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MEMORANDUM

December 18, 1970

TO: B. C. RUSCHE

FROM: P. B. PARKS PBP

COMPARISON OF ADIABATIC AND DIRECT SPACE-TIME NEUTRONICS MODELS

INTRODUCTION

Current SRP reactor safety analyses are based on the so-called "adiabatic method" of treating neutron kinetics. In this method, space and time are treated separately during the course of a reactor transient[1]; thus, the solution only approximates the actual reactor behavior since the interdependence of space and time is not treated exactly. The recently available WIGLE code is based on an "exact" method of solution and now allows a comparison of the adiabatic and direct space-time solutions. WIGLE is limited to one dimensional geometry (slab) and cannot be used for comparisons directly applicable to SRP loadings. However, certain conclusions with regard to the distribution of delayed neutrons during a reactor transient can be made. This study is part of a joint effort by EPD and TPD in the field of reactor kinetics.

In pure point kinetics, the flux shape is assumed to remain unaltered during a transient, and for small, tightly coupled reactor cores this is generally true. The adiabatic method is a refinement of the point kinetics model in an attempt to take into account flux shape changes that occur during the reactor response to a local perturbation (such as fuel or target meltdown, rod withdrawal, etc.). Static reactor codes are employed to calculate the flux shape and k_{eff} at discrete points in time. At SRP the reactivity as a function of time forms part of the input for BURP[2], which is used to calculate the total reactor power as a function of time.

Were it not for the presence of delayed neutrons, the adiabatic method, in the limit of a large number of discrete time points, would closely approximate an "exact" solution of the space-time response of the reactor to localized perturbations. However, the static solutions do not take into account the differences in the delayed and prompt neutron source distributions as functions of time. The delayed neutrons have a distribution similar to that of earlier generations of prompt neutrons, rather than the current prompt distribution.

In recent years, many approaches to neutron kinetics, other than the adiabatic method, have been reported. Only one, the direct space-time or "exact" solution has received widespread attention. In this method, the time dependent, group diffusion theory equations, together with feedback equations, are reduced to difference equations and solved by numerical techniques. This method is often used as the basis for evaluating the accuracy of other, more approximate methods.

From a practical point of view, the direct space-time treatment requires too much computer capability to be considered as a candidate for three dimensional analysis with feedback. One dimensional treatments with simple feedback approximations are widely used, and two dimensional treatments with no feedback are available at sites with large computer facilities. At SRP, there is available a revised version of WIGLE-40[3,4], a one dimensional, 2 energy group, direct space-time code, employing up to six delayed neutron groups.

It was the purpose of this study to examine the magnitude of errors in the adiabatic method caused by the neglect of prompt and delayed neutron flux shape differences and attempt to evaluate the effect of these errors on SRP safety analysis. In particular, these errors were determined for a slab reactor model with nuclear properties similar to those of heavily absorbing SRP production reactors. The reactor response to a variety of reactivity perturbations similar to those encountered in safety analyses were calculated with both the direct space-time and adiabatic treatments. Detailed feedback effects were ignored in both sets of calculations. Of necessity, only a limited number of perturbations were studied; but the results can serve as a guide when considering the relative accuracy of the adiabatic method when applied to SRP safety analyses.

SUMMARY

The space and time dependent responses to local perturbations in a reactor model using nuclear parameters characteristic of a heavy SRP lattice have been determined with both the adiabatic and direct methods of analysis. Initial results of this study indicate that, when compared to the results of direct space-time calculations,

- the adiabatic calculations overestimate the magnitude of the total reactor power change;

- the error in the total reactor power can be as large as 44% (at prompt critical), depending only on the magnitude of the perturtion;

- the space dependent power in the immediate vicinity of the perturbation is overestimated for increasing power transients but can be underestimated when the perturbation subsequently is changed to cause a decreasing power transient.

These effects are due in most part to the failure of the adiabatic method of analysis to properly treat the space and time dependence of the delayed neutrons.

The importance of this error to SRP safety analyses cannot be determined unambiguously for two reasons. In the first place, the calculations had to be made using a one-dimensional reactor model. Direct space-time reactor codes capable of treating more than one dimension were not available at the onset of this program. Secondly, temperature feedback effects could not be included consistently in the calculations due to the limited feedback capabilities of the WIGLE code. Nevertheless, the results suggest that the inherent error of the adiabatic method in the treatment of delayed neutrons leads to non-negligible effects. Thus a more sophisticated method of space dependent kinetics, to properly account for delayed neutron effects, should be included in any future revision of the SRP safety analysis codes.

DISCUSSION

A. Mockup of a Heavily Absorbing Production Reactor

A one dimensional, two zone reactor of the same width as the outer diameter (~ 494 cm) of an SRP D_2O moderated production reactor was used in the calculation. (The use of a one dimensional model to mockup a three dimensional reactor is discussed in the next section.) The width of the buckled zones was fixed at 36.3 cm. Figure 1 shows the divisions of the reactor into 2 buckled zone regions and 5 flat zone regions. Region 3, 17.8 cm wide beginning 17.8 cm from the buckled zone - flat zone boundary, was the region in which all of the perturbations were made. The cell-homogenized, two-group parameters were adjusted until the unperturbed radial flux shape was flat and k_{eff} was equal to 1.0. These adjusted values are given in Table I.

WIGLE can handle up to 6 delayed neutron groups and so the 15 group Keepin[5] ^{235}U fission and photofission set was collapsed to 6 groups using standard techniques and a γ effectiveness factor for photoneutron production of .783. The MINIBURP code[6] was used to check the accuracy of the results with 6 groups against the results with 15 groups for a step change in Δk of .0001. At 400 sec. after the increase in reactivity, the relative power levels differed by less than 1%. The final values of β_i and λ_i for the six groups are tabulated in Table I. This set of values was used for both the adiabatic and direct space-time calculations.

B. Comparison of Flux Tilting in One and Two Dimensional Reactors

An inherent problem in using a one-dimensional representation to approximate a two-dimensional lattice is determining just how closely the unperturbed and perturbed flux shapes in the slab representation approximate the flux shapes in the two-dimensional lattice.

SCRAM-1[7], a two-group, one-dimensional static diffusion theory code was used to determine the flux shape and reactivity changes in the one-dimensional representation. Changes in the thermal capture cross section, Σ_c^{th}, were made in region 3 and flux shapes such as that shown in Figure 1 were computed. To check the adequacy of this one-dimensional representation, calculations were performed with a two-dimensional GAUGE[8] representation of the reactor, assuming axial uniformity. Standard flat zone, buckled zone, and reflector dimensions were used[9]. The few group parameters listed in Table I, modified slightly to give a flat flux in the absence of any perturbation, were used as input to GAUGE. A hexagonal region or block, equivalent in size to a fuel cell and 17.8 cm in from the flat zone - buckled zone boundary, was perturbed by changing the thermal capture cross section, Σ_c^{th}. This region is in a position analogous with region 3 of the one dimensional representation used in SCRAM-1.

The flux shapes obtained with the two techniques for various perturbations can be compared by defining a single parameter called the "tilt ratio", the ratio of flux integrals over equivalent regions on opposite sides of the reactor. However, there is no completely consistent way to define the tilt ratio for the two methods of calculation. The SCRAM-1 definition was chosen to be $\Phi_{reg\ 3}/\Phi_{reg\ 5}$. For the GAUGE problem, the perturbed block plus the six adjacent blocks was designated patch 1. The patch on the exact opposite side of the reactor was designated patch 2. The "patch to patch" definition of flux tilt was defined as $\Phi_{patch\ 1}/\Phi_{patch\ 2}$. Still another expression for the GAUGE tilt was the "line to line" definition where the tilt ratio was the integral of the flux along the radius where the perturbation occurred divided by the integral of the flux along the exact opposite radius.

A plot of tilt ratio versus the change in k_{eff} caused by a perturbation in the GAUGE and SCRAM-1 representations is shown in Figure 2. The magnitude of the SCRAM-1 tilts agree reasonably well with the results obtained using the two definitions given for the GAUGE tilt ratio. Thus it appears that the one-dimensional reactor problem is a valid representation of the real reactor.

C. Adiabatic and Direct Space-Time Solutions

Two types of perturbations were considered. The first was a simple linear ramp change in Σ_c^{th} in region 3 to a maximum change of $-\Delta\Sigma_c^{th}{}_{max}$. The second can be described as a "sawtooth" in that the first linear ramp to $-\Delta\Sigma_c^{th}{}_{max}$ was followed by a second linear ramp of opposite slope to $+\Delta\Sigma_c^{th}{}_{max}$, and again followed by a third linear ramp with the original slope back to $\Delta\Sigma_c^{th} = 0$.

These perturbations were input directly into the WIGLE code; the output was a record of the flux levels as functions of both space and time. BURP cannot accept perturbations in the form of changes in Σ_c^{th} directly, but requires instead an input table of k_{eff} values as a function of time. To obtain these values, $\Delta\Sigma_c^{th}$ in region 3 of the SCRAM-1 mockup was varied over the range from -.0019 to +.0019. The resulting changes in k_{eff} are plotted in Figure 3. It was found that a minimum of 5 entries of k_{eff} vs time into BURP was sufficiently detailed, but generally 10 k_{eff} values were prescribed between Δk_{eff} = 0 and Δk_{eff} (max). Thus a sawtooth perturbation was described using 40 entries.

BURP also requires a value for the prompt neutron lifetime ℓ, as input. The usual definition

$$\ell = \left[v\ \Sigma_a\ (1 + L^2B^2) \right]^{-1}_{\text{Flat Zone}}$$

may be used for systems with fairly uniform nuclear properties or when k_{eff} does not depart strongly from unity. However, a more general definition of the prompt neutron lifetime may be found from perturbation theory as the reactivity coefficient of a uniform addition of 1/v absorber.

$$\ell = \frac{\Delta k_{eff}/k_{eff}}{\Delta\ (\Sigma_a\ v)}$$

where $\Delta\ (\Sigma_a\ v)$ and Δk_{eff} refer to the effect of the addition of the absorber(10). The calculation is performed by adding $\Delta\Sigma_a$ to all regions of a SCRAM-1 problem which has already been perturbed to a flux shape appropriate to k_{eff}. The slope, $\Delta k/\Delta\Sigma_a$, times $1/v \cdot k_{eff}$ is the lifetime. This latter definition was used in all problems in which k_{eff} approached prompt critical at some time during the postulated transient.

The adiabatic space-time solutions come from a combination of BURP and SCRAM-1 results. The flux shapes were found from SCRAM-1 while the amplitude of the space integrated flux at any given time was obtained from BURP output.

Three perturbations were selected for study: Δk_{eff} = .00718 (prompt critical) at $\Delta\Sigma_{c\ max}^{th}$ = -.0019, Δk = .00369 at $\Delta\Sigma_{c\ max}^{th}$ = -.00125, and Δk_{eff} = .0020 at $\Delta\Sigma_{c\ max}^{th}$ = -.000825. The perturbations in Σ_c^{th}, when of opposite sign, gave values of Δk_{eff} of = -.00137, -.00110 and -.00087 respectively.

1. Response to Linear Ramps

The length of the linear ramps was varied from ~ 0 (step function) to 15 sec. Figure 4 shows a comparison of flux shapes calculated using adiabatic and direct space-time methods at various times during and following a 5 sec linear change in Σ_c^{th}, resulting in a total change in k_{eff} of +.00369. Figure 5 compares the total reactor power predicted by the two methods for the same ramp. Figure 6 shows the power predictions in the perturbed region, region 3, and the power in an unperturbed region, region 5, on the opposite side of the pile.

In general the adiabatic method over-predicts the total reactor power at all times as shown in Figure 5. The power on the perturbed side of the reactor is over-predicted while the power on the opposite side is underpredicted, and at times even decreases as a function of time during a positive transient, which is obviously aphysical. The region 5 power response in Figure 6 shows these effects clearly. About 1.5 seconds beyond the end of the ramp the power in region 5 as predicted by the adiabatic method begins to exceed the power calculated with WIGLE. The asymptotic flux shapes become the same, and the adiabatic method everywhere overpredicts the power density by the same percentage.

Comparisons of WIGLE and BURP-SCRAM-1 results for this and other perturbations described above are presented in Tables II and III. Table II summarizes the results of perturbations in which Σ_c^{th} was changed linearly to produce maximum changes in k_{eff} of +.0020, +.00369, and +.00718. The change in Σ_c^{th} were made over intervals of 0, 2, 5, and 15 seconds. The results are presented in terms of differences in total reactor power (BURP - WIGLE) at the end of the ramp and 5 seconds after the reactivity reaches its maximum value. The 15 sec ramp problem was not solved at prompt critical because of computer overflow. The 2 sec ramp problem was solved only for Δk_{eff} = .00369.

The results given in Table II show that the difference in the total power predicted by the two methods is insensitive to the length of the perturbation. However, as expected, the difference is strongly dependent on the magnitude of the perturbation. The power differences for the 5 sec ramps have been plotted in Figure 7.

The region dependent power responses at the end of the perturbation are considerably more sensitive to the length of the perturbation. Table III lists the differences in the power in regions 3 and 5 as predicted by BURP-SCRAM-1 and WIGLE. In the perturbed region, which is the region of greatest interest, BURP overpredicts the power by as much as 46% at prompt critical. However, when the perturbation in k_{eff} is reduced to +.0020, the error is no more than 16%. The under-predictions by BURP in region 5 (unperturbed) at the end of the ramps are generally much smaller, the absolute value never greater than 14%. Nevertheless, the power differences in region 5 approach those in region 3 as the perturbation approaches prompt critical. This is a reflection of the much shorter flux shape stabilization times, i.e., the response is dominated by prompt neutrons.

2. Response to Sawtooth Perturbations

The length of the ramps in the "sawtooth" perturbations covered the same range as for the single ramps. Figure 8 shows the total reactor power as a function of time as predicted by BURP and WIGLE for a perturbation in which k_{eff} is changed to 1.00369, back to 1.00, then to .9989 and then back to 1.00 in 5 second intervals. Again BURP overpredicts the total power at all times. The BURP-WIGLE power differences are listed in Table IV for the other sawtooth perturbations considered.

The errors at the node ($k_{eff} = 1.0$) and at minimum k_{eff} are sensitive to the rate at which the perturbation is introduced, the greatest error occurring for the relatively slow 15 sec ramp. Again the error is strongly sensitive to the magnitude of the perturbations.

Figure 9 shows the power response in regions 3 and 5 to the 5 sec sawtooth perturbation described in Figure 8. These results demonstrate that the adiabatic method does not always overpredict the power response in the perturbed region even though the overall reactor power may be overpredicted. The adiabatic (BURP-SCRAM-1) and direct-space-time (WIGLE) region dependent power differences for the sawtooth problems are listed in Table V.

D. Implications for SRP Accident Analysis

On the basis of the transients studied, the adiabatic method would appear to consistently overestimate the magnitude of the total reactor power change during a transient (assuming the direct space-time calculations to be "exact"). The power changes in the regions adjacent to and including the perturbed regions, which are of more importance in reactor safety analysis, can be considerably overestimated in some cases but underestimated in others.

The reason for this discrepancy is the failure of the adiabatic method to treat properly the delayed neutrons. In the direct space-time calculation, the delayed neutron contribution to the flux tilt lags behind that of the prompt neutrons; whereas, in the adiabatic calculation, the delayed neutron flux tilt is assumed improperly to be the same as that of the prompt neutrons. Thus, during a transient, the total flux tilt (and hence instantaneous power) predicted by the direct space-time calculation is smaller than that predicted by the adiabatic calculation.

The shape and the magnitude of the perturbations discussed above cover the range of those normally encountered in SRP safety analysis. However, since feedback treatments comparable to those in BURP cannot be included in the WIGLE calculations, it is impossible at present to perform safety analyses with "exact" methods that would be directly comparable to problems now being routinely calculated with BURP in combination with two-dimensional static codes. For this reason, it is not possible to determine unambiguously the importance of errors incurred through the use of the adiabatic method in reactor safety analyses. However, the results presented above do indicate that future refinements in SRP analytical methods should include a more sophisticated neutronics model.

E. Further Studies and Experiments

A two-dimensional version of a direct space-time code, DISCOTHEQUE (based on the calculation method in TWIGL[11]), has recently become available and the applicability of this code to the problems discussed above will be studied. If warranted, a few checks will be made to ensure that the one-dimensional studies with WIGLE have been interpreted properly.

The Advisory Committee on Reactor Physics has recently recommended that experiments be performed to provide benchmark data against which neutron kinetics codes may be tested[12]. In particular, it recommended the study of a uniform (as homogeneous as possible) reactor with simple geometry that can be easily analyzed - possibly a cylinder or a square prism. The dimensions should be such as to allow appreciable flux tilting in at least one dimension. This type of experiment is now under consideration. In addition, a series of space-time dependent experiments is being planned as a part of the upcoming Mark 14-30 program in the PDP.

PBP:pph

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11. J. B. Yasinsky, et al, "TWIGL - A Program to Solve the Two Dimensional, Two Group, Space-Time Neutron Diffusion Equations with Temperature Feedback," WAPD-TM-743 (1968).

12. Minutes of the Advisory Committee on Reactor Physics, meeting Fayetteville, Arkansas, May 4-6, 1970, ACRP-41.

Table I

Two Group Parameters for Production Reactor Mockup
(transverse buckling = 50 μB)

	Group	D	$\Sigma_c + \Sigma_f$	Σ_r	$\nu\Sigma_f$
Buckled Zone	1	1.385	.010796	.008547	.0020927
	2	.8792	.018100	--	.026304
Flat Zone	1	1.385	.010796	.008547	.0020917
	2	.8792	.019778	--	.020348

Delayed Neutron Parameters

Group	β_i	λ_i (sec^{-1})
1	.0009856	1.55
2	.0043140	.209
3	.0017650	.0247
4	.00008052	.00280
5	.00003679	.000147
6	.00000119	.00000141

Table II

Difference in Total Reactor Power Response to Linear Change in Σ_c^{th} (BURP - WIGLE)

Maximum Δk	Length of Ramp (sec.) 0	2	5	15
	Percent Difference at End of Ramp			
.0020	-	-	4.8	5.2
.00369	-	12.5	13.7	14.8
.00718 (prompt critical)	-	-	43.6	-
	Percent Difference 5 Sec. After End of Ramp			
.0020	6.7	-	6.6	6.4
.00369	18.4	18.4	17.9	17.2

Table III

Difference in Region Dependent Response to Linear Change in Σ_c^{th} (BURP, SCRAM-1 - WIGLE)

Maximum Δk		Length of Linear Ramp: 0	2	5	15
		Percent Difference at End of Ramp			
.0020	Reg 3	-	-	15.5	12.1
	Reg 5	-	-	-10.2	- 5.8
.00369	Reg 3	-	27.1	24.7	20.6
	Reg 5	-	-13.7	- 8.0	1.3
.00718	Reg 3	-	-	45.7	-
	Reg 5	-	-	34.7	-
		Percent Difference 5 Sec. After End of Ramp			
.0020	Reg 3	13.6	-	12.1	10.4
	Reg 5	- 4.3	-	- 2.7	- .7
.00369	Reg 3	22.9	22.3	21.1	19.1
	Reg 5	7.2	8.6	9.6	11.9

Table IV

Difference in Total Reactor Response to Sawtooth Perturbations in Σ_c^{th} (BURP - WIGLE)

		Length of Ramp (sec.)		
	k_{eff}	2	5	15
Maximum Δk	Percent Power Difference at Specified k_{eff}			
+.0020	1.00200	-	4.8	5.2
	1.00000	-	1.7	2.7
	.99913	-	4.6	5.7
+.00369	1.00369	12.5	13.7	14.8
	1.00000	3.5	6.5	10.3
	.99890	8.2	12.0	16.9
+.00718	1.00718	-	43.6	-
	1.00000	-	37.7	-
	.99861	-	60.5	-

Table V

Difference in Region Dependent Response to Sawtooth Changes in Σ_c^{th} (BURP-SCRAM-1 - WIGLE)

		Length of Ramp (sec.)		
	k_{eff}	2	5	15
Maximum Δk	Percent Power Difference at Specified k_{eff}			
Region 3				
.0020	1.00200	-	15.5	12.1
	1.00000	-	- 3.9	- 3.8
	.99913	-	-10.4	- 6.2
.00369	1.00369	27.1	24.7	20.6
	1.00000	- 3.9	- 4.1	- 1.7
	.99890	-17.1	-13.5	- 4.9
.00718	1.00718	-	45.7	-
	1.00000	-	6.7	-
	.99861	-	- 1.8	-
Region 5				
.0020	1.00200	-	-10.2	- 5.8
	1.00000	-	6.0	8.1
	.99913	-	14.7	13.4
.00369	1.00369	-13.7	- 8.0	1.3
	1.00000	0.5	16.1	22.0
	.99890	24.2	28.5	29.6
.00718	1.00718	-	34.7	-
	1.00000	-	80.0	-
	.99861	-	108.8	-

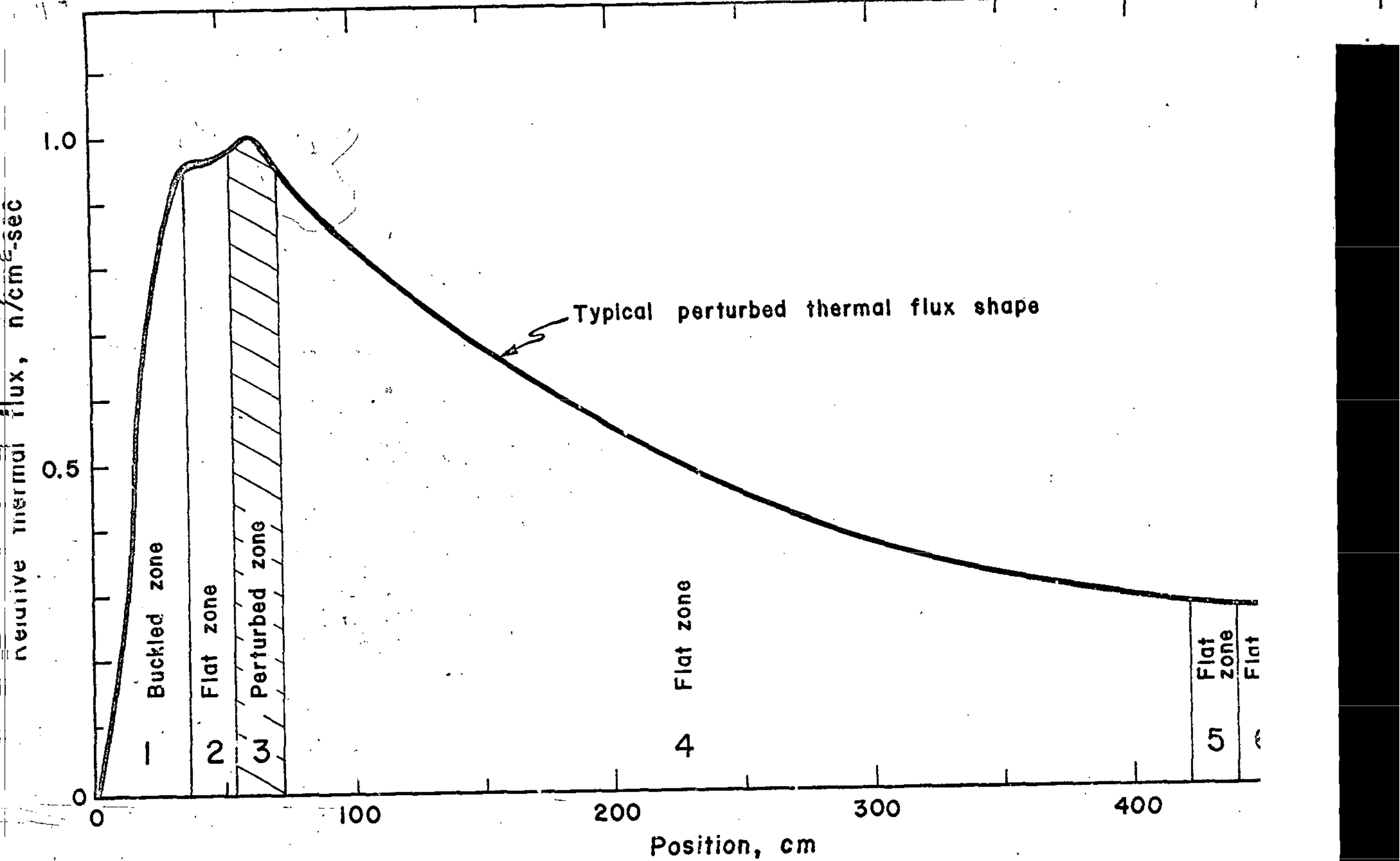

Figure 1. Slab Representation of SRP Production Reactor

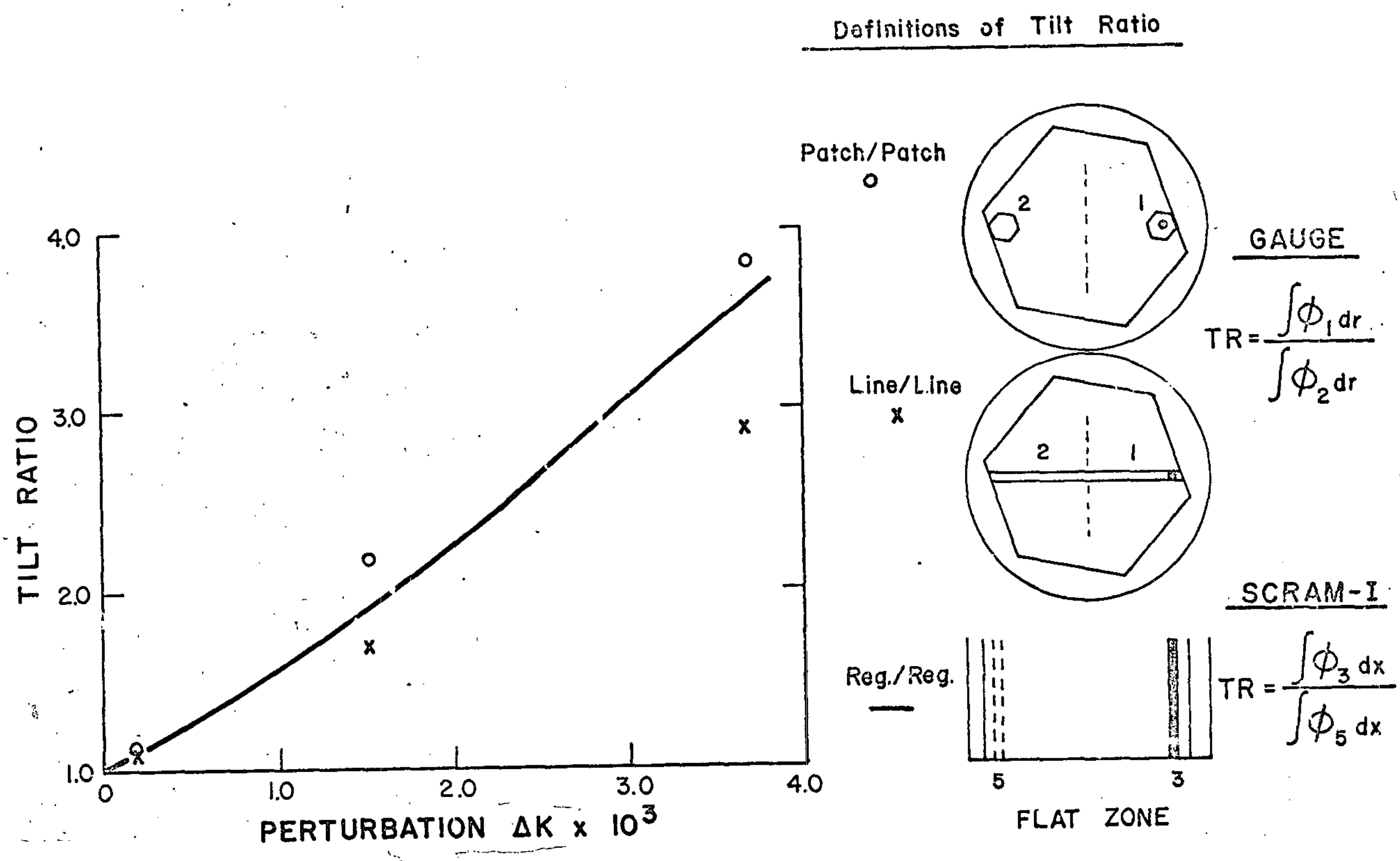

Fig. 2. Tilt Ratio vs Δk.

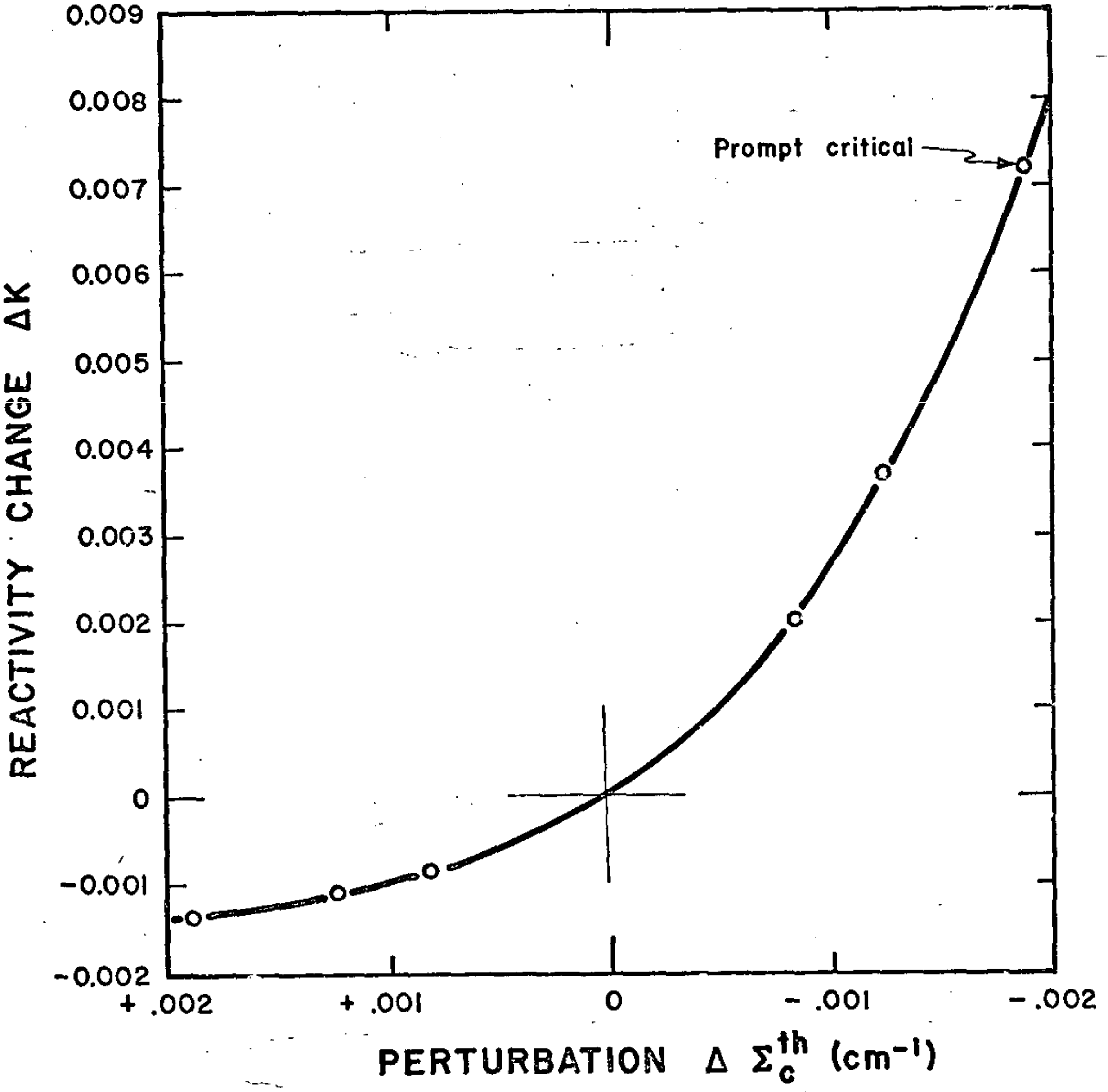

Figure 3. Calculated Relationship Between Δk and $\Delta\Sigma_c{}^{th}$ in Region 3 (from SCRAM-1)

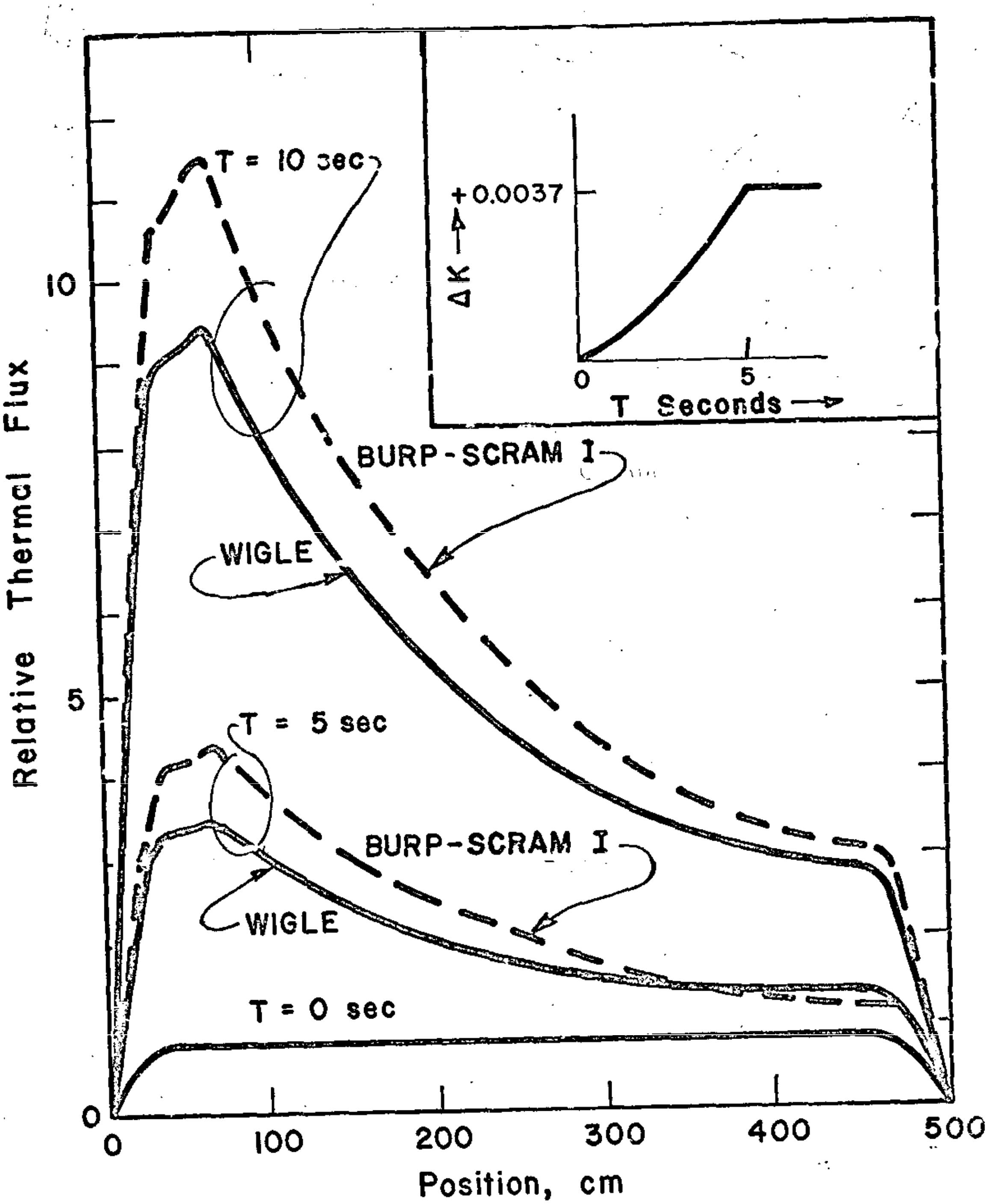

Fig. 4. Thermal Flux Shapes After 5 Sec. Linear Ramp In Σ_c^{th}

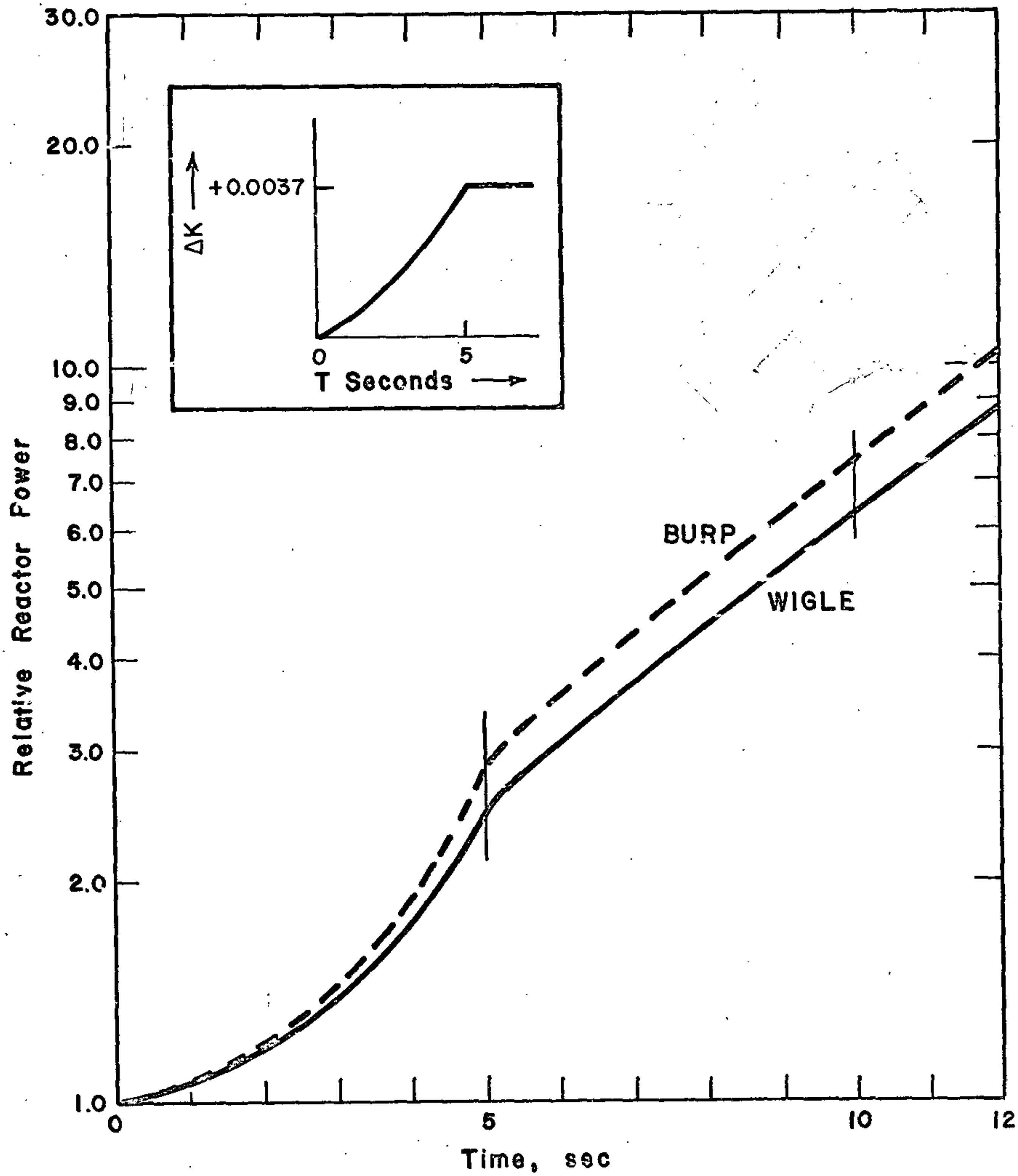

Figure 5. Total Reactor Power Response to Linear Ramp in Σ_c^{th}

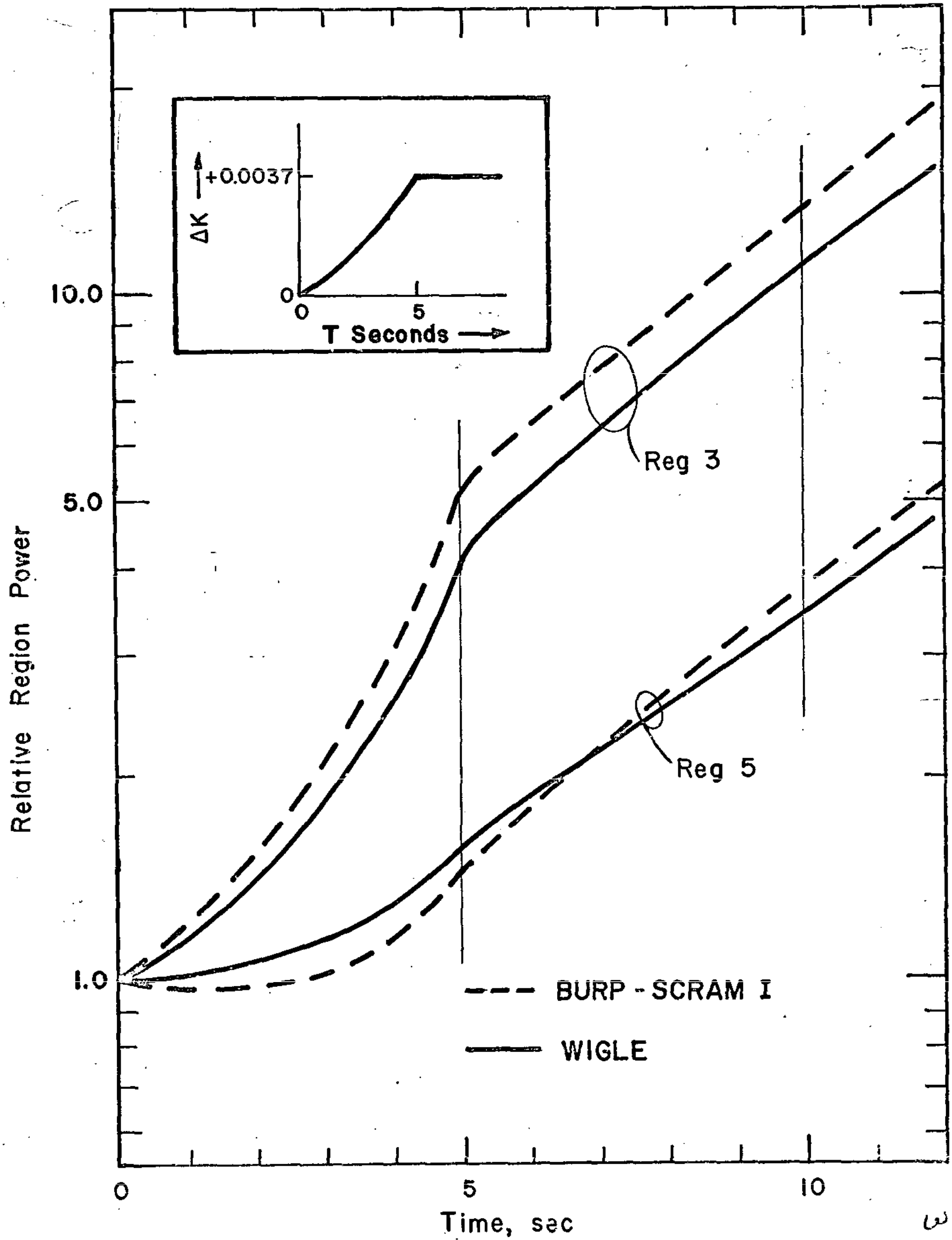

Figure 6. Region Power Response to Linear Ramp in Σ_c^{th} in Region 3

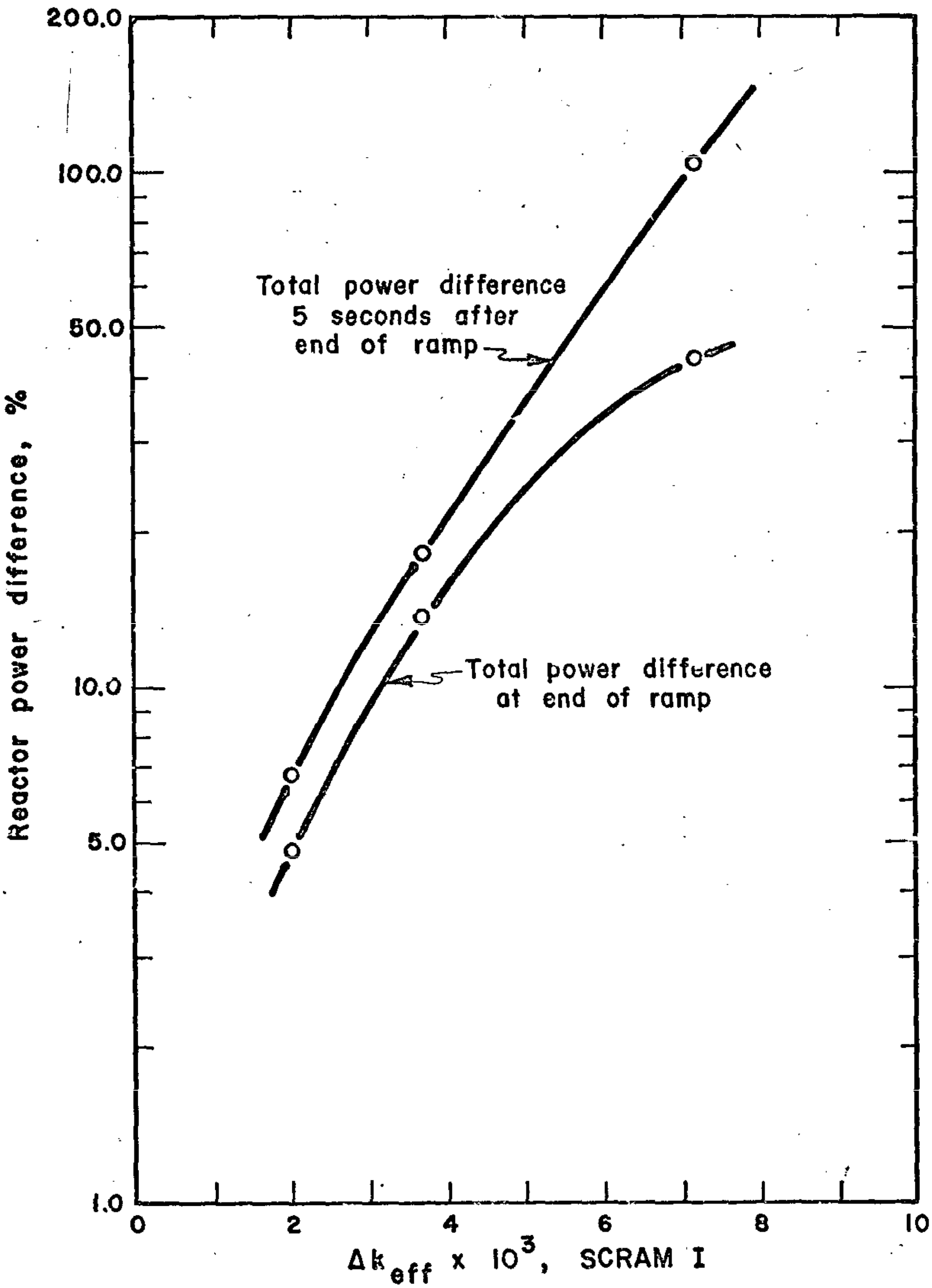

Figure 7. Difference in Total Reactor Power (BURP-WIGLE) Following 5 Sec. Linear Ramp in $\Sigma_c{}^{th}$

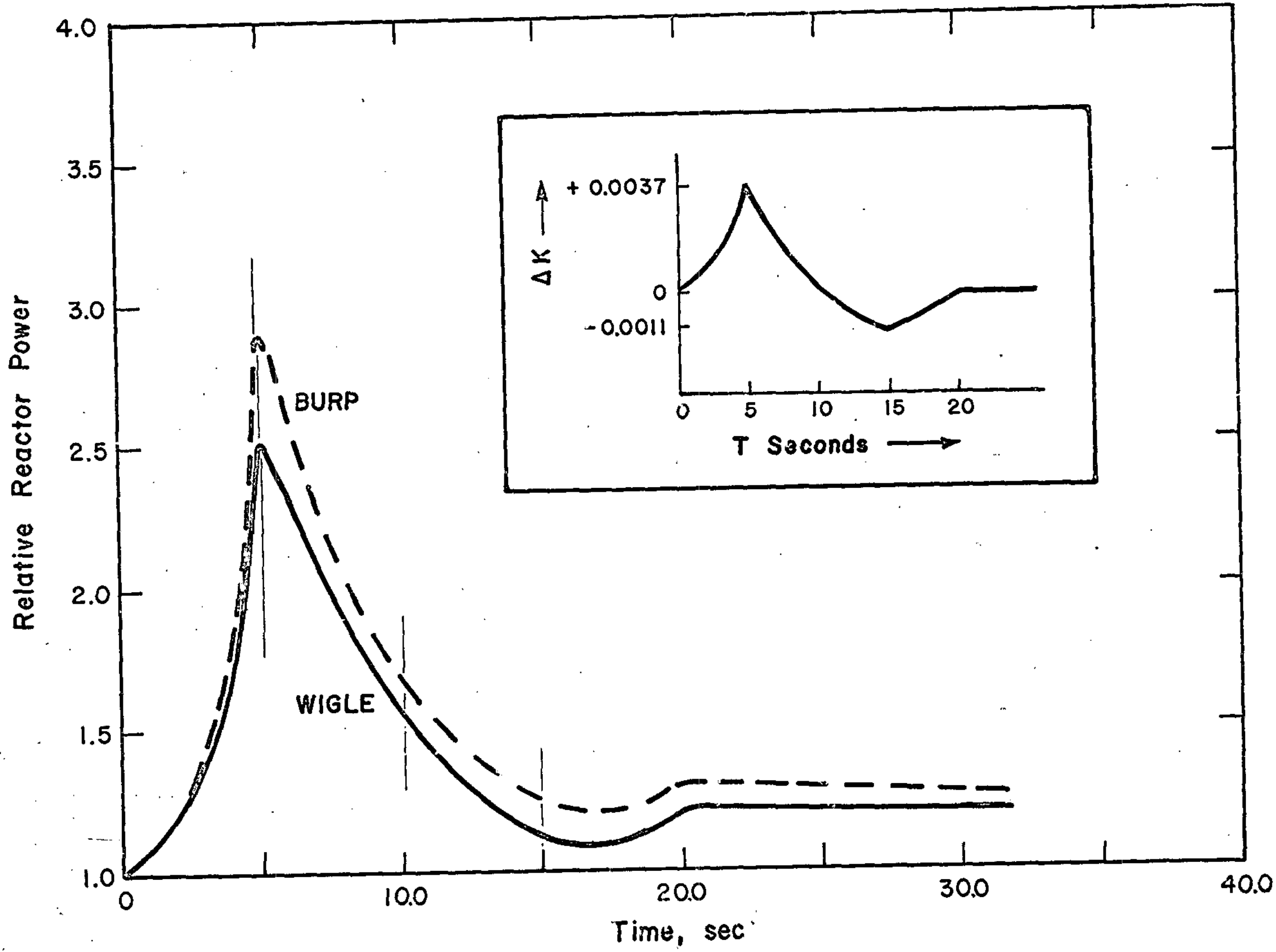

Fig. 8. Total Reactor Power Response to Sawtooth Perturbation in Σ_c^{th}

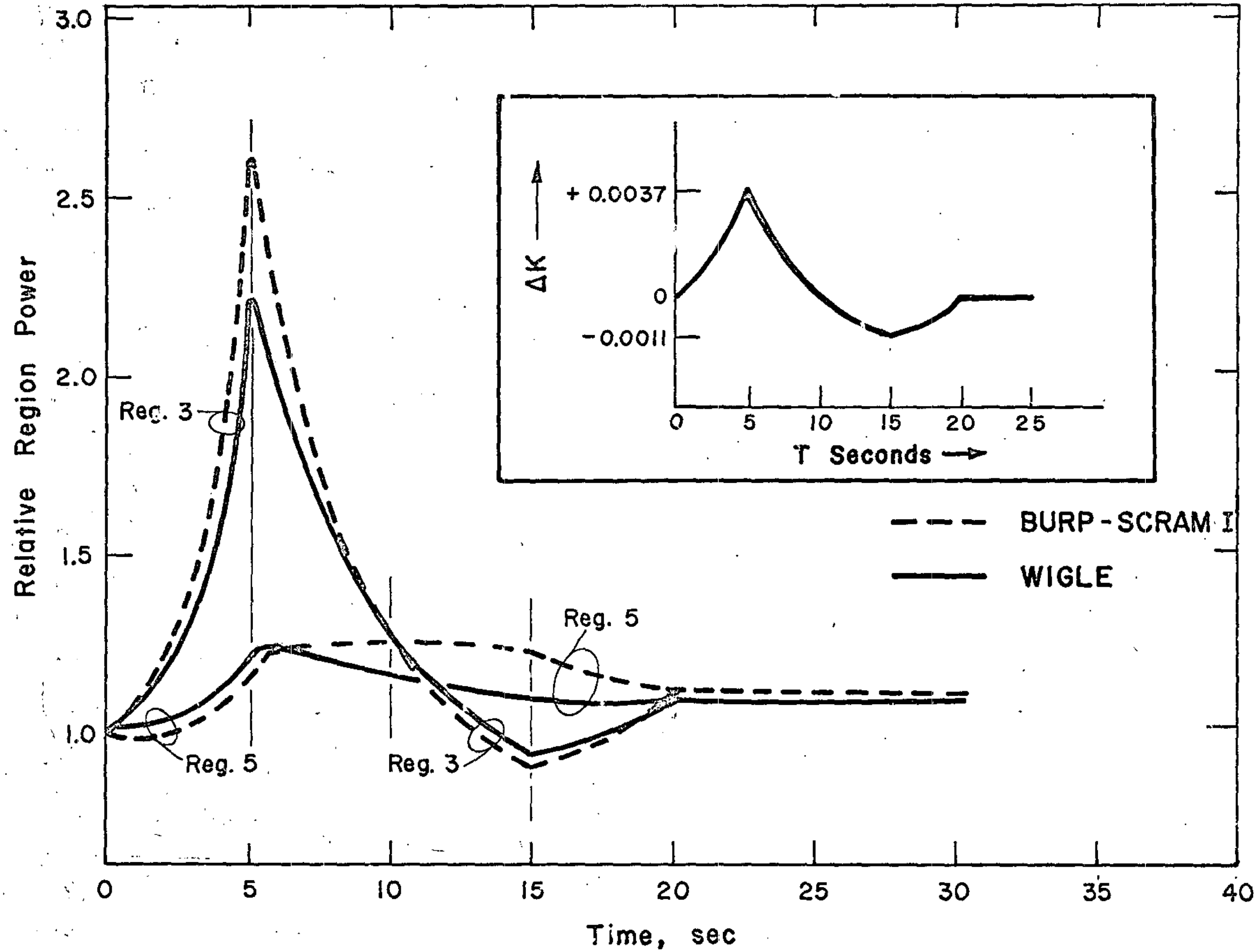

Fig. 9. Region Power Response to Sawtooth Perturbation in $\Sigma_c{}^{th}$