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Prediction of surface settlement due to the displacement of soft zones

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ABSTRACT: In areas composed of coastal plain sediments, soft zones subjected to partial overburden may be present in the subsurface. During or after a seismic event, these soft zones may be compressed. The resulting displacement due to the deformation of the soft zones will propagate to the ground surface and cause the surface to settle.

This paper presents a method to predict the settlement at the surface due to the propagation of the displacement from the soft zones. This method is performed by discretizing the soft zones into multiple clusters of finite sub-areas or subspaces. Settlement profile at the ground surface due to the displacement of each sub-area or subspace is computed assuming the shape is a normal distribution function. Settlement due to the displacement of the soft zones can then be approximated by adding the settlements computed for all the sub-areas or subspaces. This method provides a simple and useful tool for the prediction of the settlement profile and the results are consistent with those obtained from the finite difference analysis.

1 INTRODUCTION

Soft zones are underconsolidated sediment deposits at depth where the overburden from the upper strata has been redistributed to the more competent surrounding soils. Only partial overburden is distributed to the soft zones. During or after a seismic event, overburden will be redistributed again and the soft zones will be subjected to the full overburden pressure. Consequently, the soft zones will be compressed and the displacement will be propagated to the ground surface and cause the surface settle.

Analytical methods have been used to quantify the surface settlement due to the displacement of soft zones at depth. These analytical methods include two approaches: (1) computation of stresses, strains, and displacements by solving a system of equations containing equilibrium, compatibility, and constitutive equations; and (2) computation of surface displacements using kinematic relations of displacement propagation in the strata based on empirical data. The system of equations in the first approach can be solved using finite element or finite difference method while the kinematic relations in the second approach can be performed utilizing empirical data from soft ground tunneling construction.

Numerous literatures documented investigations on surface settlement due to mining subsidence, tunneling construction, and excavation. Peck, R.B. (1969) observed that the settlement trough over a single tunnel could usually be represented within reasonable limits by the normal distribution curve. Cording, E.J. et al. (1976); O’Reilly, M.P. and New, B.M. (1982); and Mair, R.J. et al. (1993) also used normal distribution curve to describe the surface settlement.

Peck, R.B. (1969) also observed that the influence of a second, parallel tunnel may sometimes be approximated by adding the ordinates of the two separate settlement curves. Suwansawat, S. and Einstein, H.H. (2007) provided settlement over twin tunnels using superposition technique. Chapman, D.N. et al. (2007) studied settlement caused by multiple tunnels using laboratory model tests.

This paper computes the surface settlement due to the displacement of soft zones using kinematic relations.

2 METHODOLOGY

Settlement data from soft ground tunneling construction (Peck, R.B. 1969; Cording E.J. et al. 1976) indicate that:

1. The vertical displacement occurring in an area at depth will propagate to a larger area at the ground surface.
2. The surface settlement profile due to the displacement at depth will be in the shape of a normal distribution curve.
3. The width of the settlement depends on the subsurface conditions.
4. The volume of the settlement depends on the volume of ground lost at depth and property of the soil.

Empirical data from tunneling construction have related the angle of propagation to the soil type. These data have also related the volume of the settlement trough to the volume lost at depth. This paper predicts the settlement due to the displacement of the soft zones using kinematic relations and the empirical data from tunneling construction.

Figure 1 shows the surface settlement due to the vertical displacement *c* at depth. Consider the surface settlement profile is in the shape of a normal distribution curve. Surface settlement *z(x)* at any point *x* from the center of the settlement can then be expressed as

$$z(x) = z_0 e^{-x^2/2i^2} \tag{1}$$

where z_o is the maximum settlement at ground surface, located at the center of the settlement; *i* is the distance from the center of the settlement to the point of inflection; and

$$i = W/(2\pi)^{1/2} \tag{2}$$

W is the half width of the settlement and may be estimated as

$$W = \sum_{k=1}^{m} z_k \tan \beta_k + a \tag{3}$$

where *m* is the total number of strata above the soft zones, Z_k is the depth of the Stratum *k*, β_k is the angle of propagation of Stratum *k*, and *a* is the half width of the soft zone with vertical displacement *c*.

As an example, the site shown in Figure 1 consists of two soil strata, stratum 1 and stratum 2. Where Z_1 and Z_2 are the thicknesses of strata 1 and 2, respectively; β_1 and β_2 are the angles of propagation for strata 1 and 2, respectively.

The vertical displacement *c* is computed considering:

$$c = \varepsilon_z t \tag{4}$$

where *t* is the thickness of the soft zone and ε_z is the vertical strain of the soft zone after compression and obtained from the one-dimensional consolidation equation:

$$\varepsilon_z = \{C_c/(1 + e_0)\} \log\{(P'_0 + \Delta P)/P'_0\} \tag{5}$$

where C_c is the compression index, e_0 is the initial void ratio, P'_0 is the effective vertical stress, and ΔP is the change in pressure.

For soils in soft zones, P'_o is equivalent to preconsolidation pressure and ΔP is the load required to increase the vertical effective to the full effective geostatic stress. Equation (5) thus reduces to:

$$\varepsilon_z = \{C_c/(1 + e_0)\} \log(1/OCR) \tag{6}$$

where *OCR* is the overconsolidation ratio. C_c, e_0, and *OCR* can be obtained from the laboratory test data.

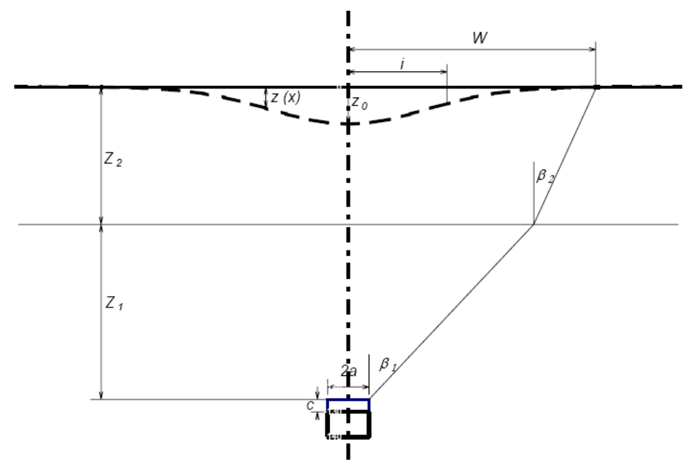

Fig. 1. Surface settlement due to the displacement at-depth

The following sections provide the methodology of the 2-D and 3-D analyses using kinematic relations.

2.1 2-D Analysis

For a 2-D analysis, surface settlement is computed by considering a vertical slice of subsurface with unit length perpendicular to the length of the soft zone. Consider the cross-section of the soft zone is in the shape of a rectangular as shown in Figure 1. When the soft zone is compressed, the vertical displacement *c* at the top of the soft zone will propagate to the ground surface and cause the surface settle. Equation (1) provides the surface settlement *z(x)*. Substituting Equation (2) into Equation (1)

$$z(x) = z_0 e^{-\pi x^2/W^2}. \quad (7)$$

Consider the volume of the settlement profile, V_S is

$$V_S = R_V V_L \quad (8)$$

where V_L is the volume lost at-depth and R_V is the ratio of the volume of settlement profile to volume lost at-depth. The volume of the settlement profile is

$$V_S = z_0 \int_{-\infty}^{\infty} e^{-\pi x^2/W^2} dx. \quad (9)$$

Note that

$$\int_0^{\infty} e^{-(\pi^{1/2}/W)^2 x^2} dx = W/2 \text{ for } \pi^{1/2}/W > 0 \quad (10)$$

Substituting Equation (10) into Equation (9)

$$V_S = z_0 W. \quad (11)$$

The volume lost at-depth due to the deformation of a soft zone is

$$V_L = 2ca. \quad (12)$$

Substituting Equations (11) and (12) into Equation (8)

$$z_0 W = 2R_V ca. \quad (13)$$

Consequently, the surface settlement at the center of the normal probably function is

$$z_0 = 2R_V ca/W. \quad (14)$$

Substituting Equation (14) into Equation (7), surface settlement *z(x)* became

$$z(x) = (2R_V ca/W) e^{-\pi x^2/W^2}. \quad (15)$$

Consider the center of the soft zone is at x_0

$$z(x) = (2R_V ca/W) e^{-\pi (x-x_0)^2/W^2}. \quad (16)$$

Equation (16) applies to a soft zone in the shape of a rectangular with a relatively narrow width compare to the depth. In a general 2-D case, the shape of the soft zone is not limited to a rectangular shape and the width of the soft zone may be relatively larger compared to the depth. Elevation and thickness of the soft zone may also vary. Furthermore, multiple soft zones may exist.

For a general 2-D case, the results can be obtained by discretizing the soft zones into number of sub-areas. These sub-areas are rectangular in shape and adjacent each other on the horizontal direction. For each sub-area, the width is the same while the depth and vertical displacement are modeled to be the same as the elevation and vertical displacement of the soft zone at the corresponding horizontal location. As shown in Figure 2, surface settlement *z(x)* at any horizontal location *x* due to the displacement of each sub-area *i* can be computed using Equation (16).

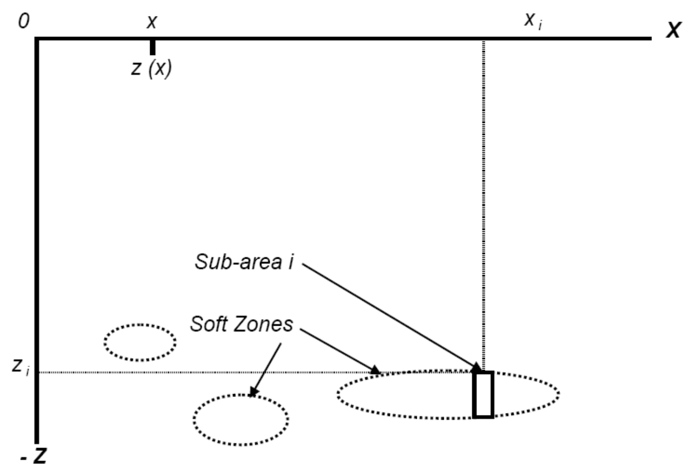

Fig. 2. 2-D analysis

Surface settlement due to the displacement of all the soft zones can then be approximated by adding the settlements computed for all the sub-areas as follows:

$$z(x) = 2R_V a \sum_{i=1}^{n} (C_i/W_i) e^{-\pi (x-x_i)^2/W_i^2} \quad (17)$$

where n is the total number of sub-areas, c_i is the vertical displacement of the sub-area *i*, *a* is the half width of the sub-area, W_i is the estimated half width of the settlement at the ground surface due to the vertical displacement of sub-area *i*, and x_i is the center of the sub-area *i*.

2.2 3-D Analysis

For a 3-D case, consider a soft zone similar to the shape of a circular plate. The surface settlement *z(r)* at any point *r* from the center of the normal distribution curve is

$$z(r) = z_0 e^{-r^2/(2r^2)}. \quad (18)$$

Similar to Equation (1), where *i* is a function of *W* as defined in Equation (2), *W* is a function of Z_k, β_k, and *a* as defined in Equation (3) except that in this case, *a* in Equation (3) is the radius of the soft zone instead of the half width of the soft zone in a 2-D case. Substituting Equation (2) into Equation (18)

$$z(r) = Z_0 e^{-\pi r^2 / W^2}. \tag{19}$$

Consider

$$V_S = R_V V_L. \tag{20}$$

The volume of the settlement profile is

$$V_S = 2\pi Z_0 \int_0^\infty e^{-\pi r^2/W^2} dr. \tag{21}$$

Let

$$r = (W/\pi^{1/2})x, \tag{22}$$

then

$$dr = (W/\pi^{1/2})dx. \tag{23}$$

Substitute Equations (22) and (23) to Equation (21)

$$V_S = 2\pi z_0 (W/\pi^{1/2})^2 \int_0^\infty e^{-x^2} dx. \tag{24}$$

Since

$$\int_0^\infty e^{-x^2} dx = 1/2, \tag{25}$$

substituting Equation (25) to Equation (24) gives

$$V_S = z_0 W^2. \tag{26}$$

Since the volume lost at-depth due to the deformation of a soft zone is

$$V_L = \pi c a^2. \tag{27}$$

Substituting Equations (26) and (27) into Equation (20)

$$z_0 W^2 = \pi R_V c a^2. \tag{28}$$

Consequently, the surface settlement at the center of the normal probably function is

$$z_0 = \pi R_V c (a/W)^2. \tag{29}$$

Substituting Equation (29) into Equation (19), surface settlement *z (r)* becomes

$$z(r) = \pi R_V c (a/W)^2 e^{-\pi r^2/W^2}. \tag{30}$$

In a rectangular coordinate system, consider the center of the soft zone is located at (x_0, y_0), Equation (30) becomes

$$z(x,y) = \pi R_V c (a/W)^2 e^{-\pi\{(x-x_0)^2+(y-y_0)^2\}/W^2}. \tag{31}$$

Equation (31) provides the settlement due to the displacement of a soft zone in the shape of a circular plate.

Consider a soft zone similar to the shape of a square plate with horizontal dimension of *2a* by *2a*, rather than a circular plate. The volume lost at-depth due to the deformation of a soft zone becomes

$$V_L = 4ca^2. \tag{32}$$

Assume the volume lost at-depth is the same as the for a soft zone in the shape of a circular column with radius of *a'*, from Equations (32) and (27)

$$4ca^2 = \pi c a'^2. \tag{33}$$

Therefore,

$$a' = (2/\pi^{1/2})a. \tag{34}$$

Based on Equations (31) and (20) the surface settlement *z(x,y)* at any point *(x, y)* is

$$z(x,y) = \pi R_V c (a'/W)^2 e^{-\pi\{(x-x_0)^2+(y-y_0)^2\}/W^2}. \tag{35}$$

where

$$W = \sum_{k=1}^{m} z_k \tan\beta_k + a'. \tag{36}$$

Substituting Equation (34) into Equations (35)

$$z(x,y) = 4R_V c (a/W)^2 e^{-\pi\{(x-x_0)^2+(y-y_0)^2\}/W^2}. \tag{37}$$

Where the half width of settlement *W* can be found by substituting Equation (34) in Equation (36),

$$W = \sum_{k=1}^{m} z_k \tan\beta_k + (2/\pi^{1/2})a. \tag{38}$$

Equation (37) applies to a soft zone in the shape of a square column; it also applies only to a soft zone with relatively narrow width compared to the depth. For a general 3-D case, similar to a general 2-D case, the results can be obtained by discretizing the soft zones into number of subspaces. These subspaces are in the shape of square columns and adjacent each other on the horizontal plane. For each subspace, the horizontal dimension is the same while the depth and vertical displacement are modeled to be the same as the elevation and vertical displacement of the soft zone at the corresponding horizontal location. As shown in Figure 3, surface settlement *z(x,y)* at any horizontal location *(x,y)* due to the displacement of each subspace *i* can be computed using Equation (37).

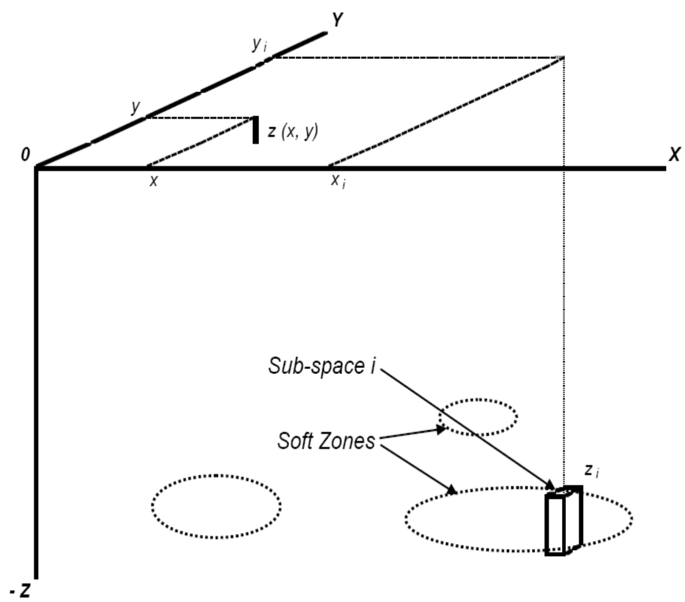

Fig. 3. 3-D analysis

Surface settlement due to the displacement of all the soft zones can then be approximated by adding the settlements computed for all the subspaces as follows:

$$z(x,y) = 4R_V a^2 \sum_{i=1}^{n} (C_i / W_i^2) e^{-\pi\{(x-x_i)^2+(y-y_i)^2\}/W_i^2} \qquad (39)$$

where n is the total number of subspaces, c_i is the vertical displacement of the subspace i, u is the half width of the subspace, W_i is the estimated half width of the settlement at the ground surface due to the vertical displacement of subspace i, and (x_i, y_i) is the center of the subspace i.

3 NUMERICAL MODELLING

2-D and 3-D analyses were performed for a project site where soft zones were found. Configurations of the soft zones were delineated based on Cone Penetration Test data and substantiated by Standard Penetration Test data as well as laboratory test results. Figures 4 and 5 show the layout of the building as well as soft zones near Elevations 175 and 150 feet, mean sea level (MSL), respectively. The elevation of the foundation will be at 258 feet, MSL.

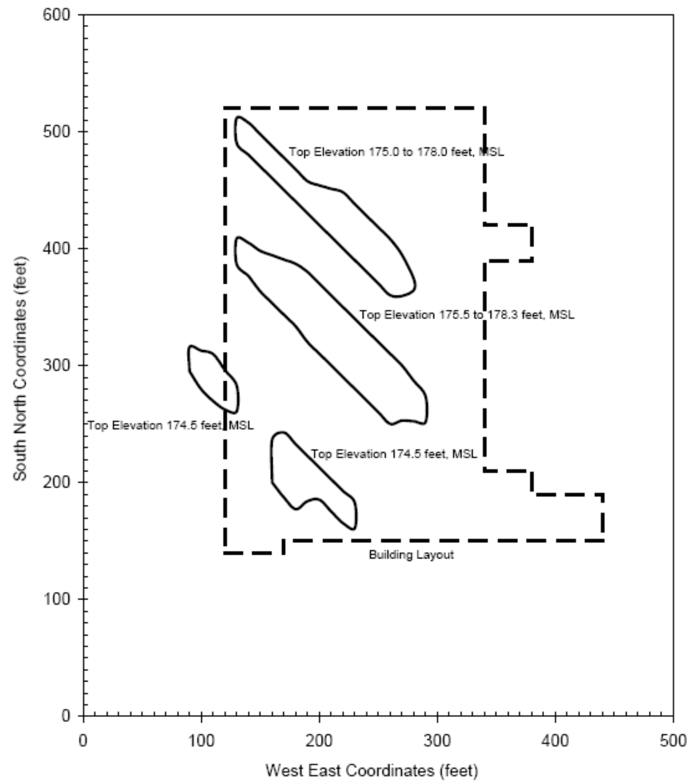

Fig. 4. Soft zones near Elevations 175 feet, MSL

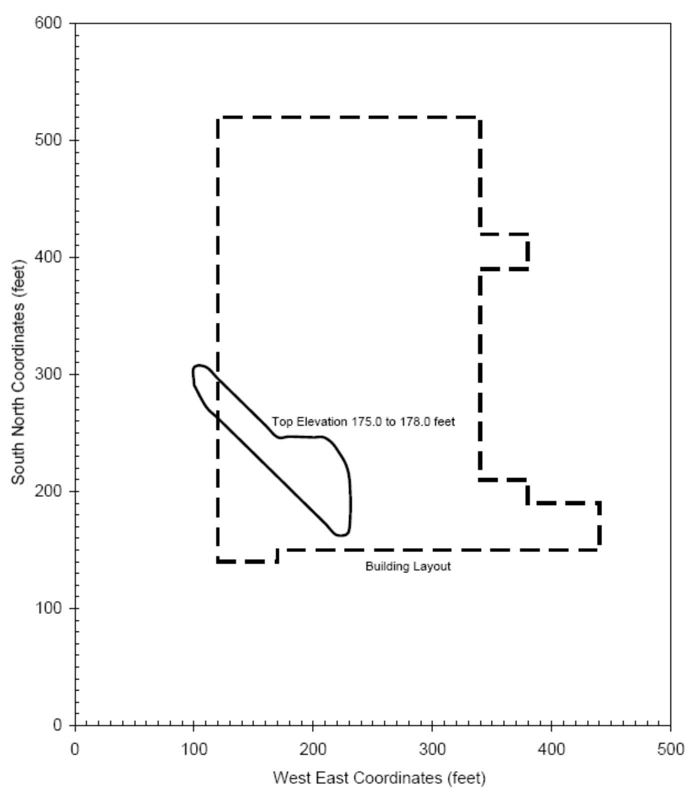

Fig. 5. Soft zone near Elevation 150 feet, MSL

Soils at the project site above the soft zones are essentially classified as clayey sand (SC) and for the most part, are overconsolidated and relatively strong. Data from tunneling construction (Cording et al. 1976) indicate that the angle of propagation is generally between 11 to 33 degrees for hard clay and sands above groundwater, 26 to 50 degrees for soft to stiff clays, and over 50 degrees for sands below the groundwater table. Angles of propagation were conservatively chosen as 33 and 50 degrees, above and below the groundwater, respectively.

Data from tunneling construction (Cording et al. 1976) also indicate that for medium to dense sends, the total volume of settlement, V_S, is generally less than the volume lost at-depth, V_L, while for loose, disturbed sands and clays, V_S is generally grater than V_L. Laboratory tests on samples from the project site indicate that the soils above the soft zones are medium sands. Therefore, the ratio of the volume of settlement profile to volume lost at-depth, R_V, was conservatively considered to be 1.0. Based on laboratory test results, the vertical strain, ε_z, for the soft zones at the project site is found to be 3.7%. The results are presented in the following sections.

3.1 2-D Analysis

2-D analyses were performed choosing a representative north-south cross-section. Inputs of the analyses include the configuration and strain of the soft zones, contour of the ground surface, configuration of the strata with various angles of propagation, and ratio of the volumes R_V. The width of the sub-areas was chosen to be 10 feet.

Fig. 6 shows the resulting 2-D settlement profile using the kinematic method. The maximum surface settlement is approximately 2.1 inches.

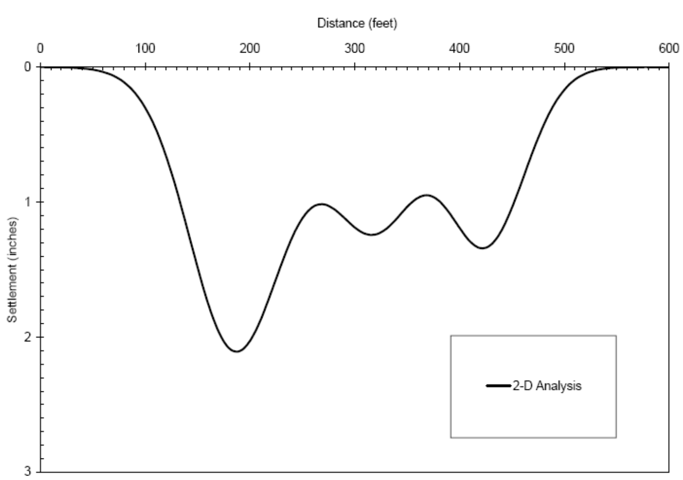

Fig. 6. Settlement profile computed using 2-D kinematic relations

3.2 3-D Analysis

A 3-D analysis was performed for the same site. Inputs such as the configuration and properties of the strata and soft zones are the same as those used for the 2-D analysis except that 3-D configurations of the entire site, rather than the representative cross-section were considered. Horizontal dimension of the subspaces was chosen to be 10 feet by 10 feet.

Figure 7 shows the resulting 3-D settlement profile using the kinematic method. The maximum surface settlement is approximately 1.1 inches.

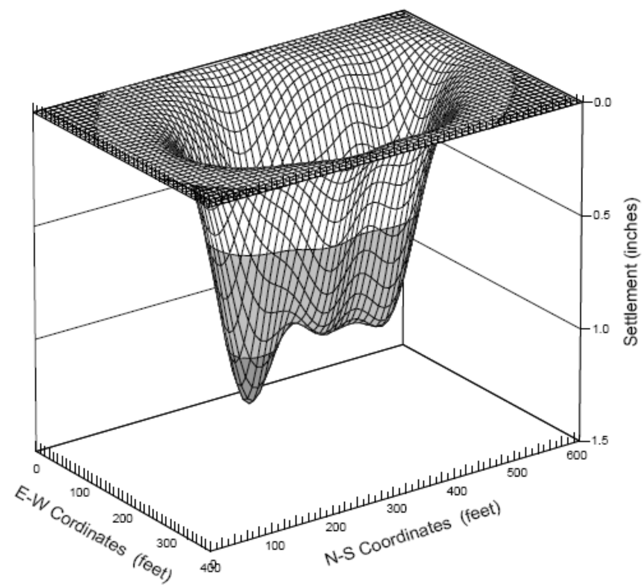

Fig. 7. Settlement profile computed using 3-D kinematic relations

Results obtained from 3-D analysis were compared with the results obtained from the 2-D analysis. Figure 8 presents the settlement profile at the north-south cross-section using both the 2-D and 3-D analyses.

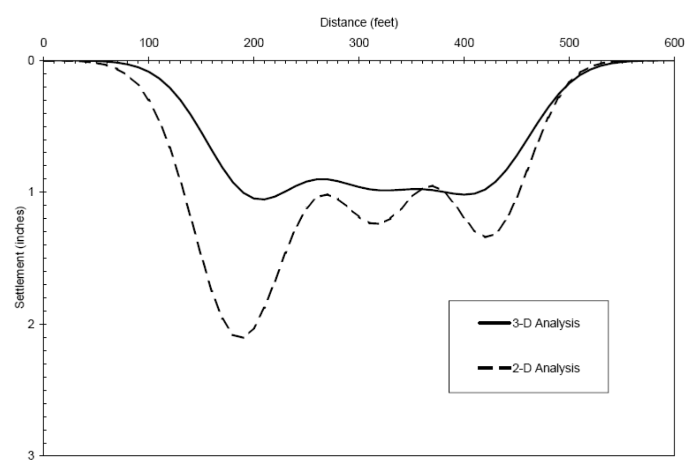

Fig. 8. Comparison of settlement profiles computed using 2-D versus 3-D kinematic relations, north-south cross-section

Comparison of the results indicates that the settlement computed using 3-D analysis is less than the settlement computed using 2-D analyses. For the north-south cross-section shown in Figure 8, the maximum settlement obtained from 3-D analysis is approximately 1.0 inches while the maximum settlement from the 2-D analyses is approximately 2.1 inches.

The effort required for performing 3-D analysis is about the same as performing 2-D analysis with the added benefit of providing a complete profile of the settlement rather than a representative cross-section.

3.3 *Finite Difference Analysis*

For a comparison, finite difference method was also used to perform the analysis. Commercially available software was used for the analysis. Numerical models were generated to simulate subsurface strata at representative cross-sections. Mohr-Coulomb elasticity/plasticity model with dilation angles was considered to simulate the soil behavior. Based on the characteristics of the soils, 11 strata were identified to represent the site stratigraphy. Soil parameters including the wave velocities, friction angle, cohesion, dilation angle, and unit weight for each stratum were obtained from field and laboratory tests. Table 1 provides the parameters used for various strata as well as the soft zones.

Table 1. Numerical model input Parameters

Stratum	Min top elev (ft)	Max top elev (ft)	Shear wave velocity (fps)	Comp wave velocity (fps)	Effective friction angle (deg)	Dilation angle (deg)	Unit weight (pcf)
1	282	295	1400	2400	33	14	118
2	252	265	1256	2200	32	13	122
3	223	239	1254	2200	31	13	123
4	225	225	1254	5200	31	13	123
5	197	217	1074	5200	30	13	108
6	195	209	1500	5200	36	15	124
7	170	180	1140	5200	14	11	118
8	164	176	1353	5200	31	13	116
9	146	155	1353	5200	31	13	116
10	134	142	1675	5200	28	12	121
11	130	140	1350	5200	35	15	125
soft zone	140	170	237	1000	5	0	100

Shear modulus, Poisson's ratio, and bulk modulus for each stratum were derived from wave velocities and soil density. Numerical models were generated to simulate representative cross-sections. Each numerical model contains approximately 10,000 grid points. Each cross section is 200 feet deep and 600 feet wide.

Vertical displacements of the soft zones were pre-calculated using the same method as used in the kinematic method. To ensure the result will properly converge; displacements were divided into 1,000 increments with each increment equaling to 1/1,000 of the total displacement. Total volume of the settlement profile could not be prescribed but was computed for the purpose of verification using the resulting surface settlement.

Figure 9 shows the comparison of the results using 2-D kinematic relations versus finite difference method for the same north-south cross-section. As shown in Figure 9, the shapes of the settlement profiles from the two different approaches are similar. However, the maximum settlement computed using kinematic relations is approximately 30 percent less than the maximum settlement computed using finite difference method. Furthermore, the total volume of the settlement profile at the ground surface using the kinematic relations was set to be the same as the volume lost at-depth, while the total volume of settlement profile using the finite difference program was approximately 50 percent more than the volume lost at-depth. Similar variations were found from other sites using finite difference method.

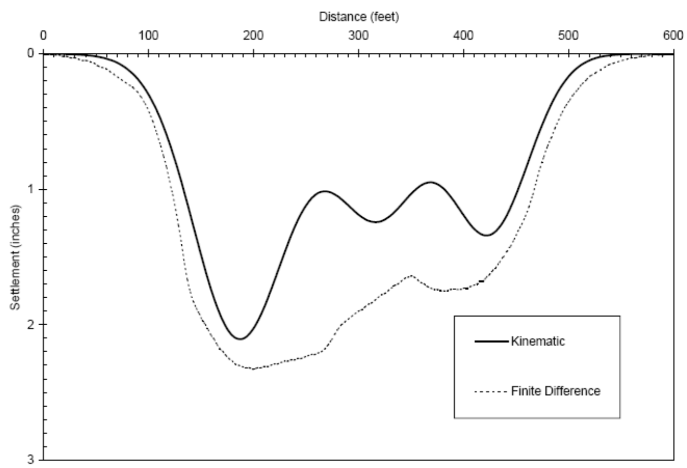

Fig. 9. Comparison of settlement profile using kinematic relations versus finite difference method.

Due to the complexity of the modeling preparation, finite deference method was not used for the 3-D analysis.

4 CONCLUSIONS

Settlement profiles due to the displacement of soft zones can be predicted using kinematic relations. Kinematic relations incorporate empirical data from tunneling construction and therefore the results are consist with the empirical data including the shape of the profile and the ratio of the volume of settlement profile to volume lost at-depth.

Finite difference solution provides complete solution on displacement as well as stress-strain distribution over the entire model. However, it is difficult to obtain parameters of soil properties and constitutive soil models; considerable time is also required to prepare the numerical models, especially for a 3-D model. The resulting settlement may significantly be deviated from the empirical data.

The concern about the soft zones is generally the resulting surface settlement profile. The method presented in this paper provides a simple and useful tool for the prediction of the settlement profile.

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