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M E M O R A N D U M

TO: G. F. MERZ

FROM: J. R. TAYLOR JT

FEASIBILITY OF USING A RADIAL OUTFLOW REACTION TURBINE (HERO HEAT ENGINE) FOR PRODUCING ELECTRICITY FROM SRP WASTE STREAMS

INTRODUCTION

A radial outflow reaction turbine (RORT) to produce electricity from SRP waste streams was investigated as part of an ongoing SRL program to identify and evaluate new initiatives to improve plant operations. The RORT would function as the expander in a total flow heat engine; its main advantages are simplicity of design and high thermodynamic efficiency. This memorandum reports the results of a study to analyze the RORT for thermodynamic, mechanical, and economic feasibility.

SUMMARY

A system of RORT's appears to be mechanically feasible and would probably be capable of producing between 100 and 150 MW of electricity. However, the system would be uneconomical because of the equipment requirements. A system containing 1000 RORT's would cost nearly 100 million dollars for materials alone (no charge for fabrication). Each RORT in the system would be ∿ 22 feet in diameter. Material costs for systems with fewer RORT's would be higher.

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RORT costs diminish rapidly with increasing fluid temperature so that at a high enough temperature it becomes competitive with more conventional alternatives (e.g., binary Rankine and flash-separator engines). However, calculations show that even if heat exchanger effluent were to leave the reactors at its boiling point (212°F), the RORT would still be too expensive; hence no additional work will be pursued on this concept.

DISCUSSION

Background

About 170,000 gpm of heated effluent (H_2O) flows from each of the three SRP reactors when they are operating. Each waste stream carries about 2000 MW of heat energy relative to the surrounding environment. Typical temperatures for these streams range from $\sim$ 130°F (55°C) to 150°F (65°C) with a yearly average of around 140°F (60°C). Yearly average ambient temperature is around 60°F (15°C).

The heated effluent from two of the three reactors (K and C) is ultimately returned to the Savannah River after being cooled to near ambient temperature in the swamps. The effluent from P reactor is returned to PAR cooling pond. Because all three reactors are at a higher elevation than their sinks, the effluent streams also contain energy in the form of hydraulic head (or kinetic energy of the flowing fluid). Both K and C Areas are located at an elevation $\sim$ 200 feet above the river, while P Area is located $\sim$ 100 feet above PAR pond. For a total flow of 170,000 gpm per area, the waste streams contain 16 MW of hydraulic potential energy relative to their sinks.

With all three reactors operating, SRP uses about 150 MW of electricity. The total plant electric bill for 1978 was nearly $27 million. Several schemes have been proposed for generating all or part of our electric needs from reactor effluent streams. Some of these schemes have proposed recovering only the hydraulic energy in the waste streams. However, a number have proposed recovering part of the enthalpy in the streams by means of a heat engine.

Proposed heat engines generally fall into one of three categories.

- Binary Rankine engines in which the heated effluent is used as a heat source for a closed Rankine cycle using some working fluid of higher volatility than water.
- Flash separator engines in which heated effluent is flashed to sub-atmospheric pressure, the liquid and vapor phases separated, and the vapor phase expanded through a conventional vapor turbine.
- Total flow engines in which the heated effluent is flashed and both phases are expanded through a two phase turbine of non-conventional design.

Recently a proposal was made to use a radial outflow reaction turbine (RORT) in a total flow heat engine for producing electricity. This memorandum documents the results of a study to determine the thermodynamic and economic feasibility of a total flow cycle using a RORT.

Description of the RORT

One of the first known steam turbines was the reaction turbine developed as a toy by the Greek geometer Hero (circa. 200 AD). The proposal discussed in this memorandum is to use a modification of the Hero turbine to produce electricity from SRP waste streams.

A sketch of the proposed RORT is shown in Figure 1. The RORT consists of a rotating disk and shaft attached via a graphite seal to a stationary inlet line. The disk contains four long narrow chambers radiating outwards from the central hub and terminating at the disk circumference. A nozzle is located at the outer terminus of each chamber. The nozzles are oriented so that exiting fluid will cause the disk and shaft to rotate. The entire turbine is contained inside a chamber maintained at subatmospheric pressure (a few tenths of a psia) by means of a barometric condenser.

Water enters the disk via the inlet pipe and shaft. The water flows radially outward through the chambers and is flashed to a two phase fluid in the nozzles. The flashing process adds considerable kinetic energy to the fluid because of the reduced density of the vapor phase. This kinetic energy is imparted to the rotating disk and ultimately converted to electricity by means of a conventional generator attached to the shaft.

Because the RORT transforms heat energy (enthalpy) to kinetic energy, it acts as a heat engine. However, the turbine also reclaims some of the hydraulic energy in the stream. Thus the RORT is a combination water wheel and heat engine.

Engine Analysis

Equations are derived in the appendix to analyze the RORT as a simple water wheel and as a combination water wheel and heat engine. For the RORT acting as a simple water wheel, engine output varies with tip speed (see Figure 2). There is an optimum tip speed which maximizes engine efficiency. Optimum tip speed depends on the pressure of water entering the RORT and the pressure just downstream of the RORT. The RORT efficiency at this optimum speed depends only on the efficiency of the liquid nozzle (typically $\sim$ 95%).

Typical waste stream flows at SRP are about 170,000 gpm per reactor area. Assuming water entering the RORT under 100* feet of head, total available hydraulic energy is 3.2 MW. Optimal efficiency for the RORT is just over 58% (Figure 2). Thus a system of RORT's would reclaim about 1.9 MW of hydraulic energy per reactor area.

Assuming a vapor nozzle efficiency of 75%, optimal efficiency for a RORT acting as a combination heat engine and water wheel is nearly 47% (Figure 3). If water enters the RORT at 140°F and is expanded to 60°F, the quality of fluid exiting the RORT is 7%. The enthalpy drop across the RORT is nearly 5.8 BTU/lb_m. Thus for a total flow of 170,000 gpm, the available thermal power driving the RORT is around 149 MW per area. In addition, there is about 3 MW of power available from hydraulic energy in the streams. Thus a 47% efficient RORT would have a gross power output of around 71.5 MW per reactor area.

Note that at the optimum speed for a combination RORT, the calculated efficiency of the water wheel is negative. This seemingly anomalous result is due to the way in which the turbine is analyzed. All of the energy required to accelerate water in the chambers to tip velocity was charged to the water wheel.

In fact, this same energy is required whether the RORT acts as water wheel or not. However, the hydraulic energy is only capable of turning the RORT at ∿ 100 ft/sec tip speed. The energy required to turn the RORT at the higher speeds is provided by the heat engine portion. The water wheel portion of the RORT always acts to increase output of the engine.

Mechanical Design

The primary mechanical design problems identified for the RORT are

- High stresses near the hub because of the high angular momentum
- Design of the graphite seal
- Design of the two phase nozzle

Stresses near the hub center have not been calculated explicity. However, the design is such that this area can be strengthened to almost any desired level with negligible effect on engine output. Thus, it is assumed that stresses in the disk can be reduced to acceptable levels.

A graphite seal has been designed by Lawrence Livermore Laboratories (LLL) for a small experimental RORT they have built. Their seal is too small to be of practical use to us; however, the design should scale up with no difficulty.

*Part of the total available hydraulic energy is expended in frictional losses.

Design of the two-phase nozzle probably represents a time consuming but fairly straightforward task. The nozzle should be nonaxisymmetric to minimize losses from the liquid phase striking the nozzle walls. Furthermore the nozzle should be curved to fit the contour of the disk circumference. In addition, a small pintle should be provided in the nozzle throat to help break up the liquid droplets and minimize slip. (Slip is the ratio of vapor velocity to liquid velocity. In a high slip system, much of the kinetic energy of the vapor phase is spent in accelerating the liquid and thus is not available for generating useful power.)

LLL has designed RORT nozzles with the above features. However, the nozzles were designed for higher temperature and higher pressure service (up to 550^{O}F and 350 psia). Thus, the basic LLL nozzle would probably have to be tuned for our service; LLL has developed design procedures to reduce the level of effort for nozzle design.

Because of the low pressures needed for SRP operation, nozzle efficiencies may not be as high as those measured at LLL.

Barometric Condensers

Each RORT would be contained inside an evacuated chamber maintained at a few tenths of a psia. Vacuum would be maintained inside the chambers by barometric condensers (an alternative would be to use a ground level jet condenser; however, for purposes of this report, the two function identically.) A barometric condenser is essentially a conventional condenser (either direct contact or shell and tube), located 34 feet or more above the ground. Condensate is drained through a drain leg and seal pot to atmospheric pressure. As long as the drain leg remains full of condensate, pressure inside the condenser is just the vapor pressure of the condensate plus a contribution from non-condensible gases.

Barmetric condensers can be purchased off-the-shelf from several vendors, though none carries units large enough for our service as a stock item. A representative of the Schutte and Koerting Company (makers of barometric condensers) was contacted and estimated that a condenser capable of 100,000 gpm condenser flow would cost 35 to 50 thousand dollars. About 35 such condensers would be required for a system of RORT's capable of producing 100-150 MW.

The largest barometric condenser Schutte and Koerting has made at present will handle only $\sim$ 70,000 gpm condenser flow*; however, there are no expected scale-up problems in going to 100,000 gpm. Experience with condensers in similar service has shown that accumulation of non-condensible gasses can be held to acceptable levels. The representative from Schutte and Koerting anticipated no problems in providing a few tenths of a psia inside the chambers.

*Schutte and Koerting has made at least two such units for the electrical power industry.

System Design

The efficiency of a RORT system is a strong function of both hot water temperature entering the RORT and cold side temperature. Furthermore for a given river water temperature, cold side temperature is a strong function of condenser water flow.

As condenser flow is increased, cold side temperature of the RORT is decreased and approaches ambient (river water) temperature asymptotically. Thus gross system output increases with increasing condenser flow. However, required pumping power also increases for increasing condenser flow. Thus there is some optimum condenser flow which maximizes net system output (gross output from the RORT's less pumping costs).

A system of RORT's was analyzed using models developed in the appendix to estimate gross system output. Required pumping power was estimated assuming that condenser water had to be lifted 32 feet and then pumped through an additional 10 psi for the barometric condenser. A pumping efficiency of 85% was assumed. Net RORT output is shown as a function of cooling water flow in Figure 4. Note that optimum condenser water flow is $\sim$ 7 times our normal reactor cooling water flow. This represents 80% of the total flow in the Savannah River.

Because optimum condenser water flow to the RORT is high compared to the flow of heat exchanger effluent, a system of RORT's should be located as near to the river as possible (to minimize piping costs and pumping requirements). Furthermore, because RORT efficiency falls off rapidly with high side temperature, the heat exchanger effluent should be piped to the RORT's. These considerations are not RORT specific but would apply to almost any type of heat engine. A recent study by the South Carolina Energy Research Institute estimated the cost of piping heated effluent to be $\sim$ 47 million dollars (total for all three areas).

Engine Size and Cost

The effective density of two-phase fluid exiting the RORT is

$$\bar{\rho} = \alpha \rho_v + (1-\alpha)\rho_\ell \qquad (1)$$

where $\bar{\rho}$ = effective fluid density, lb/ft^3

ρ_v = density of vapor phase (at saturation), lb/ft^3

ρ_ℓ = density of liquid phase, lb/ft^3

α = void fraction of fluid, dimensionless

The void faction α is related to steam quality X through

$$\alpha = \frac{1}{1 + \sigma \frac{\rho_v}{\rho_\ell} \frac{1-x}{x}} \tag{2}$$

where x = steam quality, dimensionless

σ = slip ratio, dimensionless

Physically σ is the ratio of vapor velocity to liquid velocity. The homegeneous model assumes that $\sigma = 1.0$ or that vapor and liquid both travel at the same velocity. This model is known to underpredict effective density for most cases of practical importance. However, it is widely used for initial estimates and is reasonably accurate for flows involving small liquid droplets. Because of its mathematical simplicity and because water droplets in the RORT would be kept small by the pintle (see section on mechanical design), size of the RORT is estimated using the homogeneous model.

Using $\sigma = 1.0$ in the above relationships results in

$$\bar{\rho} = \frac{\rho_\ell}{1 + x(\rho_\ell/\rho_v - 1)} \tag{3}$$

Let A_T be the total area for flow through all the RORT's in the system and let V_n be the velocity of fluid leaving the RORT nozzles. Then the total mass flow of liquid through all the RORT's is

$$G_T = \bar{\rho} V_n A_T \tag{4}$$

But G_T is the total mass flow of cooling water through all the reactors. For typical SRP operation G_T has a value of $\sim 7.1 \times 10^4$ lb/sec when all three reactors are operating. Using methods given in the appendix, V_n is 645 ft/sec at maximum efficiency. Finally, $\bar{\rho}$ can be evaluated from equation 3 (using phase densities evaluated at 60°F and a steam quality of 7% as calculated using equation 28 in the appendix). The fluid density is 0.012 lb/ft^3. The total area required can now be calculated from equation 4. Nearly 9200 ft^2 of nozzle area are required to handle the total SRP waste stream flow (all three reactors areas).

Limited data from Lawrance Livermore Laboratories has shown that for a RORT containing 4 nozzles, disk diameter must be ∿ 12 nozzle diameters to minimize interaction between the effluent streams from individual nozzles. Figure 5 shows the number of 4 nozzle RORT's required to handle our effluent as a function of disk diameter. Even if a total of 1000 RORT's are installed, each must be ∿ 21 feet in diameter.

Material costs for the RORT's were estimated assuming each disc to be the same thickness as the nozzle diameter. The RORT's were assumed to be made of aluminum. Figure 6 shows material costs only (no charge for fabrication) for RORT disks for a system of RORT's capable of utilizing all of the plant waste heat. Note that disc costs decrease with increasing number of units; however, even for a system of 1000 RORT's, material costs are nearly $100 million. Assuming fabrication costs equal to material costs, a system of 1000 RORT's capable of utilizing the total plant waste heat would cost ∿ $200 million. The cost of housings is not included in this estimate but is probably small.

A recent study[1] estimated the cost for a system of staged evaporation Rankine-cycle heat engines to produce electricity from the SRP waste streams. Total costs for a system of RORT's were estimated based on the results of that study (Table I). Total system costs for the RORT's are at least twice those for the staged-evaporation Rankine cycle. The RORT estimate is probably optimistic because the large number of units required would likely entail substantially greater process piping costs.

Because specific costs (capital cost per unit power output) for a system of RORT's is so sensitive to fluid temperature, and because SRP waste stream temperatures may be increased in the future, material costs were calculated for RORT's operating at fluid temperatures up to 212^{o}F. Although system costs were greatly reduced at the higher temperatures, RORT is still not competitive with more conventional alternatives (binary Rankine, or flash-separator engines).

JRT:bj

REFERENCES

1. Low Level Waste Heat Utilization Project Savannah River Plant (Preliminary Analysis), South Carolina Energy Research Institute, November 1978.

Table I. Estimated System Costs for RORT Compared to a Staged-Evaporation Rankine Cycle* (All Three Areas)

Item	Cost, Millions of Dollars: Staged-Evaporation Rankine Cycle	Cost, Millions of Dollars: RORT
Total installed engine	64	>200
Pumping from reactor discharge	29	29
New cooling pond** (C Area only)	13	13
Spent cooling water ditch	5	5
TOTAL DIRECT COST	111	>247
Contingency (15%)	17	37
Indirects (including spare parts)	20	20
TOTAL CURRENT COST	148	>304
Escalation	22	>45
TOTAL ESCALATED COST	170	>349

*Staged-evaporation Rankine cycle costs estimated by South Carolina Energy Research Institute[1].

**A new cooling pond for C Area is not a part of this study. However it is included for purpose of comparison with the referenced study.

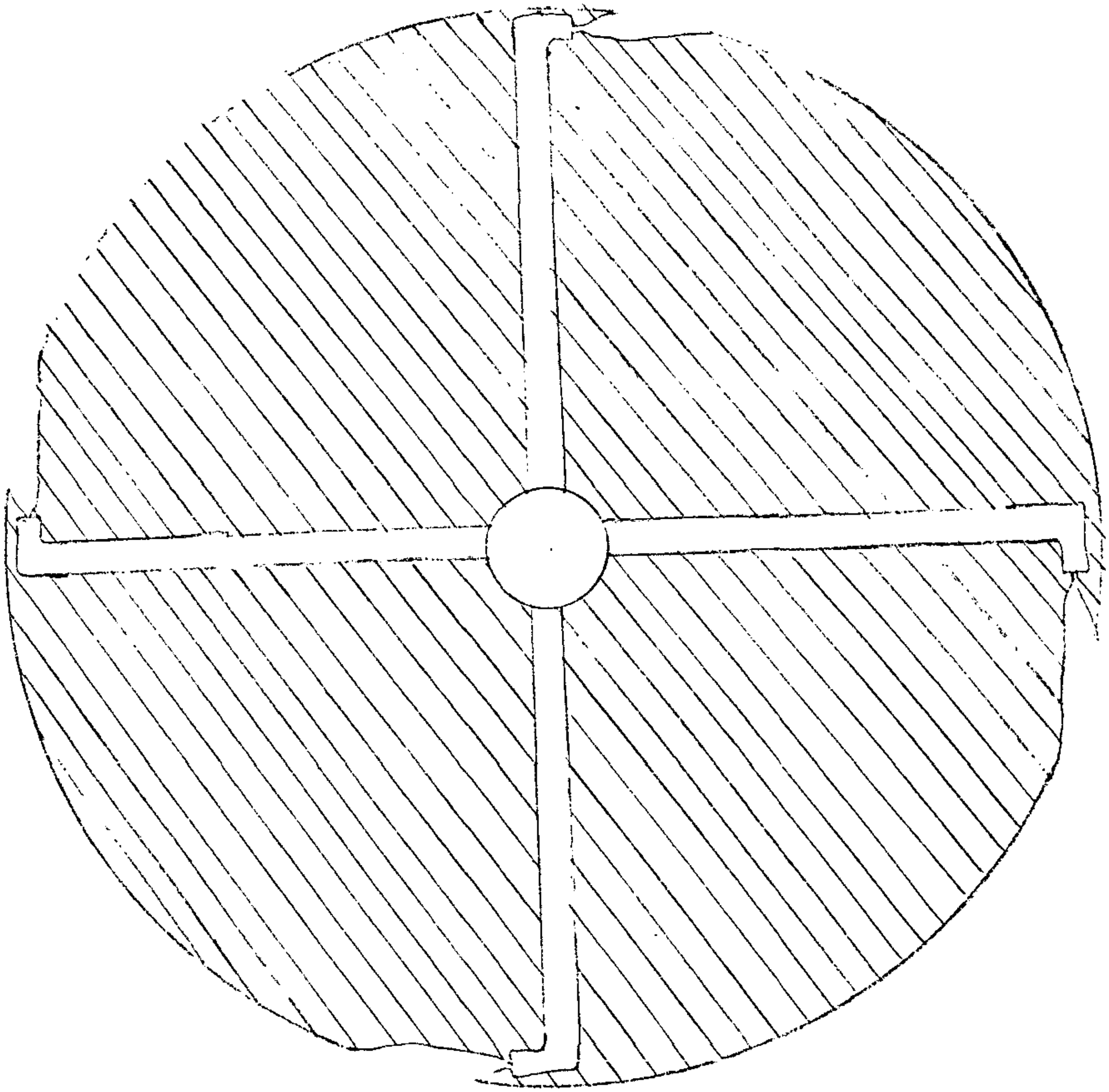
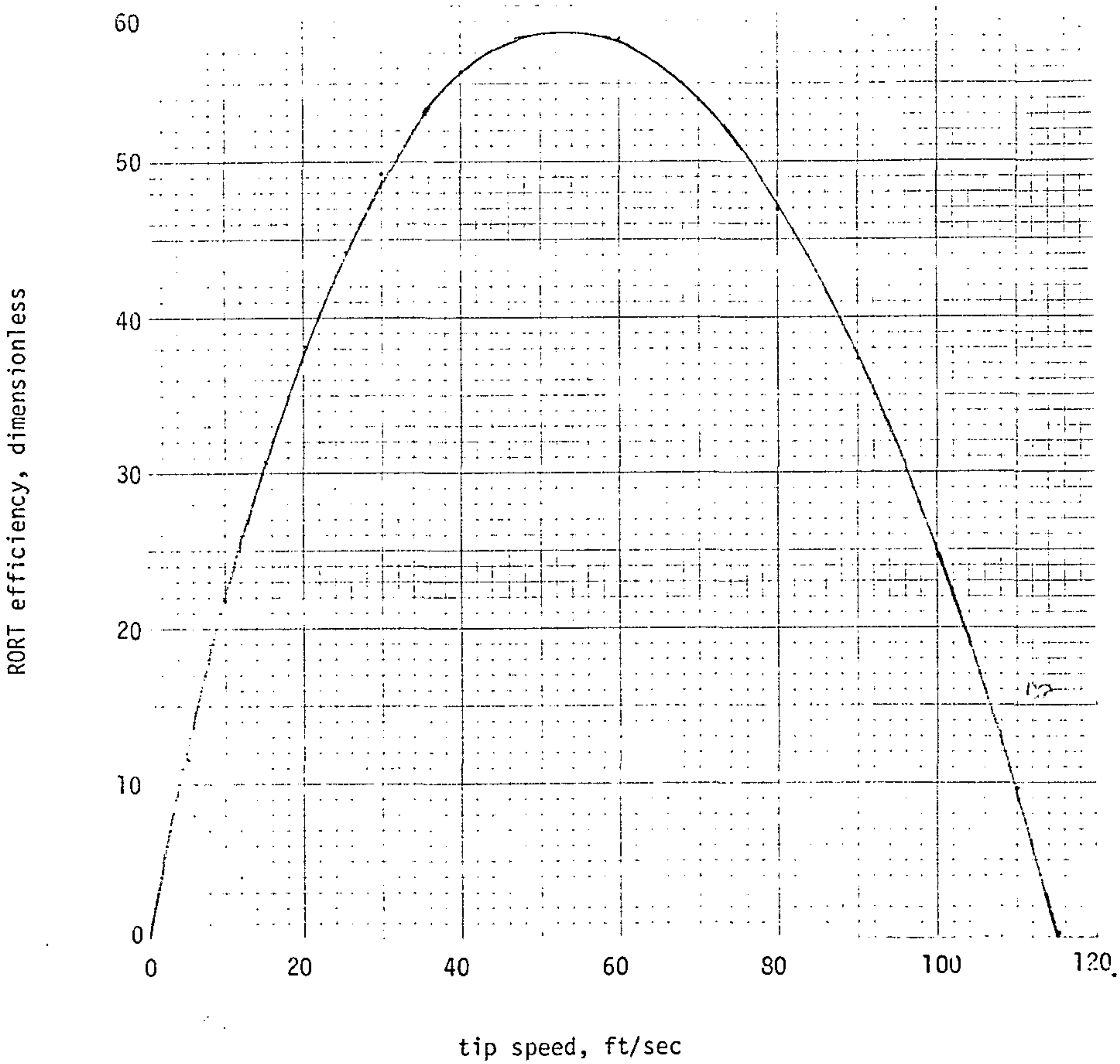


Figure 1. Schematic of the Proposed RORT

Figure 2. Efficiency of the RORT as a Simple Water Wheel



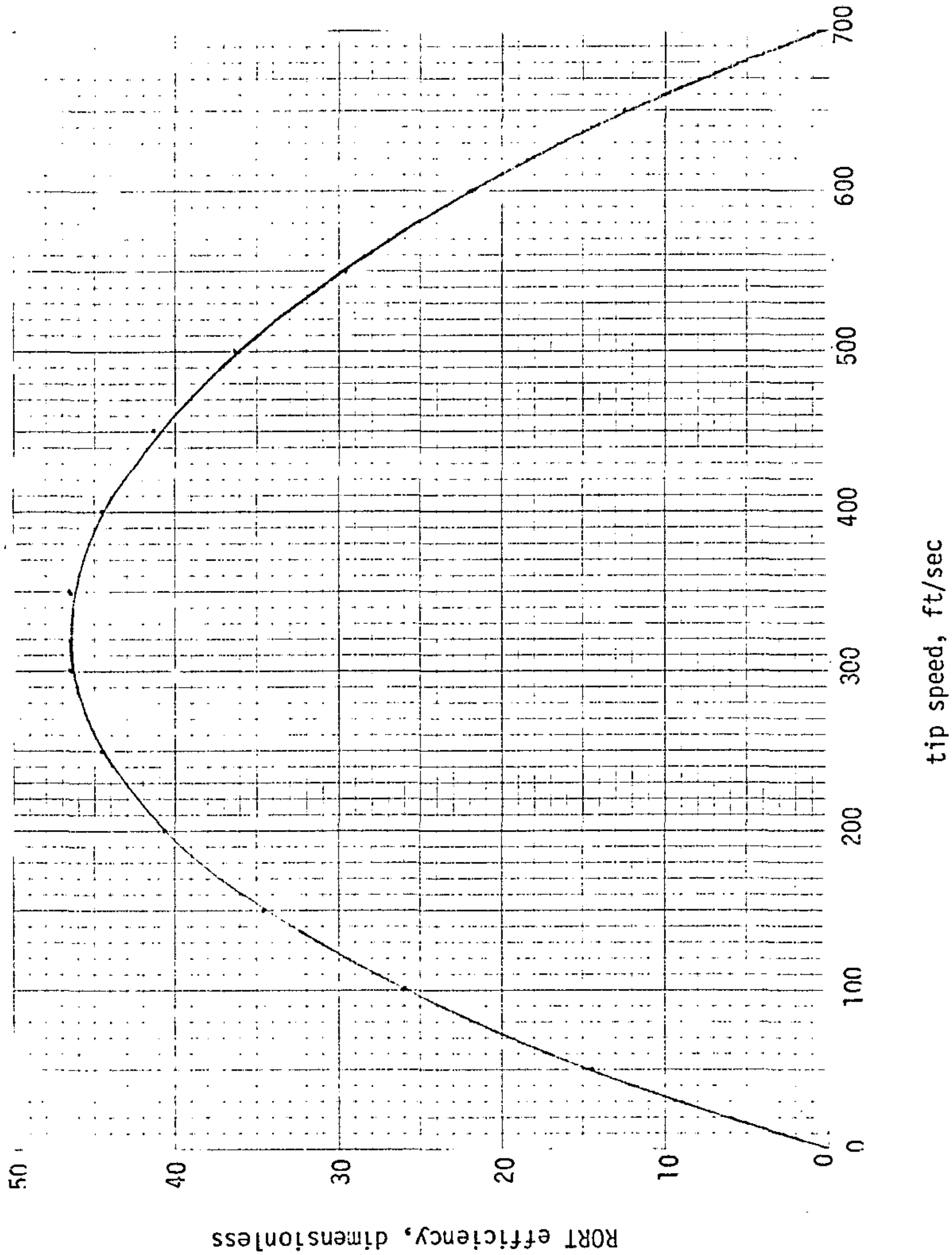


Figure 3. Efficiency of the RORT as a Combination Water Wheel and Heat Engine

Figure 4. Net Power Output per Reactor Area (Corrected for Condenser Water Pumping Power) for a System of RORT's

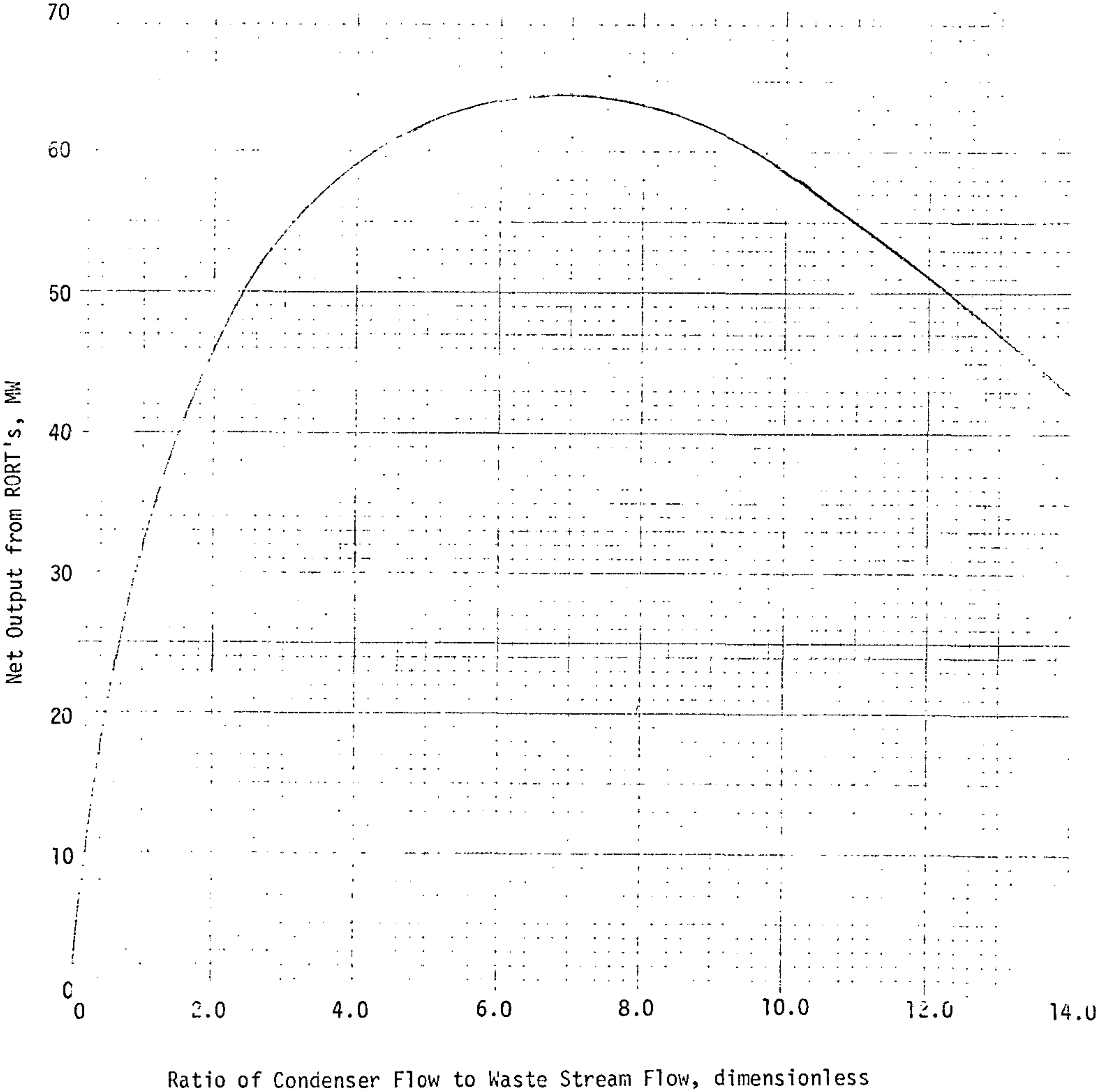


Figure 5. Size Requirements for a System of RORT's to Produce 100-150 MW Electricity

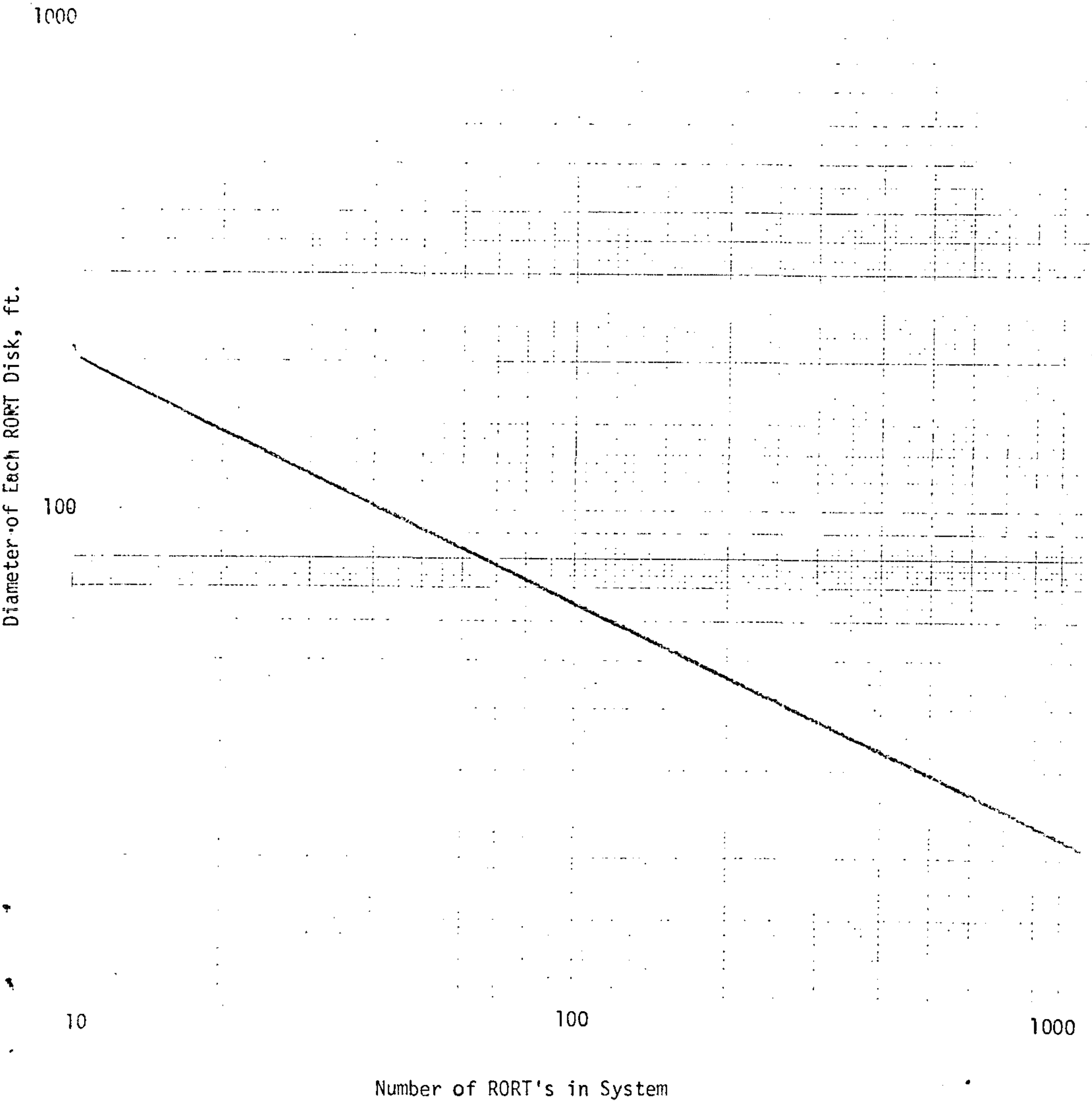
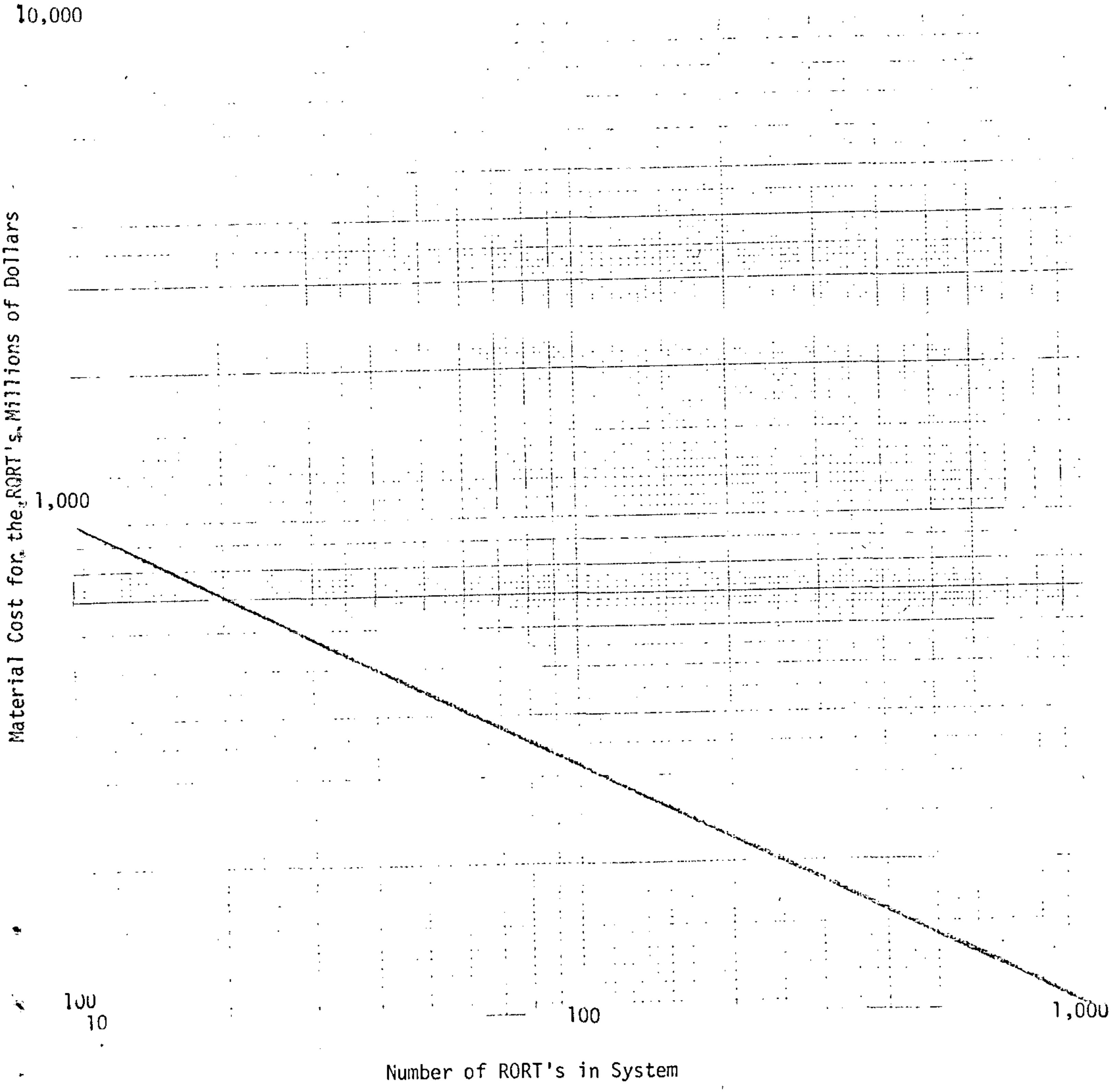


Figure 6. Material Costs for a System of RORT's (Turbines Only) to Produce 100-150 MW of Electricity



APPENDIX: THERMODYNAMIC DESIGN OF RADIAL OUTFLOW REACTION TURBINE

NOMENCLATURE

A_o = cross sectional area of turbine chamber, ft^2

g_c = dimensional conversion factor, lb_m-ft/lb_f-sec^2

G = mass flow of fluid, lb_m/sec

h = fluid enthalphy, BTU/lb_m

J = dimensional conversion factor, 778.16 ft-lb_f/BTU

P_o = pressure of fluid entering turbine, lb_f/ft^2

P_T = pressure of fluid in nozzle throat, lb_f/ft^2

R = radius of turbine disk, ft

S = fluid entropy, BTU/lb_m ^oR

u = tip speed, ft/sec

V_1 = velocity of fluid in the nozzle throat, ft/sec

V_N = velocity of fluid leaving the nozzle, ft/sec

W_N = total power output from the turbine as a water wheel, ft-lb_f/sec

W_V = power generated by the vapor nozzle, ft-lb_f/sec

W_T = total power output from the combined water wheel and heat engine, ft-lb_f/sec

X = vapor quality, dimensionless

ϵ = engine efficiency, dimensionless

ρ = fluid density, lb_m/ft^3

η = nozzle efficiency, dimensionless

σ = slip ratio, dimensionless

ν = velocity of the separate phases leaving the vapor nozzle, ft/sec

ω = angular velocity of turbine, rads/sec

μ = tip speed of turbine at maximum efficiency, ft/sec

APPENDIX: THERMODYNAMIC DESIGN OF A RADIAL OUTFLOW REACTION TURBINE

Consider the radial outflow reaction turbine shown in Figure 1 of the text. The turbine nozzles consist of a sharply converging section to reclaim kinetic energy from the liquid plus a more gradually diverging section for flashing the liquid. Thus the turbine acts as a combination water wheel and heat engine. Both modes of operation are analyzed below. Gross output from the turbine is the sum of water wheel power and heat engine power. For purposes of this appendix the converging section of the nozzle will be referred to as the liquid nozzle and the diverging section as the vapor nozzle.

The Turbine as Water Wheel

Consider a single arm of the turbine shown in Figure 1. The arm is rotating at angular velocity ω relative to the central hub. If the arm has length R, the tip is traveling at a velocity u given by

$$u = R\omega \tag{1}$$

Because the tip speed u is generally much greater than the radial velocity in the arms, liquid will be assumed to enter the arm at zero velocity. Relative to a fixed point on the hub, water enters the nozzle at near zero velocity and exits with velocity V_e. However, relative to the nozzle, water enters with velocity u and exists with velocity V_1. V_1 and V_e are related by

$$V_1 = V_e + u \tag{2}$$

A free body diagram of the liquid nozzle is shown below:

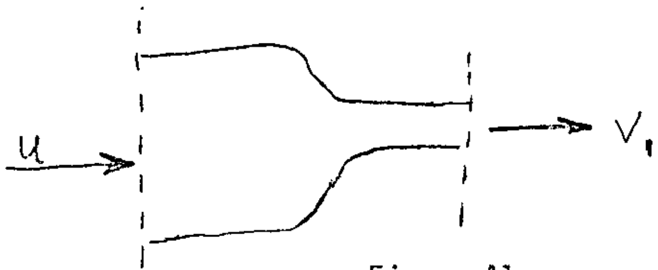


Figure A1

Now let P_u be the pressure just upstream of the nozzle and let P_o be the pressure at the entrance to the arm. Because the rotating arms act like a centrifugal pump $P_u >> P_o$.

Consider a small element of volume inside the arm of length dr located at position r relative to the hub. This element of volume has mass $\frac{\rho A_o dr}{g_c}$ and is subjected to a centrifugal accleration $\gamma\omega^2$. Thus the total centrifugal force acting on the element is

$$dF = \frac{\rho A_o dr}{g_c} r\omega^2$$

The total force acting on the arm is the sum of the forces acting on all such elements from r = o to r = R. Mathematically,

$$F_c = \int_0^R \frac{\rho A_o dr}{g_c} r\omega^2$$

or

$$F_c = \frac{\rho A_o}{g_c} \frac{R^2\omega^2}{2} \qquad (3)$$

Using equation (1) in (3), this force can be written in terms of the rectilinear velocity u:

$$F_c = \frac{\rho A_o}{g_c} \frac{u^2}{2}$$

The pressure differential due to this force is

$$\Delta P_c = F/A_D = \rho \frac{u^2}{2g_c} \tag{4}$$

Then the total pressure just upstream of the nozzle (P_u) is the sum of the entrance pressure (P_0) and the pressure differential due to centrifugal acceleration (ΔP_c). Thus

$$P_u = P_o + \rho \frac{u^2}{2g_c} \tag{5}$$

If the pressure in the throat of the nozzle is P_T, then the total pressure drop across the liquid nozzle is

$$\Delta P = P_o + \rho \frac{u^2}{2g_c} - P_T \tag{6}$$

This pressure drop is related to the entrance and exit velocities by

$$\eta_\ell \Delta P = \rho \frac{V_1^2 - u^2}{2g_c} \tag{7}$$

where η_ℓ is the efficiency of the liquid nozzle (typically ∿ 95%).

Combining (6) and (7) results in

$$\rho \frac{V_1^2 - u^2}{2g_c} = P_o - P_T + \rho \frac{u^2}{2g_c}$$

From which

$$V_1 = \sqrt{\frac{2\eta_\ell g_c}{\rho}(P_o - P_T) + u^2(1+\eta_\ell)} \tag{8}$$

Now the work done on the nozzle by the liquid can be calculated. Let G_T be the mass flow of water through the nozzle. Then the force exerted by the liquid nozzle on water entering it (refer to Figure A1) is given by:

$$F_L = G_T \frac{u}{g_c} \tag{9}$$

and the force exerted on the nozzle by the exiting water is

$$F_R = G_T \frac{V_1}{g_c} \tag{10}$$

Using (8) in this last expression

$$F_R = G_T \frac{\sqrt{\frac{2\eta_\ell g_c}{\rho}(P_0 - P_T) + u^2(1+\eta_\ell)}}{g_c} \tag{11}$$

Then the net force acting on the liquid nozzle is

$$F_N = F_R - F_L$$

or

$$F_N = \frac{G_T}{g_c}\left[\sqrt{\frac{2\eta_\ell g_c}{\rho}(P_0 - P_T) + u^2(1+\eta_\ell)} - u\right] \tag{12}$$

This force acts through a distance ds in time dt so that the gross power generated by liquid leaving the liquid nozzle is

$$dW_{gross} = \frac{d}{dt}(F_N \cdot ds)$$

where the dot indicates the dot product of vectors is to be taken. But since F acts along the same line as s,

$$dW_{gross} = \frac{d}{dt} F_N ds$$

Furthermore, since F is constant

$$dW_{gross} = F_N \, d\left(\frac{ds}{dt}\right)$$

or

$$dW_{gross} = F_N \, du$$

and the total power is

$$W_{gross} = \int_0^u F_N \, du$$

from which (13)

$$W_{gross} = \frac{G_T}{g_c}\int_0^u (V_1 - u) \, du$$

Using (12) in this expression and intergrating

$$W_{gross} = \frac{G_T}{g_c}\left[\frac{uV_1}{2} + \frac{V_1^2 - u^2(1+\eta_c)}{2\sqrt{1+\eta_c}} \ln \frac{V_1 + u\sqrt{1+\eta_c}}{\sqrt{V_1^2 - u^2(1+\eta_c)}} - \frac{u^2}{2}\right] \quad (14)$$

Part of the gross power developed by the liquid nozzle is invested in accelerating fluid to tip velocity. From equation (4), the pressure differential due to centrifugal acceleration is

$$\Delta P_c = \rho \frac{u^2}{2g_c}$$

The head H due to this pressure differential is

$$H = \frac{\Delta P_c}{\rho} = \frac{u^2}{2g_c} \quad (15)$$

The total power invested in pumping G_T lb/hr of water through this head is

$$W_{loss} = G_T \, H$$

or

$$W_{loss} = G_T \frac{u^2}{2g_c} \quad (16)$$

Then the net power generated by the turbine acting as a water wheel is

$$W_N = W_{gross} - W_{loss} \tag{17}$$

From which

$$W_{net} = G_T \frac{uV_1}{2g_c} + \frac{G_T}{g_c} \frac{V_1^2 - u^2(1+\eta_e)}{2\sqrt{1+\eta_e}} \ln \frac{V_1 + u\sqrt{1+\eta_e}}{\sqrt{V_1^2 - u^2(1+\eta_e)}} - \frac{G_T u^2}{2g_c} - \frac{G_T u^2}{2g_c}$$

or

$$W_N = \frac{G_T}{g_c}\left[u\left(\frac{V_1}{2} - u\right) + \frac{V_1^2 - u^2(1+\eta_e)}{2\sqrt{1+\eta_e}} \ln \frac{V_1 + u\sqrt{1+\eta_e}}{\sqrt{V_1^2 - u^2(1+\eta_e)}}\right] \tag{18}$$

The efficiency of the turbine as a water wheel is the ratio of net power output to available power where available power is the power equivalent of G_T lb/sec of water falling through a pressure drop of $(P_O - P_T)$. Thus

$$\epsilon_\ell = \frac{W_{net}}{G_T(P_o - P_T)/\rho}$$

from which

$$\epsilon_\ell = \frac{\frac{\rho u}{g_c}\left(\frac{V_1}{2} - u\right)}{P_o - P_T} + \frac{\rho}{P_o - P_T} \frac{V_1^2 - u^2(1+\eta_e)}{2\sqrt{1+\eta_e}} \ln \frac{V_1 + u\sqrt{1+\eta_e}}{\sqrt{V_1^2 - u^2(1+\eta_e)}}$$

This equation can be simplified by use of equation (8) to give

$$\epsilon_\ell = \frac{\frac{\rho u}{g_c}\left(\frac{V_1}{2} - u\right)}{P_o - P_T} + \frac{\eta_e}{\sqrt{1+\eta_e}} \ln \frac{V_1 + u\sqrt{1+\eta_e}}{\sqrt{V_1^2 - u^2(1+\eta_e)}} \tag{19}$$

The maximum efficiency for the wheel occurs when $\frac{d\epsilon_\ell}{du} = 0$. But since the total energy available is independent of u, this maximum efficiency also occurs where $\frac{dW_n}{du} = 0$.

W_n can be conveniently calculated from equations (13), (16), and (17).

Thus

$$W_N = \frac{G_T}{g_c} \int_0^u (V_1 - u)\, du - G_T \frac{u^2}{2g_c} \tag{20}$$

Differentiating (20),

$$\frac{dW_N}{du} = \frac{G_T}{g_c}(V_1 - u) - \frac{G_T u}{g_c} \tag{21}$$

from which

$$\frac{dW_N}{du} = \frac{G_T}{g_c}(V_1 - 2u) \tag{22}$$

The derivative $\frac{dW_n}{du}$ vanishes wherever

$$V_1 = 2u \tag{23}$$

Thus the maximum efficiency for the RORT can be evaluated by using (23) in (19).

$$\epsilon_\ell \Big|_{max} = \frac{\eta_\ell}{\sqrt{1+\eta_\ell}} \; \frac{2 + \sqrt{1+\eta_\ell}}{\sqrt{3-\eta_\ell}} \tag{24}$$

The tip speed at maximum efficiency can be determined by using (23) in (8).

$$2\mu = \sqrt{\frac{2\eta_e g_c}{\rho}(P_o - P_T) + \mu^2(1+\eta_e)}$$

from which

$$\mu = \sqrt{\frac{2\eta_e g_c}{3-\eta_e} \frac{P_o - P_T}{\rho}} \qquad (25)$$

The Turbine as Heat Engine

The vapor nozzle can and should be designed to expand the fluid nearly isentropically. For the two phase fluid leaving the vapor nozzle, entropy is defined by

$$S_{2\phi} = x S_s + (1-x) S_\ell \tag{26}$$

where subscripts S and ℓ refer to saturation properties for the vapor and liquid phase respectively. These properties are to be evaluated at the pressure at the exit of the vapor nozzle.

Let the entropy of fluid entering the nozzle be S_0 (evaluated for saturated liquid at the temperature of the hot heat exchanger effluent). Then, since the expansion is isentropic

$$S_0 = S_{2\phi}$$

or

$$S_0 = x S_s + (1-x) S_\ell \tag{27}$$

Equation (27) can be solved for x to give

$$x = \frac{S_0 - S_\ell}{S_s - S_\ell} \tag{28}$$

Total enthalpy drop can now be evaluated for the nozzle. The enthalpy of the two phase fluid at the nozzle exit is

$$h_{2\phi} = x h_s + (1-x) h_\ell \tag{29}$$

where the subscripts are consistent with those used in equation (27). Now let h_o be the enthalpy of liquid entering the vapor nozzle so that the total enthalpy drop through the nozzle is

$$\Delta h = h_o - h_{2\phi}$$

or

$$\Delta h = h_o - \left[h_\ell + x\,(h_s - h_\ell)\right] \tag{30}$$

A free body diagram of the vapor nozzle is shown below:

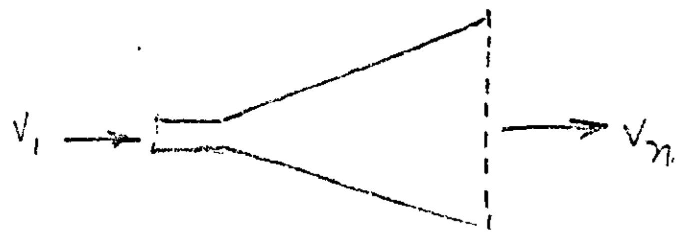


Figure A2

Water enters the vapor nozzle at velocity V_1 (relative to the nozzle) and leaves as a two-phase fluid with effective velocity V_n. Let η_V be the efficiency of the vapor nozzle. Physically, η_V is the fraction of total available heat energy which is converted to kinetic energy in the flashed fluid. Then,

$$\eta_V J \Delta h = \frac{V_n^2 - V_1^2}{2 g_c} \tag{31}$$

where J is a conversion factor relating ft-lbs and BTU's of energy.

Equation (31) can be solved for V_n to give

$$V_n = \sqrt{2\eta_v g_c J \Delta h + V_1^2} \tag{32}$$

The effective velocity V_n represents the combined effects of vapor velocity V_s and liquid velocity V_l. From the Law of Conservation of Energy, the total kinetic energy of the mixture must equal the sum of the kinetic energies of each phase.

Mathematically

$$G_T \frac{V_n^2}{2g_c} = G_\ell \frac{V_\ell^2}{2g_c} + G_s \frac{V_s^2}{2g_c}$$

from which

$$G_T V_n^2 = G_\ell V_\ell^2 + G_s V_s^2 \tag{33}$$

where G is the mass flow in lbs/sec. Mass flows for the separate phases are related to total mass flow by means of vapor quality, x.

Thus,

$$G_s = x\, G_T \tag{34a}$$

and

$$G_\ell = (1-x)\, G_T \tag{34b}$$

Using these relationships in (33) gives

$$V_n^2 = (1-x)\nu_\ell^2 + x\,\nu_s^2 \tag{35}$$

In general, the liquid and vapor velocities are different. Let σ be the slip ratio defined by

$$\sigma = \nu_s/\nu_\ell$$

from which (36)

$$\nu_s = \sigma\,\nu_\ell$$

Then equation (35) becomes

$$V_n^2 = (1-x)\nu_\ell^2 + \sigma^2 x\,\nu_\ell^2$$

from which

$$\nu_\ell = \frac{V_n}{\sqrt{1 + x(\sigma^2 - 1)}} \tag{37}$$

and

$$\nu_s = \frac{\sigma V_n}{\sqrt{1 + x(\sigma^2 - 1)}} \tag{38}$$

The force acting by the vapor nozzle on entering water is

$$F_\ell = G_T \frac{v_1}{g_c}$$

The force exerted on the nozzle by the exiting water is the sum of forces due to exiting liquid and exiting vapor. Thus

$$F_R = G_\ell \frac{V_\ell}{g_c} + G_s \frac{V_s}{g_c}$$

or

$$F_R = \frac{G_T}{g_c}\left[(1-x)V_\ell + xV_s\right] \tag{39}$$

The net force acting on the vapor nozzle is

$$F_N = F_R - F_a$$

or

$$F_N = \frac{G_T}{g_c}\left[(1-x)V_\ell + xV_s - V_1\right] \tag{40}$$

Using equations (37) and (38) in this results in

$$F_N = \frac{G_T}{g_c}\left[\frac{(1-x)V_n}{\sqrt{1+x(\sigma^2-1)}} + \frac{\sigma x V_n}{\sqrt{1+x(\sigma^2-1)}} - V_1\right]$$

or

$$F_N = \frac{G_T}{g_c}\left[\frac{1+x(\sigma-1)}{\sqrt{1+x(\sigma^2-1)}}V_n - V_1\right] \tag{41}$$

Let*

$$\beta = \frac{1 + x(\sigma - 1)}{\sqrt{1 + x(\sigma^2 - 1)}} \tag{42}$$

So that equation (41) becomes

$$F_N = {}^{G_T}\!/_{g_c}\,(\beta V_N - V_1) \tag{43}$$

Then the net power generated by the vapor nozzle is

$$W_V = {}^{G_T}\!/_{g_c} \int_0^u F_N \, du$$

from which

$$W_V = {}^{G_T}\!/_{g_c} \int_0^u (\beta V_N - V_1)\, du \tag{44}$$

(C.F. equation (13) from the previous section on the turbine as water wheel).

The total power generated by the turbine is the sum of contributions from the water wheel and heat engine, ie

$$W_T = W_V + W_N \tag{45}$$

Using equation (17) in the above result gives

$$W_T = W_V + W_{gross} - W_{loss} \tag{46}$$

*Note that for the homogeneous model $\sigma = 1.0$ so that $\beta = 1.0$.

Substituting for the various terms from equations (13), (16) and (44) yields

$$W_T = \frac{G_T}{g_c}\int_0^u (\beta V_N - V_1)\,du + \frac{G_T}{g_c}\int_0^u (V_1 - u)\,du - G_T\frac{u^2}{2g_c}$$

from which

$$W_T = \frac{G_T}{g_c}\int_0^u (\beta V_N - u)\,du - G_T\frac{u^2}{2g_c} \qquad (47)$$

Combining equations (8) and (32) gives V_n in terms of U, ie

$$V_N = \sqrt{2\eta_v g_c J \Delta h + \frac{2\eta_e g_c}{\rho}(P_0 - P_T) + u^2(1+\eta_e)} \qquad (48)$$

substituting (48) in (47) gives

$$W_T = \frac{G_T}{g_c}\int_0^u\left(\beta\sqrt{2\eta_e g_c J \Delta h + \frac{2\eta_e g_c}{\rho}(P_0 - P_T) + u^2(1+\eta_e)} - u\right)du$$

from which

$$W_T = \frac{G_T}{g_c}\beta\left[\frac{uV_N}{2} + \frac{V_N^2 - u^2(1+\eta_e)}{2\sqrt{1+\eta_e}}\ln\frac{V_N + u\sqrt{1+\eta_e}}{\sqrt{V_N^2 - u^2(1+\eta_e)}}\right] - G_T\frac{u^2}{2g_c} - G_T\frac{u^2}{2g_c}$$

or

$$W_T = \frac{G_T}{g_c}\left[u\left(\frac{\beta V_N}{2} - u\right) + \beta\frac{V_n^2 - u^2(1+\eta_e)}{2\sqrt{1+\eta_e}}\ln\frac{V_N + u\sqrt{1+\eta_e}}{\sqrt{V_N^2 - u^2(1+\eta_e)}}\right] \qquad (49)$$

The efficiency of the combination turbine is the ratio of total power output to total available power where total available power is the sum of available hydraulic power and available heat power. Thus,

$$\epsilon_o = \frac{G_T/g_c \left[u\left(\frac{\beta V_N}{2} - u\right) + \beta \frac{V_N^2 - u^2(1+\eta_e)}{2\sqrt{1+\eta_e}} \ln \frac{V_N + u\sqrt{1+\eta_e}}{\sqrt{V_N^2 - u^2(1+\eta_e)}} \right]}{G_T \frac{P_o - P_T}{\rho} + G_T J \Delta h} \tag{50}$$

Equation (50) can be simplified somewhat by dividing top and bottom by G_T and by using equation (48). Thus

$$\epsilon_o = \frac{u\left(\frac{\beta V_N}{2} - u\right)}{\left(J\Delta h + \frac{P_o - P_T}{\rho}\right) g_c} + \frac{\beta}{\sqrt{1+\eta_e}} \frac{\eta_v J \Delta h + \eta_e \frac{P_o - P_T}{\rho}}{J\Delta h + \frac{P_o - P_T}{\rho}} \ln \frac{V_N + u\sqrt{1+\eta_e}}{\sqrt{V_N^2 - u^2(1+\eta_e)}} \tag{51}$$

Maximum efficiency for the engine occurs where $\frac{dW_T}{du} = 0$. Differentiating equation (47) gives

$$\frac{dW_T}{du} = G_T/g_c (\beta V_N - u) - \frac{G_T u}{g_c}$$

from which

$$\frac{dW_T}{du} = G_T/g_c (\beta V_N - 2u) \tag{52}$$

The derivative vanishes whenever

$$V_N = \frac{2u}{\beta} \tag{53}$$

Using this relationship in equation (51)

$$\epsilon_o\Big|_{max} = \frac{\beta}{\sqrt{1+\eta_\ell}} \frac{\eta_v J\Delta h + \eta_\ell \left(\frac{P_o - P_T}{\rho}\right)}{J\Delta h + \frac{(P_o - P_T)}{\rho}} \ln \frac{2 + \beta\sqrt{1+\eta_\ell}}{\sqrt{4 - (1+\eta_\ell)\beta^2}} \tag{54}$$

The tip speed corresponding to maximum efficiency can be calculated by combining equation (48) and (53). Thus

$$\frac{4\mu^2}{\beta^2} = 2\eta_v g_c J\Delta h + \frac{2\eta_\ell g_c}{\rho}(P_o - P_T) + u^2(1+\eta_\ell)$$

from which

$$\mu = \beta\sqrt{\frac{2\eta_v g_c J\Delta h + \frac{2\eta_\ell g_c}{\rho}(P_o - P_T)}{4-(1+\eta_\ell)\beta^2}} \tag{55}$$

Note that for the special case when β = 1.0 (the homogeneous model) and when the hydraulic power is small compared to the heat power

that equations (54) and (55) reduce to

$$\epsilon_o\Big|_{max} = \frac{\eta_v}{\sqrt{1+\eta_\ell}} \ln \frac{2 + \sqrt{1+\eta_\ell}}{\sqrt{3-\eta_\ell}}$$

and

$$\mu = \sqrt{\frac{2\eta_v g_c J \Delta h}{3 - \eta_c}}$$

Comparison of these last two equations with equations (24) and (25) from the previous section shows the close similarity between the water wheel and heat engine.