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EMPIRICAL PREDICTION OF DIMENSIONAL STABILITY OF NATURAL URANIUM FUEL ELEMENTS

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J. P. LeGeros and M. D. Altschuler
E. I. du Pont de Nemours and Company
Savannah River Plant

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EMPIRICAL PREDICTION OF DIMENSIONAL STABILITY OF NATURAL URANIUM FUEL ELEMENTS

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INTRODUCTION

Natural uranium fuel elements normally change dimensions during irradiation. This change in dimensions, termed instability or growth, may cause unequal flow splits in the cooling water, difficulty in unloading irradiated elements, and fuel element failures.

A reliable, inexpensive, out-of-pile test which predicts the dimensional instability to be experienced during irradiation is needed. The in-pile methods presently used are costly, since they require fabrication and finishing of large numbers of test elements, and they may be detrimental to safe reactor operation.

Since a theoretical mechanism to explain the change in dimensions of fuel elements during irradiation has not been proven to date, an empirical approach was evaluated. This involved studying, by multiple correlation techniques, the dimensional behavior of 956 fuel elements that were divided into six different groups representing different heat treatments.

This study was confined to changes in only one dimension, the length of the fuel elements. Changes in diameter were not studied at this time because texture gradients along the radius would have had to be included, thus increasing the scope of the problem (and hence the computer calculation time) beyond the time available for this first study. Therefore, in this paper the term "instability" refers only to changes in slug length (growth or shrinkage). Diameter changes will be considered in subsequent studies.

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The factors considered in this study were:

- Physical characteristics of fuel elements.
- Reactor geometry and operating characteristics.

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The main purposes of this study were:

1. To determine the principal factors causing instability during irradiation.
2. To develop criteria for testing theoretical mechanisms of instability.
3. To derive a formula which can be used to predict the type and magnitude of irradiation-induced instability.

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SUMMARY

Among the fuel-element factors considered in this investigation, burnup, preferred orientation, and temperature were found to be the most important in determining irradiation-induced instability.

This study showed no correlation between gross or over-all irradiation-induced instability and grain sixe as determined metallographically.

A formula was developed to predict irradiation-induced axial instability. The formula, given below in symbolic form, accounts for about 47.6% of the variance in instability observed; or, expressed in another way, gives a coefficient of determination of 47.6%. The unexplained variance is believed to be due to factors not yet considered.*

$$G_c = f_1(g,B) + f_2(B,\tau) + f_3(Z,\rho) + f_4(T) + f_5(B,\lambda) \quad (1)$$

where: G_c - Predicted instability

B - Burnup

τ - Exposure rate (power per unit element length)

Z - Axial position of fuel element in reactor

ρ - Radial position of fuel element in reactor

T - Temperature of fuel element during irradiation

λ - Fuel element crystallographic preferred orientation texture

g - Grain size*

** A function, f (g,B), used in the original correlation study did not contribute significantly to the calculated instability; therefore, this function is not included in formula (1).

FUTURE WORK

The empirical approach described in this paper will be expanded to:

1. Improve the precision of the instability formula by using additional data and new theoretical functions.

2. Repeat the study considering diameter instability and radial texture gradients.

3. Evaluate proposed theoretical mechanisms of dimensional instability by determining the amount of variance explained by these mechanisms.

4. Further analyze the principal factors affecting dimensional instability.

5. Re-evaluate the effect of grain size upon instability by repeating the study using grain size data obtained by other methods and using different theoretical functions than the ones considered in this paper.

III

DISCUSSION

METHOD USED

An irradiation growth prediction formula was developed and correlated to actual fuel element instability data so that the error of estimate would be at a minimum. The error of estimate, s^2, is the sum of the squares of the deviations which exist between the data and the function, as given in formula (2).

$$s^2 = \frac{\Sigma (G_i - G_{ci})^2}{N} \qquad (2)$$

where: G_{ci} = Estimated instability (value of function) for ith sample

G_i = Actual instability value of ith sample (see appendix A)

N = Number of samples considered

FUNCTION DEVELOPED

The function developed for this correlation study is as follows:

$$G_c = a_1 + a_2 \beta g^3 + a_3 \beta g^2 + a_4 \beta g + a_5 \beta^2 + a_6 \beta + a_7 \ln(\beta+1) + a_8 \tau^2 + a_9 \tau + a_{10} \ln(\tau+1) + a_{11} \rho^2 + a_{12} \rho + a_{13} Z^4 + a_{14} Z^3 + a_{15} Z^2 + a_{16} Z + a_{17} (\beta/g) + a_{18} g^2 + a_{19} g + a_{20} T^2 + a_{21} T + a_{22} \lambda_1 \beta + a_{23} \lambda_2 \beta + a_{24} \lambda_3 \beta + a_{25} \lambda_4 \beta + a_{24} \lambda_5 \beta \quad (3)$$

where: G_c - Calculated or predicted instability

g - Grain size

τ - Rate of irradiation (power per unit length)

ρ - Radial position of fuel element in reactor

Z - Axial position of fuel element in reactor

T - Temperature of fuel element during irradiation

β - Total atom percent burnup of fuel element

λ - Crystallographic preferred orientation texture

Values for the coefficients, a_i, are given in appendix D.

The terms of the function were classified in five groups, as follows:

Group I, grain size dependency:

βg^3, βg^2, βg, β/g, g^2, g

Group II, radiation characteristics:

β^2, β, $\ln(\beta+1)$, τ^2, τ, $\ln(\tau+1)$

Group III, reactor position dependency:

ρ^2, ρ, Z^4, Z^3, Z^2, Z

Group IV, fuel element temperature dependency:

T^2, T

Group V, crystallographic preferred orientation dependency:

$\lambda_1 \beta$, $\lambda_2 \beta$, $\lambda_3 \beta$, $\lambda_4 \beta$, $\lambda_5 \beta$

Group I. The terms of group I were chosen to evaluate the effect of grain size on fuel element instability. The first three terms of this group are concerned with the effect due to the grain volume, area, and diameter, respectively. The fourth term was inserted in the event that an inverse effect of grain size upon instability might exist. These four terms are weighted with the burnup, so that (1) the data will be unique for each sample, and (2) the function will comply with proposed mechanisms. The last two terms of group I consider the effect due to the unweighted grain size values. The polynomial was limited to the second power since there was only one grain size value for each of the six heat treatments.

Group II. The terms of group II were chosen to evaluate the effect of radiation characteristics - total atom percent burnup and flux level - independent of all other factors. The logarithmic terms were included because a gradual decrease in the rate of instability was observed at higher burnups and flux levels.

Group III. The terms of group III were chosen to evaluate the possibility that some function was overlooked and might be included in the reactor coordinates. Such functions might be flow, hydraulic pressure on a fuel element, radial flux gradient, pressure due to the weight of fuel elements above, etc. A fourth degree polynomial for column position was used, to correspond to that of the flux shape.

Group IV. The terms of group IV were chosen to evaluate the effect of temperature on fuel element instability. The calculation of fuel element temperature is described in appendix C.

Group V. The terms of group V were chosen to evaluate the effect of preferred orientation on fuel element instability. The textures were weighted with burnup for the same reasons as given in group I.

The terms chosen for this function were considered as fundamental for a first investigation of the factors contributing to irradiation instability. It must be emphasized that the introduction of an additional term to the correlation function cannot increase the error of estimate; or, expressed in another way, cannot decrease the coefficient of determination (see appendix A). If the term contributes very little information concerning the growth mechanism, then the error of estimate will increase only a negligible amount when the term is removed from the function. For every additional term, however, there is one less degree of freedom.

IV

THE DATA

Grain Size, g. One value available for each heat treatment. (Each heat treatment characterized by five determinations, using the metallographic method currently standard at SRP.)

Exposure Rate, . One value available for each sample, from reactor data.

Reactor Radial Position, p. Calculated from the X and Y coordinates of each fuel element by formula (4).

$$\rho^2 = (x-28)^2 + \frac{1}{3}(y-48)^2 \qquad (4)$$

Reactor Axial Position, Z. Available for each sample, from reactor data.

Fuel Element Temperature, T. Calculated for each sample (see appendix C).

Total Atom Percent Burnup, . Calculated for each sample from exposure rate, , and duration of irradiation cycle, t, by formula)(5).

$$B = \tau t \qquad (5)$$

Crystallographic Preferred Orientation Texture, . Available from X-ray diffraction studies for each heat treatment.

Fuel Element Instability (Growth or Shrinkage) During Irradiation, G. Available for each sample, from preirradiation and postirradiation measurements.

Duration of Irradiation, t. Available for each heat treatment, from reactor data.

RESULTS

The coefficients of determination* obtained (ie, the amount of the variance accounted for) were as follows:

For the entire function:	47.588%
For all terms except those of group I:	47.582%
For all terms except those of group II:	47.074%
For all terms except those of group III:	47.181%
For all terms except those of group IV:	45.327%
For all terms except those of group V:	42.402%

* Also called coefficient of multiple determination, or improvement.

The coefficient of determination, R^2, is defined as follows:

$$R^2 = \frac{\sigma^2 - s^2}{\sigma^2} \qquad (6)$$

where: σ^2 = Variance from the mean.

s^2 = Error of estimate from correlation function (see formula (2)).

A significant difference of the coefficient of determination is 0.1.

CONCLUSIONS

1. Among the factors considered, the ones contributing most to the instability of fuel elements are the radiation characteristics (flux level and total atom percent burnup).

   Since the removal of any one of the five groups of terms from the function does not cause any great decrease in the coefficient of determination, it can be concluded from this function that the primary factor involved with irradiation growth is contained in each of the groups. This factor is burnup and/or flux rate. Burnup, , is proportional to flux rate, , for all heat treatments but one. Thus the only group not having burnup explicitly is group III, which is concerned with reactor position. Since the column position polynomial (fuel element axial position function) is of the same order as the flux shape, the burnup factor is contained implicitly.

2. Preferred orientation is a primary factor among the ones considered in fuel element instability during irradiation.

   Since the removal of group V, preferred orientation function, lowers the coefficient of determination by 5%, it must be concluded that the preferred orientation contains information not included in any of the other factors.

3. The temperature of the fuel element is an important factor among the ones considered in fuel element instability during irradiation.

   Since the removal of group IV lowers the coefficient of determination by 2%, it must be concluded that the fuel element temperature during irradiation contributes to instability in a manner not explained by any of the other factors considered.

4. Among the factors considered, ones dependent on reactor position offer no additional information to the problem of fuel element instability during irradiation.

   Since the removal of group III causes only a minor drop in the coefficient of determination, it can be concluded that factors which depend on reactor position offer no unique information to the problem and are of negligible importance to theoretical considerations.

5. Grain size (group I functions) cannot be evaluated at this time.

   Since the removal of group I does not affect the coefficient of determination, these are the possibilities with regard to the importance of grain size:

   (1) Grain size as determined metallographically is not a factor in fuel element axial instability.

   (2) The information offered by this group of terms is also contained in the preferred orientation group, group V.

   (3) The functions used in this study to emphasize the effect of grain size do not apply.

   (4) The grain size method of measurement currently being used at SRP does not give values which can be related to irradiation-induced axial instability.

A review of the literature shows that possibilities (1) and (3) seem unlikely.

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APPENDIX A

MULTIPLE CORRELATION APPROACH

This study was made using the multiple correlation approach. A least-mean-square fit of formula (2) to all of the data was made so that G_{ci} for any particular fuel element could be evaluated. Using data for the growth of each fuel element, G_i, and the mean growth of all fuel elements studied, $\bar{G}$, the coefficient of determination, R^2, was calculated.

The coefficient of determination is defined as

$$R^2 = \frac{\Sigma(G_i - \bar{G})^2 - \Sigma(G_i - G_{ci})^2}{\Sigma(G_i - \bar{G})^2} = \frac{\sigma^2 - s^2}{\sigma^2}$$

where: σ^2 = Variance = $\Sigma(G_i - \bar{G})^2$

s^2 = Error of estimate = $\Sigma(G_i - G_{ci})^2$

The error of estimate of a correlation function can never be greater than the variance.

Since slug growth is dependent upon more than one factor, it is necessary to use multiple correlation techniques. Multiple correlation reveals relationships that may exist when several independent factors act simultaneously. If each independent factor is correlated separately with instability, these relationships may remain hidden.

If there are more than two factors say $X_1, X_2, \ldots, X_n$, than a function $G_c = a + b_1X_1 + b_2X_2 + \ldots + b_nX_n$ could be chosen for correlation.

APPENDIX B

FORMULATION

The actual values for the coefficients of formula (3) were obtained by correlating that function to the data. The method used can best be understood by examining the following simple example.

Suppose the function to be correlated is

$$G_c = a_1 + a_2 X + a_3 Y$$

Then, to solve for a_1, a_2, a_3, we minimize

$$F = \sum_{i=1}^{m} (G_i - G_{ci})^2 = \sum_{i=1}^{n} (G_i - a_1 - a_2 X_i - a_3 Y_i)^2$$

and obtain

$$\frac{\partial F}{\partial a_1} = -2 \sum (G_i - a_1 - a_2 X_i - a_3 Y_i) = 0$$

$$\frac{\partial F}{\partial a_2} = -2 \sum X_i (G_i - a_1 - a_2 X_i - a_3 Y_i) = 0$$

$$\frac{\partial F}{\partial a_3} = -2 \sum Y_i (G_i - a_1 - a_2 X_i - a_3 Y_i) = 0$$

or rearranging:

$$\Sigma G_i = N a_1 + a_2 \Sigma X_i + a_3 \Sigma Y_i$$

$$\Sigma G_i X_i = a_1 \Sigma X_i + a_2 \Sigma X_i^2 + a_3 \Sigma X_i Y_i$$

$$\Sigma G_i X_i = a_1 \Sigma Y_i + a_2 \Sigma X_i Y_i + a_3 \Sigma Y_i^2$$

where N is the number of samples considered. Solving these three equations simultaneously will give the values of a_1, a_2, a_3. The coefficient of determination can then be found from the definition given in appendix A.

The difference between this simple example and the actual problem in this study is that in the actual problem, 26 instead of 3 equations were developed and solved simultaneously.

The cross-products that must be evaluated for formula (3) are summarized in figure 4.

APPENDIX C

DERIVATION OF ELEMENT TEMPERATURE

Let s - Specific heat of the water

f - Flow of water (mass per unit time)

ℓ - Length of slug

τ_i - Flux rate (power per unit length) for the ith slug of a column

Then Power for ith Slug $= \tau_i \cdot \ell = s \cdot f \cdot \Delta T_i$

where ΔT_i - Difference in temperature of water at ends of ith slug

Rearranging: $$\Delta T_i = \frac{\tau_i \ell}{s \cdot f}$$

If the inlet temperature of the water is T_o, then the temperature of the ith slug averages:

$$T_i = \sum_{i=1}^{i} \Delta T_i - \frac{1}{2} \Delta T_i + T_o$$

or

$$T_i = \sum_{i=1}^{i} \frac{\tau_i \ell}{f \cdot s} - \frac{1}{2} \frac{\tau_i \ell}{f \cdot s} + T_o$$

Since ℓ, f, and s were constant,

$$kT_i = \sum_{i=1}^{i} \tau_i - \frac{1}{2} \tau_i + kT_o \qquad \text{(approximately)}$$

To find k, several columns were examined. If T_b is the effluent temperature of the cooling water, then

$$k \, \Delta T_b = k(T_b - T_o) = \sum_{i=1}^{n} \tau_i$$

$$k = \left[\frac{\sum_{i=1}^{n} \tau_i}{T_b - T_o} \right]_{Avg.}$$

The column for column variation of k was less than 1.0%

[redacted]

APPENDIX D

I. Values for the coefficients of formula (3)

a_1	is	674
a_2		-15.867
a_3		- 1.36
a_4		- 3.1
a_5		- 4.6
a_6		-452.
a_7		762.
a_8		10.95
a_9		-243.5
a_{10}		208.
a_{11}		- .98
a_{12}		-12.68
a_{13}		- .0108
a_{14}		.490
a_{15}		--7.82
a_{16}		51.7
a_{17}		15.7
a_{18}		-26.41
a_{19}		31.58
a_{20}		629.
a_{21}		-1502.
a_{22}		20.4
a_{23}		81.5
a_{24}		338.
a_{25}		-54.2
a_{26}		59.8

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APPENDIX D (continued)

II. Data characterized for each of six fuel element categories is given in the table below:

Heat Treatment	I	II	III	IV	V	VI
Grain size, g,	.24 mm	.23 mm	.19 mm	.03 mm	.17 mm	.23 mm
Texture $_1$ (020)	1.24 ± .01	1.08	1.10	1.82	1.39	1.30
Texture $_2$ (111)	.85	.89	.89	.91	.87	.89
Texture $_3$ (112)	.83	.94	.97	.31	.97	.85
Texture $_4$ (131)	.94	.91	.95	1.77	.86	.98
Texture $_5$ (200)	1.90	1.58	1.16	2.08	1.30	.15
No. of slugs irradiated and measured	239	238	79	80	80	240
No. of samples characterized	100	100	80	80	80	100

III. The ~~cladding~~ Bulk water temperature range at which the fuel elements were irradiated is:

36°C - 97°C

IV. The range of power per unit length dissipated by the fuel elements during irradiation (this quantity is proportional to neutron flux level) is:

16 to ~~100~~ 101 kilowatts/foot

V. The range of percent growths exhibited by the fuel elements is:

Heat Treatment I	~~-.078%~~	-.78%
Heat Treatment II	~~-.075%~~	-.75%
Heat Treatment III	~~-.012%~~	-.12%
Heat Treatment IV	~~-.013%~~	-.13%
Heat Treatment V	~~+.062%~~	+.62%
Heat Treatment VI	~~-.032%~~	-.32%

VI. Mean growth of all fuel elements is ~~-.0044%~~ -.44%

Texture was computed using

$$\lambda_i = \frac{I_i/I_{0i}}{\sum_{i=1}^{n} A_{wi}\, I_i/I_{0i}}$$

Where $\sum_{i=1}^{n} A_{wi}\, I_i/I_{0i}$ included

the 020, 110, 021, 002, 111, 112, 131, 023, 200, and 113 textures.

I_i: is the integrated diffraction intensity for each plane.

I_{0i}: is the theoretical random intensity for each plane.

A_{wi}: is the Morris area weight factor for each plane. [redacted]

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