#202 60-07-08

664071

DP-1031

OL 1000-0307-92-202
N 60-07-08

AEC RESEARCH AND DEVELOPMENT REPORT

INTERACTION OF FISSILE UNITS

A Computer Code - INTERACT

H. K. CLARK

DU PONT
REG. U.S. PAT. OFF.

Savannah River Laboratory

Aiken, South Carolina

Printed in USA. Price $3.00
Available from the Clearinghouse for Federal Scientific and Technical Information, National Bureau of Standards, U. S. Department of Commerce, Springfield, Va. 22151

664071

DP-1031

Criticality Studies
(TID-4500)

INTERACTION OF FISSILE UNITS
A Computer Code - INTERACT

by

Hugh K. Clark

Parts of this work were presented at the Symposium on Criticality Control of Fissile Materials, sponsored by the International Atomic Energy Agency, at Stockholm, Sweden, November 1-5, 1965.

Approved by

P. L. Roggenkamp, Research Manager
Theoretical Physics Division

June 1966

E. I. DU PONT DE NEMOURS & COMPANY
SAVANNAH RIVER LABORATORY
AIKEN, S. C. 29801

CONTRACT AT(07-2)-1 WITH THE
UNITED STATES ATOMIC ENERGY COMMISSION

ABSTRACT

A FORTRAN code is described which computes the k_{eff} for a group of fissile units surrounded by a reflector and the spacing between units required to make k_{eff} unity. A number of approximations are made to simplify the calculations. Comparisons with experiment, of which many are included, serve to normalize the calculations. The code may be used with confidence for nuclear safety applications. In addition to the dimensions and spacings of units and reflectors, the required input includes the material buckling, the bare extrapolation distance, the migration area of the fissile units, and the albedo of the reflector.

CONTENTS

LIST OF TABLES

INTERACTION OF FISSILE UNITS
A Computer Code - INTERACT

INTRODUCTION

Fissile material encountered in operations performed outside of reactors usually exists as discrete units. In some cases, for example in storage, units may be identical and may be regularly spaced. In other cases, for example in a process line, units may have different sizes, shapes, and compositions and may be irregularly spaced. In all cases, reflectors may be actually or potentially present. A survey of normal and credible abnormal conditions, such as is required to establish the nuclear safety of an operation, therefore generally requires reliable estimates of the interaction within groups of units and reflectors.

Although the general calculation of interaction is a complicated problem, a few reasonable, simplifying approximations can be made that greatly reduce the complexity. Absolute accuracy is not of prime importance since, regardless of the degree of exactness incorporated in a method, its reliability should be established by comparison with experiment. Discrepancies between experiment and calculations can be factored into later calculations as a normalizing parameter.

A simple, practical method for computing interaction was developed several years ago at the Savannah River Laboratory, and has been in use ever since. Many comparisons have been made with experiment. Although computer codes were written to perform the calculations, they were restricted to the geometrical part of the calculations. These codes, which have not previously been described, have now been incorporated in a single over-all code, which performs not only the geometrical part of the calculation, but which also for many cases of interest computes k_{eff} for a specified group of fissile units and adjusts the spacing between units so as to make k_{eff} unity.

SUMMARY

The INTERACT code for computing k_{eff} for groups of fissile units and for computing "critical" spacings corresponding to a k_{eff} of unity is described; the FORTRAN listing is also given. The basic assumptions in the code are:

- For calculating the probability of transmission from one unit to another, the current transmitted from a surface is assumed to be uniform over the surface and to have an angular distribution per

unit solid angle proportional to the cosine of the angle between the direction of neutron travel and the normal to the surface.

- For calculating k_{eff}, transmitted and received currents are assumed to have the same surface distributions, the calculation is made one-dimensional, and the same spatial distribution is assumed for neutrons of all energies (one-group approximation).

The code can perform calculations for groups of slabs, cylinders, or spheres. Options are available for several groupings of slabs. The required input consists of dimensions and spacings of units, the reflector albedo, the material buckling, the bare extrapolation distance, and the migration area. The code can be limited solely to the calculation of transmission probabilities, if desired.

The code is satisfactory for use in nuclear safety calculations provided care is taken not to underestimate the reactivity of a unit or the albedo of a reflector. Tables I-XII give numerous comparisons with experimental critical configurations, expressed in terms of the k_{eff}'s that the experiments are calculated to have. A study of the tables serves to indicate calculated values of k_{eff} that should be considered safely subcritical for situations similar to those included in the tables. Some general observations may be made as follows:

- No values of $k_{eff} < 0.9$ have been calculated and only a few < 0.95.
- The low values occur with small bare groups at close spacings and presumably result chiefly from the nonconservatism of the assumption of uniform current. There is also some indication that the calculated values of k_{eff} decrease as the size of units relative to the critical size of a bare critical unit decreases.
- k_{eff} tends to increase with group size, presumably in part because of the conservatism in the manner of allowing for shielding within a group.
- For slabs, k_{eff} increases and then decreases as the spacing increases.
- For pairs of cylinders, except in some cases at contact, $k_{eff} \cong 1.00$.
- Except at contact the interaction between a 121-cm-long, 15.1-cm-thick slab and a 25.4-cm-diameter cylinder, parallel to the slab at its center and having the same height, is calculated quite well.

DISCUSSION

Description of Method

The problem is divided into:

- Purely geometrical calculation of the probabilities of neutron transmission from each unit in a group to each of the other units (including reflectors).

- Calculation of the criticality factor of a unit from a boundary condition expressed in terms of the ratio of the neutrons entering the unit to the neutrons leaving the unit, as determined by these probabilities.

This division of the problem can always be made without introducing any approximation, but it results in a simplification only if approximations are made regarding (1) distribution of neutron current over transmitting and receiving surfaces, and (2) angular distribution of neutrons transmitted from an element of surface.

Approximations made here for performing the geometrical calculation of probability of transmission from one unit to another are:

- Transmitted current is treated as uniform over the entire surface of a sphere or cylinder and over either of the two principal surfaces of a slab.

- Probability per unit solid angle of neutrons being transmitted from an element of surface at an angle θ with respect to the normal to the element is $\frac{\cos\theta}{\pi}$.

The reasonableness of these approximations has been discussed elsewhere[1]. For a slab and along the length of a cylinder, the assumption of a uniform current tends to compensate for ignoring the actual skewness of the angular distribution.

Criticality calculations are made in one dimension only, i.e., radial for a sphere or cylinder, or perpendicular to the two principal surfaces for a slab. If a slab is so oriented that neutrons are transmitted to an end as well as to its principal surfaces, the neutrons are treated as though they all enter the principal surfaces; neutrons transmitted from the end to other units are treated as though they were transmitted from the principal surfaces. Separability is assumed in slabs and cylinders, and finite transverse dimensions are allowed for by subtracting $B_z^2 = \frac{\pi^2}{(H+2S_H)^2}$ and $B_g^2 = \frac{\pi^2}{(L+2S_L)^2}$ from the material buckling, where S_H and S_L are appropriate extrapolation distances. Transmitted and received currents are assumed to have the same surface distributions.

Some care must be exercised in applying these approximations. For closely spaced units, current is clearly greater on portions of surfaces facing each other than elsewhere. Conservative approximations (such as subdividing units) could be introduced to allow for this effect. There are, however, sufficient experimental data now available to permit estimates to be made of its magnitude.

Geometrical Calculation

On the basis of these approximations, the probability, ρ_{jk}, that neutrons emitted from a surface, k, reach a surface, j, is

$$\rho_{jk} = \int_{A_j} \int_{A_k} \frac{\cos \theta_j \cos \theta_k \, dA_j \, dA_k}{\pi R^2 A_k} \qquad (1)$$

where A_k is the entire area of transmitting surface and $\frac{dA_j \cos \theta_j}{R^2}$ is the element of solid angle subtended by an element of receiving surface, dA_j, at dA_k. Integration is carried out over the entire transmitting and receiving surfaces. This quadruple integral has been evaluated for a number of cases analytically, and for others has been reduced to a double integral, which can be evaluated numerically by quadrature.

Analytical Results[1]

For two infinitely long parallel rectangular surfaces having height, h, and separation, s,

$$\rho = \sqrt{1 + (s/h)^2} - s/h \qquad (2)$$

For two parallel coaxial discs having transmitter and receiver radii, a and b, and separation, s,

$$\rho = \tfrac{1}{2}\left\{1 + (b/a)^2 + (s/a)^2 - \sqrt{\left[1 - (b/a)^2 - (s/a)^2\right]^2 + 4(s/a)^2}\right\} \qquad (3)$$

For two perpendicular rectangular surfaces having transmitter height, 2a, and length, 2h; having receiver width, 2d, and length, 2g; having a separation, b, between the lower edge of the transmitter and the plane of the receiver; having a separation, f, between the centers of the transmitter and receiver measured in the common direction of 2h and 2g; and having a separation, e, between the projection

$$\rho = 1 + \frac{h}{d} - \sqrt{1 + \frac{h^2}{d^2}} \qquad (5)$$

A similar approach can be used when a reflector surrounds an array. The probability of reaching the reflector is unity minus the sum of the probabilities of reaching other units; the probability of transmission from the reflector to the unit is the product of this probability and the ratio of the area of the unit to that of the reflector.

Double Integrals

For a pair of spheres having radius, a, and center-to-center separation, d, Equation (1) can be reduced[2] to

$$\rho = \frac{1}{2\pi} \int_0^1 \int_{x_1(y)}^{x_2(y)} \left[v(x,y) + w(x,y)\right] dx\, dy \qquad (6)$$

where

$$v(x,y) = \tan^{-1} \sqrt{\frac{\left[(y-a/d)\sqrt{1-x^2} + x\sqrt{1-y^2}\right]^2}{1-2ay/d} - 1}$$

$$w(x,y) = \frac{\sqrt{1-2ay/d}\sqrt{a^2(x^2+y^2)/d^2 - \left[\sqrt{(1-x^2)(1-y^2)} - x(y-a/d)\right]^2}}{\left[(y-a/d)\sqrt{1-x^2} + x\sqrt{1-y^2}\right]^2}$$

The lower limit is

$$x_1(y) = \frac{-a(y-a/d)/d + \sqrt{1-y^2}\sqrt{1-2ay/d}}{a^2/d^2 + 1-2ay/d}$$

When $y \leqslant 2a/d$, $x_2(y) = 1$; when $y > 2a/d$,

$$x_2(y) = \frac{a(y-a/d)/d + \sqrt{1-y^2}\sqrt{1-2ay/d}}{a^2/d^2 + 1-2ay/d}$$

For a pair of cylinders having height, h, radius, a, and axis-to-axis separation, d

$$\rho = \frac{1}{2\pi^2} \int_{-1}^{1} \int_{-1}^{1} \frac{\tan^{-1}\left[\dfrac{h/a}{\sqrt{d^2/a^2 - (u+v)^2} - \sqrt{1-u^2} - \sqrt{1-v^2}}\right]}{\sqrt{d^2/a^2 - (u+v)^2}}\, du\, dv \qquad (7)$$

of the transmitter on the plane of the receiver and the center of the receiver,

$$\rho = \frac{1}{8\pi ah} \sum \left[G(z,z')H(x,y') \tan^{-1} \frac{G(z,z')}{H(x,y')} + \frac{G(z,z')^2}{4} \log \frac{G(z,z')^2 + H(x,y')^2}{G(z,z')^2} - \frac{H(x,y')^2}{4} \log \frac{G(z,z')^2 + H(x,y')^2}{H(x,y')^2} \right] \quad (4)$$

where

$$G(z,z') = z + f - z'$$

$$H(x,y')^2 = (x+e)^2 + (y'+a+b)^2$$

Summation (with proper regard to sign) is over the 16 terms resulting from evaluating this quadruple integral at the limits

$-a$ and a for y'

$-h$ and h for z'

$-e$ (or $-d$ if $e > d$) and d for x

$-g$ and g for z

The probability of neutrons being transmitted from a rectangular surface having height, 2a, and length, 2h, to an identical parallel rectangular surface separated from it by a distance, 2d, is clearly obtained by subtracting from unity twice the sum of the probabilities of reaching perpendicular rectangular surfaces, one having a height, 2a, and a width, 2d, and the other a width, 2d, and a length, 2h. For each case, Equation (4) reduces to only three terms.

Evaluation of Equation (1) by subtracting from unity the probabilities of neutrons reaching surfaces other than the receiving surface of interest is frequently useful. For example, the probability of transmission from the inner surface of a cylinder to itself is obtained by subtracting from unity twice the probability of reaching the disc capping either end. From the reciprocity inherent in Equation (1), this latter probability is obtained by subtracting from unity the probability of transmission from one disc to the other and multiplying the result by the ratio of the areas of the disc and cylinder. The resulting self-transmission probability for the inner surface of a cylinder of height, h, and diameter, d, is

If the cylinders are infinitely long, the integral can be evaluated analytically, as Carlvik and Pershagen[9] show, yielding

$$\rho = \frac{1}{\pi}\left[\sin^{-1}\frac{2a}{d} - \tan\left(\frac{1}{2}\sin^{-1}\frac{2a}{d}\right)\right] \qquad (8)$$

Approximation

In cases where evaluations of Equation (1) are not available, various approximations can be made. From symmetry considerations, the probability of transmission from a sphere to another surface must be at least as great as the fraction of 4π steradians subtended by the other surface at the center of the sphere and from an infinite cylinder, at least as great as the fraction of 2π radians subtended at the axis. Spheres can be approximated by cubes and cylinders by parallelepipeds, and Equation (4) can be used to obtain the transmission probabilities. For pairs of identical cylinders or spheres at small separations, this procedure overestimates[1] the transmission probabilities.

In arrays, nearer neighbors may partially block the path to more distant neighbors. Various schemes can be used to estimate the resulting reduction in transmission probability. The scheme used here is to extend the array conceptually to infinity and to assume no blockage for successively more distant neighbors (except for those completely blocked from view) until the sum of the probabilities totals unity, after which more distant neighbors are assumed to be totally blocked from view. A reduction of the last probability incorporated in the sum is generally required to make the sum exactly unity.

Criticality Calculations

There is no restriction on the number of neutron energy groups that may be employed with this method. In view of the approximations being made, however, use of many groups can hardly be justified. Where reflectors are involved that alter the energy spectrum, there would be some advantage in using at least two groups. In the present treatment a single group is used, i.e., the spatial shape of the flux is assumed to be the same for all energies.

The total current J_i^- (i.e., the current integrated over the surface) received by each surface, i, is given in terms of total transmitted currents, J_j^+, and transmission probabilities, ρ_{ij}, by

$$J_i^- = \sum_{j=1}^{N} \rho_{ij} J_j^+ \qquad i = 1,N \qquad (9)$$

where N is the number of surfaces involved. (Unless the surface is concave, ρ_{ii} is zero.)

For a unit to be critical the ratio of J_i^- to J_i^+ must have a definite value depending on the composition and dimensions of the unit. A reasonable approximation[1] is

$$\frac{J^-}{J^+} = \frac{\phi + \frac{2}{3\Sigma}\nabla\phi}{\phi - \frac{2}{3\Sigma}\nabla\phi} \qquad (10)$$

where ϕ and $\nabla\phi$ are the flux and its gradient at the surface and Σ is the transport cross section. It is convenient to express Σ in terms of the bare extrapolation distance, S_o, by making use of the fact that $J^- = 0$ for an isolated surface and to introduce the critical extrapolation distance, S, to obtain a symmetrical expression for $\beta_i = J_i^-/J_i^+$, where for a sphere $S = \pi/B_r - R$, for a cylinder $S = 2.4048/B_r - R$, and for a slab $S_1 = \pi/B_x - T - S_2$ and where R is the critical radius, T the critical slab thickness, and S_2 the extrapolation distance at the opposite surface. The resulting expressions for slab, cylinder, and sphere are:

$$\beta_1 = \frac{\sin B_x(S_1 - S_o)}{\sin B_x(S_1 + S_o)} \qquad (11a)$$

$$\beta_1 = \frac{\dfrac{J_o(2.4048 - B_rS)}{J_1(2.4048 - B_rS)} - \dfrac{J_o(2.4048 - B_rS_o)}{J_1(2.4048 - B_rS_o)}}{\dfrac{J_o(2.4048 - B_rS)}{J_1(2.4048 - B_rS)} + \dfrac{J_o(2.4048 - B_rS_o)}{J_1(2.4048 - B_rS_o)}} \qquad (11b)$$

$$\beta_1 = \frac{\dfrac{\pi - B_rS}{1 + (\pi - B_rS)\cot B_rS} - \dfrac{\pi - B_rS_o}{1 + (\pi - B_rS_o)\cot B_rS_o}}{\dfrac{\pi - B_rS}{1 + (\pi - B_rS)\cot B_rS} + \dfrac{\pi - B_rS_o}{1 + (\pi - B_rS_o)\cot B_rS_o}} \qquad (11c)$$

For a slab, the β_i at its two surfaces are necessarily related so that if β_1 is specified β_2 is given by

$$\beta_2 = \frac{\sin 2B_x(\bar{S} - S_o) - \beta_1 \sin 2B_x \bar{S}}{\sin 2B_x \bar{S} - \beta_1 \sin 2B_x(\bar{S} + S_o)} \qquad (12)$$

where $2\bar{S} = \frac{\pi}{B_x} - T$.

With the introduction of β_i, Equations (9) become

$$\sum_{j=1}^{N} (\beta_i \delta_{ij} - \rho_{ij}) J_j^+ = 0 \qquad i = 1,N \tag{13}$$

which are homogeneous in the J_i^+; hence a solution is obtained by finding the appropriate values of β_i that make the determinant of the coefficients of the J_i^+ zero. When identical units occupy symmetrically equivalent positions, their J_i^+ are all equal, as are their J_i^-; hence the number of equations can be reduced to the number of symmetrically different positions. If all units are different, the β_i for all units except one can be evaluated from their compositions and sizes by Equation (11) and the value that β for this unit must have in order that the group be critical is determined from the requirement that the determinant be zero. For a group of identical spheres, cylinders, or slabs treated as though they are symmetrical by setting $\beta_1 = \beta_2 = \dfrac{\sin B_x(\bar{S}-S_0)}{\sin B_x(\bar{S}+S_0)}$, the β_i are all the same; β_i is then the maximum eigenvalue of the matrix of coefficients.

By definition, β_i is the albedo that the medium surrounding a surface must have if the unit is to be critical and $\alpha_i = \beta_i^{-1}$ is the albedo of the surface as determined by the size and composition of the unit. For reflectors, the appropriate albedo may be difficult to calculate. If, however, the extrapolation distance is known when the reflector is in contact with a unit, the albedo of the reflector can be obtained from the relation

$$\alpha_r = \beta u \tag{14}$$

Although the extrapolation distance is insensitive to the radius of curvature of the reflector, the albedo is not; and corrections for changes in curvature should be made. Where a reflector surrounds an array, it is simplest to assume a flat surface and to use Equations (11a) and (14) to determine its albedo.

In general, β as obtained by Equation (13) will not equal β as calculated by Equation (11) from the size and composition of the unit, i.e., S as obtained from Equations (11) and (13) will not be consistent with the actual size of the unit. It is convenient to calculate a geometric buckling from this S and the actual dimensions and to calculate k_{eff} as

$$k_{eff} = \frac{1 + M^2 B_m^2}{1 + M^2 B_g^2} \tag{15}$$

where B_m^2 is the material buckling and M^2 is an appropriate migration area.

By comparing the k_{eff} so calculated for a unit within a group with values calculated for units in similar groups found experimentally to be critical, a judgment can be made as to whether the group would be subcritical. Dimensions, spacings, and compositions can be adjusted until k_{eff} has the desired value.

Description of Code

The INTERACT code listing is given in Appendix A. The main program serves to exercise input and output options and to call the appropriate subroutine. It has three principal subroutines: SLAB, CLNDR, and SPHERE. As the names imply, calculations for groups of slabs, cylinders, or spheres are made in these subroutines.

In SLAB there are options for twelve different groupings of slabs; a thirteenth option simply evaluates Equation (4) for a general case. Other groupings of slabs can easily be added with additional coding. Evaluations of Equation (4) are performed in Subroutine NTRCT. Modifications in the initial spacing to make k_{eff} unity are made with Subroutine SEP.

CLNDR performs calculations for a group of identical cylinders, which may or may not be surrounded by a reflector. The evaluations of Equation (7) are performed in CLDRCT by 16-point Gaussian quadrature or, where appropriate, by Equation (8). The maximum eigenvalue of the matrix of coefficients of the currents transmitted from the cylinders is calculated by Subroutine MAX. Subroutine SEP is used in modifying the initial spacings so as to make k_{eff} unity. A Bessel function subroutine is required for k_{eff} calculations.

SPHERE performs calculations for a group of identical spheres, which may or may not be surrounded by a reflector. The evaluations of Equation (6) are performed in SPHRCT by 11-point Lagrange interpolation of the amounts by which the ratios of Gaussian quadratures of Equation (6) to the corresponding solid angles subtended by one sphere at the center of another exceed unity. This approach is taken because many evaluations of the integral may be required and double 16-point Gaussian quadrature (256 evaluations of the integrand) consumes too much machine time. The maximum eigenvalue and adjustments in the initial spacing are obtained with MAX and SEP.

Input to INTERACT

As many cards are used as are required by the information being supplied. Each card is begun at the left. Cards that apply only to certain types of problems should be omitted for other types. The cards must be read onto an input data tape (Tape 5). The code is designed to be run on a monitor system on the IBM 704.

Format	Data
14I5	IC(I), I = 1,5

IC(1) denotes unit type

- IC(1) = 1 denotes a group of slabs
- IC(1) = 2 denotes a group of cylinders
- IC(1) = 3 denotes a group of spheres
- If IC(1) = 0, the program calls EXIT

IC(2) describes type of problem

For IC(1) = 1, IC(2) = 1 denotes two bare, parallel, identical slabs

IC(2) = 2 denotes two bare, parallel, slabs, one twice as thick as the other

IC(2) = 3 denotes three bare, parallel, identical slabs

IC(2) = 4 denotes two bare, perpendicular identical slabs in L configuration

IC(2) = 5 denotes two bare, perpendicular slabs, one twice as thick as the other in L configuration

IC(2) = 6 denotes two bare, perpendicular identical slabs in T configuration

IC(2) = 7 denotes two bare, perpendicular slabs, one twice as thick as the other, in T configuration

IC(2) = 8 denotes three bare, parallel slabs, center slab twice as thick as the others

IC(2) = 9 denotes two bare, parallel, identical discs

Format	Data
	IC(2) = 10 denotes three bare, parallel, identical discs
	IC(2) = 11 denotes a slab parallel to a reflector having equal area
	IC(2) = 12 denotes a slab parallel to a reflector having nine times the area
	IC(2) = 13 denotes general evaluation of Equation (4)
	For IC(1) = 2, IC(2) = 0 denotes an array of cylinders
	For IC(1) = 3, IC(2) = 0 denotes an array of spheres
	IC(2) = 1 denotes an array of cylinders to be treated as spheres
	IC(3) denotes number of comment cards (maximum of 9)
	IC(4) denotes number of axis-to-axis or center-to-center spacings, expressed as squares of multiples of the minimum spacing, to be read for arrays of cylinders or spheres. If IC(4) = 0, the spacings from the previous problem are used.
	IC(5) denotes format to be used for reading WUF. If IC(5) = 0, format is 6F2.0, 60F1.0; otherwise it is 24F3.2
12A6	Problem description or other comments. The number of cards to be read is IC(3)
14F5.0	(SDU(I), I = 1, IC(4)), (WDI(I), I = 1, IC(4)) <u>Applies only for cylinders and spheres</u>. SDU denotes squares of the ratios of axis-to-axis or center-to-center spacings between units to the minimum spacing arranged in increasing order. WDI denotes the number of units at each spacing in an infinite lattice formed by extending the lattice under consideration.
8F5.2, 2F10.5	DH, DL, SP, DW, AR, SO, SH, SL, B, AM <u>Applies only for slabs</u>. For IC(2) < 13, DH = slab height, DL = slab length, SP = surface-to-surface separation,

Format	Data
	DW = slab thickness, AR = reflector albedo, SO = bare extrapolation distance, SH and SL = extrapolation distances on height and length, B = material buckling, and AM = migration area. For IC(2) = 13, DH = transmitter height, DL = transmitter length, SP = receiver width, DW = separation between lower edge of transmitter and plane of receiver, AR = length of receiver, SO = separation between centers of transmitter and receiver measured in the common direction of their length, and SH = separation between projection of transmitter on plane of receiver and center of receiver. If DH = 0, IC card is read next.
2I5, 7F5.2, 2F10.5	NN, NX, DIA, HU, SR, SP, AR, SO, SH, B, AM Applies only for cylinders. NN is number of symmetrically different positions, NX is number of units in group, DIA is diameter, HU is height (if height is infinite, HU = 0), SR is surface area of any surrounding reflector, SP is minimum axis-to-axis spacing, AR is reflector albedo, SO is bare extrapolation distance, SH is axial extrapolation distance, B is material buckling, AM is migration area. If NN = 0, IC card is read next.
2I5, 3F10.5, 2F5.2, 2F10.6	NN, NX, DIA, SP, SR, AR, SO, B, AM Applies only for spheres. NN is number of symmetrically different positions, NX is number of units in group, DIA is diameter, SP is minimum center-to-center spacing, SR is surface area of any surrounding reflector, AR is reflector albedo, SO is bare extrapolation distance, B is material buckling, AM is migration area. If NN = 0, IC card is read next.
2I5, 7F5.2, 2F10.5	NN, NX, DIA, HU, VOL, TH, STS, AR, SO, B, AM Applies only for cylinders to be treated as spheres. NN is number of symmetrically different positions. NX is number of units in group, DIA and HU are external diameter and height of cylinder, VOL is internal volume, TH is wall thickness, STS is surface-to-surface separation, AR is reflector albedo, SO is bare extrapolation distance, B is material buckling, AM is migration area. If NN = 0, IC card is read next.
14F5.0	(WST(I), I = 1, NN) Applies only for cylinders and spheres and only if NX differs from its value in the previous problem. WST is number of units in each of NN symmetrically different positions.

Format	Data
6F2.0, 60F1.0, or 24F3.2 (Depends on IC(5))	(WUF(I,J,K), I = 1, IC(4)) Applies only for cylinders and spheres and only if NX differs from its value in the previous problem. WUF is the number of units in position K transmitting to units in symmetrically equivalent positions J separated from K by spacing I. There are NN x NN cards in the deck to be read. For each K, starting with 1, J runs from 1 through NN. Appendices B, C, and D list values of WUF for several arrays of spheres and cylinders.

All dimensions are in cm, buckling is in cm^{-2}, and migration area in cm^2. If B = 0, only the geometrical calculation is performed; and parameters not needed (DW, SO, SH, SL, B, AM) may be left blank. Following a problem, the next card read is either an IC card as indicated above or a card for another problem of the same type.

Output from INTERACT

The output consists first of the code title followed by any comments that may have been read in. Next is a statement of the type of problem being run.

For slabs for which IC(2) < 13 the dimensions are listed, the initial and critical separations, four transmission probabilities corresponding to the critical separation, the material buckling, the migration area, the bare extrapolation distance, the critical extrapolation distance (average value for two principal surfaces; refers to larger slab when thicknesses differ), the extrapolation distance corresponding to the critical separation (should equal the critical extrapolation distance if problem converged), reflector albedo, and k_{eff} corresponding to initial separation.

For IC(2) = 1, 2, 3, 8, the first transmission probability is for transmission from one parallel slab to another, the next two are for transmission to the perpendicular rectangles enclosing the space between the slabs, and the fourth is zero. For IC(2) = 9, 10, the first probability is for transmission from one disc to another, and the others are zero. For IC(2) = 4, 5, the first probability is for transmission from the slab having two surfaces involved to the other slab (having only a principal surface involved) as though all neutrons were transmitted from the larger (perpendicular) surface, the second probability is for transmission from the larger surface only, the third probability is for transmission from the smaller (parallel) surface treated as though the neutrons came from the larger surface, and the fourth probability is zero. For IC(2) = 6, 7, the first probability is $\sqrt{2}$ times the sum of the next two and represents the square root of the product of the β's for the principal surfaces of the two slabs involved in the interaction; the second probability is for transmission from either large (perpendicular) surface of the slab having three surfaces involved to the other slab (having only a principal surface involved); the third is one-half the probability for transmission from the small (parallel) surface treated as though the neutrons came from a large surface, and the fourth probability is zero. For IC(2) = 11, the first probability is for transmission from the slab to the reflector and back; the second probability is for transmission from the slab to the reflector, and the third and fourth are for transmission to the perpendicular rectangles enclosing the space between the slab and the reflector. For IC(2) = 12, the first probability is for transmission from the slab to the reflector and back, the next two are for transmission to a reflecting slab off-set by the slab height and by the slab length, and the fourth is for transmission to a reflecting slab off-set by both the height and the length.

For slabs for which IC(2) = 13, the output consists of transmitter length, transmitter height, receiver length, receiver width, separation between lower edge of transmitter and plane of receiver, separation between projection of transmitter on plane of receiver and center of receiver, separation between centers of transmitter and receiver measured in common direction of their lengths, and the transmission probability.

For cylinders the output consists of the number of cylinders, the ratio of height to diameter, the initial ratio of diameter to minimum axis-to-axis spacing, the final ratio (critical ratio if problem converged), the ratio of the area of the reflector to the area of the curved surface of a cylinder, the reflector albedo, the final eigenvalue for the group, the material buckling, the migration area, the bare extrapolation distance, the critical extrapolation distance, the final extrapolation distance, and k_{eff} for the initial spacing.

For spheres (or for cylinders treated as spheres) the output consists of the number of units, the initial and final (critical if converged) ratios of diameter to minimum center-to-center spacing, the ratio of the area of the reflector to that of a sphere, the reflector albedo, the final eigenvalue for the group, the material buckling, the migration area, the bare, critical, and final extrapolation distances, and k_{eff} for the initial spacing.

If the buckling is zero so that only the geometrical part of the calculation is performed, all unused parameters are left blank.

Comparison with Experiment

A large number of critical experiments have been performed with groups of interacting units and most of the data have been compiled in a recent publication[4]. Comparison is made here (Tables I-XII) in terms of calculated and experimental critical separations (taken in some cases from the original reference rather than from the compilation) and in terms of k_{eff} calculated for the experimentally critical group. The value of k_{eff} is somewhat dependent on the value of M^2 used. A different migration area, M'^2, would give a different k'_{eff},

$$k'_{eff} = \frac{1 + \dfrac{(k_{eff}-1)M'^2B_m^2}{k_{eff} + M'^2B_m^2}}{1 - \dfrac{(k_{eff}-1)M'^2}{(k_{eff} + M'^2B_m^2)M^2}} \qquad (15)$$

but for k_{eff} near unity the dependence on M^2 is small.

Calculations are of course dependent on the reactivities assumed for individual units. A careful attempt was made to choose B_m^2 and S_o consistent with experiments performed with bare isolated units so that most of the discrepancy between calculation and experiment results from the method of calculating interaction. The choice is not a unique one, however. Once S_o is selected, B_m^2 is determined; but various S_o may be chosen. One would hope to choose S_o such that B_m^2 would be independent of shape, but this is probably an impossible goal. The dependence on S_o is fairly small, however, as can be seen from Tables I and X. A larger value of S_o tends to give greater k_{eff}'s and critical spacings.

In calculations for solutions the actual separations between solutions were used and vessel walls were ignored (i.e., treated as vacuum). Groups involving thick and thin slabs were assumed to contain slabs of a particular thickness and slabs of exactly half this thickness. No comparisons of calculations and experiments with three dimensional arrays of units are included since such comparisons have recently been published[2,3]. The cylinders of solution in Table X were all assumed to have diameters of 15.24 cm although in the experiments this was the outer diameter of some of the containers. In the calculations for the interaction between a slab and a cylinder (Table XII), the cylinder was treated as a square cylinder having the same volume in computing the probability of transmission to the slab; this probability was multiplied by the ratio of the actual slab and cylinder areas in computing the probability of transmission from slab to cylinder.

TABLE I

Parallel 15.1- and 7.6-cm-Thick Slabs of Solution Containing 76 g $^{235}U/l$[4]

$B_m^2 = 0.023306\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 3.0$ cm

Configuration	Ht, cm	Separation, cm Exptl.	Separation, cm Calc.	k_{eff}
1 - ½	24.9	0.6	0.4	0.997
	32.4	5.7	7.1	1.010
	44.6	15.9	20.9	1.017
	45.0	15.9	21.4	1.019
	58.2	31.1	39.3	1.013
	58.6	31.1	39.8	1.014
	59.7	31.1	41.5	1.016
	65.8	38.7	50.7	1.014
	68.3	46.4	55.0	1.008
	83.1	76.8	81.5	1.002
	86.7	76.8	88.6	1.004
	92.5	76.8	99.8	1.008
	93.5	76.8	101.9	1.008
	113.8	107.3	163.2	1.007
½ - 1 - ½	19.6	0.6	0.1	0.994
	44.2	26.0	32.4	1.019
	62.5	51.4	63.9	1.015
	81.5	81.9	102.1	1.010
1 - 1	25.4	5.7	8.0(8.2)[a]	1.016(1.021)[a]
	32.8	15.9	21.5(21.3)	1.023(1.024)
	44.9	38.7	51.4(50.7)	1.020(1.020)
	50.3	51.4	67.5(66.5)	1.018(1.017)
	59.7	76.8	99.6(98.1)	1.014(1.013)
	73.2	122.6	155.5(151.8)	1.009(1.008)
	82.1	168.3	201.9(201.9)	1.005(1.005)
½ - ½ - ½	25.5	0.6	0.5(0.8)[a]	0.998(1.001)[a]
	34.4	3.2	3.8(3.8)	1.006(1.007)
	58.8	8.3	10.6(9.9)	1.016(1.014)
	85.5	12.1	15.7(14.4)	1.020(1.015)
	107.4	14.6	18.8(17.1)	1.020(1.015)
	120.4	15.9	20.3(18.4)	1.020(1.014)

(a) Values in parentheses calculated with $S_o = 2.5$ cm, $B_m^2 = 0.025585\ cm^{-2}$

TABLE II

Parallel 7.6-cm-Thick Slabs of Solution Containing 480 g $^{235}U/l$ (4)

$B_m^2 = 0.031330\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 2.5\ cm$

Configuration	Ht, cm	Separation, cm Exptl.	Separation, cm Calc.	k_{eff}
½ - ½	33.4	0.9	0	0.988
	44.5	3.2	2.3	0.992
	59.4	5.7	4.8	0.993
	~67.0	7.0	5.8	0.992

TABLE III

Perpendicular 15.1- and 7.6-cm-Thick Slabs of Solution Containing 79 g $^{235}U/l$ (4)

$B_m^2 = 0.023438\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 3.0\ cm$

Configuration	Ht, cm	Separation, cm Exptl.	Separation, cm Calc.	k_{eff}
1 - 1 (T)	45.2	3.4	4.9	1.003
	53.8	9.1	20.1	1.015
	67.4	24.4	48.6	1.017
	82.8	47.2	89.7	1.014
	84.1	52.3	94.3	1.013
	87.8	62.5	107.8	1.010
½ - 1 (T)	79.3	9.1	6.0	0.998
	94.9	24.4	24.4	1.000
	105.7	47.2	47.2	1.000
1 - 1 (L)	56.9	1.5	25.7	1.020
	71.3	16.8	90.0	1.026
	76.6	29.5	129.6	1.023
	79.6	37.1	168.7	1.022
	84.6	57.4	285.3	1.018
	88.9	77.7	-	1.015
½ - 1 (L)	97.9	1.5	28.3	1.006
	102.4	16.8	40.8	1.004

TABLE IV

Parallel Slabs of Uranium Metal[4]

$B_m^2 = 0.08258$, $M^2 = 15.7$, $S_o = 2.1$ cm

Surface Dimensions, cm	Thickness, cm	Separation, cm Exptl.	Separation, cm Calc.	k_{eff}
20.3 x 25.4 (Two slabs)	4.76	0.3	0.5	1.003
	5.08	1.0	1.4	1.008
	5.40	1.6	2.4	1.014
	5.71	2.4	3.4	1.018
	6.03	3.2	4.6	1.022
	6.67	5.2	7.2	1.026
	6.98	6.4	8.8	1.027
	7.30	8.0	10.6	1.026
	7.62	9.9	12.8	1.024
	7.94	12.5	15.7	1.020
38.1 (dia) (Two slabs)	4.15	1.2	1.9	1.009
	4.46	2.5	3.8	1.017
	4.77	3.9	5.9	1.023
	5.08	5.6	8.0	1.028
	5.38	7.4	10.5	1.032
	5.72	9.6	13.5	1.035
	6.00	12.3	16.3	1.032
	6.34	15.7	20.6	1.031
27.94 (dia) (Two slabs)	4.46	0.3	0.6	1.005
	4.77	1.2	1.8	1.011
	5.08	1.9	3.1	1.018
	5.38	2.9	4.3	1.022
	5.74	3.9	5.9	1.029
	6.00	5.1	7.2	1.028
	6.34	6.5	9.1	1.031
	6.69	8.2	11.3	1.032
	7.00	10.2	13.7	1.030
	7.31	12.9	16.7	1.025
17.78 (dia) (Two slabs)	6.69	0.3	0.5	1.004
	6.95	0.6	0.9	1.005
	7.31	0.9	1.4	1.010
	7.61	1.2	1.8	1.012
	7.92	1.5	2.2	1.013
	8.28	1.9	2.8	1.016
38.1 (dia) (Three slabs)	3.17	2.2	3.3	1.017
	3.81	4.5	6.6	1.032
	4.44	7.2	10.3	1.042
	4.75	8.8	12.3	1.043
	5.08	10.7	14.6	1.045

TABLE V

15.1-cm-Thick Slab of Solution Containing 79 g ^{235}U/l Parallel to a Reflector[5,6]

Reflector Albedo	Ht, cm	Separation, cm Exptl.	Calc.(1)[a]	Calc.(9)[b]	$k_{eff}(1)$[a]	$k_{eff}(9)$[b]
0.475 (concrete)	32.3	0	0	0	1.000	1.000
	47.8	15.2	13.2	14.1	0.995	0.997
	69.6	45.7	40.5	47.6	0.997	1.001
	80.3	68.6	58.7	72.8	0.997	1.001
	92.2	106.7	83.5	114.3	0.997	1.001
	107.2	228.6	-	228.6	0.999	1.000
0.154 (1.27 cm thick steel)	55.3	0	0		1.000	
	63.2	7.6	8.2		1.001	
	70.5	15.2	17.7		1.002	
	81.3	30.5	36.6		1.002	
	92.3	61.0	73.2		1.001	
	97.1	91.4	109.7		1.001	

(a) For (1), reflector surface dimensions assumed the same as those of solution surface;

(b) For (9), reflector assumed to consist of 9 such rectangles forming a rectangle 3H x 3L; neutrons reflected to the ends were ignored.

TABLE VI

Pairs of Cylinders[4]

(Does not include the pairs listed in Tables VII and IX)

$M^2 = 32\ cm^2$, $S_o = 3$ cm

Diameter, cm	B_m^2, cm^{-2}	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
25.4	0.027892	28.7	25.7	25.4[a]	0.989
		30.7	27.7	26.7	0.996
		32.8	31.5	31.2	0.999
		34.3	35.3	35.8	1.001
		35.8	41.3	42.0	1.001
		37.2	50.3	50.3	1.000
		38.2	59.6	59.6	1.000
		39.1	75.6	73.0	0.999
	0.024258	40.8	25.7	25.4[a]	0.991
		44.9	27.6	26.5	0.996
		50.0	30.5	30.6	1.000
		54.7	33.7	35.2	1.003
		64.6	42.3	47.9	1.005
		74.5	57.0	66.3	1.004
		80.1	69.0	81.2	1.003
	0.023878	42.7	26.0	25.4[a]	0.989
		58.4	33.3	34.5	1.002
		67.6	41.0	43.2	1.002
		76.7	48.6	53.9	1.003
		90.7	66.3	75.8	1.003
30.0	0.024830	26.4	30.3	30.0[a]	0.994
		27.7	33.6	33.0	0.999
		28.6	36.7	37.7	1.002
		29.5	42.7	45.0	1.002
		30.9	60.7	74.8	1.003
		31.4	90.4	131.0	1.002
		31.5	120.5	179.6	1.001
		31.6	135.1	370.4	1.001
		31.6	150.0	411.0	1.001
	0.022242	32.9	30.3	30.4	1.002
		37.4	37.8	47.5	1.010
		38.2	45.3	54.0	1.005
		39.5	60.2	71.3	1.003
		40.9	90.4	132.2	1.003
		41.4	120.5	-	1.002
		41.4	150.0	-	1.001
	0.019886	40.9	30.3	30.0[a]	0.995
		48.6	37.8	40.2	1.003
		52.3	45.3	50.2	1.004
		56.4	60.2	69.3	1.003
		60.0	90.4	104.9	1.002
		61.3	120.5	137.6	1.001
		61.9	150.0	150.0	1.000
38.1	0.028336	17.3	38.7	38.1[a]	0.989
		17.8	43.4	39.6	0.996
		18.0	53.4	44.0	0.995
		18.3	88.4	59.4	0.997
	0.024757	20.1	38.6	38.1[a]	0.991
		20.8	43.4	41.5	0.998
		21.0	48.1	45.2	0.998
		21.3	69.7	55.1	0.997
		21.5	88.4	70.7	0.999
50.8	0.030167	14.7	51.1	104.1	1.007
		14.8	56.1	-	1.008
		14.8	71.1	-	1.005
	0.025380	16.7	51.1	50.8[a]	0.992
		17.0	61.1	52.4	0.996
		17.3	76.0	-	1.003

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE VII

Groups of Cylinders of
Solution Containing about 500 g $^{235}U/l$[4]

$B_m^2 = 0.030367\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 2.7$ cm

L = Linear, T = Triangular, S = Square

Diameter, Cm	Configuration	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
12.7	7-T	28.7	13.6	12.7[a]	0.924
		66.3	15.9	14.1	0.947
15.2	7-T	22.6	15.9	15.2[a]	0.935
		33.0	18.1	16.6	0.958
		51.6	20.6	19.1	0.973
		83.8	23.2	21.7	0.981
20.3	2-L	68.3	21.0	20.3[a]	0.979
	3-L	45.7	21.0	20.3[a]	0.981
		124.5	28.2	26.9	0.995
	4-L	41.9	21.0	20.3[a]	0.988
		96.5	28.2	28.0	0.999
	5-L	40.1	21.0	20.4	0.992
		78.7	28.2	27.6	0.996
20.3	3-T	27.2	21.0	20.3[a]	0.938
		35.1	23.2	20.3[a]	0.967
		45.2	25.7	23.5	0.982
		55.9	28.2	26.3	0.988
		68.8	30.8	28.9	0.991
		106.7	35.9	33.9	0.993
	3-T	36.1	21.0	20.3[a]	0.967
	(90°)	71.3	28.2	26.8	0.992
	3-T	42.4	21.0	20.3[a]	0.969
	(120°)	87.4	28.2	26.1	0.993
	3-S	36.1	21.0	20.3[a]	0.967
		71.4	28.2	26.8	0.992
20.3	7-T	18.3	21.0	20.3[a]	0.943
		21.6	23.2	21.1	0.962
		25.7	25.7	24.1	0.977
		29.7	28.2	27.1	0.988
		33.5	30.8	30.1	0.994
		41.9	35.9	35.4	0.997
		55.9	43.5	42.7	0.996
25.4	2-L	25.5	26.0	25.4[a]	0.977
		29.9	35.7	31.8	0.993
		32.6	55.8	47.8	0.996
		33.6	∞		0.992

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE VIII

Groups of Cylinders of Solution Containing 84 g $^{235}U/l$[4]

$B_m^2 = 0.024384\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 3.0\ cm$

Dia, cm	Configuration	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
15.2	7-T	31.0	16.3	15.2[a]	0.947
		56.9	18.1	17.2	0.978
		195.6	20.6	20.0	0.991
20.3	3-T	41.4	21.0	20.3[a]	0.945
		79.3	23.2	20.3[a]	0.978
20.3	7-T	28.7	23.2	22.2	0.982
		45.2	28.2	28.8	1.006
		89.9	35.9	37.9	1.015
		119.1	38.4	40.6	1.014

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE IX

Groups of 24.1-cm-Diameter Cylinders of Solution Containing 87 g $^{235}U/l$[4]

$B_m^2 = 0.024483\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 3.0$ cm
L = Linear, T = Triangular, S = Square

		Pitch, cm		
Configuration	Ht, cm	Exptl.	Calc.	k_{eff}
2-L	61.2	27.0	26.4	0.998
	80.5	32.1	32.8	1.002
	113.0	39.7	42.1	1.003
	137.2	44.8	47.5	1.002
3-L	56.6	29.5	30.8	1.005
	84.3	39.7	43.4	1.006
	111.3	49.8	53.7	1.004
	152.7	62.5	66.3	1.002
4-L	57.7	32.1	34.1	1.007
	76.2	39.7	43.6	1.007
	97.8	49.8	53.6	1.004
5-L	55.1	32.1	34.2	1.008
	71.9	39.7	43.4	1.007
	92.0	49.8	53.5	1.004
6-L	54.1	32.1	34.5	1.009
	88.4	49.8	53.3	1.004
2 x 2 - S	40.1	32.1	32.4	1.002
	69.1	49.8	53.7	1.007
	120.4	80.3	85.9	1.003
	158.8	100.6	104.0	1.001
3-T	34.0	27.0	24.1[a]	0.981
	51.6	34.6	35.2	1.002
	71.4	44.8	47.4	1.005
	92.2	54.9	59.1	1.004
	126.2	70.2	74.9	1.003
	152.7	80.3	84.7	1.002
7-T	30.7	32.1	32.7	1.005
	51.1	49.8	53.7	1.011
	83.6	80.3	85.9	1.005

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE X

Groups of 15.24-cm-Diameter Cylinders of Solution Containing 380 g $^{235}U/l$ [4,7]

$B_m^2 = 0.026822\ cm^{-2}$, $M^2 = 35\ cm^2$, S = 3.1 cm
T = Triangular, S = Square

Configuration	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
2 x 2 - S	66.9	15.9	15.2[a]	0.942
	108.7	16.5	15.2[a]	0.954
3 x 3 - S	51.5	19.4	19.6	1.006
	55.0	19.9	20.0	1.003
	77.9	21.4	21.8	1.007
	101.6	22.4	23.0	1.009
	126.0	23.2	23.8	1.009
4 x 4 - S	50.1	22.1	23.4	1.025
	76.2	25.1	27.2	1.035
	101.8	27.0	29.2	1.035
	128.0	28.2	30.6	1.033
7-T	24.8	15.9	15.2[a]	0.948
	39.1	18.1	17.2	0.978
	69.8	20.6	20.2	0.994
	99.5	21.9	21.8	0.999
19-T	50.1	24.5	26.5	1.041
	76.2	28.1	30.8	1.038
	101.6	30.6	33.7	1.035
	127.0	32.4	35.4	1.035

(a) Calculations indicate cylinders would be subcritical at contact.

TABLE XI

Groups of 24.13-cm-Diameter Cylinders of Solution Containing about 890 g U(4.9)/l[a]

$B_m^2 = 0.011970\ cm^{-2}$, $M^2 = 31\ cm^2$, $S_o = 3.00$ cm
T = Triangular, S = Square

Configuration	Ht, cm	Pitch, cm Exptl.	Pitch, cm Calc.	k_{eff}
3 x 3 - S	61.0	26.1	24.1[a]	0.964(0.942)[b]
	122.0	29.6	27.3(24.3)[b]	0.978(0.955)
	142.2	30.3	28.1(24.9)	0.977(0.954)
4 x 4 - S	61.0	28.3	28.2(26.3)	0.998(0.979)
	122.0	33.9	33.9(31.6)	1.002(0.979)
	142.2	35.1	35.1(32.5)	1.000(0.979)
5 x 5 - S	61.0	30.0	30.7(29.4)	1.008(0.991)
	122.0	37.1	39.5(36.5)	1.015(0.996)
	142.2	38.9	41.3(38.2)	1.015(0.996)
7-T	61.0	26.6	24.1[a]	0.949(0.926)
	122.0	29.2	25.4(24.1)[a]	0.962(0.939)
	142.2	29.8	25.9(24.1)[a]	0.963(0.939)
19-T	122.0	37.9	39.4(37.2)	1.013(0.994)
	142.2	39.5	40.7(38.4)	1.010(0.989)

(a) Calculations indicated cylinders would be subcritical at contact.
(b) Values in parentheses calculated with $S_o = 2.50$ cm, $B_m^2 = 0.012536$.

TABLE XII

Cylinder (25.4-cm-dia) and
Slab (15.1-cm-thick, 120.6-cm-long)
of Solution Containing 78 g $^{235}U/l$ [4]

$B_m^2 = 0.024793\ cm^{-2}$, $M^2 = 32\ cm^2$, $S_o = 2.70$ cm

Ht, cm	Separation, cm	ρ_{sc}	ρ_{cs}	β_c	β_s	k_{eff}
30.7	0.7	0.355	0.235	0.360	0.231	0.934
47.8	15.7	0.246	0.163	0.169	0.237	1.001
58.2	31.0	0.183	0.121	0.113	0.196	1.010
66.5	46.2	0.140	0.093	0.085	0.153	1.010
79.3	76.7	0.087	0.058	0.056	0.089	1.006
89.2	107.2	0.059	0.039	0.039	0.058	1.004
115.8	∞	Isolated slab				1.000
147.3	∞	Isolated cylinder				1.000

APPENDIX A - FORTRAN Listing

```
C       PROGRAM FOR COMPUTING INTERACTION   H. K. CLARK     8211-1K
C
C      ALL DIMENSIONS MUST BE IN CM.
C
C      IC(I) CONTROLS THE TYPE OF PROBLEM AND THE INPUT REQUIRED
C      IC(1)=1 DENOTES AN ARRAY OF SLABS, IC(1)=2 DENOTES CYLINDERS, AND
C      IC(1)=3 DENOTES SPHERES.
C
C      IC(2) DENOTES THE INPUT AND TYPE OF PROBLEM
C
C      FOR IC(1)=1, IC(2)=I WHERE I INDICATES OPTION USED FOR SLABS
C      I=1 - 2  PARALLEL SLABS OF EQUAL THICKNESS
C      I=2 - 2 PARALLEL SLABS, ONE TWICE AS THICK AS THE OTHER
C      I=3 - 3 PARALLEL SLABS OF EQUAL THICKNESS
C      I=4 - 2 PERPENDICULAR SLABS OF EQUAL THICKNESS-L
C      I=5 - 2 PERPENDICULAR SLABS, ONE TWICE AS THICK AS THE OTHER-L
C      I=6 - 2 PERPENDICULAR SLABS OF EQUAL THICKNESS-T
C      I=7 - 2 PERPENDICULAR SLABS, ONE TWICE AS THICK AS THE OTHER-T
C      I=8 - 3 PARALLEL SLABS, CENTER TWICE AS THICK AS OTHER 2
C      I=9 - 2 PARALLEL DISCS
C      I=10 - 3 PARALLEL DISCS
C      I=11 - SLAB PARALLEL TO REFLECTOR OF EQUAL AREA
C      I=12 - SLAB PARALLEL TO REFLECTOR OF 9 TIMES THE AREA
C      I=13 - GENERAL CASE OF PERPENDICULAR SLABS (REFER TO FIG 2 OF NSE
C      15, PP 20-28(1963) - DH=2A, DL=2H, DW=B, SO=F, SP=2D, SH=E, AR=2G)
C      OTHERWISE SO IS BARE EXTRAPOLATION DISTANCE, DW IS SLAB THICKNESS,
C      SP IS SURFACE TO SURFACE SPACING, B IS MATERIAL BUCKLING, AM IS
C      MIGRATION AREA. UNLESS B EXCEEDS ZERO, KEFF AT SP AND CRITICAL
C      SPACING (IF IT EXISTS) WILL NOT BE CALCULATED.
C      FOR DISC DL MUST BE ZERO. FOR INFINITELY LONG SLAB DL MUST BE
C      NEGATIVE
C
C      FOR IC(1)=2, IC(2)=0 INDICATES A REFLECTED ARRAY OF CYLINDERS. THE
C      INPUT CONSISTS OF THE NUMBER OF SYMMETRICALLY EQUIVALENT POSITIONS
C      (NN), NUMBER OF UNITS, DIAMETER, HEIGHT, AREA OF ENCLOSING SURFAC-
C      ES, MINIMUM AXIS-TO-AXIS SPACING, ALBEDO OF REFLECTOR, BARE AND
C      AXIAL EXTRAPOLATION DISTANCES, MATERIAL BUCKLING, MIGRATION AREA.
C      IF CYLINDER HEIGHT IS INFINITE, HU IS MADE ZERO OR NEGATIVE AND SR
C      IS THE REFLECTOR AREA PER UNIT HEIGHT.
C
C      FOR IC(1)=3, IC(2)=0 INDICATES A REFLECTED ARRAY OF SPHERES. THE
C      INPUT CONSISTS OF THE NUMBER OF SYMMETRY TYPES, NUMBER OF UNITS,
C      DIAMETER, MINIMUM CENTER-TO-CENTER SEPARATION, TOTAL AREA OF
C      REFLECTING SURFACES, ALBEDO OF REFLECTOR, BARE EXTRAPOLATION DIS-
C      TANCE, MATERIAL BUCKLING, AND MIGRATION AREA.
C      FOR IC(1)=3, IC(2)=1 INDICATES A REFLECTED ARRAY OF CYLINDERS TO
C      BE TREATED AS SPHERES. INPUT IS NN, NO, UNITS,   CYLINDER DIAMETER
C      HEIGHT, INTERNAL VOLUME, WALL THICKNESS, SURFACE-TO-SURFACE SEPA-
C      RATION, ALBEDO OF REFLECTOR, BARE EXTRAPOLATION DISTANCE, MATERIAL
C      BUCKLING AND MIGRATION AREA.              (THIS OPTION IS FOR
C      TREATING THOMAS EXPERIMENTS AS CUBIC ARRAYS OF SPHERES)
C
C      IC(3) DENOTES NUMBER OF COMMENT CARDS
C
C      IC(4) DENOTES NUMBER OF SPACINGS TO BE READ IN FOR CYL AND SPHERE
C      IF IC(4)=0 NUMBER IS ASSUMED NOT TO CHANGE AND SDU AND WDI ARE NOT
C      READ IN.
C
C      IC(5) DENOTES FORMAT FOR WUF. IF IC(5)=0, FORMAT IS 6F2.0,60F1.0
C      OTHERWISE FORMAT IS 24F3.2
C
```

```
C     THE WST AND WUF CARDS ARE READ ONLY IF NM CHANGES. NM=0 CALLS FOR
C     THE IC CARD TO BE READ NEXT
C
C     THE SDU ARE THE SQUARES OF THE RATIOS OF THE INTER-UNIT SPACINGS
C     TO THE MINIMUM SPACING IN AN INFINITE LATTICE LISTED IN INCREASING
C     ORDER. THE WDI ARE THE NUMBERS OF UNITS AT THIS SPACING IN THE
C     INFINITE LATTICE. THE WST ARE THE NUMBERS OF UNITS OF EACH SYM-
C     METRY TYPE IN THE ACTUAL LATTICE. THE WUF(I,J,K) GIVE THE NUMBER
C     OF UNITS OF TYPE K TRANSMITTING NEUTRONS TO A UNIT OF TYPE J SEP-
C     ARATED FROM IT BY SPACING I.
C
  301 FORMAT(14I5)
  302 FORMAT(8F5.2,2F10.5)
  303 FORMAT(2I5,3F10.5,2F5.2,2F10.6)
  304 FORMAT(2I5,2F5.2, F10.5,4F5.2,2F10.5)
  305 FORMAT(2I5,7F5.2,2F10.5)
  306 FORMAT(24F3.2)
  307 FORMAT(14F5.0)
  308 FORMAT(6F2.0,60F1.0)
  309 FORMAT(24X12A6)
  310 FORMAT(1H1,40X,41HARRAYS OF INTERACTING UNITS   H. K. CLARK)
  311 FORMAT(/////)
  312 FORMAT(12A6)
  321 FORMAT(120H1ARRAYS OF CYLINDERS. INTERACTION CALCULATED WITH COSIN
     1E DISTRIBUTION FOR CURRENT EMITTED FROM CYLINDRICAL SURFACE       )
  322 FORMAT(120H0 NO.              DIA/MIN PITCH  REFL/CYL  REFL      FINAL
     1     MATERIAL    MIGRATION    EXTRAPOLATION DISTANCE     INITIAL  /
     2120H UNITS  DIA/HT  INITIAL FINAL     AREA   ALBEDO    ALBEDO    BU
     3CKLING       AREA           BARE   CRIT   FINAL      KEFF    )
  323 FORMAT(I5,F8.3,F9.3,F7.3,F8.2, F9.4,F10.4,F11.6,F11.2,F13.2,F7.2,F
     18.2,F11.4)
  324 FORMAT(120H1ARRAYS OF SPHERES. INTERACTION CALCULATED WITH COSINE
     1DISTRIBUTION FOR CURRENT EMITTED FROM SPHERICAL SURFACE           )
  325 FORMAT(120H0 NO.   DIA/MIN PITCH   REFL/SPH   REFL      FINAL      M
     1ATERIAL     MIGRATION     EXTRAPOLATION DISTANCE      INITIAL     /
     2120H UNITS   INITIAL FINAL       AREA     ALBEDO     ALBEDO     BUCKLING
     3       AREA           BARE   CRIT   FINAL       KEFF           )
  326 FORMAT(I5,2F8.3,F9.2,  F13.4,F10.4,F11.6,F11.2,F13.2,F7.2,F8.2,F11
     1.4)
  327 FORMAT(96H1ARRAYS OF SLABS.  COSINE DISTRIBUTION ASSUMED FOR CURRE
     1NT EMITTED FROM SURFACE.             TYPE I2)
  328 FORMAT(120H0          DIMENSIONS             SEPARATION   FRACTIONS REACHI
     1NG OTHER SURFACES                  EXTRPLTN DISTANCES  REFL       /
     2120H      H         L         W     INITIAL  FINAL  RHO1     RHO2     RHO3
     3   RHO4     BM**2    M**2    BARE  CRIT   FINAL ALBEDO  KEFF)
  329 FORMAT(F7.2,F8.2,F7.2,F8.2,F8.2,4F8.4,F10.6,F7.2,F6.2,2F7.2,F6.2,F
     17.3)
  330 FORMAT(120HOREFER TO NUCLEAR SCIENCE AND ENGINEERING, VOLUME 15, N
     1O. 1, JANUARY 1963,FIG 2, P 23 FOR MEANING OF SYMBOLS            /
     2120H          2H              2A              2G              2D
     3         B              E              F              RHO      )
  331 FORMAT(1P7E15.4,0PF12.4)
      DIMENSION IC(10),CW(108),SDU(50),WDI(50),WUF(50,10,10),WST(10)
      IF(SENSE LIGHT 1)1,2
    1 CALL EXIT
    2 CALL EFTM(16)
      NTIN=5
      NTOUT=6
    3 READ INPUT TAPE NTIN,301,(IC(I),I=1,5)
      NM=0
      IP=0
      IF(IC(1))1,1,4
```

```
  4 IF(IC(3))6,6,5
  5 NCW=12*IC(3)
    READ INPUT TAPE NTIN,312,(CW(I),I=1,NCW)
    WRITE OUTPUT TAPE NTOUT,310
    WRITE OUTPUT TAPE NTOUT,311
    WRITE OUTPUT TAPE NTOUT,309,(CW(I),I=1,NCW)
  6 IF(IC(1)-1)9,9,7
  7 IF(IC(4))9,9,8
  8 NU=IC(4)
 16 READ INPUT TAPE NTIN,307,(SDU(I),I=1,NU),(WDI(I),I=1,NU)
  9 IF(IC(1)-2)50,100,150
 50 I=IC(2)
    WRITE OUTPUT TAPE NTOUT,327,I
    IF(I-13)52,51,52
 51 WRITE OUTPUT TAPE NTOUT,330
    GO TO 53
 52 WRITE OUTPUT TAPE NTOUT,328
 53 READ INPUT TAPE NTIN,302,DH,DL,SP,DW,AR,SO,SH,SL,B,AM
    Y=0.
    IP=IP+1
    Z=0.
    U=0.
    UU=0.
    IF(DH)3,3,54
 54 CALL SLAB(I,DH,DL,SP,DW,AR,B,SO,AM,EK,SC,SA,SQ,Y,Z,U,UU,SH,SL)
    IF(I-13)56,55,56
 55 WRITE OUTPUT TAPE NTOUT,331,DL,DH,AR,SP,DW,SH,SO,Y
    GO TO 59
 56 IF(B)57,57,58
 57 WRITE OUTPUT TAPE NTOUT,329,DH,DL,DW,SP,SP,Y,Z,U,UU
    GO TO 59
 58 WRITE OUTPUT TAPE NTOUT,329,DH,DL,DW,SP,SQ,Y,Z,U,UU,B,AM,SO,SC,SA,
   1AR,EK
 59 IF(IP-55)53,60,60
 60 IP=0
    GO TO 50
100 WRITE OUTPUT TAPE NTOUT,321
    WRITE OUTPUT TAPE NTOUT,322
101 READ INPUT TAPE NTIN,305,NN,NX,DIA,HU,SR,SP,AR,SO,SH,B,AM
    IF(NN)3,3,102
102 IF(NX-NM)103,108,103
103 NM=NX
    XNU=FLOATF(NM)
    READ INPUT TAPE NTIN,307,(WST(I),I=1,NN)
    IF(IC(5))104,104,106
104 DO 105 K=1,NN
    DO 105 J=1,NN
105 READ INPUT TAPE NTIN,308,(WUF(I,J,K),I=1,NU)
    GO TO 108
106 DO 107 K=1,NN
    DO 107 J=1,NN
107 READ INPUT TAPE NTIN,306,(WUF(I,J,K),I=1,NU)
108 CALL CLNDR(NN,HU,DIA,SP,SR,AR,SDU,WDI,WUF,WST,XNU,B,SO,AM,EK,SC,SA
   1,SQ,SH,Y,NU)
    IF(HU)111,111,112
111 RDH=0.
    RSR=SR/(3.1416*XNU*DIA)
    GO TO 113
112 RDH=DIA/HU
    RSR=SR/(3.1416*XNU*DIA*HU)
```

```
113 RDP=DIA/SP
    RDQ=DIA/SQ
    IP=IP+1
    IF(B)109,109,110
109 WRITE OUTPUT TAPE NTOUT,323,NM,RDH,RDP,RDP,RSR,AR,Y
    GO TO 114
110 WRITE OUTPUT TAPE NTOUT,323,NM,RDH,RDP,RDQ,RSR,AR,Y,B,AM,SO,SC,SA,
   1EK
114 IF(IP-55)101,115,115
115 IP=0
    GO TO 100
150 WRITE OUTPUT TAPE NTOUT,324
    WRITE OUTPUT TAPE NTOUT,325
151 IF(IC(2))152,152,153
152 READ INPUT TAPE NTIN,303,NN,NX,DIA,SP,SR,AR,SO,B,AM
    IF(NN)3,3,155
153 READ INPUT TAPE NTIN,305,NN,NX,DIA,HU,VOL,TH,STS,AR,SO,B,AM
    IF(NN)3,3,154
154 SP=((DIA+STS)**2*(HU+STS))**(1./3.)
    DIA=2.*(VOL**(1./3.)/1.611991+TH)
    SR=FLOATF(NX)**(1./3.)*SP
    SR=6.*SR**2
155 IF(NX-NM)156,161,156
156 NM=NX
    XNU=FLOATF(NM)
    READ INPUT TAPE NTIN,307,(WST(I),I=1,NN)
    IF(IC(5))157,157,159
157 DO 158 K=1,NN
    DO 158 J=1,NN
158 READ INPUT TAPE NTIN,308,(WUF(I,J,K),I=1,NU)
    GO TO 161
159 DO 160 K=1,NN
    DO 160 J=1,NN
160 READ INPUT TAPE NTIN,306,(WUF(I,J,K),I=1,NU)
161 CALL SPHERE(NN,DIA,SP,SR,AR,SDU,WDI,WUF,WST,XNU,B,SO,AM,EK,SC,SA,S
   1Q,Y,NU)
    RDP=DIA/SP
    RDQ=DIA/SQ
    RSR=SR/(3.1416*XNU*DIA**2)
    IP=IP+1
    IF(B)162,162,163
162 WRITE OUTPUT TAPE NTOUT,326,NM,RDP,RDP,RSR,AR,Y
    GO TO 164
163 WRITE OUTPUT TAPE NTOUT,326,NM,RDP,RDQ,RSR,AR,Y,B,AM,SO,SC,SA,EK
164 IF(IP-55)151,165,165
165 IP=0
    GO TO 150
    END(1,0,0,0,0)
```

```
      SUBROUTINE SLAB          H. K. CLARK     8211-1K
      SUBROUTINE SLAB(I,DH,DL,SP,DW,AR,B,SO,AM,EK,SC,SA,SQ,Y,Z,U,UU,SH,S
     1L)
      SQ=SP
      IF(B)2,2,1
    1 IF(DL)9,3,4
    3 BW=SQRTF(B-(4.8096/(DH+2.*SH))**2)
      GO TO 5
    4 BW=SQRTF(B-9.8696044*(1./(DH+2.*SH)**2+1./(DL+2.*SL)**2))
      GO TO 5
    9 BW=SQRTF(B-9.8696044*(1./(DH+2.*SH)**2))
    5 SC=1.5707963/BW-.5*DW
      SA=SC
      SM=1.5707963/BW
      SB=0.
      IK=0
      AL=2.*BW*SO
      BK=1.+AM*B
      TS=SINF(AL)
      TC=COSF(AL)
    2 P=0.
      Q=0.
      IF(DL)6,6,7
    6 R=0.
      GO TO 8
    7 R=DH/DL
    8 T=.5
      V=1.
      GO TO (60,60,60,61,62,64,65,60,67,67,68,69,73),I
   60 S=SQ/DL
      CALL NTRCT(P,Q,R,S,T,V,U,Z,Y,ZX)
      GO TO 10
   61 PW=DW
      GO TO 63
   62 PW=.5*DW
   63 P=SQ/DL
      R=DL/DH
      S=(DL-PW)/DH
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,Z)
      P=0.
      Q=.5*(DL-PW)/PW
      R=DH/PW
      S=SQ/DL
      V=DL/PW
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,ZZ)
      Q=0.
      R=PW/DH
      S=SQ/DH
      V=1.
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,ZY)
      P=(DL-PW)/PW
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,ZX)
      U=1.-2.*ZZ-ZY-ZX
      U=U*PW/DL
      Y=Z+U
      GO TO 10
   64 PW=DW
      GO TO 66
   65 PW=.5*DW
   66 P=SQ/DL
      R=DL/DH
```

```
      S=.5*(DL-PW)/DH
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,Z)
      P=0.
      R=DH/PW
      S=SQ/DL
      V=DL/PW
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,ZZ)
      P=.5*(DL-PW)/PW
      R=PW/DH
      S=SQ/DH
      V=1.
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,ZY)
      U=1.-2.*(ZZ+ZY)
      U=.5*U*PW/DL
      Y=1.4142136*(U+Z)
      GO TO 10
   67 W=SQ/DH
      Y=1.+2.*W**2-2.*W*SQRTF(1.+W**2)
      GO TO 10
   68 S=SQ/DL
      CALL NTRCT(P,Q,R,S,T,V,U,UU,Z,Y)
      Y=AR*Z**2
      GO TO 10
   69 S=SQ/DL
      P=0.
      V=1.
      CALL NTRCT(P,Q,R,S,T,V,W,X,YY,Y)
      P=1.
      CALL NTRCT(P,Q,R,S,T,V,ZW,ZX,ZY,ZZ)
      R=DL/DH
      S=SQ/DH
      CALL NTRCT(P,Q,R,S,T,V,ZW,ZX,Y,ZY)
      P=0.
      Q=.5
      S=.5*SQ/DH
      V=2.
      CALL NTRCT(P,Q,R,S,T,V,ZW,U,Y,ZX)
      R=DH/DL
      S=.5*SQ/DL
      CALL NTRCT(P,Q,R,S,T,V,UU,U,Y,ZW)
      IF(DH-DL)70,70,71
   70 Z=1.-W-ZZ-2.*ZX-YY
      U=1.-X-ZY-2.*ZW-YY
      GO TO 72
   71 Z=1.-X-ZZ-2.*ZX-YY
      U=1.-W-ZY-2.*ZW-YY
   72 P=1.
      Q=0.
      S=SQ/(3.*DH)
      V=3.
      R=DL/DH
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,ZW)
      R=DH/DL
      S=SQ/(3.*DL)
      CALL NTRCT(P,Q,R,S,T,V,W,X,Y,ZX)
      UU=.25*(1.-2.*(ZW+ZX+Z+U)-YY)
      Y=AR*(YY**2+2.*(U**2+Z**2+2.*UU**2))
      GO TO 10
   73 P=DW/DH
      Q=SD/DL
      S=SP/AR
```

```
      T=SH/SP
      V=AR/DL
      CALL NTRCT(P,Q,R,S,T,V,W,X,Z,Y)
      IF(ABSF(V-1.)+Q+ABSF(T-.5)+P)80,74,80
   74 IF(R-1.)75,76,76
   75 Y=W
      GO TO 80
   76 Y=X
      GO TO 80
   10 IF(B)80,80,11
   11 GO TO (12,13,13,12,13,13,13,13,12,13,12,12),I
   12 FA=TS/(TC-Y)
      IF(FA)21,22,22
   21 SA=(1.5707963+ATANF(-1./FA))/(2.*BW)
      GO TO 50
   22 SA=ATANF(FA)/(2.*BW)
      GO TO 50
   13 SD=SA
   14 GO TO (12,15,16,12,15,16,15,15,12,16,12,12),I
   15 FS=SINF(BW*SA)
      FC=COSF(BW*SA)
      GO TO 17
   16 FS=SINF(2.*BW*SA)
      FC=COSF(2.*BW*SA)
   17 GO TO (12,18,19,12,18,19,18,20,12,19,12,12),I
   18 FA=TC-(FC**2-FS**2)*TS/(2.*FS*FC)
      FAD=.5*BW*TS/(FC*FS)**2
      GO TO 25
   19 FA=(FS*TC-FC*TS)/(FS*TC+FC*TS)
      FAD=4.*BW*TS/(TC*(FS+TS*FC/TC)**2)
      GO TO 25
   20 DEN=2.*TC*FC*FS+TS*(FC**2-FS**2)
      FA=(2.*FS*FC-TS)/DEN
      FAD=2.*BW*TS*(1.+TC*(FC**2-FS**2)-2.*TS*FS*FC)/DEN**2
      GO TO 25
   25 GO TO (12,26,27,12,26,27,28,26,12,27,12,12),I
   26 FB=TC+FS*TS/FC
      FBD=TS*BW/FC**2
      GO TO 35
   27 FB=(FS-TS)/FS
      FBD=2.*BW*FC*TS/FS**2
      GO TO 35
   28 FB=(FC-TS)/(TC*FC-TS*FS)
      FBD=BW*TS*(1.-TC*FS-TS*FC)/(TC*FC-TS*FS)**2
      GO TO 35
   35 SA=SD+(Y**2-FA*FB)/(FA*FBD+FB*FAD)
      IF(SA-SO)36,58,58
   36 SA=.5*(SO+SD)
      GO TO 13
   58 IF(SM-SA)59,37,37
   59 SA=.5*(SM+SD)
      GO TO 13
   37 IF(ABSF(SA-SD)-.005)50,13,13
   50 IF(IK)51,51,52
   51 EK=BK/(1.+AM*(B-BW**2+9.8696044/(DW+2.*SA)**2))
   52 IF(SC-SO)57,57,56
   57 IK=10
   56 IK=IK+1
      IF(IK-10)55,55,80
   55 IF(SA-SB)53,80,53
   53 IF(ABSF(SA-SC)-.005)80,54,54
   54 CALL SEP(0.,SC,SA,SB,SP,SQ,IK)
      GO TO (60,60,60,63,63,66,66,60,67,67,68,69),I
   80 RETURN
      END(1,0,0,0,0)
```

```
C       SUBROUTINE NTRCT        H. K. CLARK
        SUBROUTINE NTRCT(P,Q,R,S,T,V,RHO1,RHO2,RHO3,RHO)
C       REFER TO FIGURE 5.3 OF DP-532. P=B/2A,Q=F/2H, R=A/H, S=D/G,
C       T=E/2D, V=G/H. FOR PARALLEL RECTANGLES P=0, Q=0, T=.5, V=1, AND
C       RHO3=1-2*(RHO1+RHO2)
        FSTF(W,Z)=W+Q-Z
        SNDF(Y,X)=(Y+.5*A+P*A)**2+(X+V*B*T)**2
        RHQF(W,Z,Y,X)=(FSTF(W,Z)*SQRTF(SNDF(Y,X))*ATANF(FSTF(W,Z)/SQRTF(SN
       1DF(Y,X)))+.25*FSTF(W,Z)**2*LOGF((FSTF(W,Z)**2+SNDF(Y,X))/FSTF(W,Z)
       2**2)+.25*SNDF(Y,X)*LOGF(SNDF(Y,X)/(FSTF(W,Z)**2+SNDF(Y,X))))/(6.28
       331854*A)
        A=R
        B=S
        IF(ABSF(V-1.)+P+Q+ABSF(T-.5))17,5,17
     5  IF(A)50,50,6
    50  RHO2=0.
        RHO3=SQRTF(1.+(S/R)**2)-S/R
        RHO1=.5*(1.-RHO3)
        GO TO 4
     6  IF(S)13,13,14
    13  RHO1=0.
        RHO2=0.
        RHO3=1.
        GO TO 4
    14  W=.5
        Z=-.5
        Y=.5*A
        X=.5*B
        SUMA=-RHQF(W,Z,Y,X)
        X=-.5*B
        SUMA=SUMA+RHQF(W,Z,Y,X)
        Y=-.5*A
        X=.5*B
        SUMA=SUMA+RHQF(W,Z,Y,X)
        IF(A-1.)7,9,8
     7  RHO1=2.*SUMA
        A=1./A
        B=B*A
        GO TO 6
     9  RHO1=2.*SUMA
        RHO2=RHO1
        RHO3=1.-4.*RHO1
        GO TO 4
     8  RHO2=2.*SUMA
        RHO3=1.-2.*(RHO1+RHO2)
        GO TO 4
    17  IF(S)11,11,12
    11  RHO=0.
        GO TO 4
    12  IF(T-.5)18,18,19
    18  U=V*S*T
        GO TO 20
    19  U=.5*V*S
    20  W=.5*V
        Z=.5
        Y=.5*R
        X=.5*S*V
    21  IF(FSTF(W,Z))22,28,22
    22  RHO=RHQF(W,Z,Y,X)
        Y=-.5*R
        RHO=RHO-RHQF(W,Z,Y,X)
```

```
      X=-U
      IF(SNDF(Y,X))39,40,39
   39 RHO=RHO+RHQF(W,Z,Y,X)
   40 Y=.5*R
      RHO=RHO-RHQF(W,Z,Y,X)
      GO TO 29
   28 RHO=0.
   29 Z=-.5
      X=.5*S*V
      RHO=RHO-RHQF(W,Z,Y,X)
      X=-U
      RHO=RHO+RHQF(W,Z,Y,X)
      Y=-.5*R
      IF(SNDF(Y,X))30,31,30
   30 RHO=RHO-RHQF(W,Z,Y,X)
   31 X=.5*S*V
      RHO=RHO+RHQF(W,Z,Y,X)
      W=-.5*V
      IF(FSTF(W,Z))32,34,32
   32 RHO=RHO-RHQF(W,Z,Y,X)
      Y=.5*R
      RHO=RHO+RHQF(W,Z,Y,X)
      X=-U
      RHO=RHO-RHQF(W,Z,Y,X)
      Y=-.5*R
      IF(SNDF(Y,X))33,34,33
   33 RHO=RHO+RHQF(W,Z,Y,X)
   34 Z=.5
      X=-U
      Y=-.5*R
      IF(FSTF(W,Z))35, 4,35
   35 IF(SNDF(Y,X))36,37,36
   36 RHO=RHO-RHQF(W,Z,Y,X)
   37 X=.5*S*V
      RHO=RHO+RHQF(W,Z,Y,X)
      Y=.5*R
      RHO=RHO-RHQF(W,Z,Y,X)
      X=-U
      RHO=RHO+RHQF(W,Z,Y,X)
    4 RETURN
      END(1,0,0,0,0)
*     BIN
C     SUBROUTINE SEP       H. K. CLARK     8211-1K
      SUBROUTINE SEP(DIA,SC,SA,SB,SP,SQ,IK)
      IF(IK-1)1,1,7
    1 IF(SP-DIA)2,2,3
    2 SQ=10.+DIA
      GO TO 6
    3 SQ=SP*(1.+.2*(SA-SC)/ABSF(SA-SC))
      IF(SQ-DIA)5,6,6
    5 SQ=DIA
    6 SR=SP
      GO TO 10
    7 A=(SQ*(SC-SB)-SR*(SC-SA))/(SA-SB)
      SR=SQ
      IF(A-DIA)8,9,9
    8 A=DIA
    9 SQ=A
   10 SB=SA
      RETURN
      END(1,0,0,0,0)
```

```
C      SUBROUTINE CLNDR          H. K. CLARK     8211-1K
       SUBROUTINE CLNDR(NM,HU,DIA,SP,SR,AR,SDU,WDI,WUF,WST,XNU,B,SO,AM,EK
      1,SC,SA,SQ,SH,Y,NU)
       DIMENSION SDU(50),WDI(50),WUF(50,10,10),XIN(10,10),WST(10),H(50)
       DIMENSION RRU(10),RUR(10),A(10),TT(113)
       SQ=SP
       SMA=0.
       SMB=0.
       IF(HU)2,2,1
     1 IF(SR)2,2,24
    24 SR=SR-1.5708*DIA**2*XNU
     2 RU=.5*DIA
       DO 47 I=1,NU
       DO 45 J=1,NM
       DO 45 K=1,NM
    45 SMA=SMA+WUF(I,J,K)
       IF(ABSF(SMA-SMB)-.000001)47,47,46
    46 N=I
       SMB=SMA
    47 CONTINUE
       IF(B)48,48,51
    51 IF(HU)53,53,52
    52 BR=SQRTF(B-9.8696044/(HU+2.*SH)**2)
       GO TO 54
    53 BR=SQRTF(B)
    54 SC=2.4048/BR-RU
       SM=2.4048/BR
       SB=0.
       IK=0
       AL=BR*SO
       BK=1.+AM*B
       SA=SC
       DUM=BESJF(2.4048-AL,0.,1,113,XLOCF(TT(1)))
       TJ=TT(1)/TT(2)
    48 SUM=0.
       DO 3 I=1,N
       S=SQRTF(SDU(I))*SQ
       AA=RU/S
       H(I)=CLDRCT(AA,0.)
    34 SUM=SUM+H(I)*WDI(I)
       IF(1.-SUM)5,4,3
     3 CONTINUE
       L=N
       GO TO 6
     4 L=I
       GO TO 6
     5 L=I
       H(L)=(1.-SUM+H(L)*WDI(L))/WDI(L)
     6 IF(HU)10,10,11
    10 SE=0.
       GO TO 12
    11 SE=RU/HU
    12 AA=RU/(SQRTF(SDU(L))*SQ)
       H(L)=H(L)*CLDRCT(AA,SE)/CLDRCT(AA,0.)
       L=L-1
       IF(L)7,7,8
     8 DO 9 I=1,L
       S=SQRTF(SDU(I))*SQ
       AA=RU/S
     9 H(I)=CLDRCT(AA,SE)
     7 L=L+1
```

```
13 DO 14 J=1,NM
   DO 14 K=1,NM
14 XIN(J,K)=0.
   DO 15 J=1,NM
   DO 15 K=1,NM
   DO 15 I=1,L
   IF(WUF(I,J,K))15,15,16
16 XIN(J,K)=XIN(J,K)+WUF(I,J,K)*H(I)
15 CONTINUE
   IF(AR)21,30,21
21 SUM=0.
   DO 19 I=1,NM
19 SUM=SUM+WST(I)
   DO 27 J=1,NM
   RRU(J)=0.
   DO 17 K=1,NM
17 RRU(J)=RRU(J)+XIN(J,K)
   RRU(J)=1.-RRU(J)
18 IF(HU)25,25,26
25 RUR(J)=RRU(J)*6.2832*RU/SR
   GO TO 27
26 RUR(J)=RRU(J)*6.2832*RU*HU/SR
27 CONTINUE
   SUM=0.
   DO 20 I=1,NM
20 SUM=SUM+RUR(I)*WST(I)
   RRR=1.-SUM
   DO 22 J=1,NM
22 A(J)=RRU(J)*AR*WST(J)/(1.-RRR*AR)
   DO 23 J=1,NM
   DO 23 K=1,NM
23 XIN(J,K)=XIN(J,K)+RUR(J)*A(K)
30 CALL MAX(XIN,NM,5,Y)
   IF(B)80,80,66
66 TA=TJ*(1.+Y)/(1.-Y)
55 SD=SA
   AT=2.4048-SA*BR
   DUM=BESJF(AT,0.,1,113,XLOCF(TT(1)))
   FA=TT(1)/TT(2)
   FAD=BR*(1.-FA/AT+FA**2)
   SA=SD+(TA-FA)/FAD
   IF(SA-SO)56,67,67
56 SA=.5*(SO+SD)
   GO TO 55
67 IF(SM-SA)68,57,57
68 SA=.5*(SM+SD)
   GO TO 55
57 IF(ABSF(SA-SD)-.005)58,55,55
58 IF(IK)59,59,60
59 EK=BK/(1.+AM*(B-BR**2+5.7831/(RU+SA)**2))
60 IF(SC-SO)64,64,65
64 IK=10
65 IK=IK+1
   IF(IK-10)63,63,80
63 IF(SA-SB)61,80,61
61 IF(ABSF(SA-SC)-.005)80,62,62
62 CALL SEP(DIA,SC,SA,SB,SP,SQ,IK)
   GO TO 48
80 RETURN
   END(1,0,0,0,0)
```

```
C     FUNCTION CLDRCT(R,S)              H. K. CLARK      8211-IK
      FUNCTION CLDRCT(R,S)
C     COMPUTES INTERACTION INTEGRAL FOR FINITE CYLINDERS AS A FUNCTION
C     OF R=RADIUS/(AXIS-TO-AXIS SEPARATION) AND OF S=RADIUS/HEIGHT
      RADF(U,V)=SQRTF(1./R**2-(U+V)**2)
      GRDF(U,V)=ATANF(1./(S*(RADF(U,V)-SQRTF(1.-U**2)-SQRTF(1.-V**2))))/
     1RADF(U,V)
      DIMENSION X(8),H(8)
      N=8
      X(1)=.98940093
      X(2)=.94457502
      X(3)=.86563120
      X(4)=.75540441
      X(5)=.61787624
      X(6)=.45801678
      X(7)=.28160355
      X(8)=.095012510
      H(1)=.02715246
      H(2)=.06225352
      H(3)=.09515851
      H(4)=.12462897
      H(5)=.14959599
      H(6)=.16915652
      H(7)=.18260342
      H(8)=.18945061
      IF(S)1,5,1
    1 SUM=0.
      DO 4 I=1,N
      DO 4 J=1,I
      IF(I-J)3,3,2
    2 SUM=SUM+4.*H(I)*H(J)*(GRDF(X(I),X(J))+GRDF(X(I),-X(J)))
      GO TO 4
    3 IF(R-.5)10,11,10
   10 SUM=SUM+2.*H(J)**2*(GRDF(X(J),X(J))+GRDF(X(J),-X(J)))
      GO TO 4
   11 SUM=SUM+2.*H(J)**2*(1.5707963/RADF(X(J),X(J))+GRDF(X(J),-X(J)))
    4 CONTINUE
      CLDRCT=.050660591*SUM
      GO TO 8
    5 B=2.*R
      E=1.-B**2
      IF(E)6,6,7
    6 CLDRCT=.18169012
      GO TO 8
    7 C=ATANF(B/SQRTF(E))
      CLDRCT=.31830988*(C-SINF(.5*C)/COSF(.5*C))
    8 RETURN
      END(1,0,0,0,0)
```

```
C     SUBROUTINE MAX(A,N,M,R)       H. K. CLARK         8211-1K
C     CALCULATES DOMINANT EIGENVALUE R OF A SYMMETRIC OR NON-SYMMETRIC
C     MATRIX A OF ORDER N TO AN ACCURACY OF 10**(-L). THE EIGENVALUE
C     MUST BE POSITIVE. A NEGATIVE EIGENVALUE OF EQUAL MAGNITUDE IS
C     ALLOWED.
      SUBROUTINE MAX(A,N,L,R)
      DIMENSION A(10,10),B(10),C(10),D(10)
      IF(N-2)34,35,36
   34 R=A(1,1)
      GO TO 33
   35 R=.5*(A(1,1)+A(2,2)+SQRTF((A(1,1)-A(2,2))**2+4.*A(1,2)*A(2,1)))
      GO TO 33
   36 DO 1 I=1,N
      B(I)=1.
    1 C(I)=1.
      DO 2 I=1,N
      DO 2 J=1,N
      IF(A(I,J)-A(J,I))3,2,3
    2 CONTINUE
      IS=0
      GO TO 4
    3 IS=1
    4 IB=0
    5 DO 6 I=1,N
      D(I)=0.
      DO 6 J=1,N
    6 D(I)=D(I)+A(I,J)*B(J)
      S=0.
      DO 7 I=1,N
    7 S=S+ABSF(D(I))
      T=D(1)
      DO 8 I=2,N
      IF(ABSF((B(I)*T-D(I))*FLOATF(N)/S)-.00001)8,8,9
    8 CONTINUE
      IB=IB+1
      J=N-IB+1
      B(J)=0.
      GO TO 5
    9 DO 10 I=1,N
   10 B(I)=D(I)
      IF(IS)19,19,11
   11 IC=0
   12 DO 13 I=1,N
      D(I)=0.
      DO 13 J=1,N
   13 D(I)=D(I)+A(J,I)*C(J)
      S=0.
      DO 14 I=1,N
   14 S=S+ABSF(D(I))
      T=D(1)
      DO 15 I=2,N
      IF(ABSF((C(I)*T-D(I))*FLOATF(N)/S)-.00001)15,15,16
   15 CONTINUE
      IC=IC+1
      J=N-IC+1
      C(J)=0.
      GO TO 12
   16 DO 17 I=1,N
   17 C(I)=D(I)
      S=0.
      DO 18 I=1,N
```

```
18 S=S+B(I)*C(I)
   GO TO 21
19 S=0.
   DO 20 I=1,N
20 S=S+B(I)**2
21 Q=SQRTF(FLOATF(N-XMAXOF(IB,IC)))
22 T=S
   DO 23 I=1,N
   D(I)=0.
   DO 23 J=1,N
23 D(I)=D(I)+A(I,J)*B(J)
   DO 24 I=1,N
24 B(I)=D(I)
   IF(IS)29,29,25
25 DO 26 I=1,N
   D(I)=0.
   DO 26 J=1,N
26 D(I)=D(I)+A(J,I)*C(J)
   DO 27 I=1,N
27 C(I)=D(I)
   S=0.
   DO 28 I=1,N
28 S=S+B(I)*C(I)
   GO TO 31
29 S=0.
   DO 30 I=1,N
30 S=S+B(I)**2
31 R=SQRTF(S/T)
   IF(ABSF(Q-R)-10.**(-L))33,32,32
32 Q=R
   GO TO 22
33 RETURN
   END(1,0,0,0,0)
```

```
C      SUBROUTINE SPHERE          H. K. CLARK     8211-1K
       SUBROUTINE SPHERE(NM,DIA,SP,SR,AR,SDU,WDI,WUF,WST,XNU,B,SO,AM,EK,S
      1C,SA,SQ,Y,NU)
       DIMENSION SDU(50),WDI(50),WUF(50,10,10),XIN(10,10),H(50),
      1RRU(10),RUR(10),WST(10),A(10)
       SMA=0.
       SMB=0.
       SQ=SP
       RU=.5*DIA
       DO 37 I=1,NU
       DO 35 J=1,NM
       DO 35 K=1,NM
    35 SMA=SMA+WUF(I,J,K)
       IF(ABSF(SMA-SMB)-.000001)37,37,36
    36 N=I
       SMB=SMA
    37 CONTINUE
       IF(B)40,40,41
    41 BR=SQRTF(B)
       BK=1.+AM*B
       SC=3.1415927/BR-RU
       SA=SC
       SM=SC+RU
       SB=0.
       IK=0
       AL=BR*SO
       TS=SINF(AL)
       TC=COSF(AL)
       TT=3.1415927-AL
    40 SUM=0.
       DO 3 I=1,N
       S=SQRTF(SDU(I))*SQ
       C=RU/S
       H(I)=SPHRCT(C)
    34 SUM=SUM+H(I)*WDI(I)
       IF(1.-SUM)5,4,3
     3 CONTINUE
       L=N
       GO TO 11
     4 L=I
       GO TO 11
     5 L=I
       H(L)=(1.-SUM+H(L)*WDI(L))/WDI(L)
    11 DO 12 J=1,NM
       DO 12 K=1,NM
    12 XIN(J,K)=0.
       DO 13 J=1,NM
       DO 13 K=1,NM
       DO 13 I=1,L
       IF(WUF(I,J,K))13,13,14
    14 XIN(J,K)=XIN(J,K)+WUF(I,J,K)*H(I)
    13 CONTINUE
       IF(AR)10,30,10
    10 DO 16 J=1,NM
       RRU(J)=0.
       DO 15 K=1,NM
    15 RRU(J)=RRU(J)+XIN(J,K)
       RRU(J)=(1.-RRU(J))
    16 RUR(J)=RRU(J)*12.5664*RU**2/SR
       SUM=0.
       DO 17 J=1,NM
```

```
17 SUM=SUM+RUR(J)*WST(J)
   RRR=1.-SUM
   DO 18 J=1,NM
18 A(J)=RRU(J)*AR*WST(J)/(1.-RRR*AR)
   DO 19 J=1,NM
   DO 19 K=1,NM
19 XIN(J,K)=XIN(J,K)+RUR(J)*A(K)
30 CALL MAX(XIN,NM,5,Y)
   IF(B)80,80,53
53 TA=(1.+Y)*TT/((1.-Y)*(1.+TT*TC/TS))
42 SD=SA
   FS=SINF(SA*BR)
   FC=COSF(SA*BR)
   FT=3.1415927-SA*BR
   FA=FT/(1.+FT*FC/FS)
   FAD=BR*(FT**2-FS**2)/(FS+FT*FC)**2
   SA=SD+(TA-FA)/FAD
   IF(SA-SO)43,54,54
43 SA=.5*(SO+SD)
   GO TO 42
54 IF(SM-SA)55,44,44
55 SA=.5*(SM+SD)
   GO TO 42
44 IF(ABSF(SA-SD)-.005)45,42,42
45 IF(IK)46,46,47
46 EK=BK/(1.+AM*9.8696044/(RU+SA)**2)
47 IF(SC-SO)51,51,52
51 IK=10
52 IK=IK+1
   IF(IK-10)50,50,80
50 IF(SA-SB)48,80,48
48 IF(ABSF(SA-SC)-.005)80,49,49
49 CALL SEP(DIA,SC,SA,SB,SP,SQ,IK)
   GO TO 40
80 RETURN
   END(1,0,0,0,0)
```

```
FUNCTION SPHRCT      H. K. CLARK     8211-1K
FUNCTION SPHRCT(X)
DIMENSION PL(5),RF(5),FP(5),FM(5)
FZ=.0590
FP(1)=.0724
FP(2)=.0857
FP(3)=.1034
FP(4)=.1254
FP(5)=0.1674
FM(1)=.0465
FM(2)=.0354
FM(3)=.0238
FM(4)=.0119
FM(5)=0.
RFZ=6.9444444E-05
RF(1)=5.7870370E-05
RF(2)=3.3068783E-05
RF(3)=1.2400794E-05
RF(4)=2.7557319E-06
RF(5)=2.7557319E-07
P=20.*X-5.
DO 1 I=1,5
1 PL(I)=P**2-FLOATF(I)**2
SUM=0.
DO 6 I=1,6
PROD=1.
DO 3 J=2,6
IF(J-I)2,3,2
2 PROD=PROD*PL(J-1)
3 CONTINUE
IF(I-1)4,4,5
4 SUM=SUM-PROD*RFZ*FZ
GO TO 6
5 SUM=SUM+RF(I-1)*PROD*FLOATF((-1)**I)*P*((P+FLOATF(I-1))*FP(I-1)+(P
1-FLOATF(I-1))*FM(I-1))
6 CONTINUE
SPHRCT=(1.+SUM)*.5*(1.-SQRTF(1.-X**2))
RETURN
END(1,0,0,0,0)
```

APPENDIX B - Cubic Arrays of Spheres

SDU(I)	1	2	3	5	6	9	10	11	13	14	17	18	19	21
	22	25	26	27	29	30	33	34	35	37	38	41	42	43
	45	46	49	50	51	53	54	57	58	59	61	62	65	66
WDI(I)	6	12	8	24	24	24	24	24	24	48	48	24	24	48
	24	24	72	24	72	48	48	48	48	24	72	96	48	24
	48	48	48	72	48	72	72	48	24	72	72	96	96	96

2 x 2 x 2

WST(I)	8		
WUF(I,J,K)	3	3	1

3 x 3 x 3

WST(I)	1	6	12	8		
WUF(I,J,K)	0	0	0	0	0	0
	1	0	0	0	0	0
	0	1	0	0	0	0
	0	0	1	0	0	0
	6	0	0	0	0	0
	0	4	0	0	0	0
	2	0	2	2	0	0
	0	3	0	0	3	0
	0	12	0	0	0	0
	4	0	4	4	0	0
	0	4	0	0	4	0
	3	0	0	6	0	3
	0	0	8	0	0	0
	0	4	0	0	4	0
	2	0	0	4	0	2
	0	0	0	0	0	0

4 x 4 x 4

WST(I)	8	24	24	8											
WUF(I,J,K)	3	3	1	0	0	0	0	0	0	0	0	0	0	0	0
	1	2	1	2	1	0	0	0	0	0	0	0	0	0	0
	0	1	1	2	2	1	0	0	0	0	0	0	0	0	0
	0	0	1	0	3	3	0	0	0	0	0	0	0	0	0
	3	6	3	6	3	0	0	0	0	0	0	0	0	0	0
	2	3	2	4	4	2	2	1	0	0	0	0	0	0	0
	2	2	1	2	3	3	2	2	2	2	0	0	0	0	0
	0	3	0	6	0	0	0	3	0	6	3	0	0	0	0
	0	3	3	6	6	3	0	0	0	0	0	0	0	0	0
	2	2	1	2	3	3	2	2	2	2	0	0	0	0	0
	1	2	0	4	0	0	2	2	0	4	2	0	1	0	0
	3	0	0	0	0	0	6	0	6	0	0	0	3	0	3
	0	0	1	0	3	3	0	0	0	0	0	0	0	0	0
	0	1	0	2	0	0	0	1	0	2	1	0	0	0	0
	1	0	0	0	0	0	2	0	2	0	0	0	1	0	1
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

5 x 5 x 5

WSTI(I) 1 6 12 8 6 24 24 12 24 8

WUF(I,J,K)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	3	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	2	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	2	0	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	3	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	4	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	6	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	4	0	0	4	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	2	3	0	2	0	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	2	0	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	4	0	2	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	2	0	2	0	2	2	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	3	0	0	0	0	6	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0
0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	4	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	4	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	2	0	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	0	1	0	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	0	0	0	0	4	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	2	0	0	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	3	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	3	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	2	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	2	0	0	0	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	2	0	1	0	0	0	0	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	4	0	0	8	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	4	6	0	4	0	4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	6	0	0	6	0	6	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	4	0	8	0	0	4	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	3	0	0	4	0	2	0	0	4	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	2	4	0	2	0	4	2	0	4	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	4	0	4	0	0	0	0	6	0	0	4	0	2	0	0	0	0	0	0	0	0	0	0
0	2	0	0	2	0	2	0	0	4	0	2	0	0	2	0	2	0	0	0	0	0	0	0	0	0
0	0	0	6	0	0	0	0	6	0	0	0	0	6	0	0	0	0	6	0	0	0	0	0	0	0
0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	4	8	0	8	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	4	0	0	4	0	4	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	6	0	3	0	0	6	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	4	0	0	8	0	0	0	0	8	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	2	4	0	2	0	4	2	0	4	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	2	0	4	0	0	4	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0
0	4	0	0	2	0	4	0	0	4	0	4	0	0	2	0	4	0	0	0	0	0	0	0	0	0
2	0	1	2	0	0	0	3	2	0	2	0	3	2	0	2	0	0	2	0	0	0	0	0	0	0
0	3	0	0	0	0	6	0	0	0	0	3	0	0	0	0	6	0	0	0	0	3	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	4	0	4	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	4	0	2	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	3	0	0	0	0	6	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	0	2	0	0	0	0	3	0	0	2	0	1	0	0	0	0	0	0	0	0	0	0
0	2	0	0	1	0	2	0	0	2	0	2	0	0	1	0	2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	2	0	0	0	0	2	0	2	0	0	2	0	0	0	0	2	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	8	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	4	0	4	0	4	4	0	4	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
0	3	0	0	3	0	6	0	0	6	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0
0	0	0	8	0	4	0	0	0	0	4	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	2	0	2	0	0	4	0	2	0	0	2	0	2	0	0	0	0	0	0	0	0	0
2	0	1	2	0	0	0	3	2	0	2	0	3	2	0	2	0	0	2	0	0	0	0	0	0	0
2	0	0	4	0	0	0	0	4	0	4	0	0	4	0	0	0	0	4	0	2	0	0	0	0	0
0	2	0	0	0	0	4	0	0	0	0	2	0	0	0	0	4	0	0	0	0	2	0	0	0	0
3	0	0	0	0	0	0	0	0	0	6	0	0	0	0	6	0	0	0	0	3	0	0	0	0	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	4	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	2	0	0	0	0	4	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	3	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	2	0	0	0	0	2	0	0	0	0	2	0	0	0	0	2	0	0	0	0	0	0	0
0	1	0	0	0	0	2	0	0	0	0	1	0	0	0	0	2	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	2	0	0	0	0	2	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX C - Square Arrays of Cylinders

SDU(I)	1	2	5	10	13	17	25	26	29	34	37	41	50	53
	58	61	65	73	74	82	85							
WDI(I)	4	4	8	8	8	8	8	8	8	8	8	8	8	8
	8	8	16	8	8	8	16							

1 x 2

WST(I)	2	
WUF(I,J,K)	1	

1 x 3

WST(I)	1	2
WUF(I,J,K)	0	
	1	
	2	
	0	

1 x 4

WST(I)	2	2
WUF(I,J,K)	1	
	1	
	1	
	0	

1 x 5

WST(I)	1	2	2
WUF(I,J,K)	0		
	1		
	0		
	2		
	0		
	1		
	0		
	1		
	0		

1 x 6

WST(I)	2	2	2
WUF(I,J,K)	1		
	1		
	0		
	1		
	0		
	1		
	0		
	1		
	0		

2 x 2

WST(I)	4	
WUF(I,J,K)	2	1

3 x 3

WST(I)	1	4	4
WUF(I,J,K)	0	0	0
	1	0	0
	0	1	0
	4	0	0
	0	2	0
	2	0	2
	0	4	0
	2	0	2
	0	0	0

4 x 4

WST(I)	4	8	4		
WUF(I,J,K)	2	1	0	0	0
	1	1	1	0	0
	0	1	2	0	0
	2	2	2	0	0
	1	1	2	1	0
	2	0	0	2	2
	0	1	2	0	0
	1	0	0	1	1
	0	0	0	0	0

5 x 5

WST(I)	1	4	4	4	8	4	
WUF(I,J,K)	0	0	0	0	0	0	0
	1	0	0	0	0	0	0
	0	1	0	0	0	0	0
	0	0	0	0	0	0	0
	0	0	1	0	0	0	0
	0	0	0	0	0	0	0
	4	0	0	0	0	0	0
	0	2	0	0	0	0	0
	2	0	2	0	0	0	0
	1	0	2	0	0	0	0
	0	1	0	1	0	0	0
	0	0	2	0	2	0	0
	0	4	0	0	0	0	0
	2	0	2	0	0	0	0
	0	0	0	0	0	0	0
	0	2	0	2	0	0	0
	1	0	1	0	1	0	0
	0	1	0	2	0	0	0
	0	0	0	0	0	0	0
	1	0	2	0	0	0	0
	0	2	0	2	0	0	0
	0	0	0	0	0	0	0
	1	0	1	0	1	1	0
	0	0	0	0	0	0	0
	0	0	8	0	0	0	0
	0	2	0	2	0	0	0
	2	0	2	0	2	0	0
	2	0	2	0	2	2	0
	0	1	0	2	0	0	0
	2	0	0	0	0	2	2
	0	0	0	0	0	0	0
	0	0	2	0	2	0	0
	0	1	0	2	0	0	0
	0	0	0	0	0	0	0
	1	0	0	0	0	1	1
	0	0	0	0	0	0	0

6 x 6

WST(I)	4	8	4	8	8	4						
WUF(I,J,K)	2	1	0	0	0	0	0	0	0	0	0	0
	1	1	1	0	0	0	0	0	0	0	0	0
	0	1	2	0	0	0	0	0	0	0	0	0
	0	0	1	1	0	0	0	0	0	0	0	0
	0	0	1	1	1	0	0	0	0	0	0	0
	0	0	0	0	2	0	0	0	0	0	0	0
	2	2	2	0	0	0	0	0	0	0	0	0
	1	1	2	1	0	0	0	0	0	0	0	0
	2	0	0	2	2	0	0	0	0	0	0	0
	1	1	1	1	1	1	0	0	0	0	0	0
	0	1	1	0	1	1	0	0	0	0	0	0
	0	0	2	2	0	0	2	0	0	0	0	0
	0	1	2	0	0	0	0	0	0	0	0	0
	1	0	0	1	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	1	0	0	1	0	0	0	0	0	0
	1	0	0	1	0	0	1	0	0	0	0	0
	0	1	0	0	0	2	0	0	0	0	0	0
	0	0	2	2	0	0	0	0	0	0	0	0
	1	1	1	1	1	1	0	0	0	0	0	0
	0	2	2	0	0	2	0	0	0	0	0	0
	1	0	0	0	2	0	0	1	0	0	0	0
	1	0	1	1	0	0	1	1	1	0	0	0
	0	0	0	0	0	0	0	0	2	2	0	0
	0	0	2	2	2	0	0	0	0	0	0	0
	0	1	1	0	1	1	0	0	0	0	0	0
	2	0	0	2	0	0	2	0	0	0	0	0
	1	0	1	1	0	0	1	1	1	0	0	0
	0	1	0	0	0	2	0	0	0	1	0	0
	2	0	0	0	0	0	0	2	0	0	0	2
	0	0	0	0	2	0	0	0	0	0	0	0
	0	0	1	1	0	0	1	0	0	0	0	0
	0	1	0	0	0	2	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	1	0	0
	1	0	0	0	0	0	0	1	0	0	0	1
	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX D - Triangular Arrays of Cylinders

SDU(I)	1	3	7	13	19	21	28
WDI(I)	6	6	12	12	12	12	12

3

WST(I)	3
WUF(I,J,K)	2

7

WST(I)	1	6
WUF(I,J,K)	0	0
	1	0
	6	0
	2	2

19

WST(I)	1	6	6	6
WUF(I,J,K)	0	0	0	0
	1	0	0	0
	0	1	0	0
	0	0	0	0
	6	0	0	0
	2	2	0	0
	2	0	2	0
	1	2	2	0
	0	6	0	0
	2	0	2	0
	0	2	0	0
	2	0	2	2
	0	0	0	0
	1	2	2	0
	2	0	2	2
	0	0	0	0

REFERENCES

1. H. K. Clark. "Interaction of Fissionable Units." Nucl. Sci. Eng. 15, 20-28 (1963).

2. H. K. Clark. Comparison of a Simple Treatment of Critical Arrays of Fissionable Units with Experiments. USAEC Report DP-868, E. I. du Pont de Nemours and Co., Savannah River Laboratory, Aiken, S. C. (1964).

3. H. K. Clark. "Application of a Simple, Practical Method for Computing Interaction to Arrays Found Experimentally to be Critical." Nucl. Sci. Eng. 20, 307-313 (1964).

4. H. C. Paxton, J. T. Thomas, Dixon Callihan, and E. B. Johnson. Critical Dimensions of Systems Containing U^{235}, Pu^{239}, and U^{233}. USAEC Report TID-7028, Los Alamos Scientific Lab., N. Mex. and Oak Ridge National Lab., Tenn. (1964).

5. J. K. Fox and L. W. Gilley. "Critical Parameters of Aqueous Solutions of U^{235}." Applied Nuclear Physics Division, Annual Progress Report for Period Ending September 1, 1957. USAEC Report ORNL-2389, Oak Ridge National Laboratory, Tenn., pp 71-83 (1957).

6. J. K. Fox and L. W. Gilley. Preliminary Report of Critical Experiments in Slab Geometry. USAEC Report ORNL-CF-56-7-148, Oak Ridge National Laboratory, Tenn. (1956) (declassified April 1, 1957).

7. L. W. Gilley, D. F. Cronin, J. K. Fox, and J. T. Thomas. "Critical Arrays of Neutron Interacting Units." Neutron Physics Division Annual Progress Report for Period Ending September 1, 1961. USAEC Report ORNL-3193, Oak Ridge National Laboratory, Tenn., pp 159-167 (1961).

8. E. B. Johnson and D. F. Cronin. "Critical Dimensions of Aqueous UO_2F_2 Solutions Containing 4.9% ^{235}U-Enriched Uranium." Neutron Physics Division, Annual Progress Report for Period Ending August 1, 1964. USAEC Report ORNL-3714, Oak Ridge National Laboratory, Tenn., pp 31-33 (1964).

9. I. Carlvik and B. Pershagen. The Dancoff Correction in Various Geometries. Aktiebolaget Atomenergi, Stockholm. Report AE-16 (1959).

EXTERNAL RELEASE OF TECHNICAL INFORMATION

Description of Material   No. DP-1031   Date: 6/16/66

Title: Interaction of Fissile Units. A Computer Code - INTERACT

Author: H. K. Clark

Type of Material

Classified DP Report ☐   Classified Paper ☐

Unclassified DP Report [x]   Unclassified Paper ☐

Letter ☐

Technical Content

Approved by /s/ P. L. Roggenkamp   Date: 6/1/66

Classification

Approved by SWO'Rear
S. W. O'Rear   Date: 6/16/66

Authority:
Topics 4 and 5.2.3 CG-UF-2
Topics 701 and 702.2 SROO Classification Guide

Category if DP Report

Approved by SWO'Rear
S. W. O'Rear   Date: 6/16/66

Final Du Pont Release

Approved by [signature]
Coordinating Organization Director   Date: 6/21/66

Released by
R. G. Erdley: 6-29-66