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June 20, 1974

TO: J. M. BOSWELL

FROM: W. W. F. YAU *W.W.F. Yau*

SUMMARY: WASTE TANK STRESS ANALYSIS, TYPE I, II AND III TANKS

INTRODUCTION

Results of stress analysis for the steel containers of types I and III waste tanks were reported in DPST-73-348(1), where detail stress variation was illustrated for regions of high stress with maximum stress components provided at locations above the toroidal knuckles in all cases. It was also implied that the stresses in the knuckles are not excessive though calculations were not shown. Studies made by the Engineering Department(2) at a later date indicated good agreement on magnitudes and locations of maximum stresses above the knuckles; however two sets of data were given regarding the stresses at the junction of the knuckle and bottom plate. One calculation showed a higher stress component than the maximum stresses above the knuckle, and the other, a lower value. As a consequence, the data were considered inconclusive and further work on the state of stress within the bottom knuckle was recommended by the Engineering Department.

This letter presents the findings of stress magnitudes in the bottom regions of the toroidal knuckle and detail description of the analytical approach. In supplement to DPST-73-348, results of stress analysis for type II waste tanks are also given. Thus the analysis of stress under hydrostatic loading of all three types of tank configuration is considered to be complete.

- (1) DPST-73-348, W. W. F. Yau to A. J. Schwertferger, Stress Analysis of Type I and III Waste Tanks, May 23, 1973.  
(2) J. H. Faupel to J. G. Brewer, Analysis of Primary Steel Shells of Waste Storage Tanks, September 17, 1973.

SUMMARY

1. For all types of steel containers of waste tanks at SRP, the critical locations of stress-maximum difference between circumferential and meridional components-are all at a distance above the toroidal knuckles. Locations and magnitudes of stress components due to full capacity hydrostatic loading for the three types of steel containers are

Tank Type	Location (Inches above Tank Bottom)	Stresses in KSI ( $\gamma = 1$ ) <sup>*</sup>			
		Inside Surface		Outside Surface	
		Hoop	Meridian	Hoop	Meridian
I	26	+9.27	-2.07	+7.92	+2.83
II	56	+8.20	-0.83	+7.50	+1.62
III	46	+8.68	-0.83	+8.03	+1.40

2. At the junction of the circular base plate and the knuckle bottom, stresses are purely due to bending moment arising from local change of curvature of the meridian, commonly termed as "edge effect". For all tanks, such stresses are considered not critical, their magnitudes are

Tank Type	Stresses in KSI ( $\gamma = 1$ )			
	Inside Surface		Outside Surface	
	Hoop	Meridian	Hoop	Meridian
I	-2.01	-7.17	+2.01	+7.17
II	-1.11	-3.95	+1.11	+3.95
III	-1.08	-3.87	+1.08	+3.87

3. In terms of deformation, edge effect at the bottom of the knuckle produces an uplift on the circumferential edge of the base plate, coupled with a rotation of the outward tangent toward the axis of symmetry, so that the flat base plate assumes the shape of a saucer under hydrostatic loading. For type II tanks, there is a series of small bracings (two inches high) welded to the lower portion of the knuckle forming a supportive ring. The "dishing" effect of the base plate is sufficient to render the bracings ineffective as structural support; however, these bracings serve to provide rigidity in the

\* Specific gravity

meridional direction and alleviate bending stresses in the lower portion of the knuckle to a small extent.

4. For loading of waste liquid with specific gravity ( $\gamma$ ) other than unity, the numerical values for stress components summarized in the previous tables can be linearly scaled with respect to specific gravity. Thermal stresses due to outward heat flow are tensile on the outside, and compressive on the inside surface and their magnitudes, equal for both hoop and meridional components, are at most 1.25 Ksi, based on the 100°F differential temperature across wall thickness. Thermal stresses should be superposed on those due to hydrostatic loading to obtain the overall state of stress.

### DISCUSSION

For type II waste tanks under full capacity loading, response of the cylindrical portion of the wall is obtained by following the procedure outlined in DPST-73-348. Results are calculated in Appendix A and the stress variation in the region of high stress is shown in Figure 1.

Stress analysis for regions surrounding the junction of the knuckle and the base plate is shown in Appendices B, C, and D. Appendix B gives response of the circular plate in terms of general edge loading according to the theory of plates<sup>(3)</sup>. In Appendix C, the governing differential equations are first derived in terms of the horizontal component of the shear force and rotation of the tangent to the meridian according to basic requirements of thin shell mechanics. Then the method of solution is described and Clark's approach<sup>(4)</sup> of approximation is applied. As in Appendix B, the approximate solution is explicitly put in terms of edge loading in general forms. Appendix D shows formulation of the boundary value problem according to equilibrium and continuity, so the information provided by Appendices B and C is used to determine the stress state at the junction.

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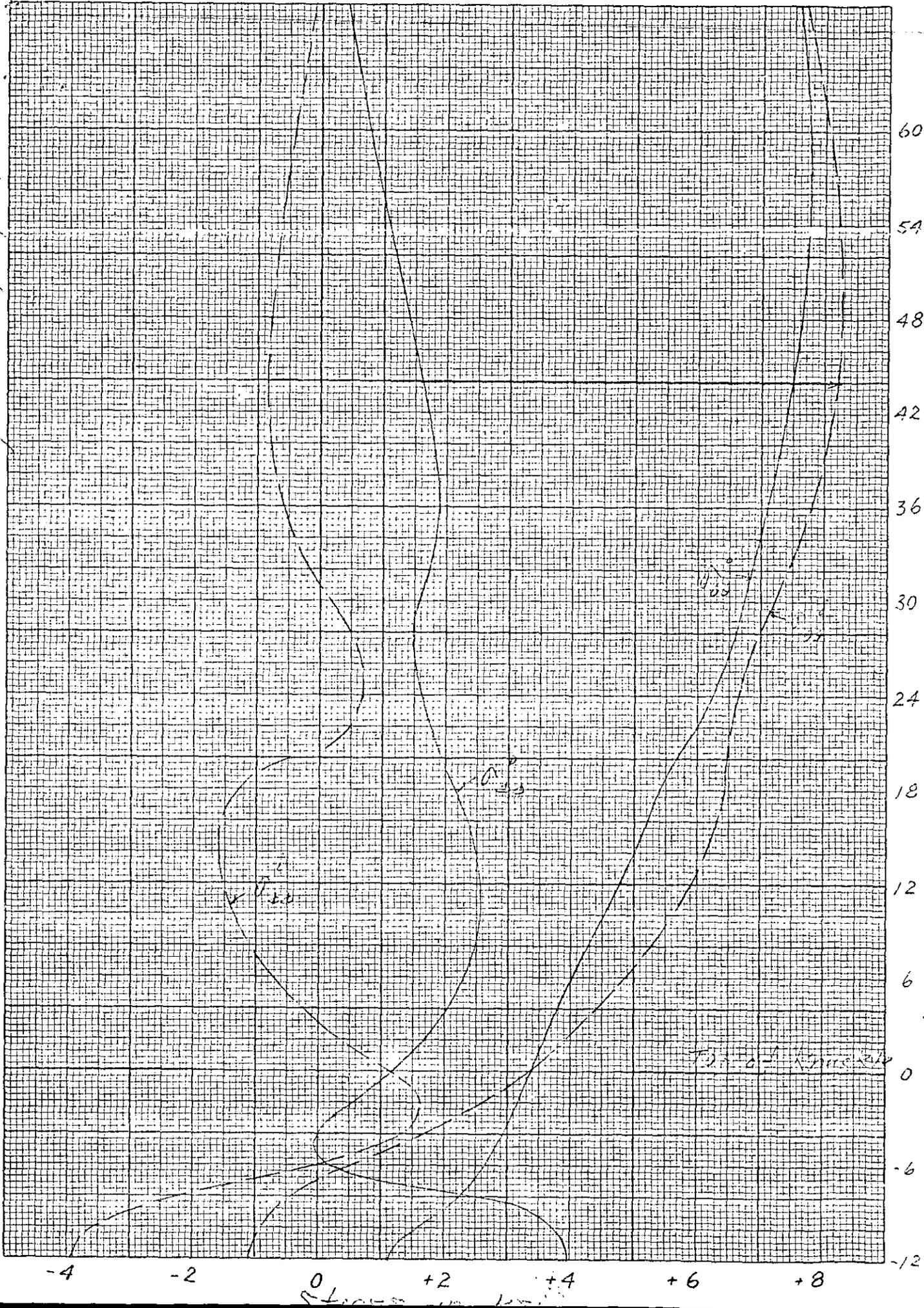
(3) Timoshenko, Woinowsky-Krieger, "Theory of Plates and Shells", pp. 308 ~ 309, McGraw-Hill, 1959.

(4) R. A. Clark, "Outer Theory of Thin Elastic Toroidal Shells", pp. 146-178, J. of Math. & Phys. 29 (1950).

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*Distance in inches from top of knuckle*



NOMENCLATURE

D	=	Flexural rigidity of plates or shells,
E	=	Modulus of elasticity
$F_0$	=	Edge force per unit length
H	=	Hydraulic head above top of knuckle, or horizontal force per unit length
h	=	Shell thickness
k	=	A constant slightly greater than 1
L	=	Second order linear operator
m	=	A constant, $(1-\nu^2) = m^2$
$M_0$	=	Edge moment per unit length
$p_0$	=	Hydrostatic pressure on the base plate
$Q_0$	=	Edge shear per unit length
R	=	Radial coordinate, or radius of the cylindrical portion
$R_0$	=	Radius of base plate
$r_0$	=	Knuckle radius
U	=	Hankel's function
u	=	Displacement
W	=	Complex function of $\alpha$ and $\beta$
x	=	Distance from axis of symmetry
y, z	=	Dummy variables
$\alpha$	=	Dimensionless horizontal force, $m \times H / (Eh^2)$
$\beta$	=	Rotational displacement
$\beta_0$	=	Edge rotation
$\beta_1, \beta_2$	=	Constants
$\gamma$	=	Specific gravity
$\delta_v$	=	Vertical displacement
c	=	A constant, $m \frac{r_0}{h} \left( 1 - \frac{\nu^2 h^2}{m^2 r_0^2} \right)^{\frac{1}{2}}$
$\chi$	=	Curvature
$\lambda$	=	$r_0 / R_0$
$\nu$	=	Poisson's ratio
$\rho_w$	=	Density of water
$\theta$	=	Circumferential coordinate
$\phi$	=	Meridional coordinate
$\sigma$	=	Stress

APPENDIX A

Type II tank: Change of thickness at  $z_1 = 18''$  above knuckle.

$$H = 324''$$

$$\rho_w = 0.03613 \text{ #/in}^3$$

$$R = 510''$$

$$E = 30(10)^6 \text{ psi}$$

$$r_0 = 12''$$

$$\nu = 0.28$$

$$h_1 = 7/8''$$

$$\beta_1 = 0.06104$$

$$h_2 = 5/8''$$

$$\beta_2 = 0.05488$$

$$k = 1.01693$$

For  $z \leq 18''$ ,  $x = \beta_1 z$

$$P_{00}(x) = 6823 \left[ 1 - 0.05056x - 0.5084 e^{-x} (\cos x \pm 0.5052 \sin x) \right]$$

$$P_{44}(x) = 6939 \left[ 0.02353 + 0.14 e^{-x} (\cos x \pm 6.4436 \sin x) \right]$$

$$u_1(z_1) = 0.09984'', \quad u_1'(z_1) = 0.001256$$

$$C_5 = 0.39775, \quad C_6 = 1.8508 - \text{See p. B8 of (1)}$$

For  $z \geq 18''$ ,  $x = \beta_2 z$

$$P_{00}(x) = 9552 \left[ 1 - 0.05624x + 0.2731 e^{-x} (\cos x - 3.8201 \sin x) - 0.6776 e^{-x} (\cos x + 1.2381 \sin x) \right]$$

$$P_{44}(x) = 228.6 \left[ 1 - 68.593 e^{-x} (\cos x - 0.3829 \sin x) + 73.325 e^{-x} (\cos x + 0.0579 \sin x) \right]$$

APPENDIX B

The governing equation for the transverse displacement  $w$  of the base circular plate(5) under pressure  $p_0$  is

$$\nabla^4 w = \frac{p_0}{D}, \quad \nabla^2 = \frac{1}{R} \frac{d}{dR} \left( R \frac{d}{dR} \right) \quad (B1)$$

and its general solution in terms of radius  $R$  is

$$w = c_1 + c_2 \ln R + c_3 R^2 + c_4 R^2 \ln R + \frac{p_0 R^4}{64D} \quad (B2)$$

where  $p_0$  is the constant pressure. Referring to Figure B, let  $M_0$  (positive counterclockwise),  $\beta_0$  (positive clockwise), and  $Q_0$  (positive downward) be the moment slope and shear at the edge,  $R = R_0$ , of the base plate respectively and at  $R = R_c$ , where the plate loses contact with the foundation, the moment and slope should vanish and the shearing force can be readily calculated by equilibrium consideration of the suspended portion of the plate, or  $R_c Q_c + p_0 (R_0^2 - R_c^2) + R_0 Q_0 = 0$ .

With these five boundary conditions, elimination of the integration constants,  $R_c$  and  $Q_c$ , obtains following relationship at the edge,

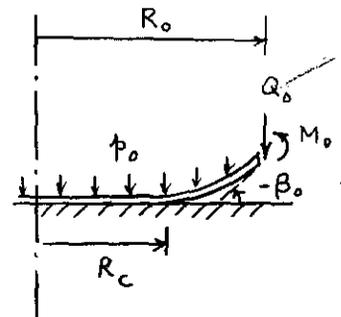


Fig. B

$$\left[ M_0 \frac{R_0}{D} - (1-\nu) \beta_0 \right] \left[ M_0 \frac{R_0}{D} + (1+\nu) \beta_0 \right] + \beta_0 Q_0 \frac{R_0^2}{D} = 0 \quad (B3)$$

provided that  $R_0 - R_c$  is small for  $R_0 > R_c$ ,  $M_0$  can be uniquely expressed as

$$M_0 \frac{R_0}{D} = \left[ \beta_0 \left( \beta_0 - Q_0 \frac{R_0^2}{D} \right) \right]^{\frac{1}{2}} - \nu \beta_0 \quad (B4)$$

Equation (B4) will be used in Appendix D to establish edge effects of the knuckle.

For the waste tanks  $Q_0$  is approximately  $p_0 r_0$  or the weight of the waste above the knuckle.

APPENDIX C

For a differential element of the knuckle between two meridional planes and two horizontal planes as shown in Figure C1, the equilibrium equations of the element under normal pressure  $p$  in toroidal coordinates  $(x, \phi)$  are

$$\frac{d}{d\phi} (xV) = x r_0 p \cos \phi \tag{C1}$$

$$\frac{d}{d\phi} (xH) = N_\theta r_0 + x r_0 p \sin \phi \tag{C2}$$

$$\frac{d}{d\phi} (xM_\phi) = M_\theta r_0 \cos \phi + x r_0 (V \cos \phi - H \sin \phi) \tag{C3}$$

Where  $H, V,$  and  $M_\phi$  are components of force and moment per unit length on the horizontal section, and  $N_\theta$  and  $M_\theta$  are force and moment per unit length on the meridional section.  $r_0$  is the knuckle radius,  $x$  is the radius of the horizontal circle and the meridional  $\phi$  is measured from the bottom of the knuckle. All quantities are shown in their positive directions.

Let  $u_T$  and  $u_N$  be the tangential and normal components of the displacement respectively, and  $\beta$  be the rotational displacement of the tangent at  $(x, \phi)$ , clockwise as positive as shown in Figure C2, then

$$\beta = \beta_N - \beta_T = \frac{1}{r_0} \frac{du_N}{d\phi} - \frac{u_T}{r_0} \tag{C4}$$

and the two curvatures, by definition, are

$$\kappa_\phi = - \frac{1}{r_0} \frac{d\beta}{d\phi} \tag{C5}$$

$$\kappa_\theta = - \frac{\beta \cos \phi}{R_0 + r_0 \sin \phi} \tag{C6}$$

The strain components become

$$\epsilon_\phi = \frac{1}{r_0} \left( \frac{du_T}{d\phi} + u_N \right) \tag{C7}$$

$$\epsilon_\theta = \frac{u_T \cos \phi + u_N \sin \phi}{R_0 + r_0 \sin \phi} \tag{C8}$$

The constitutive equations can be readily obtained from the stress-strain and moment-curvature relations as

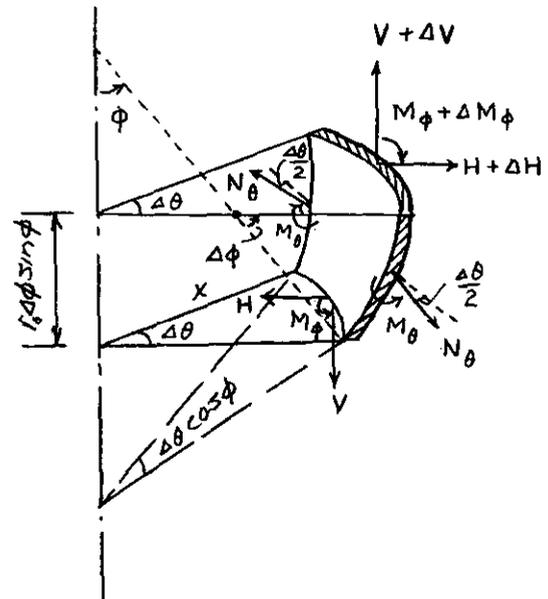


Fig. C1

$$\begin{aligned}
 E\epsilon_{\phi\phi} &= \sigma_{\phi\phi} - \nu\sigma_{\theta\theta} = \frac{1}{h} (N_{\phi} - \nu N_{\theta}) \\
 E\epsilon_{\theta\theta} &= \sigma_{\theta\theta} - \nu\sigma_{\phi\phi} = \frac{1}{h} (N_{\theta} - \nu N_{\phi}) \quad (C9) \\
 M_{\phi} &= -D(\kappa_{\phi} + \nu\kappa_{\theta}) = \frac{D}{r_0} \left( \frac{d\beta}{d\phi} + \frac{r_0 \cos\phi}{R_0 + r_0 \sin\phi} \nu\beta \right) \\
 M_{\theta} &= -D(\kappa_{\theta} + \nu\kappa_{\phi}) = \frac{D}{r_0} \left( \frac{r_0 \cos\phi}{R_0 + r_0 \sin\phi} \beta + \nu \frac{d\beta}{d\phi} \right)
 \end{aligned}$$

Where  $N_{\phi} = V \sin\phi + H \cos\phi$ ,  $D = Eh^3 / [12(1-\nu^2)]$ , and  $h$  is the thickness of the knuckle.

Substituting  $M_{\phi}$  and  $M_{\theta}$  into eq. (C3), one of the governing equations is

$$\beta'' + \frac{x'}{x} \beta' - \left[ \left( \frac{x'}{x} \right)^2 - \nu \left( \frac{x''}{x} \right) \right] \beta = \frac{r_0}{D} (x''H + x'V), \quad (C10)$$

$x = R_0 + r_0 \sin\phi$

Where the prime symbol stands for the derivative with respect to  $\phi$ .

Eliminating  $N_{\phi}$  and  $N_{\theta}$  from eqs. (C2) and (C9) the other governing equation can be shown as

$$\begin{aligned}
 (xH)'' + \frac{x'}{x} (xH)' - \left[ \left( \frac{x'}{x} \right)^2 + \nu \frac{x''}{x} \right] (xH) \\
 = -Ehr_0 \frac{x''}{x} \beta - \nu \frac{x''}{x} (xV)' - \left( \frac{x''}{x} - \nu \right) \frac{x'}{x} (xV) \\
 + r_0 \left[ (x\phi \sin\phi)' + (1+\nu) \frac{x'}{x} (x\phi \cos\phi) \right] \quad (C11)
 \end{aligned}$$

Equation (C1) can be directly integrated to obtain  $V$ , so only  $\beta$  and  $H$  are considered as unknowns in equations (C10) and (C11). To consider bending effects of the knuckle zone only, we set  $\phi = 0$ , which is equivalent to excluding the membrane effect due to  $\phi$ . Eqs (C10) and (C11) are further simplified to the following dimensionless forms:

$$L(\beta) - \nu\lambda\beta + m\lambda \frac{r_0}{h} \alpha = 0 \quad (C12)$$

$$L(\alpha) + \nu\lambda\alpha - m\lambda \frac{r_0}{h} \beta = 0 \quad (C13)$$

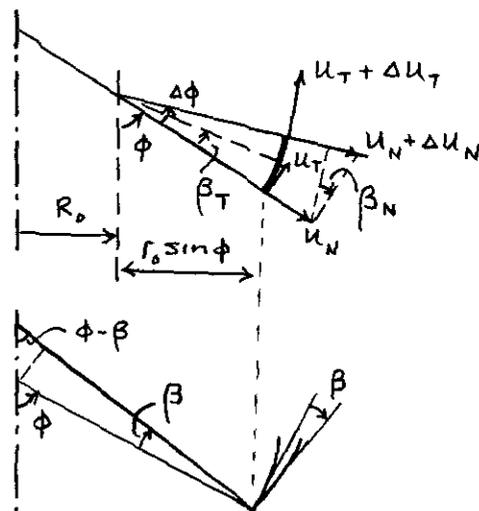


Fig. C2

Where

$$\lambda = \frac{r_0}{R_0}, \quad \alpha = \frac{m \times H}{E h^2}, \quad m^2 = 12(1-\nu^2) \quad \text{and}$$

L is the second order linear differential operator,

$$L = \frac{1 + \lambda \sin \phi}{\sin \phi} \left\{ \frac{d^2}{d\phi^2} + \frac{\lambda \cos \phi}{1 + \lambda \sin \phi} \frac{d}{d\phi} - \left( \frac{\lambda \cos \phi}{1 + \lambda \sin \phi} \right)^2 \right\}$$

Eqs. (C12) and (C13) are reducible to a single equation of fourth order for either  $\alpha$  or  $\beta$ , or a second order equation for a complex function W.

$$W'' - \left[ (ci - \frac{1}{2}) \frac{\lambda \sin \phi}{1 + \lambda \sin \phi} + \frac{3}{4} \left( \frac{\lambda \cos \phi}{1 + \lambda \sin \phi} \right)^2 \right] W = 0 \quad (C14)$$

$$\text{Where } W = (1 + \lambda \sin \phi)^{\frac{1}{2}} (\beta + ik\alpha), \quad k = \left(1 - \frac{\nu^2 h^2}{m^2 r_0^2}\right)^{\frac{1}{2}} + \left(\frac{\nu h}{m r_0}\right) i$$

$$c = m \frac{r_0}{h} \left(1 - \frac{\nu^2 h^2}{m^2 r_0^2}\right)^{\frac{1}{2}}$$

For the three types of tank material and geometry,  $\lambda$  is less than 1/40, and the value of c, between 39.9 and 79.8, is much larger than  $\frac{1}{2}$ , so eq. (C14) can be approximated by

$$W'' - ic \frac{\lambda \sin \phi}{1 + \lambda \sin \phi} W = 0 \quad (C15)$$

The solution of eq. (C15) approaches to that of the Hankel equation (5) asymptotically,

$$\frac{d^2 U}{d\phi^2} + \kappa^2 U = 0, \quad (C16)$$

by introducing transformation on  $W$  and  $\phi$ . Let

$$\tilde{W}(\phi) = g(\phi) U(iy)$$

$$y(\phi) = \left(\frac{3}{2}\right)^{\frac{1}{2}} \left(\frac{3}{2}\omega\right)^{\frac{2}{3}}, \quad g(\phi) = \left(\frac{3}{2}\omega\right)^{\frac{1}{6}} \left(\frac{d\omega}{d\phi}\right)^{-\frac{1}{2}}$$

$$\omega(\phi) = \int_0^\phi \left(\frac{\sin \phi}{1 + \lambda \sin \phi}\right)^{\frac{1}{2}} d\phi$$

eq. (C16) becomes

$$\tilde{W}'' - ic \frac{\lambda \sin \phi}{1 + \lambda \sin \phi} \tilde{W} - \frac{g''}{g} \tilde{W} = 0 \quad (C17)$$

Comparing eqs. (C15) and (C17), they differ by the term  $(g''/g) \tilde{w}$ , for  $\frac{\pi}{2} \geq \phi \geq 0$ ,  $g''/g$  is at least one order smaller than  $c\lambda$ , hence accepting the solution of eq. (C16) as that of eq. (C17) follows the same approximation scheme applied to eqs. (C14) and (C15). Reissner (6) first introduced asymptotic integration method into shell theory (6) and Clark made estimates of accuracy (4) for particular toroidal shells and found the error within 4%. Comparing the geometry parameters of the waste tanks to Clark's example, it is reasonable to expect greater accuracy for the waste tank stresses without solving eq. (C14) exactly.

Two independent solutions (5, p.74) to eq. (C16) are

$$U_i(z) = \left(\frac{2}{3} z^{\frac{3}{2}}\right)^{\frac{1}{3}} H_{\frac{1}{3}}^{(i)}\left(\frac{2}{3} z^{\frac{3}{2}}\right), \quad i=1, 2$$

Where  $H_{1/3}^{(i)}$  are Hankel functions of 1/3 order, first (i=1) and second (i=2) kind. Their series forms are

$$U_1(ix) = -2^{\frac{4}{3}} 3^{-\frac{1}{2}} i \sum_{n=0}^{\infty} \frac{i^n x^{3n}}{3^{2n} n! \Gamma(n + \frac{2}{3})} - 2^{\frac{4}{3}} 3^{-\frac{1}{2}} e^{-\frac{\pi}{3}i} \sum_{n=0}^{\infty} \frac{i^n x^{3n}}{3^{2n} n! \Gamma(n + \frac{4}{3})}$$

$$U_2(ix) = 2^{\frac{4}{3}} 3^{-\frac{1}{2}} i \sum_{n=0}^{\infty} \frac{i^n x^{3n}}{3^{2n} n! \Gamma(n + \frac{2}{3})} + 2^{\frac{4}{3}} 3^{-\frac{1}{2}} e^{\frac{\pi}{3}i} \sum_{n=0}^{\infty} \frac{i^n x^{3n}}{3^{2n} n! \Gamma(n + \frac{4}{3})}$$

Their values of  $U_i$  and  $U_i'$  at the origin are

$$U_1(0) = -1.07439i, \quad U_1'(0) = 0.3916(\sqrt{3} + i)$$

The general solution to (C15) is

$$W(\phi) = g(\phi) \left[ (A + iB) U_1(iy) + (C + iD) U_2(iy) \right]$$

From the asymptotic forms of  $U_1$  and  $U_2$  for large argument,

$$U_1(ix) = x^{-\frac{1}{4}} e^{-\frac{\sqrt{3}}{3}(1+i)x^{\frac{3}{2}}}, \quad U_2(ix) = x^{-\frac{1}{4}} e^{+\frac{\sqrt{3}}{3}(1+i)x^{\frac{3}{2}}}$$

$U_2$  increases exponentially. For edge effects near the bottom of the knuckle zone, the constants C and D must be small compared to A and B. For small  $\phi$ ,

$$W(\phi) \doteq \lim_{\phi \rightarrow 0} g(\phi) \left[ (A + iB) U_1(iy) \right] \tag{C18}$$

Using the two boundary conditions at  $\phi = 0$ , i.e.,  $M_\phi(0) = -M_0$ ,  $H(0) = -H_0$ , we obtain

$$A = 3.0953 \frac{H_0 R_0}{E h^2}, \quad B = 1.4743 \left( -\frac{1}{3} \frac{M_0 R_0}{D} - 1.28 \right) \frac{H_0 R_0}{E h^2}$$

The rotational displacement at  $\phi = 0$  is

$$\beta_0 = R_0 [W(0)] = 1.0744 B = 1.584 C \frac{M_0 R_0}{D} - 1.92 \frac{H_0 R_0}{E h^2} \tag{C19}$$

(6) E. Reissner, "On the Theory of Thin Elastic Shells", Reissner Anniversary Volume, pp. 231 ~ 247, Ann Arbor, Edwards (1949).

APPENDIX D

Considering membrane effects at the junction of the base plate and the knuckle, there is a tensile force at the junction

$$F = h \sigma_{\phi\phi}(0) = \beta_0 r_0$$

So the radial displacement with respect to the base plate is

$$u_b = \frac{\beta_0 r_0 R_0}{Eh} (1-\nu)$$

and with respect to the knuckle at  $\phi = 0$

$$u_k = \frac{R_0}{E} (\sigma_{\phi\phi} - \nu \sigma_{\theta\theta}) = \frac{\beta_0 r_0 R_0}{Eh} (1 - \frac{1}{2}\nu)$$

where  $\sigma_{\phi\phi}$  and  $\sigma_{\theta\theta}$  were obtained from p. B3 of DPST-73-348. Since  $u_b \neq u_k$ , the membrane effects alone would give rise to a discontinuity of displacement at the junction. So there must be edge effects such that, for the knuckle

$$H_0 = \beta_0 r_0 \tag{D1}$$

Substituting  $H_0 = \beta_0 r_0 = Q_0$  into eqs. (B4) and (C19) we obtain

$$\beta_0 = 13.874 \frac{\beta_0 r_0^3}{Eh^3} c^{-\frac{2}{3}} \left[ (1+0.277) \frac{R_0 h}{r_0^2} c^{\frac{2}{3}} \right]^{\frac{1}{2}} - (1+0.1385 \frac{R_0 h}{r_0^2} c^{\frac{2}{3}}) \tag{D2, D3}$$

$$M_0 = -28.512 \beta_0 r_0^2 c^{-\frac{2}{3}} \left[ (1+0.277) \frac{R_0 h}{r_0^2} c^{\frac{2}{3}} \right]^{\frac{1}{2}} - 1 \tag{D2, D3}$$

and the bending stress at the bottom of the knuckle is

$$\sigma_0 = \pm 4.752 \beta_0 \frac{r_0^2}{h^2} c^{-\frac{1}{3}} \left[ (1+0.277) \frac{R_0 h}{r_0^2} c^{\frac{2}{3}} \right]^{\frac{1}{2}} - 1 \tag{D4}$$

where the plus and minus signs apply to outside and inside surfaces respectively. The vertical deflection at  $\phi = 0$  is

$$\delta_v \approx - \frac{M_0 \beta_0}{\beta_0 r_0} > 0 \tag{D5}$$

which means the base plate is lifted upward at the rim due to hydrostatic loading.

For three types of tank configuration, under full capacity loading of waste with unit specific gravity, the following table provides the stress calculation:

Type of Tank	$R_0$ (in.)	$r_0$ (in.)	$h$ (in.)	$\frac{1}{c^{\frac{2}{3}}}$	$\sigma_0$ (psi)
I	438	12	$\frac{1}{2}$	0.7704	7172
II	498	12	$\frac{7}{8}$	0.9690	3950
III	498	12	1	1.0131	3950