destructive method sacrifices production costs for tested tubes and can only assume that these tubes are representative. Also, the requirements of sample preparation, interdivisional transfers, and accountability can be cumbersome. Typically, only a one-month turn-around for tube appraisal could be expected. Consequently, Nuclear Engineering Division sought a more attractive method to replace mass spectroscopy.

Beginning in 1978,  $\gamma$ -ray methods were examined for  $^{235}\text{U}$ -enrichment measurements.<sup>5</sup> Following experimental studies in 1980-81, a detailed method was proposed.<sup>6</sup> In this method,  $\gamma$ -rates associated with  $^{235}\text{U}$  and  $^{232}\text{U}$  are correlated with enrichment. Instrumentation for appraising fuel tubes with this method has been assembled and tested. This report describes the performance of this prototype  $\gamma$ -monitor of  $^{235}\text{U}$ -enrichment.

## II. Summary

The prototype  $\gamma$ -monitor successfully appraised inner, middle, and outer fuel tubes of Mark 14 design. Measurements of enrichment uniformity to  $\leq 0.6\%$  and  $^{235}\text{U}$  uniformity to  $\leq 1.0\%$  were demonstrated, where collimated 1/4"-diameter fuel areas were  $\gamma$ -counted for 2000 sec. The method is nondestructive, involves minimal fuel handling, and can appraise a tube in about one day. The prototype cost  $\sim \$30\text{K}$  and a final instrument would be  $\sim \$50\text{K}$ .

## III. Theoretical Basis

The  $\gamma$ -monitor of  $^{235}\text{U}$ -enrichment has a theoretical basis that addresses (A) the enrichment correlation with  $\gamma$ -rates associated with  $^{235}\text{U}$  and  $^{232}\text{U}$ , (B) the tube geometry effects on the correlation, and (C) the  $\gamma$ -rate sensitivity to various tube parameters. Each of these aspects will be discussed in detail in this section.

### A. $^{235}\text{U}$ Enrichment Correlation

Uranium oxide ( $\text{U}_3\text{O}_8$ ) fuel tubes at SRP are blended from high-enrichment ( $\sim 75\%$ ) and low-enrichment ( $\sim 40\%$ ) components, to produce a fuel of design enrichment ( $\sim 60\%$ ). Typical  $\gamma$ -spectra of such components are shown in Figure 1. The spectra show a 186 keV  $\gamma$ -ray from  $^{235}\text{U}$  and a 238 keV  $\gamma$ -ray from the  $^{232}\text{U}$  decay chain. The 186 $\gamma$ /238 $\gamma$  ratio of the high enrichment oxide is  $\sim 3$  times that of the low enrichment oxide. Thus, when these two fuels are blended, the 186 $\gamma$ /238 $\gamma$  ratio of the resulting fuel will be correlated with enrichment.

The 186 $\gamma$ /238 $\gamma$  correlation with enrichment will now be examined in detail for the case where identical sample counting geometries are assumed. This assumption eliminates the need for  $\gamma$ -attenuation corrections, which will be addressed in Section III.B., in dealing with actual fuels. For the present discussion, the sample geometry is that of the design fuel. The notation  $186\gamma/238\gamma = R_0/R_0' = F_0$  will be used for this case.

Note the following relationships:

$$R_0 = 186 \text{ keV } \gamma\text{-rate} = k\varepsilon$$

$$R_0' = 238 \text{ keV } \gamma\text{-rate} = k \frac{\varepsilon}{F_0} = k\varepsilon' \quad (1)$$

where  $k$  is constant,  $\varepsilon$  is the  $^{235}\text{U}$  enrichment, and  $\varepsilon'$  is the "effective"  $^{232}\text{U}$  enrichment. If we blend a low enrichment oxide (enrichment  $\varepsilon_L$ , fraction  $\chi_L$ ) and a high enrichment oxide (enrichment  $\varepsilon_H$ , fraction  $\chi_H = 1 - \chi_L$ ) to obtain the resulting oxide (enrichment  $\varepsilon$ , fraction 1) the following is true:

$$\begin{aligned} \varepsilon &= \chi_L \varepsilon_L + (1 - \chi_L) \varepsilon_H \\ \varepsilon' &= \chi_L \varepsilon_L' + (1 - \chi_L) \varepsilon_H' \end{aligned} \quad (2)$$

Upon eliminating  $\chi_L$  from both equations above,

$$\varepsilon' = \left[ \frac{\varepsilon_L' - \varepsilon_H'}{\varepsilon_L - \varepsilon_H} \right] \varepsilon + \left[ \frac{\varepsilon_L \varepsilon_H' - \varepsilon_H \varepsilon_L'}{\varepsilon_L - \varepsilon_H} \right]$$

or

$$\varepsilon' = b\varepsilon + c \quad (3)$$

where  $b$  and  $c$  are constants. Consequently, from (1) and (3)

$$F_0 = R_0/R_0' = \varepsilon/\varepsilon' = \frac{\varepsilon}{b\varepsilon + c} \quad (4)$$

Actual measurements of component oxides and fuels (all of similar sample geometry) are given in Figure 2, where  $\varepsilon'$  is plotted against  $\varepsilon$  to demonstrate Equation (3). The  $\varepsilon_L$  oxides cluster about  $(\varepsilon, \varepsilon') = (43, 14.5)$ ,  $\varepsilon_H$  oxides cluster about  $(\varepsilon, \varepsilon') = (76, 9.5)$ , and the  $(\varepsilon, \varepsilon')$  of fuel tubes lie on a line connecting these clusters. (In the ideal case, one would use samples of the actual component oxides for a given fuel tube to obtain better absolute measurements of  $\varepsilon$ ; however, for  $\varepsilon$ -uniformity measurements about an average, this refinement is unnecessary). The data of this plot yield  $b = -0.161$  and  $c = 21.7e\%$ .

The physical explanation of the  $F_0 = \varepsilon/\varepsilon'$  difference in the high and low enrichment component oxides, is due to the fuel cycle. SRP uranium fuels have been continually reprocessed and rebled as reused fuels for a number of years. As the  $^{235}\text{U}$  is burned out (lower  $\varepsilon$ ),  $^{232}\text{U}$  builds in (higher  $\varepsilon'$ ), in agreement with the observations in Figure 2 and the predictions of Figure 3.<sup>6</sup> Also the fuel blending itself preserves this  $\varepsilon'$  vs  $\varepsilon$  trend.

The above  $\epsilon'$  vs  $\epsilon$  trend is anticipated for uranium fuels well into the future. However, referring to Figure 3, it is noted that freshly reprocessed fuel requires some time for the  $^{232}\text{U}$  decay chain to buildup, due to the 1.9ly half-life of the immediate daughter  $^{228}\text{Th}$ . Indeed, the samples were shown to vary in  $F_0$  over a 2.5 yr examination period; however, the  $\gamma$ -intensities were never too weak to yield good counting statistics. Furthermore, it is expected that the time from freshly reprocessed fuel to fuel tube fabrication will always be long enough to permit sufficient  $^{228}\text{Th}$  buildup.

Should the above method be unworkable for future uranium fuels, an alternative method has been proposed. This method requires an external  $\gamma$ -source, which is used to measure the total uranium by attenuation/transmission. Cladding thickness measurements,<sup>7</sup> as well as 186 keV  $\gamma$ -rates for  $^{235}\text{U}$  are also required. This method is more involved than the present one; however, the  $\gamma$ -monitor should be readily adapted if necessary. A comparison of the two methods is given in Appendix A.

#### B. Tube Geometry Effects on $^{235}\text{U}$ Enrichment Correlation

In measurements with actual fuel tubes, effects of attenuation will alter  $F_0$  to some value  $F$ . Consider the  $\gamma$ -detection geometry depicted in Figure 4 for a typical tube. The 186 keV  $\gamma$ -detection rate for an unattenuated disk of fuel with area  $A$  and thickness  $dx$  is  $dR_u$ ,

$$dR_u = D \cdot \epsilon \cdot \rho_U \cdot A dx \quad (5)$$

where

$$D = (186 \text{ keV } \gamma\text{'s/sec-gm } ^{235}\text{U}_3\text{O}_8) \cdot (186 \text{ keV detection efficiency})$$

$$\rho_U = (\text{density of } \text{U}_3\text{O}_8 \text{ in fuel core, gm/cm}^3)$$

$$\epsilon = (^{235}\text{U enrichment, fraction})$$

Including attenuation effects of the cladding and fuel core, Figure 4 predicts

$$dR = e^{-\mu_A t_c} e^{-\mu_f x} dR_u$$

$$R = \int_0^{t_f} dR(x) = D \cdot A \cdot \frac{\rho_U}{\mu_f} \epsilon (1 - e^{-\mu_f t_f}) e^{-\mu_A t_c} \quad (6)$$

where

- $R$  = actual attenuated 186 keV  $\gamma$ -detection rate, count/sec  
 $\mu_f$  = atten coeff of 186 keV  $\gamma$  in fuel core,  $\text{cm}^{-1}$   
 $\mu_A$  = atten coeff of 186 keV  $\gamma$  in aluminum clad,  $\text{cm}^{-1}$   
 $t_f$  = fuel core thickness, cm  
 $t_c$  = fuel clad thickness, cm

It is useful to write  $R$  in terms of more fundamental parameters, using \*

$$\begin{aligned}
 \rho_U &= \frac{W_U}{W_U/\rho_U^* + W_A/\rho_A^*} \\
 \mu_A &= \rho_A^* g_A \\
 \mu_f &= \frac{W_U g_U + W_A g_A}{W_U/\rho_U^* + W_A/\rho_A^*} \quad (7)
 \end{aligned}$$

where

- $W_U$  = weight fraction of  $U_3O_8$  in fuel core  
 $W_A$  = weight fraction of Al in fuel core  
 $\rho_U^*$  = theoretical density of  $U_3O_8$ ,  $\text{gm}/\text{cm}^3$   
 $\rho_A^*$  = theoretical density of Al,  $\text{gm}/\text{cm}^3$   
 $g_U$  = 186 keV  $\gamma$ -mass atten coeff for  $U_3O_8$ ,  $\text{cm}^2/\text{gm}$  - tabulation <sup>8</sup>.  
 $g_A$  = 186 keV  $\gamma$ -mass atten coeff for Al,  $\text{cm}^2/\text{gm}$  - tabulation <sup>8</sup>.

Using equations (7), with equation (6),

$$\begin{aligned}
 R &= \epsilon DA \left( \frac{W_U}{W_U g_U + W_A g_A} \right) e^{-\rho_A^* g_A t_c} \left( 1 - e^{-\left( \frac{W_U g_U + W_A g_A}{W_U/\rho_U^* + W_A/\rho_A^*} \right) t_f} \right) \\
 R' &= \epsilon 'D' A' \left( \frac{W_U}{W_U g_U' + W_A g_A'} \right) e^{-\rho_A^* g_A' t_c} \left( 1 - e^{-\left( \frac{W_U g_U' + W_A g_A'}{W_U/\rho_U^* + W_A/\rho_A^*} \right) t_f} \right) \quad (8)
 \end{aligned}$$

\* All densities are set to 100% theoretical. As shown in Appendix B, this approximation has negligible effect on the analysis, although future work will include a correction for this.

where  $R'$ ,  $D'$ , .....refer to the 238 keV  $\gamma$ -ray.

Simplifying the above notations, we write

$$\begin{aligned}
 R &= \epsilon DA G(W_U, W_A, t_c, t_f) \\
 R' &= \epsilon' D'A' G'(W_U, W_A, t_c, t_f) \\
 F &= \frac{\epsilon}{\epsilon'} \left( \frac{DA}{D'A'} \right) \frac{G(W_U, W_A, t_c, t_f)}{G'(W_U, W_A, t_c, t_f)} \quad (9)
 \end{aligned}$$

The measurements of this work deal mainly with deviations about the normal values; thus, detailed knowledge of the (DA) - type factors, which are constant, is not required. However, should absolute  $\epsilon$ - values be desired, these factors are required and may be obtained using the tube design standard measurements in Section III.A, viz:

$$\begin{aligned}
 \bar{R}_0 &= k\epsilon = \epsilon(DA)G(\bar{W}_U, \bar{W}_A, \bar{t}_c, \bar{t}_f) \\
 \bar{R}_0' &= k\epsilon' = \epsilon'(D'A')G'(\bar{W}_U, \bar{W}_A, \bar{t}_c, \bar{t}_f) \quad (10)
 \end{aligned}$$

Where  $\bar{R}_0$ ,  $\bar{R}_0'$ ,  $\bar{W}_U$ ,  $\bar{W}_A$ ,  $\bar{t}_c$ , and  $\bar{t}_f$  correspond to tube design values.

Rearranging equations (10), we obtain

$$\begin{aligned}
 k &= \bar{R}_0/\epsilon \\
 (DA) &= \frac{k}{G(\bar{W}_U, \bar{W}_A, \bar{t}_c, \bar{t}_f)} \\
 (D'A') &= \frac{k}{G'(\bar{W}_U, \bar{W}_A, \bar{t}_c, \bar{t}_f)} \\
 \left( \frac{DA}{D'A'} \right) &= \frac{G'(\bar{W}_U, \bar{W}_A, \bar{t}_c, \bar{t}_f)}{G(\bar{W}_U, \bar{W}_A, \bar{t}_c, \bar{t}_f)} \quad (11)
 \end{aligned}$$

Finally, it is worth noting that in general

$$\begin{aligned}
 R &= R(\epsilon, W_U, t_c, t_f) \\
 R' &= R'(\epsilon, W_U, t_c, t_f) \\
 F &= F(\epsilon, W_U, t_c, t_f) \quad (12)
 \end{aligned}$$

because  $W_A = 100 - W_U$  (in wt%) and  $\epsilon' = b\epsilon + c$  per discussion in Section III.A.

### C. Parameter Dependence on Correlation

Of the three equations for R, R', and F derived in III.B., only two are independent. Because  $^{235}\text{U}$  enrichment is primarily correlated with F and the  $^{235}\text{U}$  mass is primarily correlated with R, the equations for R and F are most useful. Per equations (12), R and F depend on  $\epsilon$ ,  $W_U$ ,  $t_f$ , and  $t_c$ . Four equivalent parameters are more useful for the analysis, namely

$$t_c, t_f, \epsilon, \text{ and } m = \frac{W_U}{W_U/\rho_U^* + (100 - W_U)/\rho_A^*} \epsilon t_f$$

where m is the  $^{235}\text{U}$  mass/unit area for the fuel tube region examined.

The measured deviations in F and R will depend on the above parameters according to

$$\Delta F = \delta F/\delta t_c \cdot \Delta t_c + \delta F/\delta t_f \cdot \Delta t_f + \delta F/\delta \epsilon \cdot \Delta \epsilon + \delta F/\delta m \cdot \Delta m$$

$$\Delta R = \delta R/\delta t_c \cdot \Delta t_c + \delta R/\delta t_f \cdot \Delta t_f + \delta R/\delta \epsilon \cdot \Delta \epsilon + \delta R/\delta m \cdot \Delta m$$

A BASIC program was developed to calculate the partial derivatives ( $\delta X/\delta Y$  notation) from expressions in Section III.B, as described in Appendix B. The two expressions  $\Delta F$  and  $\Delta R$  include four unknowns; thus, a unique solution for  $\Delta \epsilon$  (and  $\Delta m$ ) is not possible. However, for each expression, the first two terms, which depend on  $\Delta t_c$  and  $\Delta t_f$ , have relatively little impact on  $\Delta F$  and  $\Delta R$ . By allowing  $\Delta t_c$  and  $\Delta t_c'$  to range between  $\pm 10$  mils, which defines a  $\Delta t_f = (\Delta t_c + \Delta t_c')$  range, the corresponding  $\delta F/\delta t_c \cdot \Delta t_c$  and  $\delta F/\delta t_f \cdot \Delta t_f$  are small.\* As shown in Appendix B, we may write

$$\Delta F = \pm \sigma_{F\Delta t} + \delta F/\delta \epsilon \cdot \Delta \epsilon + \delta F/\delta m \cdot \Delta m$$

$$\Delta R = \pm \sigma_{R\Delta t} + \delta R/\delta \epsilon \cdot \Delta \epsilon + \delta R/\delta m \cdot \Delta m \quad (14)$$

where  $\sigma_{F\Delta t}$  and  $\sigma_{R\Delta t}$  are calculated over the range of  $\Delta t_c$  and  $\Delta t_f$  with equal weighting. These uncertainties are absorbed into the  $\Delta F$  and  $\Delta R$  measurement error, so that two equations ( $\Delta F$ ,  $\Delta R$ ) with two unknowns ( $\Delta \epsilon$ ,  $\Delta m$ ) result. Table 1 gives the resulting  $\Delta \epsilon$  and  $\Delta m$  solutions for Mark 14 tubes examined in this work.

\* Here,  $t_c'$  is the inner clad thickness defined in Figure 4. The tube thickness  $t = t_c' + t_f + t_c$  is assumed constant.

The above treatment is primarily useful for deviations from the design value. Absolute measurements of  $\epsilon$  and  $m$  can be obtained by proper use of the formulae in Section III.B.

#### IV. Experimental Tests

##### A. Instrumentation

The instrumentation is shown in Figures 5 and 6. The high-purity germanium (HPGe) detector, electronics, and multichannel analyzer/computer have been described elsewhere.<sup>9</sup> The collimator/shield was developed based on earlier tests,<sup>6,10</sup> and its dimensions are given in Figure 7. A lead plug is placed inside the tube during the measurements, to shield the detector from gamma rays from the far side of the tube.

The HPGe detector has resolution of  $\sim 1$  keV for the 186 keV and 238 keV  $\gamma$ -rays. The collimator has a  $1/4$ "-diameter entrance and a minimum of 3" of lead shielding for the detector. Previous studies indicate that this collimator should have little leakage for the above  $\gamma$ -rays.<sup>10</sup> The fuel tube is mounted on a device designed for the cladding thickness monitor,<sup>7</sup> allowing reliable positioning for counting.

A BASIC program for the MCA/computer was developed prior to arriving at the refined analysis of Section III, and analyzes  $\Delta\epsilon$  based on  $\Delta F$  alone. Table 1 shows that this yields a reasonable estimate of  $\Delta\epsilon$ , provided that  $\Delta R$  is  $\leq 5\%$ . The program plots the  $\gamma$ -peaks to confirm the window settings, which had negligible drift. Typical plots, a program listing, and user information are detailed in Appendix C.

A summary of costs for the instrument is given in Table 2. Here, the prototype monitor ( $\sim \$30K$ ) is compared with a final version ( $\sim \$50K$ ). A more sophisticated MCA/computer system constitutes the main increase in cost for the final system.

##### B. Measurements for Mark 14 Tubes

The  $\gamma$ -monitor appraised  $^{235}\text{U}$  in Mark 14 inner, middle, and outer tubes. A cross sectional diagram of the Mark 14 fuel assembly is given in Figure 8. The data were primarily analyzed using the formulae developed in Table 1. Average values for each tube were used as references for the fluctuations. Figure 9 shows the results for the  $\epsilon$ -measurements, while Figure 10 displays the  $m$ -measurements. Both  $\epsilon$  and  $m$  were measured as a function of position along the tubes.

The  $\epsilon$ -values were measured to a counting precision of  $\sigma = \pm 0.6\%$ , which agrees well with the standard deviation of the individual measurements. Each value was obtained with 2000 sec counting time. The on-line MCA/computer results for  $\epsilon$  agree quite well with the refined  $\epsilon$  calculations, as shown in Table 3.

The R correction had only a small effect because the observed  $\Delta R$  values of  $< 3\%$  have little impact on the  $\Delta\epsilon$  formulae of Table 1.

The m-values were measured to a counting precision of  $\sigma < \pm 1.0\%$ , but the sample deviations were as large as 3.0%. Although such deviations are typical,<sup>3,4</sup> future monitor improvements are underway to assure that none of this fluctuation is due to variations in counting geometry.

Absolute measurements for  $\epsilon$  and m were also examined, although the present studies were not optimized for these. The results are summarized in Table 4, and show that agreement between known and measured values is sufficiently good to be useful for estimating large fluctuation from design values. Better agreement can be obtained if actual component oxide samples are used to calibrate the individual tubes.

## V. Conclusions and Discussion

The prototype  $\gamma$ -monitor for  $^{235}\text{U}$  uniformity is suitable for routine appraisal of Mark 14 fuel tubes. The monitor is nondestructive, yields  $^{235}\text{U}$  enrichment ( $\epsilon$ ) fluctuation to  $\pm 0.6\%$ , is relatively fast, and avoids extensive accountability/handling requirements. In addition, the monitor yields  $^{235}\text{U}$  mass/area (m) fluctuations to  $\pm 1.0\%$ , and gives useful estimates of absolute values for  $\epsilon$  and m. These results correspond to 2000 sec counting times per tested area, meaning that a single tube can be appraised in about a day. Throughput can be enhanced by developing multiple monitors, each of which would cost  $\sim \$20\text{K}$  additional relative to the  $\sim \$50\text{K}$  cost of the initial monitor, which includes a computer. The R correction had only a small effect because the observed  $\Delta R$  values of  $< 3\%$  have little impact on the  $\Delta\epsilon$  formulae of Table 1.

The  $\gamma$ -monitor is also being adapted to handle  $^{235}\text{U}$  enrichment measurements for 1-gallon cans of uranium oxides<sup>9</sup> and fuel tube billets. Furthermore, the monitor can be used to measure high density "hot spots" of fissile material, as was done in an earlier appraisal of Mark 41 fuels.<sup>10</sup> In general, a variety of nondestructive fuel interrogation applications can evolve through the use of this  $\gamma$ -monitor.

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TABLE 1

## Analyses for Mark 14 Tubes

<u>Tube</u>	<u>Parameters</u>	<u>Analysis</u>
Inner	$\bar{\epsilon} = 65.1\%$	$\Delta\epsilon = 0.319\Delta F + 0.028\Delta R$
	$\bar{t}_f = 42 \text{ mil}, U_3O_8 = 62\%$	$\Delta m = -0.102\Delta F + 1.197\Delta R$
Middle	$\bar{\epsilon} = 60.3\%$	$\Delta\epsilon = 0.315\Delta F + 0.029\Delta R$
	$\bar{t}_f = 43 \text{ mil}, U_3O_8 = 62\%$	$\Delta m = -0.111\Delta F + 1.201\Delta R$
Outer	$\bar{\epsilon} = 44.0\%$	$\Delta\epsilon = 0.280\Delta F + 0.022\Delta R$
	$\bar{t}_f = 40 \text{ mil}, U_3O_8 = 60\%$	$\Delta m = 0.118\Delta F + 1.174\Delta R$

TABLE 2

## Instrumentation Costs

<u>Item</u>	<u>Prototype*</u>	<u>Final**</u>
HPGE detector	\$11,643	\$14,000
Electronics	3,145	4,000
Multichannel Annalyzer		9,000
Computer	8,190	10,000
Data Storage	(cassette)	4,000 (floppy disk)
Printer	660	3,300
Shielding (SRL-NED)	5,000 (est)	5,000 (est)
Total	\$28,638	\$49,300

Also recommended but not included in above costs:

- 1) High Quality Line Filter for Clean Computer Power
- 2) Temperature Control of Equipment - Local Air Conditioning
- 3) Automatic Tube Positioning Device to act as Sample Changer

\* Based on 1981 purchase

\*\* Estimated per 1983 prices.

TABLE 3

## Comparison of Enrichment Measurements

<u>Tube</u>	<u>Position</u> (in)	<u>Refined <math>\epsilon</math></u> ( $\epsilon\%$ )	<u>MCA/com <math>\epsilon^*</math></u> ( $\epsilon\%$ )	<u>Diff (M-R)</u> ( $\epsilon\%$ )
Inner	22	65.14±0.58	65.10±0.47	-0.04
	32	64.28±0.56	64.44±0.47	+0.16
	42	64.27±0.58	64.49±0.48	+0.22
	52	64.61±0.58	64.73±0.48	+0.12
	62	65.67±0.62	65.66±0.49	-0.01
	72	65.35±0.59	65.32±0.48	-0.03
	82	65.85±0.61	65.72±0.48	-0.13
	99	65.09±0.58	65.05±0.47	-0.04
	109	66.13±0.61	65.92±0.48	-0.21
	109	64.68±0.58	64.67±0.48	-0.01
	119	65.11±0.57	65.02±0.46	-0.09
	119	65.03±0.57	65.05±0.48	+0.02
Middle	23.5	59.73±0.58	59.73±0.49	0.00
	32.5	59.88±0.60	59.91±0.50	+0.03
	43.0	60.23±0.61	60.18±0.50	-0.05
	52.0	59.85±0.60	59.98±0.50	+0.03
	64.0	60.18±0.61	60.18±0.50	0.00
	72.5	60.98±0.63	60.84±0.51	-0.14
	81.5	60.70±0.63	60.65±0.51	-0.05
	81.5	60.28±0.50	60.29±0.48	+0.01
	92.0	60.86±0.83	60.89±0.68	+0.03
	92.0	60.32±0.50	60.40±0.48	+0.08
Outer	24	44.22±0.51	44.30±0.46	+0.08
	39	44.42±0.49	44.47±0.45	+0.05
	54	44.75±0.52	44.74±0.45	-0.01
	69.5	43.56±0.49	43.48±0.45	-0.08
	84.5	44.77±0.52	44.78±0.45	+0.01
	98.5	43.72±0.49	43.70±0.45	-0.02
	111.5	42.95±0.47	42.90±0.45	-0.05
	111.5	44.44±0.51	44.45±0.45	+0.01
	131.5	43.19±0.49	43.19±0.45	0.00

\* The  $\sigma$  for the monitor values  $\epsilon$  is slightly smaller than that of the refined  $\epsilon$ , because of (1) the  $\Delta R$  effect is excluded, (2) the error treatment of average  $\bar{F}$  is less conservative, and (3) the  $t_c$ ,  $t_f$  errors are neglected.

TABLE 4

Absolute  $^{235}\text{U}$  Measurement

(Note: All errors are due to counting statistics alone).

Enrichments  $\epsilon$  (e%)

<u>Sample</u>	<u>Known</u>	<u>Refined Analysis</u>	<u>MCA Analysis</u>
Outer tube	44.0	43.7±0.2	-
Tube Section	56	-	56.7±0.8
Middle Tube	60.3	60.5±0.2	-
Inner Tube	65.1	66.3±0.2	
$\text{U}_3\text{O}_8$ Powder	76.9	-	73.4±1.0

Mass/Area  $m$  ( $\text{gm}/\text{cm}^3 \cdot \text{e}\% \cdot \text{mil}$ )\*

<u>Sample</u>	<u>Known</u>	<u>- Detailed Analysis</u>
Outer tube	4792	4882±27
Middle tube	7465	7289±111
Outer tube	7872	7931±59

\* Known values of  $m = \frac{W_U}{W_U/\rho_U^* + W_A/\rho_U^*} \epsilon t_f$  obtained from design information. The analysis values are relative, but have been normalized to the average factor of known/analysis. (Note,  $m$  does not include theoretical density  $\chi$  correction, which may refine the agreement below the ~2% exhibited).

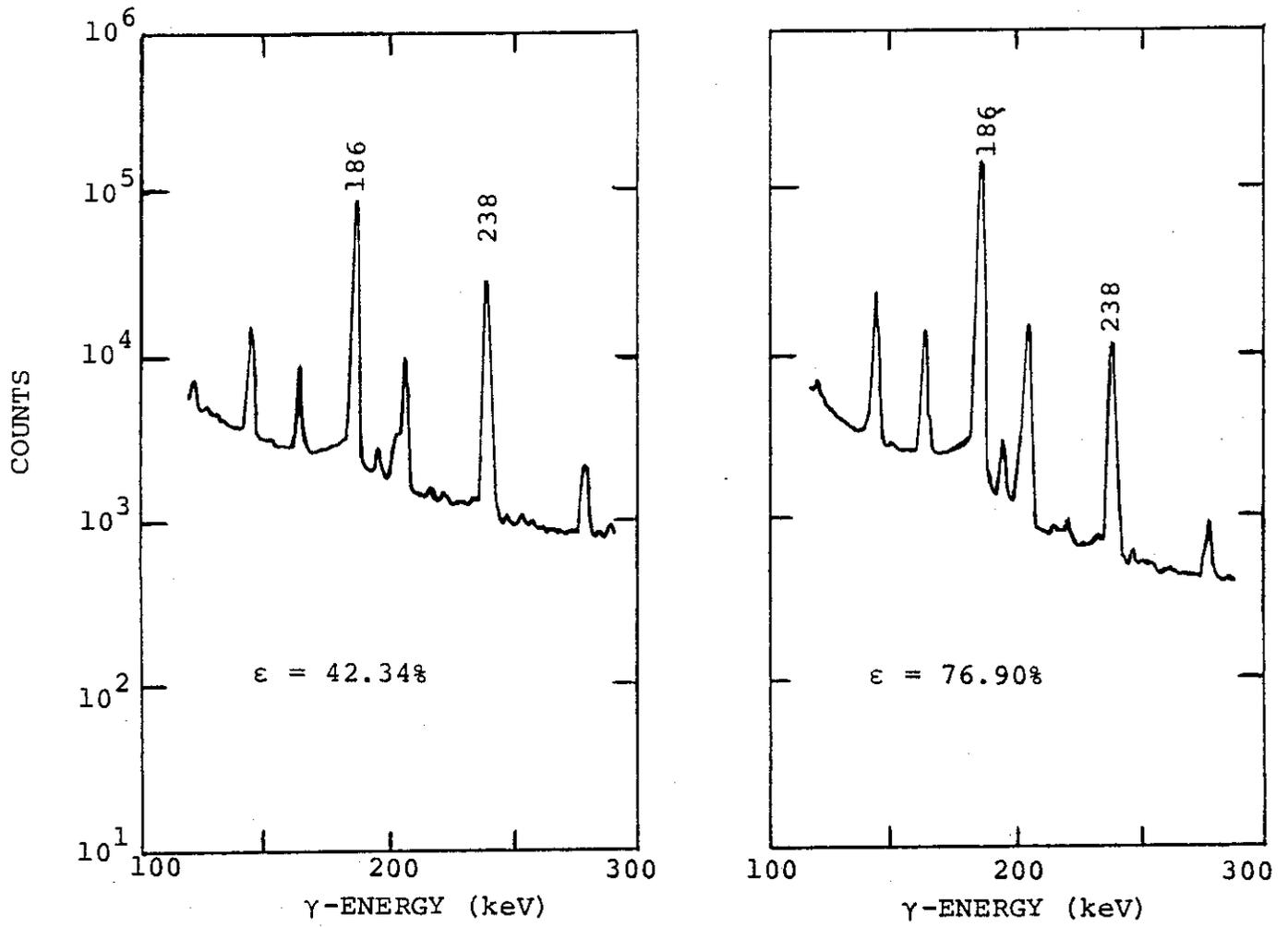


FIGURE 1.  $\gamma$ -Spectra of Typical  $U_3O_8$  for Blend Components

# U-232 vs U-235

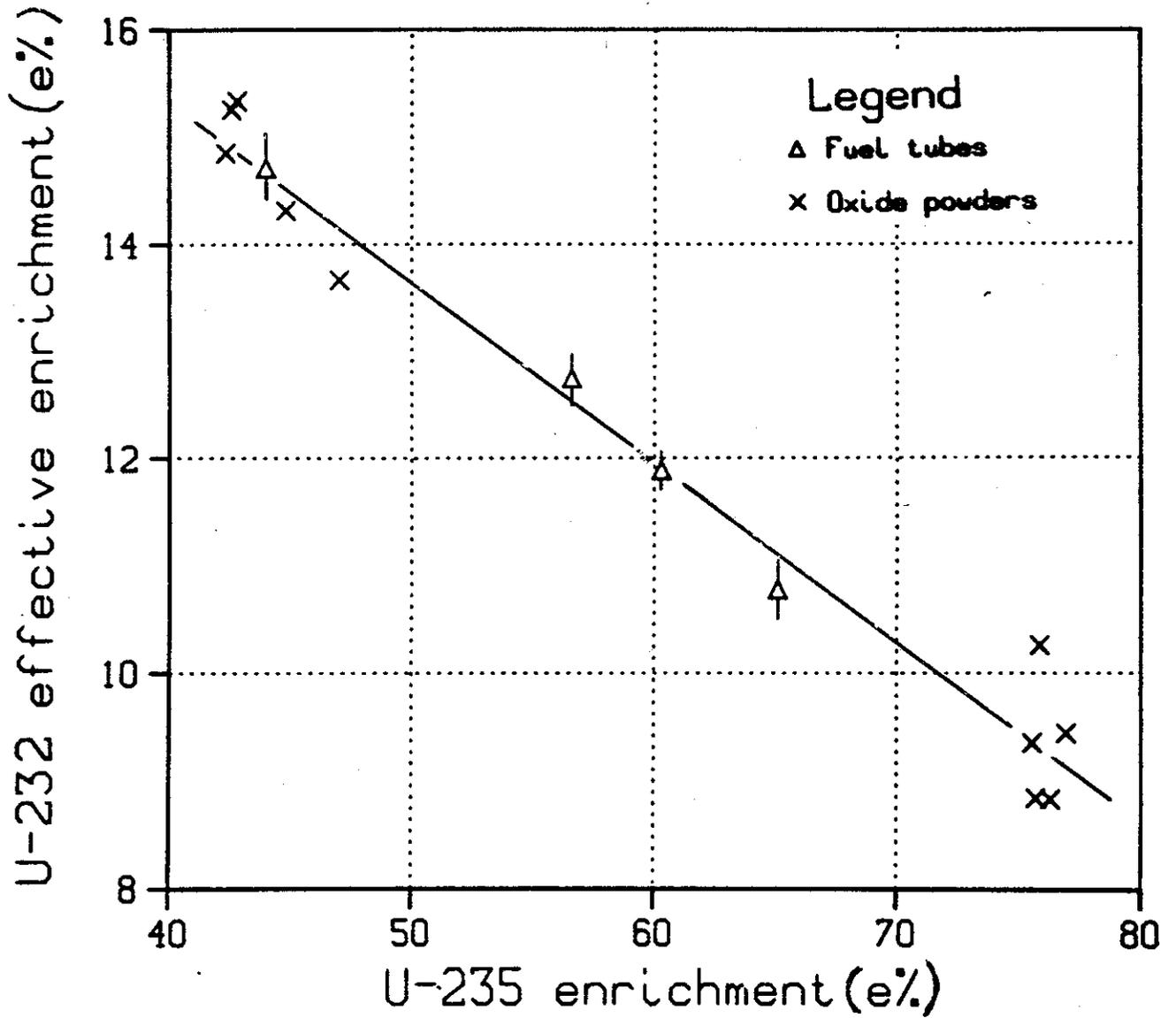


FIGURE 2. Correlation Between  $\epsilon'$  and  $\epsilon$

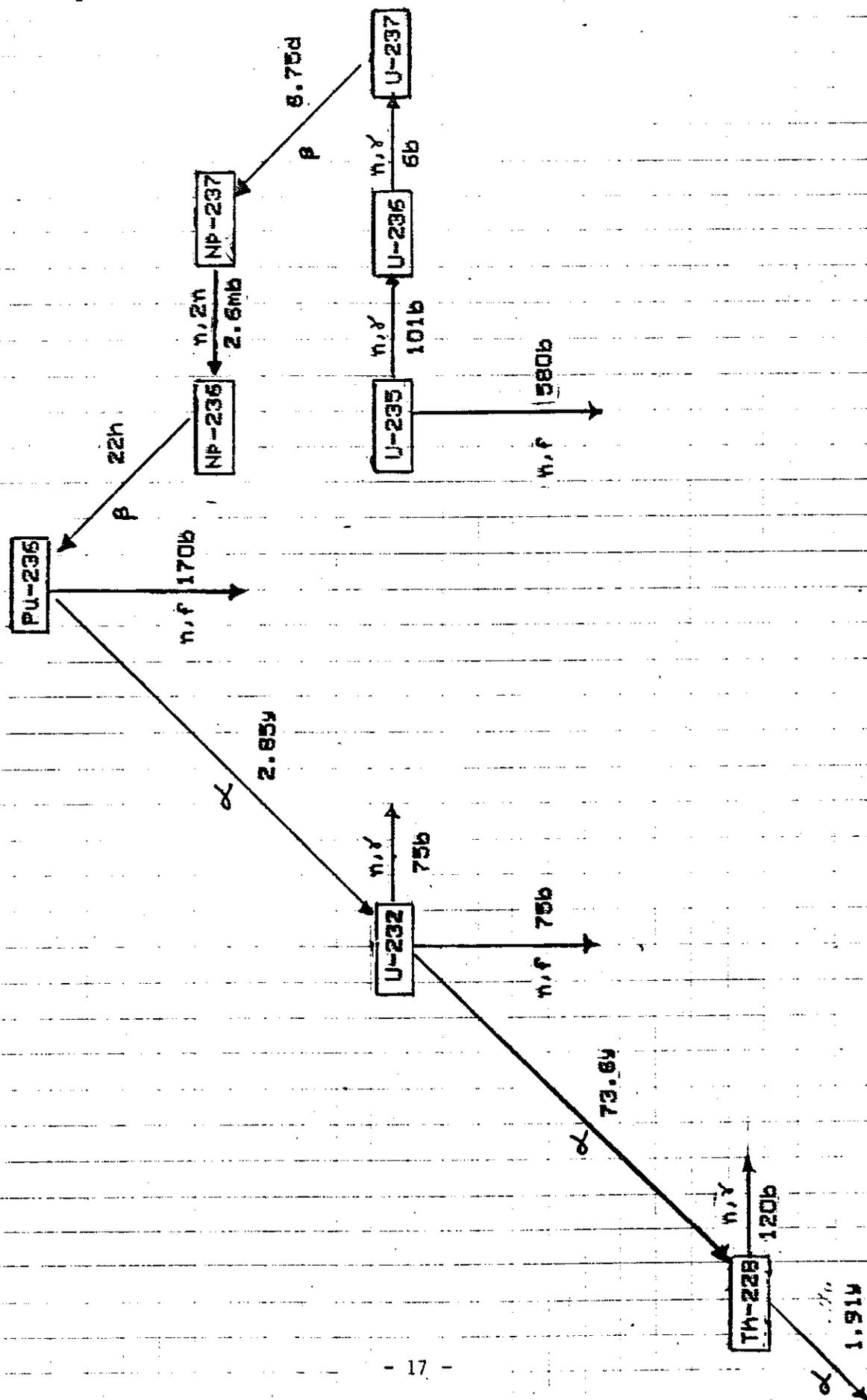


FIGURE 3. Production of U-232 per Burnup of U-235

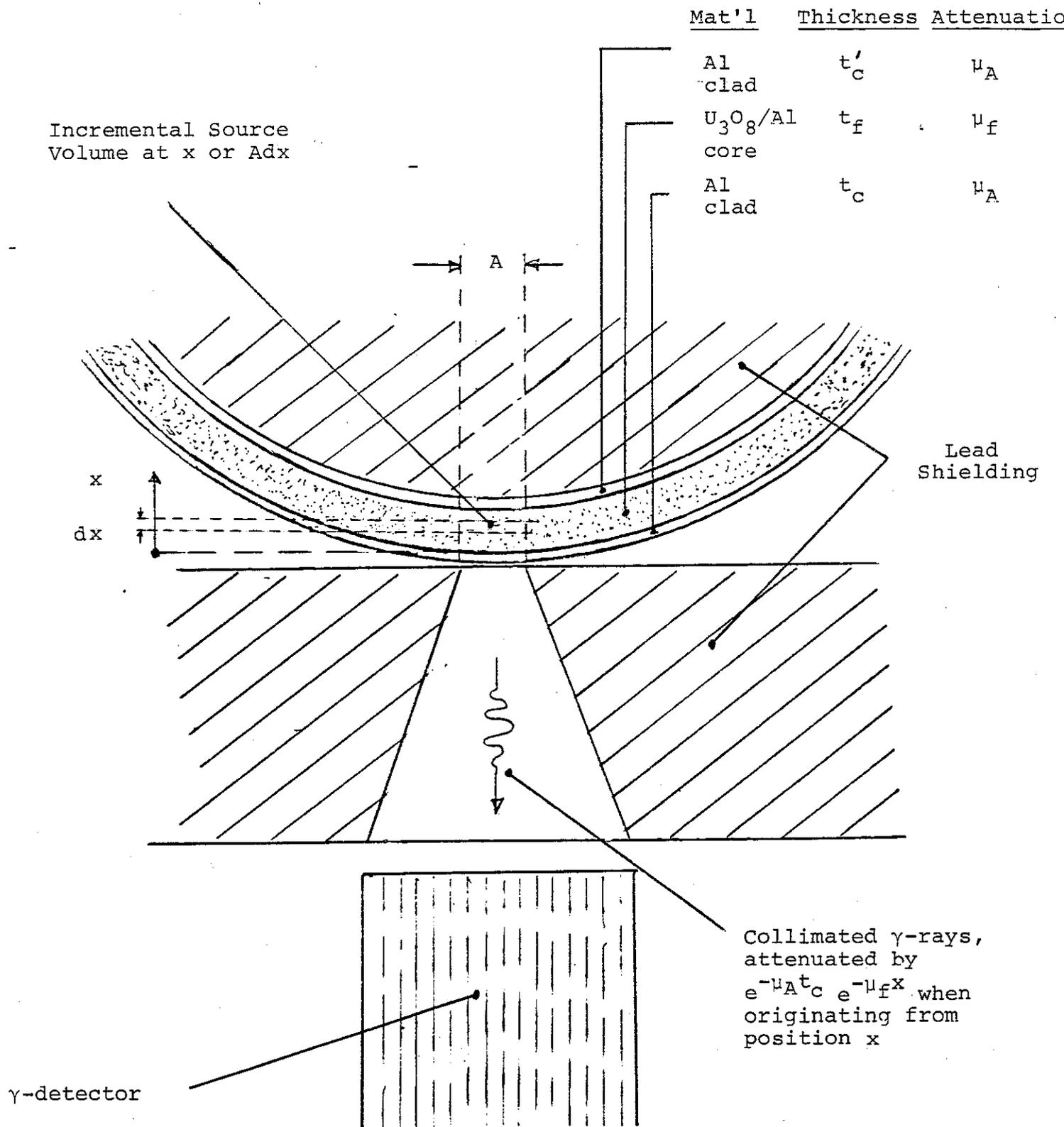


FIGURE 4. Geometry/Attenuation Effects on  $\gamma$ -Detection

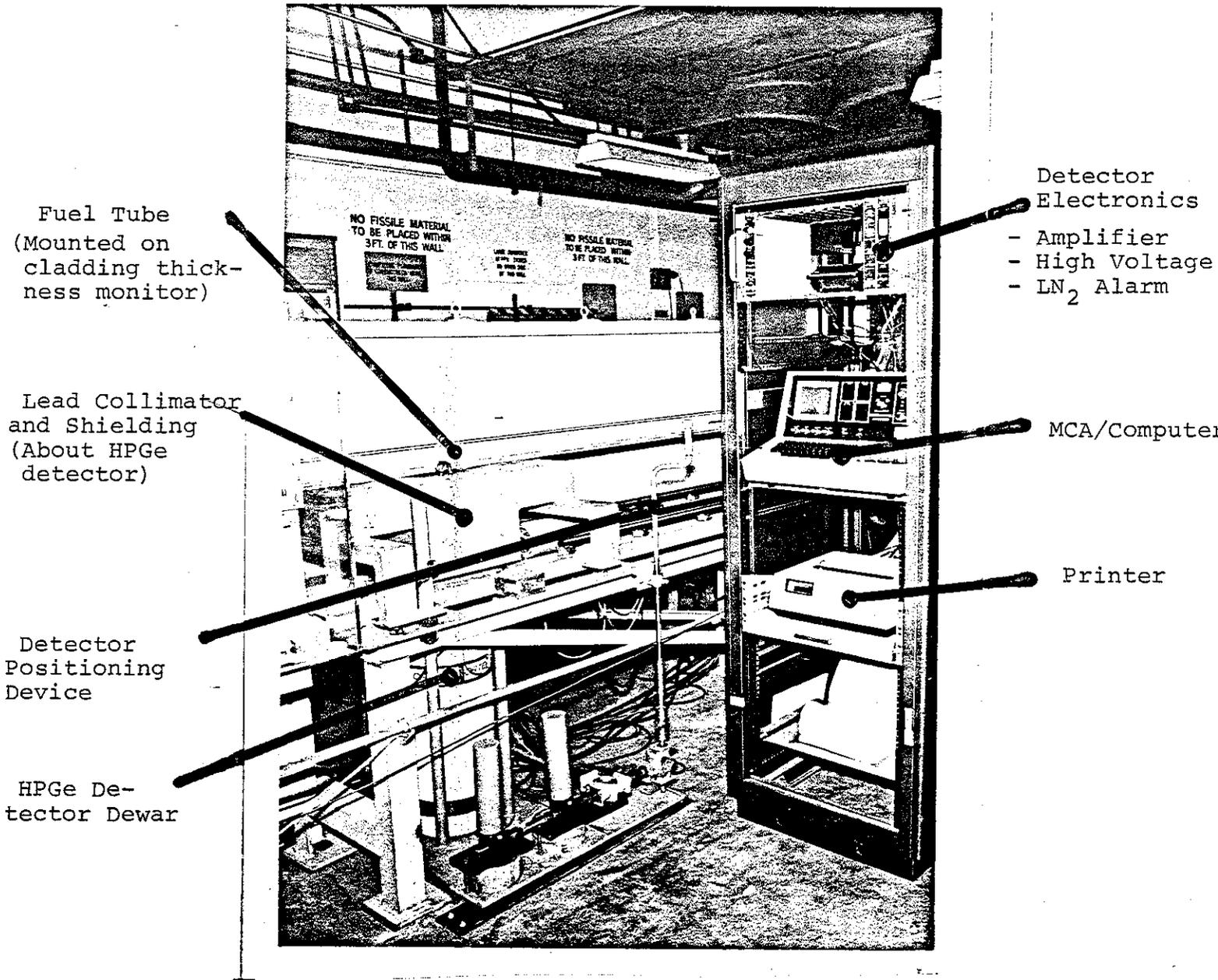


FIGURE 5.  $\gamma$ -Monitor Equipment Including

Lead Insert  
for Fuel Tube

Collimator  
Entrance

Lead Shielding  
for Detector

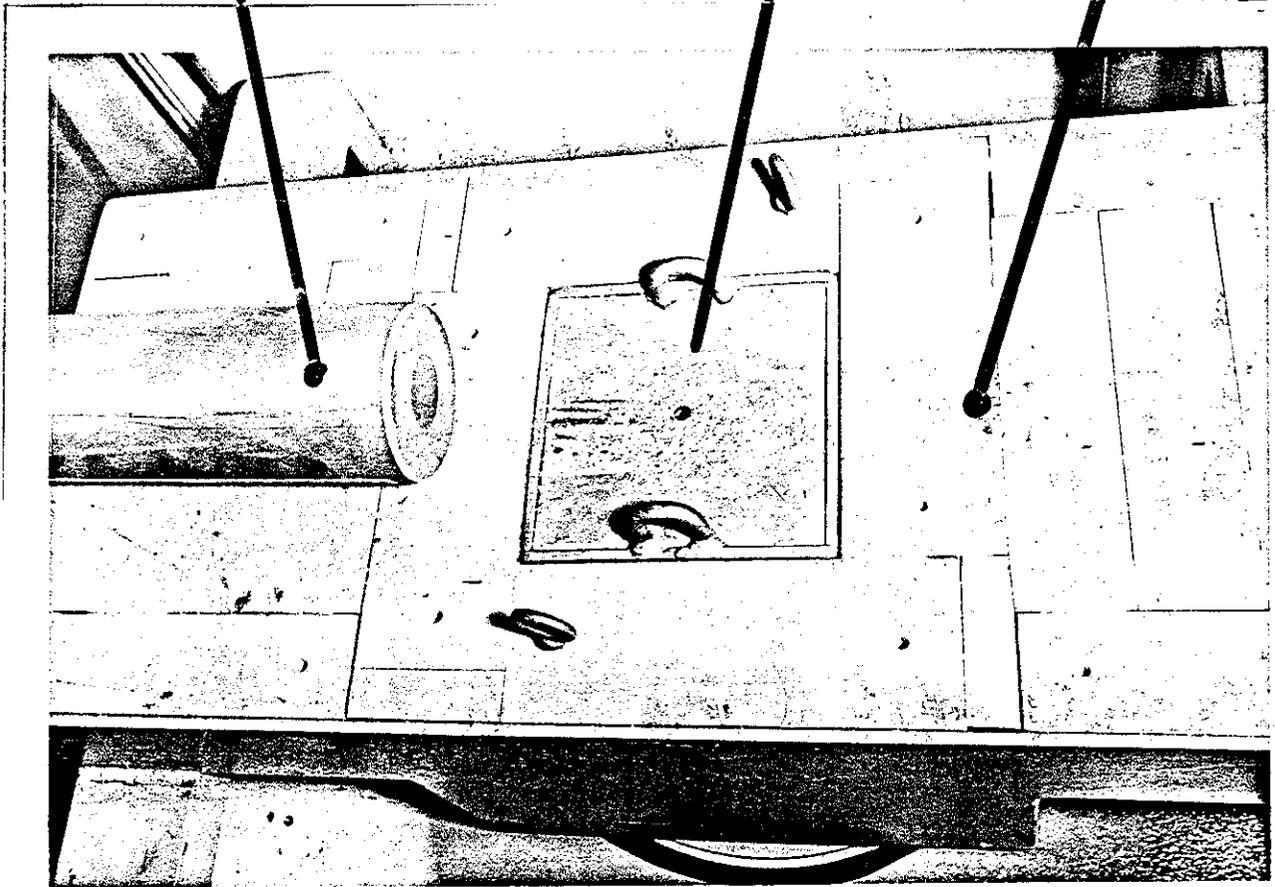


FIGURE 6. Collimator and Shielding

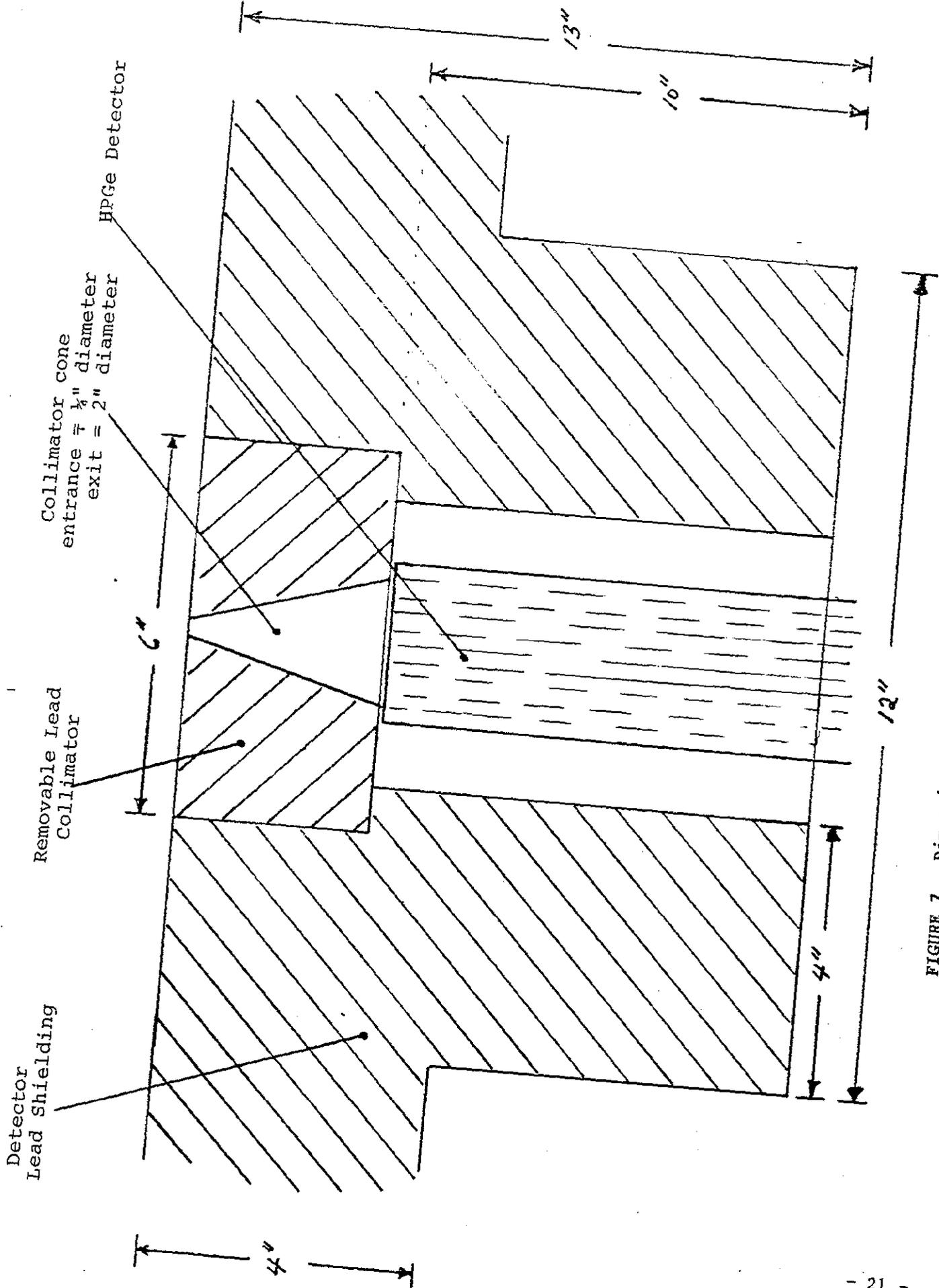


FIGURE 7. Dimensions of Collimator/Shield

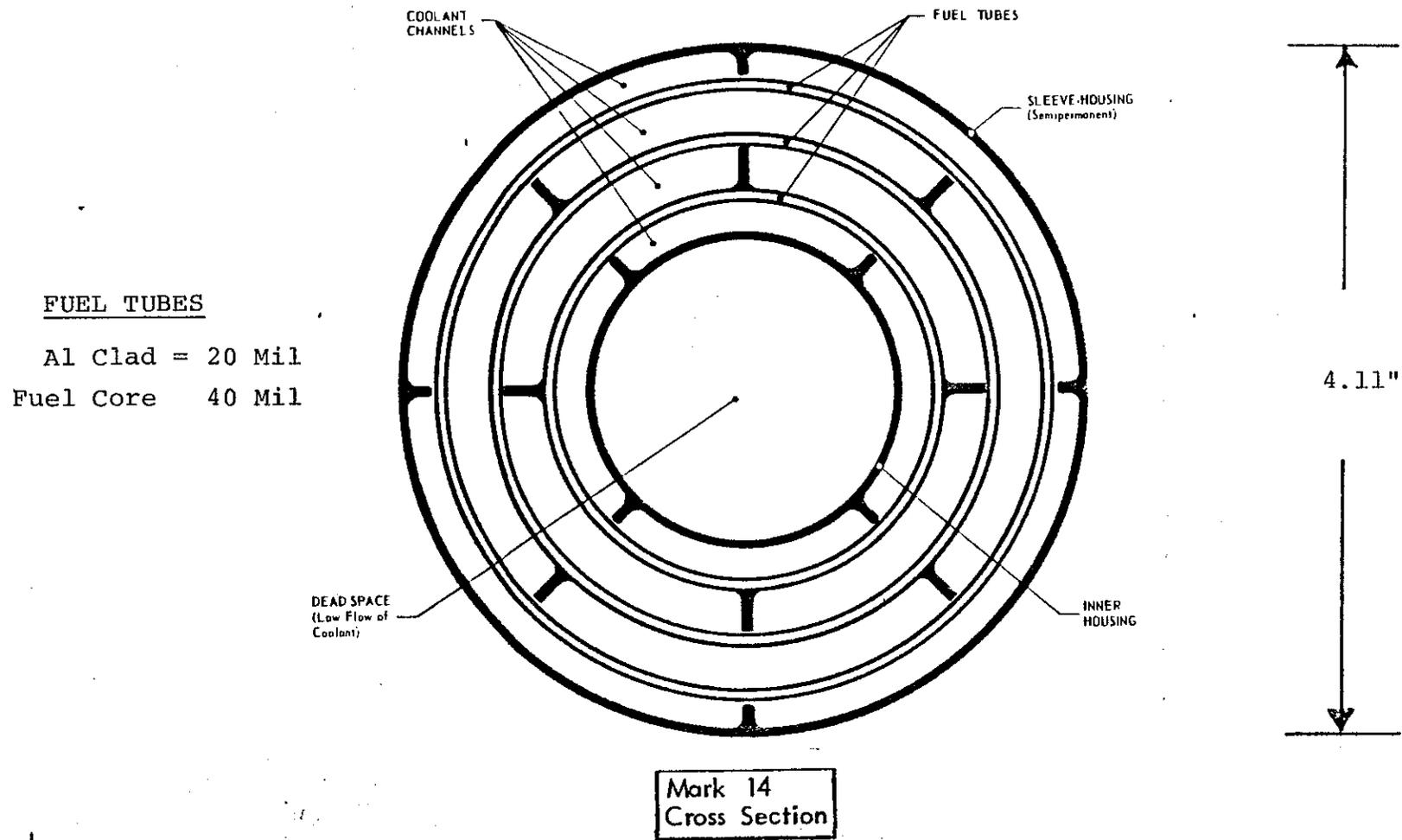


FIGURE 8. Mark 14 Fuel Assembly

# Measurement of e-Uniformity of Fuel

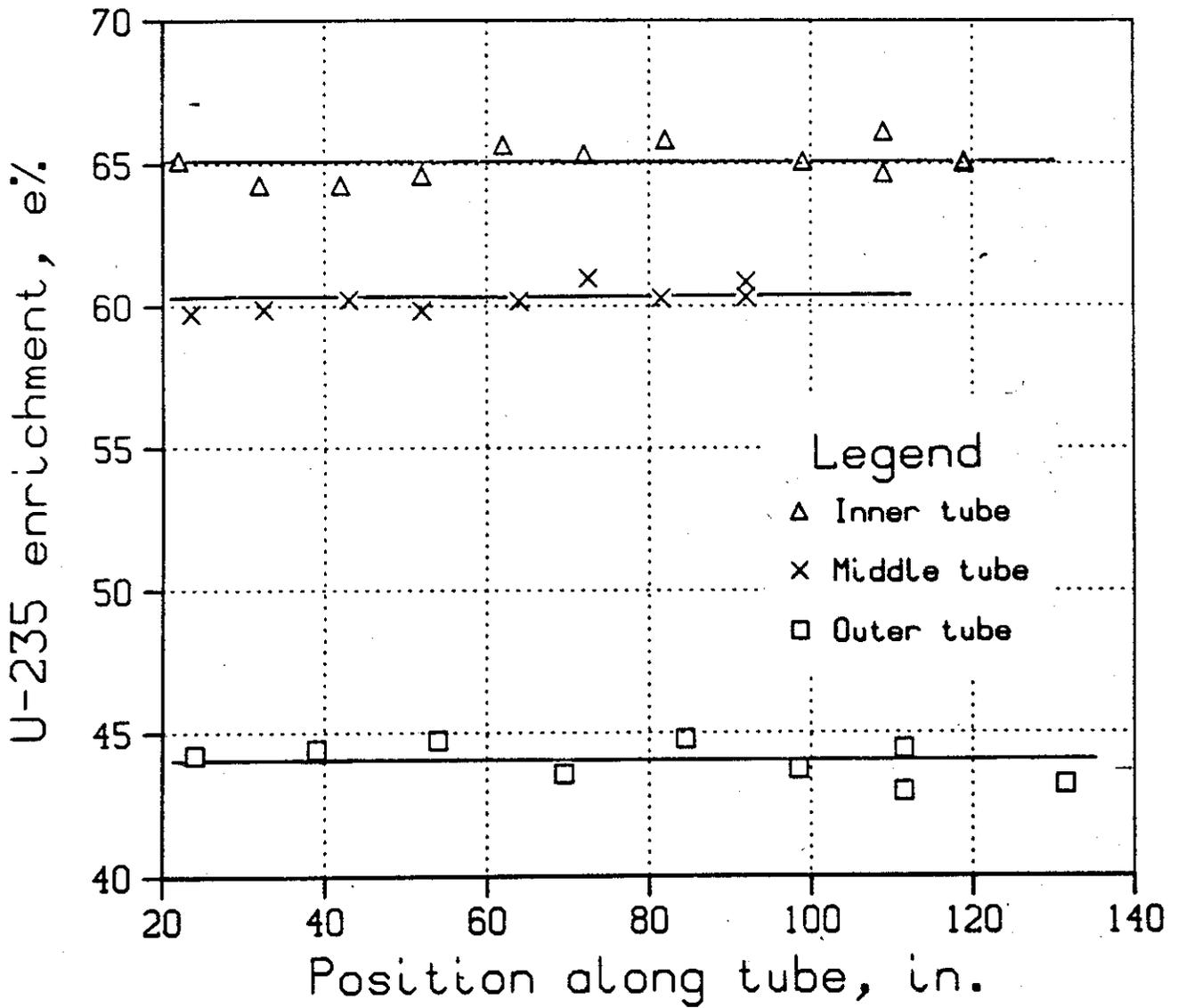


FIGURE 9.  $^{235}\text{U}$  Enrichment Vs. Tube Position

# Measurements of m-Uniformity of U-235

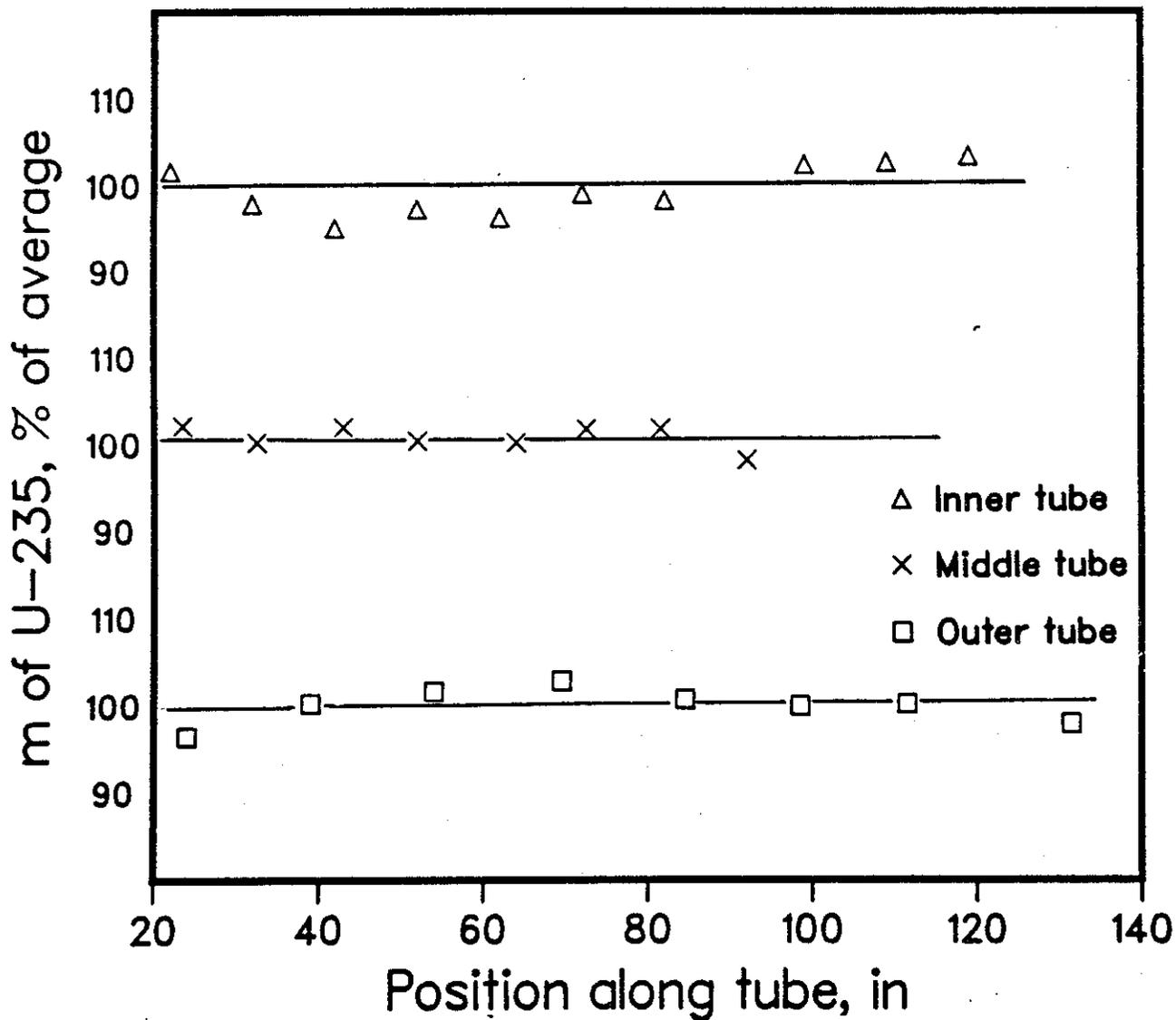


FIGURE 10.  $^{235}\text{U}$  Mass/Area Vs. Tube Position

APPENDIX A. ALTERNATIVE  $\gamma$ -MONITOR FOR  $^{235}\text{U}$  FUEL TUBES

A. Alternative  $\gamma$ -Monitor Methodology

In the event that some future uranium fuels are not amenable to  $^{235}\text{U}$  enrichment measurements via 186 $\gamma$ /238 $\gamma$  correlation described in the text, an alternative method is proposed. In addition to the basic  $\gamma$ -monitor, this method uses an external  $\gamma$ -source and the cladding thickness monitor.<sup>7</sup>

In essence, we need to obtain the attenuation correction factors for the 186 keV  $\gamma$ -rate, which is derived in the text (Equation 6) as

$$R = DA (\rho_U/\mu_f)\epsilon(1-e^{-\mu_f t})e^{-\mu_A t_c} \quad (\text{A-1})$$

Here, DA is constant for the fixed counting geometry. The factor  $(\rho_U/\mu_f)$  can vary; however, variations should be negligible per following reasoning. From the text discussion (Equation 7), we note that

$$\rho_U/\mu_f = \frac{W_U}{W_U g_U + W_A g_A}$$

$$\rho_U/\mu_f = (1.23 + 0.125 W_A/W_U)^{-1} \text{ cm}^2/\text{gm} \quad (\text{A-2})$$

Normal blending of uranium oxide and aluminum demonstrate good uniformity.<sup>11</sup> A conservative example with  $W_U = 60 \pm 2$  wt% and  $W_A = 40 \mp 2$  wt% yields  $\rho_U/\mu_f$  constant to  $\pm 0.5\%$ . For an enrichment of  $\epsilon = 60.0\text{e}\%$ , the corresponding error would be  $\leq 0.3\text{e}\%$ . Thus, this factor will be assumed constant.\* The remaining factors of (A-1) deal with  $\gamma$ -attenuation and these cannot be assumed constant.

\* Measurements for  $\mu_f$  can be obtained in this method, and these allow one to measure the constancy of  $\rho_U/\mu_f$ . For example, if we write the formulae for  $\rho_U$  and  $\mu_f$  (Equations 7), including the theoretical density  $\chi$  and using  $W_A = 100 - W_U$ , we have:

$$\rho_U = \frac{W_U}{W_U/\rho_U^* + (100 - W_U)/\rho_A^*} \chi$$

$$\mu_f = \frac{W_U g_U + (100 - W_U) g_A}{W_U/\rho_U^* + (100 - W_U)/\rho_A^*} \chi \quad (\text{A'-1})$$

The  $W_U$  can be eliminated between these two expressions to yield

$$\rho_U/\mu_f = \frac{\rho_U^*}{(\mu_U^* - \mu_A^*)} \left(1 - \frac{\mu_A^* \chi}{\mu_f}\right) \quad (A'-2)$$

where  $\mu_U^* = \rho_U^* g_U$  and  $\mu_A^* = \rho_A^* g_A$ . All starred terms are theoretical density values - which are constants of nature. The last factor governs the constancy of the expression, and it can vary only due to  $W_U$ , per equation (A'-1). For a typical case with the  $W_U = 60\%$ , and  $\chi = 0.9$ , we have

$$\left(1 - \mu_A^* \chi / \mu_f\right) = (1 - (0.338)(0.9)/3.22) = (1 - 0.094)$$

Thus, if  $\mu_f$  is measured to  $\pm 5\%$  (per attenuation data), then  $\rho_U/\mu_f$  constancy is measured to  $\pm 0.5\%$ , yielding  $\pm 0.3e\%$  at  $\epsilon = 60.0e\%$ .

---

An external  $\gamma$ -source is used in measuring the attenuation factors  $e^{-\mu_f t_f}$  and  $e^{-\hat{\mu}_A t_c}$ . This source must have  $\gamma$ -energy near (but not equal to) 186 keV. It is attached to the lead plug so that it is detected after transmission through the tube wall and collimator. The  $\gamma$ -source is counted in the following configurations:

- Unattenuated to yield rate  $R_{e0}$
- Attenuated by pure Al at tube ends for  $R_{eA}$
- Attenuated at fuel measurement area for  $R_{eF}$

The following ratios (refer to Figure 4) are governed by the attenuation factor indicated.

$$R_{eA}/R_{e0} = r_A = e^{-\hat{\mu}_A (t_c' + t_f + t_c)}$$

$$R_{eF}/R_{e0} = r_F = e^{-\hat{\mu}_A t_c' + -\hat{\mu}_f t_f - \hat{\mu}_A t_c}$$

$$R_{eF}/R_{eA} = r = e^{-(\hat{\mu}_f - \hat{\mu}_A) t_f} \quad (A-3)$$

We use  $\hat{\mu}$  in place of  $\mu$  to distinguish the external source  $\gamma$ -ray from the 186 keV  $\gamma$ -ray.

The cladding thickness monitor measures  $t_c'$ ,  $t_c$ , and  $t_f$  =  $t - t_c' - t_c$ , where  $t$  is the tube wall thickness. From this we obtain

$$e^{-\hat{\mu}_A t_c} = (r_A)^{t_c/t}$$

$$e^{-\hat{\mu}_A t_f} = (r_A)^{t_f/t}$$

$$e^{-\hat{\mu}_f t_f} = r (r_A)^{t_f/t}$$

(A-4)

Finally we write

$$e^{-\mu_A t_c} = (r_A)^{\mu_A t_c / \hat{\mu}_A t}$$

$$e^{-\mu_f t_f} = [r(r_A)^{t_f/t}]^{\mu_f / \hat{\mu}_f} \quad (A-5)$$

where we note that good estimates for  $\mu_A / \hat{\mu}_A$  and  $\mu_f / \hat{\mu}_f$  can be made since  $\mu \approx \hat{\mu}$  implies little change. Using (A-5) and the preceding observation, we write (A-1) as:

$$R = (\text{CONST}) \cdot \epsilon \cdot (1 - [r(r_A)^{t_f/t}]^{\mu_f / \hat{\mu}_f}) \cdot (r_A)^{\mu_A t_c / \hat{\mu}_A t} \quad (A-6)$$

The above equation for R is sensitive to the analysis values. The  $\mu / \hat{\mu}$  values should be close to 1 and estimated accurately. The last factor will also have a value close to 1 and the error can be made small with good counting statistics. For example, with  $t_c = 20$  mils and  $t = 80$  mils,  $(r_A)^{\mu_A t_c / \hat{\mu}_A t} \approx 0.98$ . The error for the other attenuation factor requires closer examination. For Mark 14 fuels, this factor  $(1 - [r(r_A)^{t_f/t}]^{\mu_f / \hat{\mu}_f})$  is  $(1 - 0.70) = 0.30$ , so that the %-error in the factor will be about double the %-error of the measurement for  $[r(r_A)^{t_f/t}]^{\mu_f / \hat{\mu}_f}$ . Sufficiently strong sources should be able to produce the required counting statistics; however, extreme care must be exercised, as the measurement must be accurate to  $<0.4\%$  to yield an enrichment accurate to  $0.5e\%$ . A pulser peak, counted with the spectra, would be necessary to assure that dead times are properly adjusted for all count rates.

Candidate external sources are  $^{57}\text{Co}$  (270d) with 122 keV  $\gamma$ -ray,  $^{139}\text{Ce}$  (140d) with 166 keV  $\gamma$ -ray,  $^{125}\text{Sb}$  (2.7y) with 176 keV  $\gamma$ -ray, and possibly  $^{228}\text{Th}$  (1.96y) with 238 keV  $\gamma$ -ray. Of these, the most ideal is  $^{125}\text{Sb}$ , due to its long half-life and a  $\gamma$ -energy in close proximity to 186 keV.

## B. Comparison of Proposed and Alternative Methods

In the proposed method R and  $F = R/R'$  are each a function of  $\epsilon$ ,  $m$ ,  $t_f$ , and  $t_c$ , and a differential analysis for the Mark 14 middle tube (which is representative) yielded

$$\Delta \epsilon = 0.315 \Delta F + 0.029 \Delta R \pm 0.054e\%$$

where the  $\pm$  term denotes the uncertainty of  $t_c$  and  $t_f$  due to  $\pm 10$  mils for inner and outer clad thickness. Furthermore, it was shown that for measurement of  $\sigma_F = 2\%$  and  $\sigma_R = 0.8\%$ , the  $\Delta \epsilon$  is measured to  $\sigma_\epsilon = 0.63e\%$ .

A similar analysis for the alternative method, using Equation (A-6) yields\*

$$\Delta\epsilon = 0.60\Delta R + 1.253\Delta r + 0.504\Delta r_A - 0.047 \Delta(t_f/t) - 0.490\Delta(\mu_f/\hat{\mu}_f) + 0.011\Delta(t_c/t) + 0.011 \Delta(\mu_A/\hat{\mu}_A)$$

If we assume comparable measurement precision for R (ie  $\sigma_R = 0.8\%$ ), then to get similar precision for  $\Delta\epsilon$  (i.e.  $\sigma_\epsilon = 0.63e\%$ ), the following measurement precisions are required:

- $\sigma_r < 0.33\%$ : External  $\gamma$ -source detection rate  $>50/\text{sec}$
- $\sigma_{r_A} < 0.81\%$ : External  $\gamma$ -source detection rate  $>10/\text{sec}$
- $\sigma_{t_f/t} < 8.68\%$ :  $t_f$  accurate to 4 mils
- $\sigma_{\mu_f/\hat{\mu}_f} < 0.83\%$ : Table interpolation accuracy
- $\sigma_{t_c/t} < 37.1\%$ :  $t_c$  accurate to 8 mils
- $\sigma_{\mu_A/\hat{\mu}_A} < 37.1\%$ : Table interpolation accuracy

The first two measurement requirements indicate that the external  $\gamma$ -ray detection rate must be about 10 times that of the 186 keV  $\gamma$ -ray, if counts are to be obtained in 2000 sec. Thus, reliable deadtime corrections must be assured; use of a pulser will probably be required. The  $t_f$  and  $t_c$  measurements are easily measured to less than  $\pm 1$  mil with cladding thickness monitor, so the required  $\sigma$  for these are easily met. The error in  $\mu_f/\hat{\mu}_f$  is not as severe as implied, because it will be systematic for all measurements and thus tends to cancel in making relative comparison of  $\epsilon$  for appraisal of uniformity. The required  $\sigma_{\mu_A/\hat{\mu}_A}$  is large and thus easily satisfied.

Of the two methods, the proposed one is preferred to the alternative one, because quality control requirements are much less severe. In fact, it might be worthwhile to spike the fuels with  $^{232}\text{U}$  or some other  $\gamma$ -source to assure availability of this method. Nevertheless, the alternative method, albeit more tedious, can be developed if needed.

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\* The  $\Delta F$  and  $\Delta R$  are % deviation;  $\Delta\epsilon$  is in e%.

APPENDIX B. DETAILED ANALYSIS FOR PROPOSED METHOD

The proposed method measures R and R' and then calculates F = R/R'. The complete expression for R (and a similar one for R') is given (see Section III of text) as

$$R = D \cdot A \cdot \frac{\rho_U}{\mu_f} \varepsilon (1 - e^{-\mu_f t_f}) e^{-\mu_A t_c}$$

with

$$\rho_U = \frac{W_U}{W_U/\rho_U^* + W_A/\rho_A^*} \chi$$

$$\mu_A = \rho_A^* g_A \tag{B-1}$$

$$\mu_f = \frac{W_U g_U + W_A g_A}{W_U/\rho_U^* + W_A/\rho_A^*} \chi = \mu_f^* \chi$$

In complete detail, we write

$$R = D \cdot A \cdot \varepsilon \left( \frac{W_U}{W_U g_U + W_A g_A} \right) e^{-\rho_A^* g_A t_c} (1 - e^{-\frac{(W_U g_U + W_A g_A)}{(W_U/\rho_U^* + W_A/\rho_A^*)} t_f \chi}) \tag{B-2}$$

A sensitivity analysis of expressions for R, R' and F, using the (B-2) format, constitutes the basis for the analysis.

Before proceeding with the sensitivity analysis, note that  $\chi$ , the theoretical density of the fuel core, is included in Equations (B-1) and (B-2). Typically  $\chi = 0.9$ , but  $\chi = 1$  was used in the present study. This causes no serious inconsistency because relative deviations about the average are used in the analysis. This fact is illustrated by examining the  $\chi$ -dependent factor  $(1 - e^{-\mu_f t_f}) = (1 - e^{-\mu_f^* t_f \chi})$  for  $\chi = 0.9$  and  $\chi = 1.0$ , using the Mark 14 middle tube data. For  $\chi = 0.9$ , and 1.0 a 1% variation in  $\mu_f t_f$  causes respective variations of 0.83% and 0.82% in R due to this factor. Similarly, 1% variation in  $\mu_f^* t_f$  yields respective variations of 0.90% and 0.89% in R'. Clearly, such effects on R, R', and F have very minor impact on uniformity appraisals based on  $\Delta R$  and  $\Delta F$ . Nevertheless, future work will incorporate the proper value of  $\chi$ .

Variations in F and R were examined with the BASIC program listed in Exhibit B.1. The functional forms examined were

$$R = (\text{CONST}) \epsilon \left( \frac{W_U}{W_U g_U + W_A g_A} \right) e^{-\rho_A g_A t_c} \left( 1 - e^{-\left( \frac{W_U g_U + W_A g_A}{W_U / \rho_U^* + W_A / \rho_A^*} \right) t_f} \right)$$

$$F = (\text{CONST}) \frac{\epsilon}{b\epsilon + c} \left( \frac{W_U g_U' + W_A g_A'}{W_U g_U + W_A g_A} \right) e^{-\rho_A (g_A - g_A') t_c} \frac{\left( 1 - e^{-\left( \frac{W_U g_U + W_A g_A}{W_U / \rho_U^* + W_A / \rho_A^*} \right) t_f} \right)}{\left( 1 - e^{-\left( \frac{W_U g_U' + W_A g_A'}{W_U / \rho_U^* + W_A / \rho_A^*} \right) t_f} \right)}$$

(B-3)

Both constants were set equal to 1, as relative deviations were examined. Using the relation  $W_A = 100 - W_U$  (in wt%), the expressions have functional forms of

$$R = R(\epsilon, W_U, t_f, t_c)$$

$$F = F(\epsilon, W_U, t_f, t_c)$$

(B-4)

All other parameters are constants. The BASIC code initially calculates the partial derivatives of the following differentials

$$\Delta R = \delta R / \delta t_c \cdot \Delta t_c + \delta R / \delta t_f \cdot \Delta t_f + \delta R / \delta \epsilon \cdot \Delta \epsilon + \delta R / \delta W_U \cdot \Delta W_U$$

$$\Delta F = \delta F / \delta t_c \cdot \Delta t_c + \delta F / \delta t_f \cdot \Delta t_f + \delta F / \delta \epsilon \cdot \Delta \epsilon + \delta F / \delta W_U \cdot \Delta W_U$$

(B-5)

Following this calculation, a transformation of variables is made so that  $m = {}^{235}\text{U}$  mass/area is represented. The relationship is

$$m = \frac{W_U}{W_U / \rho_U^* + W_A / \rho_A^*} \epsilon t_f \chi$$

where  $\chi$  is set to 1. In essence, we need to calculate the partial derivatives of

$$\Delta R = \delta R / \delta t_c \cdot \Delta t_c + \delta R / \delta t_f \cdot \Delta t_f + \delta R / \delta \epsilon \cdot \Delta \epsilon + \delta R / \delta m \cdot \Delta m$$

$$\Delta F = \delta F / \delta t_c \cdot \Delta t_c + \delta F / \delta t_f \cdot \Delta t_f + \delta F / \delta \epsilon \cdot \Delta \epsilon + \delta F / \delta m \cdot \Delta m$$

(B-7)

The transformation for the partial derivatives involves a change of variables  $\{A\} = \{t_c, t_f, \epsilon, W_U\}$  to a set  $\{B\} = \{t_c, t_f, \epsilon, m\}$ . The details for R partial derivatives are given as follows.

$$(\delta R / \delta t_c)_B = (\delta R / \delta t_c)_A \frac{(\delta t_c)_A}{(\delta t_c)_B} + \dots$$

$$(\delta R / \delta m)_B = (\delta R / \delta t_c)_A \frac{(\delta t_c)_A}{(\delta m)_B} + \dots$$

or in matrix notation:

$$\begin{pmatrix} \delta R / \delta t_c \\ \delta R / \delta t_f \\ \delta R / \delta \epsilon \\ \delta R / \delta m_{R/B} \end{pmatrix} = \begin{pmatrix} \delta t_c / \delta t_c & \delta t_f / \delta t_c & \delta \epsilon / \delta t_c & \delta W_U / \delta t_c \\ \delta t_c / \delta t_f & \delta t_f / \delta t_f & \delta \epsilon / \delta t_f & \delta W_U / \delta t_f \\ \delta t_c / \delta \epsilon & \delta t_f / \delta \epsilon & \delta \epsilon / \delta \epsilon & \delta W_U / \delta \epsilon \\ \delta t_c / \delta m & \delta t_f / \delta m & \delta \epsilon / \delta m & \delta W_U / \delta m_{A/B} \end{pmatrix} \begin{pmatrix} \delta R / \delta t_c \\ \delta R / \delta t_f \\ \delta R / \delta \epsilon \\ \delta R / \delta W_U / R/A \end{pmatrix}$$

$$(\delta R / \delta B) = (\delta A / \delta B) \cdot (\delta R / \delta A) \quad (B-8)$$

Per the transformation matrix, the {A} variables must be expressed in terms of the {B} variables, viz

$$\{A\} = f\{B\}$$

$$t_c = t_c \quad (B-9)$$

$$t_f = t_f$$

$$\epsilon = \epsilon$$

$$W_U = 100 / (1 + \epsilon t_f \rho_{A^*} / m - \rho_{A^*} / \rho_{U^*})$$

where the expression for  $W_U$  is derived from (B-6), with  $\chi = 1$ .  
The resulting transformation matrix is

$$(\delta A / \delta B) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -W_U^2 \left( \frac{\epsilon \rho_{A^*}}{m} \right) / 100 \\ 0 & 0 & 1 & -W_U^2 \left( \frac{t_f \rho_{A^*}}{m} \right) / 100 \\ 0 & 0 & 0 & \left( \frac{W_U}{m} \right)^2 \epsilon t_f \rho_{A^*} / 100 \end{pmatrix} \quad (B-10)$$

When the  $(\delta A / \delta B)$  matrix is calculated for the Mark 14 tubes and applied to the four calculated  $(\delta R / \delta A)$  partial derivatives to solve for (B-7), we obtain  $\Delta R$  and similarly  $\Delta F$ . The BASIC results are given in Exhibit B.2, and we will discuss the results for the Mark 14 middle tubes, for the sake of illustration. For this case,

$$\Delta R = -0.084 \Delta t_c - 0.040 \Delta t_f + 0.292 \Delta \epsilon + 0.825 \Delta m \quad (B-11)$$

$$\Delta F = -0.009 \Delta t_c - 0.003 \Delta t_f + 3.150 \Delta \epsilon - 0.075 \Delta m$$

where all  $\Delta$ 's are in %, except for  $\Delta \epsilon$  which is in e%. Letting  $t_c = 20 \pm 10$  mils,  $t_c' = 20 \pm 10$  mils and  $t_f = t - t_c - t_c'$ , the code defines %-changes in the first two terms. Upon calculating a standard deviation corresponding to these variations (evenly weighted), the first two terms are replaced by

$$\pm \sigma_{R\Delta t} = \pm 0.361\% \quad (B-12)$$

$$\pm \sigma_{F\Delta t} = \pm 0.040\%$$

Thus we may write

$$\Delta R \pm 0.361\% = 0.292 \Delta \epsilon + 0.825 \Delta m \quad (B-13)$$

$$\Delta F \pm 0.040\% = 3.150 \Delta \epsilon - 0.075 \Delta m$$

upon solving these equations for  $\Delta \epsilon$  and  $\Delta m$ , the code obtains

$$\Delta \epsilon = 0.315 \Delta F + 0.029 \Delta R \pm 0.016e\% \quad (B-14)$$

$$\Delta m = -0.111 \Delta F + 1.201 \Delta R \pm 0.434\%$$

Upon combining these results with measurement error of 2% in F and 0.8% in R, the corresponding error in  $\Delta \epsilon$  is  $\pm 0.63e\%$  and that of  $\Delta m$  is  $\pm 1.08\%$ . Actual measurements were better, as the corresponding list of code calculations indicate.

Similar results for the Mark 14 inner and outer tubes are given in Exhibit B.2. All errors and coefficients are about the same for each tube studied.

EXHIBIT B.1 BASIC Program for Proposed Method

EXHIBIT B.1 BASIC PROGRAM FOR PROPOSED METHOD

```

40 SP$="      "
50 DIM AB(4,4)
60 FOR I=1 TO 4:FOR J=1 TO 4:AB(I,J)=0:NEXTJ:NEXTI
70 AB(1,1)=1:AB(2,2)=1:AB(3,3)=1
80 FOR I=1 TO 4:FOR J=1 TO 4:PRINT AB(I,J):NEXT J:PRINT:NEXTI
85 INPUT ZZ
90 MC=2.54/1000:REM MICRONS-TO-CM
100 REM-BASIC PHYSICAL DATA
105 PA=2.702:REM THEO GM/CC AL
110 PU=8.30:REM THEO GM/CC U308
115 LAK(1)=0.125:REM-AL CM2/GM 186 KEY ATTEN
120 LAK(2)=0.112:REM-                238
125 LUK(1)=(1.43*(842.09-8*16)+0.125*(8*16))/842.09:REM U308 CM2/GM 186 KEY ATTEN
130 LUK(2)=(0.80*(842.09-8*16)+0.116*(8*16))/842.09:REM                238
135 AR(1)=1:AR(2)=1:REM REL APERTURE
200 CLS:REM-CALCULATION MENU
210 PRINT"CALCULATION MENU":PRINT
220 PRINT"  (1) SENSITIVITY EXAMINATION - ALL PARAMETERS"
230 PRINT"  (2) MEASUREMENT ACCURACY -ENRICHMENT AND U-235"
240 PRINT"  (3) FAMILIES OF EN VS GAMMA RATIO"
250 PRINT"  (4) FAMILIES OF U-235 VS GAMMA RATE"
260 PRINT"  (5) EN VS U-235 FAMILIES"
270 PRINT:INPUT"OPTION":OP
275 IF OP=2 THEN DT$="Y":OP=1
280 ON OP GOSUB 400
290 GOTO 200
400 REM - CALCULATION FOR ALL PARAMETER SENSITIVITIES
405 LPRINT"VARIABLES TESTED FOR SENSITIVITY"
410 LPRINT
412 GOSUB 10000:REM-ADDITIONAL INPUTS
415 LPRINT"VARIABLE","MIN","MAX","STEP"
420 INPUT"MAX CLAD THICKNESS (MIL)":TC(2)
425 INPUT"MIN CLAD THICKNESS (MIL)":TC(1)
430 INPUT"STEP (MIL)":TC(0)
435 LPRINT"TC(MIL)",TC(1),TC(2),TC(0):PRINT
440 INPUT"MAX FUEL THICKNESS (MIL)":TF(2)
450 INPUT"MIN FUEL THICKNESS (MIL)":TF(1)
460 INPUT"STEP (MIL)":TF(0)
470 LPRINT"TF(MIL)",TF(1),TF(2),TF(0):PRINT
475 INPUT"MAX ENRICHMENT (%)":ER(2)
480 INPUT"MIN ENRICHMENT (%)":ER(1)
485 INPUT"STEP (%)":ER(0)
495 INPUT"ENRICHMENT MODEL - (B)EST ESTIMATE/(C)ONSERVATIVE":BC$:PRINT
500 LPRINT"EN(%)",ER(1),ER(2),ER(0):" ENRICHMENT MODEL ":BC$
505 PRINT
510 INPUT"MAX U308 WT FRACTION (%)":WU(2)
515 INPUT"MIN U308 WT FRACTION (%)":WU(1)
520 INPUT"STEP (%)":WU(0)
525 LPRINT"WU(%)",WU(1),WU(2),WU(0)
526 LPRINT:LPRINT

```

```

530 REM-CALCULATION LOOPS
531 LPRINT" TC TF WU EN JF/JTC JF/JTF JF/JEN JF/JMI JR/JTC JR/JTF JR/
JEN JR/JMI"
532 LPRINT" MIL MIL WT% ER% %/MIL %/MIL %/ER% %/MI %/MIL %/MIL %/
ER% %/MI"
533 LPRINT
534 GOSUB 11000:REM ADDITIONAL TABLE ENTRIES
535 FOR TK=TC(1) TO TC(2) STEP TC(0)
540 FOR TM=TF(1) TO TF(2) STEP TF(0)
545 FOR MU=WU(1) TO WU(2) STEP WU(0)
550 FOR ER=ER(1) TO ER(2) STEP ER(0)
555 TC=TK*MC:TF=TM*MC
560 GOSUB 2000:REM PARTIAL DERIVS WRT A(TC,TF,EN,WU)
565 GOSUB 2500:REM PARTIAL DERIVS WRT B(TC,TF,EN,MI)
570 LPRINT USING "####":TK;TM;WU;ER;
575 LPRINT USING "####.###";100*MC*(FB(1)/F0);100*MC*(FB(2)/F0);100*(FB(3)/F0);M
H*(FB(4)/F0);
580 LPRINT USING "####.###";100*MC*(RB(1)/R0);100*MC*(RB(2)/R0);100*(RB(3)/R0);M
H*(RB(4)/R0)
585 GOSUB 5000:REM TC TO' ERRORS IN F AND R
590 GOSUB 6000:REM ERRORS IN EN & MI PER TC TO' ERRORS
595 GOSUB 7000:REM ERRORS IN EN & MI FOR TYP EXP ERRORS
600 NEXT ER
610 NEXT MU
620 NEXT TM
630 NEXT TK
640 RETURN
1000 REM-SUB TO CALCULATE RG(1) AND RG(2)
1010 FOR I=1 TO 2
1020 WA=100-WU:REM - NOT NECESSARY TO DIVIDE BY 100
1030 RG(I)=(EN(I)*AR(I)*(WU/(WU*LU(I)+WA*LAC(I)))*EXP(-PA*LAC(I)*TC)*(1-EXP(-(WU*
LU(I)+WA*LAC(I))/(WU/PU+WA/PA))*TF))
1040 NEXT I
1050 RETURN
2000 REM - PARTIAL DERIVITIVES WRT A(TC,TF,EN,WU)
2010 WU=MU:EN=ER:TC=TK*MC:TF=TM*MC:GOSUB 4000:REM ENRICHMENT FORMULAE
2020 GOSUB 1000
2030 R0=RG(1):F0=RG(1)/RG(2)
2040 WU=MU*(1.01):EN=ER:TC=TK*MC:TF=TM*MC:GOSUB 4000
2050 GOSUB 1000
2060 R1=RG(1):F1=RG(1)/RG(2)
2070 RAC(4)=(R1-R0)/(0.01*MU):FAC(4)=(F1-F0)/(0.01*MU)
2080 WU=MU:EN=ER*(1.01):TC=TK*MC:TF=TM*MC:GOSUB 4000
2085 GOSUB 1000
2090 R1=RG(1):F1=RG(1)/RG(2)
2100 RAC(3)=(R1-R0)/(0.01*ER):FAC(3)=(F1-F0)/(0.01*ER)
2110 EN=ER:TC=TK*MC*(3.0):TF=TM*MC:WU=MU:GOSUB 4000
2120 GOSUB 1000
2130 R1=RG(1):F1=RG(1)/RG(2)
2140 RAC(1)=(R1-R0)/(2.0*TK*MC):FAC(1)=(F1-F0)/(2.0*TK*MC)
2150 TC=TK*MC:TF=TM*MC*(1.01):EN=ER:WU=MU:GOSUB 4000
2160 GOSUB 1000
2170 R1=RG(1):F1=RG(1)/RG(2)
2180 RAC(2)=(R1-R0)/(0.01*TM*MC):FAC(2)=(F1-F0)/(0.01*TM*MC)
2185 TF=TM*MC
2190 RETURN

```

## EXHIBIT B.1 (CONT'D)

```

2500 REM-PARTIAL DERIVATIVES WRT B(TC,TF,EN,MI)
2510 WU=MU:EN=ER:TF=MC*TM
2520 MI=WU*TF*EN/(WU/PU+(100-WU)/PA):MH=MI
2540 TF=1.01*MC*TM:GOSUB 2700
2550 AB(2,4)=(WU-MU)/(0.01*MC*TM)
2560 TF=MC*TM:EN=1.01*ER:GOSUB 2700
2570 AB(3,4)=(WU-MU)/(0.01*ER)
2575 EN=ER:MI=1.01*MH:GOSUB 2700
2577 AB(4,4)=(WU-MU)/(0.01*MH)
2580 MI=MH
2585 WU=MU
2600 FOR I=1 TO 4
2610 FB(I)=0:RB(I)=0
2620 FOR J=1 TO 4
2630 FB(I)=FB(I)+AB(I,J)*FR(J)
2640 RB(I)=RB(I)+AB(I,J)*RR(J)
2650 NEXT J
2660 NEXT I
2670 GOTO 2800
2700 WU=-100/(PA/PU-1-TF*EN*PA/MI):RETURN
2800 RETURN
3000 REM- SOLVE FOR DW AND DE
3010 DD=RW*FE-FW*RE
3020 DW=(DR*FE-DF*RE)/DD
3030 DE=(RW*DF-FW*DR)/DD
3040 REM - U235 CONTENT AND ERROR BELOW
3050 U0=MU*ER
3060 U1=(MU+DW)*(ER+DE)
3070 DU=U1-U0
3080 RETURN
4000 REM - ENRICHMENT EN(I) COEFF CALCS
4010 EN(1)=EN:EN(2)=20:REM-EN(2) CONSTANT FOR CONSERVATIVE CALC
4020 IF BC$="B" THEN EN(2)=21.7-0.161*EN
4030 REM-ABOVE LINE BEST ESTIMATE - SEE P 40 OF DATA NOTES
4500 RETURN
5000 REM-CALC OF TC TC' VARIATIONS IN F, AND EN
5005 TQ=TK
5010 CS(1)=100*MC*FB(1)/F0:F0(1)=100*MC*FB(2)/F0
5020 CS(2)=100*MC*RB(1)/R0:F0(2)=100*MC*RB(2)/R0
5030 N=(2*CE+1)*(2*KE+1)
5040 FOR I=1 TO 2
5050 SD=0:MX=0
5060 FOR TC=-CE TO CE
5070 FOR TK=-KE TO KE
5080 ET=CS(I)*TC-F0(I)*(TC+TK)
5090 IF ABS(ET)>MX THEN MX=ABS(ET)
5100 SD=SD+ET*2
5110 NEXT TK
5120 NEXT TC
5130 SD=SQR(SD/N)
5140 MD(I)=MX:SD(I)=SD
5150 NEXT I
5160 LPRINT SP$;:LPRINT USING"####.###";MD(1);SD(1);:LPRINT SP$;:LPRINT USING"##
###.###";MD(2);SD(2)
5165 TK=TQ
5170 RETURN

```

EXHIBIT B.1 (CONT'D)

```

6000 REM-CALC OF EN MI VARIATION DUE TO TC TC'
6010 FE=100*FB(3)/F0:FM=MH*FB(4)/F0:RE=100*RB(3)/R0:RM=MH*RB(4)/R0
6020 QQ=ER:REM-TO RETAIN ER ORIGINAL VALUE
6030 DD=FE*RM-RE*FM
6040 EF=RM/DD:ER=-FM/DD:MF=-RE/DD:MR=FE/DD
6050 DE(1)=ABS(EF*MD(1))+ABS(ER*MD(2)):REM MAX EN DEV
6060 DE(2)=SQRT((EF*SD(1))2+(ER*SD(2))2):REM SD EN
6070 DM(1)=ABS(MF*MD(1))+ABS(MR*MD(2)):REM MAX MI DEV
6080 DM(2)=SQRT((MF*SD(1))2+(MR*SD(2))2):REM SD MI
6090 LPRINT SP$;LPRINT USING"####.###";DE(1);DE(2);LPRINT SP$;LPRINT USING"###
###.###";DM(1);DM(2)
6100 ER=QQ
6110 RETURN
7000 REM-SUB WITH TYPICAL EXP ERRORS INCLUDED
7010 SD(1)=SQRT(SD(1)2+XFC2)
7020 SD(2)=SQRT(SD(2)2+XRC2)
7030 MD(1)=0:MD(2)=0
7040 GOSUB 6000
7045 IF DT#="Y" THEN GOSUB 7100
7050 RETURN
7100 REM - SUB FOR DATA CALCULATIONS
7102 SD(1)=SQRT(SD(1)2-XFC2)
7104 SD(2)=SQRT(SD(2)2-XRC2)
7110 LPRINT:LPRINT"DATA CALCULATION MATRICIES":LPRINT
7120 LPRINT" FE      FM      ";LPRINT USING "####.###";FE;FM
7130 LPRINT" RE      RM      ";LPRINT USING "####.###";RE;RM
7140 LPRINT
7150 LPRINT" EF      ER      ";LPRINT USING "####.###";RM/DD;-FM/DD
7160 LPRINT" MF      MR      ";LPRINT USING "####.###";-RE/DD;FE/DD
7170 LPRINT:LPRINT"CALCULATIONS BELOW":LPRINT
7180 LPRINT SP$;" DEL F  ";" SIG F  ";" DEL R  ";" SIG R  ";" EN  ";" +/- EN
";" MI  ";" +/- MI  "
7190 LPRINT SP$;" %  ";" %  ";" %  ";" %  ";" EN %  ";"
";" % AVE  "
7200 LPRINT
7210 INPUT"DELTA F";YF
7220 INPUT"SIGMA F";XF:PRINT
7230 INPUT"DELTA R";YR
7240 INPUT"SIGMA R";XR:PRINT:PRINT
7250 YE=(RM*YF-FM*YR)/DD:EN=ER+YE
7260 YM=(-RE*YF+FE*YR)/DD:MA=100+YM
7270 XF=SQRT(SD(1)2+XFC2)
7280 XR=SQRT(SD(2)2+XRC2)
7290 SE=SQRT((RM*XF)2+(FM*XR)2)/ABS(DD)
7300 SM=SQRT((RE*XF)2+(FE*XR)2)/ABS(DD)
7310 LPRINT SP$;LPRINT USING "####.###";YF;XF;YR;XR;EN;SE;MA;SM
7320 GOTO 7210
7330 RETURN
8000 LPRINT:GOTO 8000
9000 PRINT 100*(1-EXP(-.333*2.54*2/1000))/2

```

EXHIBIT B.1 (CONT'D)

```

10000 REM-ADDITIONAL INPUTS
10010 PRINT
10020 INPUT"+/- OUTER CLAD VARIATION (MIL)";CE
10030 INPUT"+/- INNER CLAD VARIATION (MIL)";KE
10040 LPRINT"+/- OUTER CLAD VARIATION (MIL) = ";CE
10050 LPRINT"+/- INNER CLAD VARIATION (MIL) = ";KE
10060 INPUT"EXPERIMENTAL SIGMA OF RATIO F (%)";XF
10070 INPUT"EXPERIMENTAL SIGMA OF RATE R (%)";XR
10080 LPRINT"EXPER SIGMA OF F (%) = ";XF
10090 LPRINT"EXPER SIGMA OF R (%) = ";XR
10100 LPRINT
10110 RETURN
11000 REM-ADDED TABLE HEADINGS
11010 LPRINT SP#;" MAX DF SIG DF";SP#;" MAX DR SIG DR"
11020 LPRINT SP#;" +/- %F +/- %F";SP#;" +/- %R +/- %R"
11030 LPRINT
11040 LPRINT SP#;" MAX DE SIG DE";SP#;" MAX DM SIG DM"
11050 LPRINT SP#;" +/- eE% +/- E%";SP#;" +/- %M +/- %M"
11060 LPRINT
11070 LPRINT SP#;" EXP TP>>SIG DE";SP#;" EXP TP>>SIG DM"
11080 LPRINT SP#;" +/- E%";SP#;" +/- %M"
11090 LPRINT
11100 RETURN

```

EXHIBIT B.2 Sensitivity Calculation for Mark 14 Tubes

EXHIBIT B.2

SENSITIVITY CALCULATION FOR MARK 14 TUBES

VARIABLES TESTED FOR SENSITIVITY

INNER TUBE

+/- OUTER CLAD VARIATION (MIL) = 10  
 +/- INNER CLAD VARIATION (MIL) = 10  
 EXPER SIGMA OF F (%) = 2  
 EXPER SIGMA OF R (%) = .8

VARIABLE	MIN	MAX	STEP
TC(MIL)	20	20	10
TF(MIL)	42	42	1
EN(%)	65.1	65.1	1 ENRICHMENT MODEL B-
WU(%)	62	62	1

TC MIL	TF MIL	WU WT%	EN ER%	JF/JTC %/MIL	JF/JTF %/MIL	JF/JEN %/ER%	JF/JMI %/MI	JR/JTC %/MIL	JR/JTF %/MIL	JR/JEN %/ER%	JR/JMI %/MI
				MAX DF	SIG DF			MAX DR	SIG DR		
				+/- %F	+/- %F			+/- %R	+/- %R		
				MAX DE	SIG DE			MAX DM	SIG DM		
				+/- eE%	+/- E%			+/- %M	+/- %M		
				EXP TP>>SIG DE				EXP TP>>SIG DM			
				+/- E%				+/- %M			
20	42	62	65	-0.009	-0.003	3.112	-0.073	-0.084	-0.040	0.264	0.829
				0.009	0.040			0.843	0.361		
				0.052	0.016			1.018	0.433		
				0.000	0.639			0.000	1.070		

DATA CALCULATION MATRICIES

FE	FM	3.112	-0.073
RE	RM	0.264	0.829
EF	ER	0.319	0.028
MF	MR	-0.102	1.197

CALCULATIONS BELOW

X POSITION IN	DEL F %	SIG F %	DEL R %	SIG R %	EN EN %	+/- EN	MI % AVE	+/- MI
109	-1.517	1.822	2.066	0.651	64.675	0.582	102.627	0.801
22	-0.010	1.822	1.404	0.649	65.136	0.582	101.682	0.799
32	-2.394	1.755	-2.012	0.642	64.280	0.560	97.835	0.789
42	-2.212	1.805	-4.350	0.637	64.272	0.576	95.018	0.784
52	-1.334	1.822	-2.442	0.642	64.606	0.582	97.213	0.791
62	2.060	1.938	-2.975	0.641	65.673	0.619	96.230	0.792
72	0.835	1.855	-0.745	0.644	65.345	0.592	99.023	0.794
82	2.474	1.921	-1.244	0.644	65.854	0.613	98.259	0.795
99	-0.192	1.805	1.954	0.649	65.094	0.576	102.358	0.798
109	3.037	1.921	1.987	0.649	66.125	0.613	102.069	0.801
119	-0.291	1.788	3.603	0.653	65.109	0.571	104.342	0.803
119	-0.457	1.788	2.755	0.653	65.032	0.571	103.344	0.803

VARIABLES TESTED FOR SENSITIVITY MIDDLE TUBE

+/- OUTER CLAD VARIATION (MIL) = 10  
 +/- INNER CLAD VARIATION (MIL) = 10  
 EXPER SIGMA OF F (%) = 2  
 EXPER SIGMA OF R (%) = .8

VARIABLE	MIN -	MAX	STEP
TC(MIL)	20	20	10
TF(MIL)	43	43	10
EN(%)	60.3	60.3	1 ENRICHMENT MODEL B
WU(%)	62	62	1

TC MIL	TF MIL	WU WT%	EN ER%	JF/JTC %/MIL	JF/JTF %/MIL	JF/JEN %/ER%	JF/JMI %/MI	JR/JTC %/MIL	JR/JTF %/MIL	JR/JEN %/ER%	JR/JMI %/MI
				MAX DF	SIG DF			MAX DR	SIG DR		
				+/- %F	+/- %F			+/- %R	+/- %R		
				MAX DE	SIG DE			MAX DM	SIG DM		
				+/- eE%	+/- E%			+/- %M	+/- %M		
				EXP TP>>SIG DE				EXP TP>>SIG DM			
				+/- E%				+/- %M			
20	43	62	60	-0.009	-0.003	3.150	-0.075	-0.084	-0.040	0.292	0.825
				0.089	0.040			0.843	0.361		
				0.052	0.016			1.023	0.434		
				0.000	0.630			0.000	1.078		

DATA CALCULATION MATRICIES

FE	FM	3.150	-0.075
RE	RM	0.292	0.825
EF	ER	0.315	0.029
MF	MR	-0.111	1.201

CALCULATIONS BELOW

X POSITION IN	DEL F %	SIG F %	DEL R %	SIG R %	EN - EN %	+/- EN	MI % AVE	+/- MI
23.5	-1.962	1.853	1.571	0.719	59.727	0.584	102.106	0.888
32.5	-1.331	1.913	0.052	0.714	59.882	0.603	100.211	0.884
43.0	-0.384	1.932	1.611	0.719	60.225	0.609	101.978	0.890
52.0	-1.449	1.913	0.207	0.717	59.850	0.603	100.410	0.887
64.0	-0.384	1.932	0.144	0.714	60.183	0.609	100.216	0.885
72.5	2.001	1.992	1.676	0.719	60.978	0.628	101.791	0.891
81.5	1.311	2.011	-0.448	0.714	60.700	0.634	99.316	0.887
92.0	2.199	2.622	-4.813	0.821	60.855	0.826	93.973	1.028
92.0	0.197	1.575	-1.505	0.673	60.319	0.496	98.170	0.828
81.5	-0.197	1.575	1.505	0.682	60.281	0.496	101.830	0.838

OUTER TUBE

VARIABLES TESTED FOR SENSITIVITY

+/- OUTER CLAD VARIATION (MIL) = 10  
 +/- INNER CLAD VARIATION (MIL) = 10  
 EXPER SIGMA OF F (%) = 2  
 EXPER SIGMA OF R (%) = .8

VARIABLE	MIN	MAX	STEP
TC(MIL)	20	20	10
TF(MIL)	40	40	10
EN(%)	44	44	1 ENRICHMENT MODEL B
WU(%)	60	60	10

TC MIL	TF MIL	WU WT%	EN ER%	JF/JTC %/MIL	JF/JTF %/MIL	JF/JEN %/ER%	JF/JMI %/MI	JR/JTC %/MIL	JR/JTF %/MIL	JR/JEN %/ER%	JR/JMI %/MI
				MAX DF	SIG DF			MAX DR	SIG DR		
				+/- %F	+/- %F			+/- %R	+/- %R		
				MAX DE	SIG DE			MAX DM	SIG DM		
				+/- eE%	+/- E%			+/- %M	+/- %M		
				EXP TP>>>SIG DE				EXP TP>>>SIG DM			
				+/- E%				+/- %M			
20	40	60	44	-0.009	-0.003	3.542	-0.067	-0.004	-0.040	0.355	0.845
				0.089	0.040			0.843	0.362		
				0.044	0.014			1.000	0.424		
				0.000	0.561			0.000	1.057		

DATA CALCULATION MATRICIES

FE	FM	3.542	-0.067
RE	RM	0.355	0.845
EF	ER	0.280	0.022
MF	MR	-0.118	1.174

CALCULATIONS BELOW

% POSITION IN	DEL F %	SIG F %	DEL R %	SIG R %	EN EN %	+/- EN	MI % AVE	+/- MI
24.0	1.006	1.805	-2.977	0.827	44.216	0.506	96.387	0.994
54.0	2.544	1.838	1.638	0.843	44.749	0.515	101.623	1.013
84.5	2.678	1.838	0.890	0.840	44.770	0.515	100.729	1.009
111.5	-3.740	1.671	-0.261	0.840	42.947	0.468	100.134	1.005
111.5	1.541	1.805	0.326	0.840	44.439	0.506	100.201	1.008
98.5	-0.999	1.738	-0.129	0.837	43.717	0.487	99.966	1.004
69.5	-1.768	1.738	2.263	0.840	43.555	0.487	102.865	1.007
131.5	-2.737	1.738	-2.080	0.831	43.187	0.487	97.881	0.996
39.0	1.474	1.738	0.330	0.840	44.420	0.487	100.214	1.007

## APPENDIX C. ON-LINE ANALYSIS WITH MCA COMPUTER

The BASIC code for the Nucleus MCA/Computer is shown in Exhibit C.1. It does not use the proposed (F,R) analysis of Appendix B, as it only uses the F measurement. However, because  $\Delta\epsilon$  is only weakly dependent on  $\Delta R$ , the MCA code is quite useful for preliminary on-line results. (Actually, no significant differences in results were experienced for the two codes, as shown in Table 3). Nevertheless, the MCA code could be extended to handle the (R,F) analysis if desired.

The MCA code uses the formalism developed in the earlier study of this method.<sup>5</sup> This code assumes measurement data on the component oxides are available for instrument calibration. Then individual measurements are accumulated to obtain an average  $F = \bar{F}$  for the tube. Several measurements are required to define  $\bar{F}$  before individual enrichments may be deduced. An example set of measurements is given in Exhibit C.2, as a guide to the potential user.

Upon running the MCA BASIC program, the computer requests inputs for the 186 keV and 238 keV  $\gamma$ -ray energy windows. A spectrum is then collected using an oxide sample or fuel tube as a source. A plot of the windows allows the user to appraise the acceptability of the window settings. After setting the windows, the component oxide information is input. Count data for the components may be collected with the MCA or entered manually if available from earlier measurements (which was done in the Exhibit C.2 example). After the calibration, tube measurements commence.

Tube measurements can proceed in automatic mode after the first count is set up. Also, data from earlier measurements can be included for averaging F, as illustrated in Exhibit C.2. After each count measurement, the window regions are plotted, and peak areas and errors are calculated. One then selects the analysis option, which normally combines the present measurement with previous ones for averaging a new  $\bar{F}$ , and then calculates the enrichments. The individual F values are also printed. One also has the option to repeat and/or eliminate suspicious measurements, as well as add data from earlier measurements. In fact, the code can be used to reanalyze any set of measurements, provided that the corresponding peak counts are available as input.

The method measures deviations about the average enrichment predicted by the component oxides, and thus is best for small fluctuations. However, given a well established average, reasonably accurate absolute measurements for large fluctuations are also possible. For example, using an average  $\bar{F}$  based on 10 Mark 14 inner tube measurements ( $\epsilon = 65.1e\%$ ), a sample tube section with  $\epsilon = 56.6e\%$  was measured to have  $\epsilon = 56.7 \pm 0.8e\%$ . Using an  $\bar{F}$  based on only 4 outer tube measurements ( $\epsilon = 44.0e\%$ ), an oxide sample of known  $\epsilon = 76.9e\%$  was measured as  $\epsilon = 73.4 \pm 1.0e\%$ . Absolute  $\epsilon$ -measured can be improved with code modification if desired.

EXHIBIT C.1 MCA BASIC Code of Monitor

EXHIBIT C.1 MCA BASIC CODE OF MONITOR

```

10 DIM U(30),DU(30),T(30),DT(30)
20 DIM M(2,32,2),N(2,32)
30 DIM R(30),ER(30),DE(30)
99 REM - SETUP GAMMA WINDOWS FROM LINES 100-999
100 PRINT"SPAN GAMMA PEAKS WITH EVEN NUMBER OF CHANNELS":PRINT
110 PRINT:PRINT"186 KEV GAMMA"
120 INPUT"LOWER CHANNEL";CU(3)
130 INPUT"UPPER CHANNEL";CU(4):UH=CU(4)-CU(3)+1
140 IF(UH/2<>)INT(UH/2)THEN PRINT"NOT EVEN NUMBER OF CHANNELS":GOTO 120
145 UH=UH/2
150 CU(2)=CU(3)-2:CU(1)=CU(2)-UH+1
160 CU(5)=CU(4)+2:CU(6)=CU(5)+UH-1
170 PRINT:PRINT"237 KEV GAMMA"
180 INPUT"LOWER CHANNEL";CT(3)
190 INPUT"HIGHER CHANNEL";CT(4)
195 TH=CT(4)-CT(3)+1
200 IF TH/2<>)INT(TH/2) THEN PRINT"NOT EVEN NUMBER OF CHANNELS":GOTO 180
205 TH=TH/2
210 CT(2)=CT(3)-2:CT(1)=CT(2)-TH+1
220 CT(5)=CT(4)+2:CT(6)=CT(5)+TH-1
230 INPUT"PUT CURSOR IN CHANNEL 0 - THEN HIT 1 RETURN";ZZ
240 M=USR(47):M=USR(49)
250 FOR I=0TOCU(1)-1
260 M=USR(45)
270 NEXT I
280 M=USR(47):M=USR(47):M=USR(48)
290 FOR I=CU(1) TO CU(6)
300 M=USR(45)
310 NEXT I
320 M=USR(47):M=USR(47):M=USR(49)
330 FOR I = CU(6)+1 TO CT(1)-1
340 M=USR(45)
350 NEXT I
360 M=USR(47):M=USR(47):M=USR(48)
370 FOR I=CT(1) TO CT(6)
380 M=USR(45)
390 NEXT I
400 M=USR(47):M=USR(47):M=USR(49)
410 FOR I=CT(6)+1 TO 2047
420 M=USR(45)
430 NEXT I
440 M=USR(47)
450 INPUT "PEAKS IN WINDOW - THEN HIT 1 AND RETURN FOR PRINT CHECK":ZZ
460 PRINT
470 PRINT "186 KEV GAMMA":PRINT
480 FOR I=1TO6:CC(I)=CU(I):NEXT:GOSUB 4200
495 PRINT:PRINT"237 KEV GAMMA":PRINT
490 FOR I=1TO6:CC(I)=CT(I):NEXT:GOSUB 4200
510 PRINT:PRINT:PRINT"CALIBS OK ?":PRINT
520 PRINT" (1) YES-CONTINUE WITH DATA"
530 PRINT" (2) NO-NEED ADJUST AMP"
540 PRINT" (3) NO-NEED FIX ROIS"
550 PRINT:INPUT"CHOOSE OPTION":OP
560 ON OP GOTO 1000,450,100

```

EXHIBIT C.1 (CONT'D)

```
1000 REM SETUP FOR ANALYSIS INPUT
1005 NC=0:NM=0
1015 IF NC=0 THEN GOTO 1020
1016 PRINT "NEW FUEL CLEARS MEMORY OF PREVIOUS DATA"
1017 INPUT "SURE READY FOR NEW FUEL WITH NEW CALIBS(Y/N)";YE#
1018 IF YE# (<) "Y" THEN GOTO 3000
1020 PRINT "COMPONENT A INPUTS"
1030 INPUT "U-235 ENRICHMENT %";EA:EA=EA/100
1040 INPUT "TOTAL % IN BLENDED FUEL";XA:XA=XA/100
1050 PRINT
1060 PRINT "COMPONENT B INPUTS"
1070 INPUT "U235 ENRICHMENT %";EB:EB=EB/100
1080 INPUT "TOTAL % IN BLENDED FUEL";XB:XB=XB/100
1090 PRINT
2000 REM-DATA MEASUREMENT INPUTS
2010 REM
2020 REM
2030 PRINT "CALIBRATION MEASUREMENTS. ";NC;" SETS PRESENTLY"
2050 PRINT
2060 PRINT "(1) NEW AND ADDITIONAL"
2070 PRINT "(2) CURRENT VALUES"
2080 PRINT
2090 INPUT "SELECTION (1 OR 2)";C1:IF C1 THEN NC=NC+1
2100 IF C1=1 THEN GOSUB 6100
2110 REM UA,TA,UB,TB AND ERRORS SET UP IN 2100
2120 PRINT
2200 PRINT "TUBE MEASUREMENTS. ";NM;" SETS PRESENTLY"
2210 PRINT "(1) NEW AND/OR ADDITIONAL"
2220 PRINT "(2) CURRENT VALUES"
2225 PRINT
2230 INPUT "SELECTION (1 OR 2)";M1:IF M1=1 THEN NM=NM+1
2240 IF M1=1 THEN GOSUB 7100
2300 REM - AT THIS POINT HAVE ALL DATA FOR CALCULATIONS
2310 REM - STORE DATA IN M MATRIX FOLLOWS
2320 GOSUB 8000
2330 REM-READY FOR CALCULATION
2340 PRINT
2350 PRINT "MEASUREMENT", "      ENRICHMENT", "      186KEV/237KEV"
2360 PRINT
2370 GOSUB 9000
3000 REM - SET UP FOR NEXT MEASUREMENT
3010 PRINT
3020 PRINT "CONTINUATION OF STUDY"
3030 PRINT
3040 PRINT"(1) ADDITIONAL MEASUREMENTS"
3050 PRINT"(2) ADDITIONAL CALIBS AND/OR (1)"
3060 PRINT"(3) CHANGE CALIBS ENRICHMENTS AND/OR (2)"
3070 PRINT"(4) NEW FUEL TUBE"
3080 PRINT
3090 INPUT "SELECT(1/2/3/4)";C9
3100 ON C9 GOTO 2120, 2030, 1020, 1015
```

EXHIBIT C.1 (CONT'D)

```

4000 REM-CALCULATE COUNTS IN CHANNEL CH
4020 H0=PEEK(49152+4*CH)
4030 H1=PEEK(49153+4*CH)*256
4040 HZ=INT(PEEK(49154+4*CH)/16)
4050 H2=(PEEK(49154+4*CH)-HZ*16)*65536
4060 CT=H0+H1+H2
4070 RETURN
4200 REM-CALCULATE PEAK PLOT FROM CC(1) TO CC(6)
4210 PRINT:PRINT"CHANNEL", "COUNTS", "PLOT OF PEAK-SCALED TO PEAK"
4220 CP=0: CX=40: CM=0
4230 FOR CH=CC(1) TO CC(6)
4240 GOSUB 4000: REM COUNTS CT RETURNED
4250 IF CP<>0 THEN GOTO 4280
4260 IF CT>CM THEN CM=CT
4270 GOTO 4330
4280 PRINT CH, CT, "I";: CL=(CT/CM)*CX
4290 IF CL<1 THEN GOTO 4310
4300 FOR J=1 TO CL:PRINT"*":NEXT J
4310 FOR K=1 TO 6
4315 IF CH=CC(K) THEN PRINT "----";
4317 NEXT K
4320 PRINT " "
4330 NEXT CH
4340 IF CP=0 THEN CP=1:GOTO 4230
4350 RETURN
4400 REM - CALCULATE PEAK AREA AND ERROR FROM CC(I)
4405 PK=0: BK=0
4410 FOR K=1 TO 3
4420 KL=2*K-1: KU=2*K
4430 FOR CH=CC(KL) TO CC(KU)
4440 GOSUB 4000: REM CT COUNTS RETURNED FROM CHANNEL CH
4450 IF K<>2 THEN BK=BK+CT
4460 IF K=2 THEN PK=PK+CT
4470 NEXT CH
4472 NEXT K
4475 PC=PK-BK: PE=SQR(BK+PK)
4480 PRINT:PRINT"GROSS PEAK";PK:PRINT"BACKGROUND";BK
4490 PRINT "NET PEAK COUNTS = ";PC;" +/- ";PE:PRINT
4500 RETURN
5000 REM - COUNT SEQUENCE
5010 PRINT:PRINT"CHOOSE COUNT MODE":PRINT
5020 PRINT"(1) MANUAL - SET UP EACH TIME"
5030 PRINT"(2) AUTO - NEED SET UP INITIALLY ONLY"
5040 PRINT:INPUT"CHOOSE OPTION";OP
5050 ON OP GOTO 5060, 5090
5060 PRINT"SWITCH TO DISPLAY WITH CLT B, SET UP, AND START COUNT"
5070 PRINT"WHEN FINISHED, HIT CTL B, TYPE 1, AND RETURN":INPUT ZZ
5080 GOTO 5110
5090 PRINT "COUNTING NOW - HIT CTL B TO EXAMINE"
5100 M=USR(9):M=USR(1)
5110 RETURN

```

EXHIBIT C.1 (CONT'D)

```

6000 REM CALIB MEASUREMENTS SUBROUTINE
6100 PRINT "CALIBRATION INPUT MODE"
6110 PRINT
6120 PRINT "(1) KEYBOARD INPUT"
6130 PRINT "(2) MCA COUNT"
6140 PRINT
6150 INPUT "SELECTION (1 OR 2)";C2
6160 ON C2 GOSUB 6500,6600
6170 REM NOW HAVE LATEST UA, TA, UB, TB, AND ERRORS
6180 PRINT
6200 PRINT "CALIBRATION USED IN ANALYSIS"
6210 PRINT
6220 PRINT "(1) PRESENT INPUT ONLY"
6230 PRINT "(2) COMBINE WITH EARLIER MEASUREMENTS"
6240 PRINT "(3) PRINT CURRENT DATA FIRST"
6250 PRINT "(4) GET ADDITIONAL DATA FIRST"
6260 PRINT "(5) DONE"
6350 INPUT "SELECTION (1/2/3/4/5)";C3
6355 IF C3=4 THEN NC=NC+1:GOTO 6100
6360 ON C3 GOSUB 6700,6800,6800
6370 IF C3<5 THEN GOTO 6180
6400 RETURN
6500 PRINT
6510 PRINT "CALIBRATION DATA SETS";NC
6520 PRINT
6530 INPUT "UA COUNTS";UA(NC)
6535 INPUT "+/-";AU(NC)
6540 INPUT "TA COUNTS";TA(NC)
6545 INPUT "+/-";AT(NC)
6549 INPUT "UB COUNTS";UB(NC)
6550 INPUT "+/-";BU(NC)
6560 INPUT "TB COUNTS";TB(NC)
6565 INPUT "+/-";BT(NC)
6570 RETURN
6600 REM - SUBROUTINE FOR DATA COLLECT MODE
6602 PRINT "CALIB NUMBER ";NC
6605 PRINT :INPUT "CALIBRATIONS A OR B READY (HIT 1)";ZZ
6610 GOSUB 5000:REM - COUNT DATA
6612 INPUT "CALIB (A/B)";AB$
6615 PRINT:PRINT"CALIB";AB$:PRINT
6620 PRINT "186 KEV GAMMA":PRINT
6625 FOR I=1TO6:CC(I)=CU(I):NEXT:GOSUB 4200:REM PLOT
6630 GOSUB 4400:UX=PC:XU=PE:REM - ANALYSIS
6635 PRINT:PRINT "CALIB";AB$
6640 PRINT "237 KEV GAMMA":PRINT
6645 FOR I=1TO6:CC(I)=CT(I):NEXT:GOSUB 4200:REM - PLOT
6650 GOSUB 4400:TX=PC:XT=PE:REM - ANALYSIS
6655 IF AB$() "A" THEN GOTO 6665
6660 UA(NC)=UX:AU(NC)=XU:TA(NC)=TX:AT(NC)=XT:GOTO 6675
6665 IF AB$() "B" THEN PRINT "REDO":INPUT "CALIB (A/B)";AB$:GOTO6655
6670 UB(NC)=UX:BU(NC)=XU:TB(NC)=TX:BT(NC)=XT
6675 INPUT "BOTH A AND B STANDARDS DONE (Y/N)";YE$
6680 IF YE$() "Y" THEN INPUT "GET CALIB READY TO COUNT/HIT 1 RETURN";ZZ
6682 IF YE$() "Y" THEN GOTO 6605
6685 RETURN

```

```

6700 PRINT
6710 UA=UA(NC):AU=AU(NC):TA=TA(NC):AT=AT(NC)
6720 UB=UB(NC):BU=BU(NC):TB=TB(NC):BT=BT(NC)
6730 RETURN
6800 PRINT
6810 PRINT "CASE", "UA", "TA", "UB", "TB"
6820 FOR IC=1 TO NC
6830 PRINT IC, UA(IC), TA(IC), UB(IC), TB(IC)
6840 PRINT "+/- ERR", AU(IC), AT(IC), BU(IC), BT(IC)
6845 NEXT IC
6850 PRINT :IF C3=3 THEN RETURN
6860 PRINT "SELECT CASES TO BE USED IN CALIB"
6870 UA=0:AU=0:TA=0:AT=0:UB=0:TB=0:BT=0:CN=0
6880 FOR IC=1 TO NC
6890 PRINT "INCLUDE CASE ";IC:"INPUT" (Y/N)";YE$
6900 IF YE$(Y) THEN GOTO 6960
6910 UA=UA+UA(IC):AU=AU+AU(IC)^2
6920 TA=TA+TA(IC):AT=AT+AT(IC)^2
6930 UB=UB+UB(IC):BU=BU+BU(IC)^2
6940 TB=TB+TB(IC):BT=BT+BT(IC)^2
6950 CN=CN+1
6960 NEXT IC
6970 UA=UA/CN:AU=SQR(AU)/CN
6971 TA=TA/CN:AT=SQR(AT)/CN
6972 UB=UB/CN:BU=SQR(BU)/CN
6973 TB=TB/CN:BT=SQR(BT)/CN
6975 PRINT "AVERAGE VALUES"
6980 PRINT "UA = ";UA;" +/- ";AU
6981 PRINT "TA = ";TA;" +/- ";AT
6982 PRINT "UB = ";UB;" +/- ";BU
6983 PRINT "TB = ";TB;" +/- ";BT
6990 RETURN
7000 REM---TUBE MEASUREMENTS SUBROUTINE
7100 PRINT "TUBE MEASUREMENTS INPUT MODES"
7120 PRINT
7130 PRINT "(1) KEYBOARD INPUT"
7140 PRINT "(2) MCA COUNT"
7145 PRINT
7150 INPUT "SELECTION (1 OR 2)";M2
7160 ON M2 GOSUB 7500,7600
7170 REM---NOW HAVE LATEST INDIVIDUAL MEASUREMENTS U(NM), T(NM)
7180 PRINT "ANALYSIS OPTIONS"
7185 PRINT
7190 PRINT "(1)NORMAL - INCLUDE PREVIOUS MEASUREMENTS - DONE
7210 PRINT "(2) ABNORMAL-DELETE SUSPICIOUS MEASUREMENTS"
7230 PRINT "(3) PRINT CURRENT DATA FIRST"
7235 PRINT "(4) GET ADDITIONAL DATA"
7240 PRINT "(5) DONE"
7250 PRINT
7260 INPUT "SELECTION (1/2/3/4/5)";M3
7265 IF M3=4 THEN NM=NM+1:GOTO 7100
7270 ON M3 GOSUB 7700,7800,7800
7280 IF M3<>5 GOTO 7180
7400 RETURN
7500 PRINT
7510 PRINT"INPUT DATA SET NUMBER";NM
7520 PRINT
7530 INPUT"U COUNTS";U(NM)
7535 INPUT" +/-";DU(NM)
7540 INPUT"T COUNTS";T(NM)
7545 INPUT" +/-";DT(NM)
7550 RETURN

```

EXHIBIT C.1 (CONT'D)

```

7600 REM - TUBE SAMPLE MCA COLLECT MODE
7605 GOSUB 5000:REM - COUNT DATA:PRINT
7610 PRINT "TUBE MEASUREMENT NUMBER";NM:PRINT
7615 PRINT:PRINT "186 KEV GAMMA":PRINT
7620 FOR I=1 TO 6:CC(I)=CU(I):NEXT:GOSUB 4200:REM - PLOT
7630 GOSUB 4400:UX=PC:XU=PE:REM-ANALYSIS
7635 PRINT "TUBE MEASUREMENT NUMBER";NM:PRINT
7640 PRINT "237 KEV GAMMA":PRINT
7645 FOR I=1TO6:CC(I)=CT(I):NEXT:GOSUB 4200:REM-PLOT
7650 GOSUB 4400:TX=PC:XT=PE:REM-ANALYSIS
7655 U(NM)=UX:DU(NM)=XU:T(NM)=TX:DT(NM)=XT
7660 RETURN
7700 M3=5
7710 RETURN
7800 PRINT
7810 PRINT"MEAS NO", "U", "T"
7820 FOR IM=1 TO NM
7830 PRINT IM;U(IM); "+/-";DU(IM);T(IM); "+/-";DT(IM)
7840 NEXT IM
7850 IF M3=3 THEN RETURN
7860 PRINT "DELETE MEASUREMENT(S) "
7865 KM=0
7870 FOR IM=1 TO NM
7880 PRINT "DELETE MEASUREMENT";IM
7885 INPUT "(Y/N)";YE$
7890 IF YE$(<>)"Y"THEN GOTO 7950
7900 KM=KM+1
7910 FOR JM=IM TO NM
7920 U(IM)=U(IM+1):DU(IM)=DU(IM+1)
7930 T(IM)=T(IM+1):DT(IM)=DT(IM+1)
7940 NEXT JM
7950 NEXT IM
7955 NM=NM-KM
7960 RETURN
8000 A=NM+1:B=NM+2:U=0:T=1
8010 M(U,A,0)=UA:M(U,A,1)=UA+AU:N(U,A)=UA
8020 M(T,A,0)=TA:M(T,A,1)=TA+AT:N(T,A)=TA
8030 M(U,B,0)=UB:M(U,B,1)=UB+BU:N(U,B)=UB
8040 M(T,B,0)=TB:M(T,B,1)=TB+BT:N(T,B)=TB
8050 FOR IM=1 TO NM
8060 M(U,IM,0)=U(IM):M(U,IM,1)=U(IM)+DU(IM):N(U,IM)=U(IM)
8070 M(T,IM,0)=T(IM):M(T,IM,1)=T(IM)+DT(IM):N(T,IM)=T(IM)
8080 NEXT IM
8090 RETURN

```

EXHIBIT C.1 (CONT'D)

```

9000 FOR IM=1 TO NM
9005 SD=0
9010 FOR JM=0 TO NM+2
9020 FOR Z=0 TO 1
9030 IF JM=0 THEN Z=1:GOTO 9050
9040 N(Z, JM)=M(Z, JM, 1)
9050 RA=N(T, A)/N(U, A):RB=N(T, B)/N(U, B)
9055 QRM=(XA*EA*RA+XB*EB*RB)/(XA*EA+XB*EB)
9060 R(IM)=N(T, IM)/N(U, IM)
9065 TS=0:US=0
9070 FOR KM=1 TO NM
9080 TS=TS+N(T, KM):US=US+N(U, KM)
9090 NEXT KM
9100 RM=TS/US
9110 QR=R(IM)*QRM/RM
9120 Y=(QR-RB)/(RA-QR)
9130 E=(EA*EB*(Y+1))/(EA+Y*EB)
9140 IF JM=0 THEN ER(IM)=E:GOTO 9170
9150 SD=SD+(E-ER(IM))^2
9160 N(Z, JM)=M(Z, JM, 0)
9170 NEXT Z
9180 NEXT JM
9190 DE(IM)=SQR(SD)
9192 RZ=U(IM)/T(IM)
9194 EZ=RZ*SQR((DU(IM)/U(IM))^2+(DT(IM)/T(IM))^2)
9200 PRINT IM, 100*ER(IM); "+/-"; 100*DE(IM), RZ; "+/-"; EZ
9300 NEXT IM
9400 RETURN
10000 END

```

EXHIBIT C.2 MCA BASIC Sequence for Example Set of Measurements

EXHIBIT C.2 MCA BASIC SEQUENCE FOR EXAMPLE  
SET OF MEASUREMENTS

\*  
RUN  
SPAN GAMMA PEAKS WITH EVEN NUMBER OF CHANNELS

186 KEV GAMMA  
LOWER CHANNEL? 920  
UPPER CHANNEL? 945

237 KEV GAMMA  
LOWER CHANNEL? 1185  
HIGHER CHANNEL? 1210  
PUT CURSOR IN CHANNEL 0 - THEN HIT 1 RETURN? 1  
PEAKS IN WINDOW - THEN HIT 1 AND RETURN FOR PRINT CHECK? 1

186 KEV GAMMA

CHANNEL	COUNTS	PLOT OF PEAK-SCALED TO PEAK
906	361	I**---
907	321	I*
908	366	I**
909	300	I*
910	346	I*
911	330	I*
912	321	I*
913	382	I**
914	367	I**
915	357	I*
916	354	I*
917	346	I*
918	391	I**---
919	371	I**
920	362	I**---
921	339	I*
922	371	I**
923	358	I*
924	422	I**
925	404	I**
926	535	I**
927	786	I*****
928	1458	I*****
929	2785	I*****
930	4493	I*****
931	6328	I*****
932	7212	I*****
933	6663	I*****
934	4968	I*****
935	3213	I*****
936	1796	I*****
937	1051	I*****
938	691	I****
939	539	I**
940	460	I**
941	460	I**
942	430	I**
943	382	I**
944	378	I**
945	364	I**---
946	347	I*
947	330	I*---
948	333	I*
949	328	I*
950	334	I*
951	342	I*
952	303	I*
953	300	I*
954	333	I*
955	298	I*
956	307	I*
957	313	I*
958	305	I*
959	301	I*---

237 KEV GAMMA

CHANNEL	COUNTS	PLOT OF PEAK-SCALED TO PEAK
1171	219	I*****---
1172	229	I*****
1173	219	I*****
1174	200	I*****
1175	226	I*****
1176	228	I*****
1177	212	I*****
1178	237	I*****
1179	246	I*****
1180	210	I*****
1181	225	I*****
1182	242	I*****
1183	218	I*****---
1184	240	I*****
1185	239	I*****---
1186	233	I*****
1187	260	I*****
1188	288	I*****
1189	295	I*****
1190	377	I*****
1191	575	I*****
1192	844	I*****
1193	1169	I*****
1194	1403	I*****
1195	1469	I*****
1196	1088	I*****
1197	864	I*****
1198	587	I*****
1199	440	I*****
1200	334	I*****
1201	292	I*****
1202	280	I*****
1203	331	I*****
1204	325	I*****
1205	349	I*****
1206	347	I*****
1207	323	I*****
1208	322	I*****
1209	270	I*****
1210	267	I*****---
1211	229	I*****
1212	242	I*****---
1213	230	I*****
1214	234	I*****
1215	222	I*****
1216	225	I*****
1217	214	I*****
1218	234	I*****
1219	223	I*****
1220	227	I*****
1221	226	I*****
1222	242	I*****
1223	192	I*****
1224	190	I*****---

CALIBS OK ?

- (1) YES-CONTINUE WITH DATA
- (2) NO-NEED ADJUST AMP
- (3) NO-NEED FIX ROIS

CHOOSE OPTION? 1

COMPONENT A INPUTS

U235 ENRICHMENT %? 45

TOTAL % IN BLENDED FUEL? 50

COMPONENT B INPUTS

U235 ENRICHMENT %? 75

TOTAL % IN BLENDED FUEL? 50

CALIBRATION MEASUREMENTS. 0 SETS PRESENTLY

- (1) NEW AND ADDITIONAL
- (2) CURRENT VALUES

SELECTION (1 OR 2)? 1

CALIBRATION INPUT MODE

- (1) KEYBOARD INPUT
- (2) MCA COUNT

SELECTION (1 OR 2)? 1

CALIBRATION DATA SETS 1

UA COUNTS? 45000

+/-? 100

TA COUNTS? 15000

+/-? 100

UB COUNTS? 90000

+/-? 100

TB COUNTS? 10000

+/-? 100

CALIBRATION USED IN ANALYSIS

- (1) PRESENT INPUT ONLY
- (2) COMBINE WITH EARLIER MEASUREMENTS
- (3) PRINT CURRENT DATA FIRST
- (4) GET ADDITIONAL DATA FIRST
- (5) DONE

SELECTION (1/2/3/4/5)? 1

CALIBRATION USED IN ANALYSIS

- (1) PRESENT INPUT ONLY
- (2) COMBINE WITH EARLIER MEASUREMENTS
- (3) PRINT CURRENT DATA FIRST
- (4) GET ADDITIONAL DATA FIRST
- (5) DONE

SELECTION (1/2/3/4/5)? 3

CASE	UA	TA	UB	TB
1	45000	15000	90000	10000
+/- ERR	100	100	100	100

## CALIBRATION USED IN ANALYSIS

- (1) PRESENT INPUT ONLY
  - (2) COMBINE WITH EARLIER MEASUREMENTS
  - (3) PRINT CURRENT DATA FIRST
  - (4) GET ADDITIONAL DATA FIRST
  - (5) DONE
- SELECTION (1/2/3/4/5)? 5

- TUBE MEASUREMENTS. 0 SETS PRESENTLY
- (1) NEW AND/OR ADDITIONAL
  - (2) CURRENT VALUES

- SELECTION (1 OR 2)? 1
- TUBE MEASUREMENTS INPUT MODES

- (1) KEYBOARD INPUT
- (2) MCA COUNT

SELECTION (1 OR 2)? 1

INPUT DATA SET NUMBER 1

U COUNTS? 38579

+/-? 236

T COUNTS? 7759

+/-? 139

ANALYSIS OPTIONS

- (1) NORMAL - INCLUDE PREVIOUS MEASUREMENTS - DONE
- (2) ABNORMAL-DELETE SUSPICIOUS MEASUREMENTS
- (3) PRINT CURRENT DATA FIRST
- (4) GET ADDITIONAL DATA
- (5) DONE

SELECTION (1/2/3/4/5)? 4

TUBE MEASUREMENTS INPUT MODES

- (1) KEYBOARD INPUT
- (2) MCA COUNT

SELECTION (1 OR 2)? 2

CHOOSE COUNT MODE

- (1) MANUAL - SET UP EACH TIME
- (2) AUTO - NEED SET UP INITIALLY ONLY

CHOOSE OPTION? 1

SWITCH TO DISPLAY WITH CLT B, SET UP, AND START COUNT  
WHEN FINISHED, HIT CTL B, TYPE 1, AND RETURN

? 1

TUBE MEASUREMENT NUMBER 2

EXHIBIT C.2 (CONT'D)

MEASUREMENT	ENRICHMENT	186KEV/237KEV
1	59.5396 +/- .487568	4.97216 +/- .0941245
2	59.716 +/- .500488	5.00356 +/- .0974227
3	59.9835 +/- .496199	5.05157 +/- .0975435
4	59.6858 +/- .498375	4.99816 +/- .0968605
5	59.9863 +/- .501228	5.05207 +/- .0984616
6	60.6462 +/- .503466	5.17265 +/- .101424
7	60.4593 +/- .508549	5.1382 +/- .101621

CONTINUATION OF STUDY

- (1) ADDITIONAL MEASUREMENTS
- (2) ADDITIONAL CALIBS AND/OR (1)
- (3) CHANGE CALIBS ENRICHMENTS AND/OR (2)
- (4) NEW FUEL TUBE

SELECT(1/2/3/4)? 1

TUBE MEASUREMENTS. 7 SETS PRESENTLY

- (1) NEW AND/OR ADDITIONAL
- (2) CURRENT VALUES

SELECTION (1 OR 2)? 1

TUBE MEASUREMENTS INPUT MODES

- (1) KEYBOARD INPUT
- MCA COUNT

SELECTION (1 OR 2)? 2

CHOOSE COUNT MODE

- (1) MANUAL - SET UP EACH TIME
- (2) AUTO - NEED SET UP INITIALLY ONLY

CHOOSE OPTION? 2

COUNTING NOW - HIT CTL B TO EXAMINE

186 KEV GAMMA

CHANNEL	COUNTS	PLOT OF PEAK-SCALED TO PEAK
906	214	I*---
907	227	I**
908	216	I*
909	227	-I**
910	213	I*
911	212	I*
912	200	I*
913	229	I**
4	214	I*
915	254	I**
916	260	I**
917	233	I**
918	237	I**---
919	235	I**
920	208	I*---
921	208	I*-----
922	249	I**
923	240	I**
924	244	I**
925	313	I**
926	387	I***
927	708	I*****
9	1246	I*****
929	2341	I*****
930	3481	I*****
931	4240	I*****
932	4331	I*****
933	3623	I*****
934	2355	I*****
935	1365	I*****
936	827	I*****
937	520	I****
938	382	I***
939	323	I**
940	289	I**
941	259	I**
942	222	I**
943	243	I**
944	219	I**
945	225	I**---
946	182	I*
947	205	I*---
948	186	I*
949	208	I*
950	214	I*
951	202	I*
952	197	I*
3	233	I**
4	201	I*
955	217	I**
956	205	I*
957	178	I*
958	200	I*
959	184	I*---

GROSS PEAK 29048

BACKGROUND 5566

NET PEAK COUNTS = 23482

237 KEV GAMMA

CHANNEL	COUNTS	PLOT OF PEAK-SCALED TO PEAK
1171	151	I*****---
1172	157	I*****
1173	158	I*****
1174	156	I*****
1175	134	I*****
1176	154	I*****
1177	153	I*****
1178	147	I*****
1179	159	I*****
1180	141	I*****
1181	156	I*****
1182	151	I*****
1183	168	I*****---
1184	139	I*****
1185	135	I*****---
1186	137	I*****
1187	168	I*****
1188	188	I*****
1189	245	I*****
1190	328	I*****
1191	481	I*****
1192	670	I*****
1193	804	I*****
1194	880	I*****
1195	711	I*****
1196	593	I*****
1197	425	I*****
1198	272	I*****
1199	213	I*****
1200	169	I*****
1201	187	I*****
1202	217	I*****
1203	204	I*****
1204	226	I*****
1205	234	I*****
1206	207	I*****
1207	197	I*****
1208	159	I*****
1209	167	I*****
1210	151	I*****---
1211	160	I*****
1212	135	I*****---
1213	150	I*****
1214	137	I*****
1215	136	I*****
1216	137	I*****
1217	146	I*****
1218	161	I*****
1219	123	I*****
1220	154	I*****
1221	148	I*****
1222	153	I*****
1223	134	I*****
1224	138	I*****---

GROSS PEAK 8368  
 BACKGROUND 3837  
 NET PEAK COUNTS = 4531 +/- 110.476

## ANALYSIS OPTIONS

- 1) NORMAL - INCLUDE PREVIOUS MEASUREMENTS - DONE
- 2) ABNORMAL-DELETE SUSPICIOUS MEASUREMENTS
- 3) PRINT CURRENT DATA FIRST
- 4) GET ADDITIONAL DATA
- 5) DONE

SELECTION (1/2/3/4/5)? 1

MEASUREMENT	ENRICHMENT	186KEV/237KEV
1	59.4837 +/- .490959	4.97216 +/- .0941245
2	59.6602 +/- .50402	5.00356 +/- .0974227
3	59.9277 +/- .499704	5.05157 +/- .0975435
4	59.6299 +/- .501884	4.99816 +/- .0968605
5	59.9304 +/- .50472	5.05207 +/- .0984616
6	60.5905 +/- .506967	5.17265 +/- .101424
7	60.4036 +/- .51204	5.1382 +/- .101621
8	60.6438 +/- .679305	5.18252 +/- .132866