DP - 101

Physics

FLOATING-DECIMAL MATRIX INVERSION

by

F. M. Trantham, Jr.

Theoretical Physics Division

April 1955

E. I. du Pont de Nemours & Co.
Explosives Department — Atomic Energy Division
Technical Division — Savannah River Laboratory

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ABSTRACT

The method of matrix inversion described in IBM Technical Newsletter No. 4 is adapted to the Heising Floating-Decimal General-Purpose Board for the IBM Card-Programmed Electronic Calculator, Model 412, 605.

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INTRODUCTION

In IBM Technical Newsletter No. 4,[1] a method is described for inversion of matrices up to order 21, on the Model I Card-Programmed Calculator. This method uses floating-decimal notation and requires special-purpose wiring panels. The basic approach is that of the Crout Method described in the seminar proceedings of 1950.[2] The Crout Method is recapitulated in Appendix A.

The method described in the present report is identical in concept to the method referred to above. However, it uses an existing general-purpose setup[3] thus affording an economy of patch panels and of time spent in wiring. No additional 605 panel wiring is required and 412 panel wiring is minimized by superimposing supplementary wiring on the existing 412 general-purpose panel.

SUMMARY

The wiring for the Crout Method of matrix inversion was incorporated in the Heising General-Purpose Setup for the CPC formerly in use at the Savannah River Laboratory. Five hours are required to invert a 27th order matrix by means of the alphamerical 412 CPC which feeds 100 cards per minute. The corresponding times for inverting matrices of order 21 and 12 are three hours and one hour, respectively.

The general-purpose setup used has eight counter groups making one additional storage available. Matrices up to order 22 may be inverted with one 941 auxiliary storage unit. Matrices up to order 38 may be inverted with two 941 units.

In the floating-decimal setup that is used, a number is represented by seven significant digits and a two-digit exponent. Representation of a number by eight significant digits and a two-digit exponent is possible in floating-decimal setups that employ fewer than eight counter groups.

The 527 summary punch panel requires no operator decisions at any time because all 527 panel wiring changes, which must be made between the forward and back solutions and during the back solution, are effected automatically instead of manually.

Sorting time, both prior to the forward solution and between the forward and back solutions, is reduced by using the sorter to order only the element cards of the matrix.

After the sorting is done, the collator is used to insert the ordered element cards into the program deck.

Row identification is provided on the print-out by accumulating the check sum from row to row.

Zero matrix elements are entered as zeros instead of as 1×10^{-40}.

The zero elements to the right of the diagonal in the unit matrix are still introduced only when the back solution is started. However, the zero elements to the left of the diagonal in the unit matrix need never be introduced.

The method was adapted to the Heising Board because this board was available at the time of wiring. In principle, the method could be incorporated as a feature of any floating-decimal general-purpose board.

DISCUSSION

GENERAL PLAN OF DISCUSSION

In Appendix B, the essentials of the Heising Board Coding are quoted directly from the original article(3). Under the heading, REDUCTION CALCULATIONS, the limitations imposed by the Heising Board Coding are discussed, the basic computing scheme is illustrated, and the assignment of storage addresses to matrix indices is tabulated. Under CHECK SUM, a method is described for utilizing the check sum to identify the row. Under SORTING, matrix element indices are assigned to card columns in a manner that is of mnemonic value to the operator. The section on BLANK CARDS explains the method employed to insure that no case arises in which a number is called from storage at a time when it is inaccessible.

Under FORWARD SOLUTION CARDS, a detailed outline is given of all instructions which must be in the cards of the original elements and their associated programming cards. Also given are the instructions which must be summary punched in the "b" cards during the forward solution. Under BACK SOLUTION CARDS, an outline is given of all instructions which must be in the associated programming cards and the appropriate unit matrix zero element cards. Also given are the instructions which will be summary punched in the "c" cards during the back solution.

The section on COLLATING deals with the method by which ordered element cards are inserted into the ordered permanent program deck so that the permanent program deck, once sorted, need never be sorted again. The collating procedure employed during the forward and back solutions is exactly as described on page 38 of Technical Newsletter No. 4(1) and is not repeated here. The wiring panels for this collating procedure are shown in Figures 2 and 3, respectively. The section on the 412 ACCOUNTING MACHINE explains the use of alternate instruction fields for forward and back solutions, the use of the setup change switches, the method of control of summary punching and listing, the method of identifying summary punched cards, the use of follow cards, and the method by which punch selectors on the 527 summary punch machine are controlled by the 412 machine. The section on SPECIAL INSTRUCTIONS describes the logical meaning and the method of machine interpretation of Special Instructions. The section on the 527 SUMMARY PUNCH describes the conditions under which the various instructions are summary punched. The section on the USE OF 412 SELECTORS TO CONTROL SELECTORS ON THE 527 describes a method of insuring that the ten-degree drop-out of selectors on the 412 during idle cycles can never vitiate the control of selectors on the 527 by selectors on the 412.

The card form and all wiring diagrams appear at the end of the report.

REDUCTION CALCULATIONS

In order to form the negative product (-A x B) or the accumulative negative product (-A x B + D), either the sign of Factor A or the sign of Factor B must be reversed. In the absence of a sign reversal code for Factor B*, sign reversal of Factor A is mandatory for negative product formation. Moreover, the Heising proscription of code XX for any factor** dictates that Factor A not come from a card when an X punch in the units position of Factor A code is used to reverse the sign of Factor A. Therefore, Card Field B (card columns 21 to 29) is used for punching the "a" matrix elements and for summary punching the "b" and "c" elements.

Since the code B + D is not explicitly available, unity is punched or summary punched in Field A of all "a", "b", and "c" element cards. Then Operation Code 1 (A x B + D) will give (1 x B + D) when Factor A is read from an element card.

Operation Code 1 will give the transfer code for B, (1 x B + 0), if Factor A is unity and Factor D is zero.

The basic computing scheme for the reduction of a given element $b_{i,k}$ located to the right of the principal diagonal is as shown on the following page.

* The addition of a single wire to the Heising 412 Panel connecting the right normal hub of Pilot Selector 29 with the left transferred hub of Pilot Selector 30 would permit sign reversal of Factor B to occur when an X punch is used over the units position of Factor B (card column 4).

** An examination of the wiring diagrams of the Heising 412 Panel indicates and machine tests confirm that the code XX can be used.

Card	Program Number	Factor A (from storage)	Factor B (from card)	Factor D (from previous result)	Operation	Result	Store
1	1	$-b_{i,1}$	$b_{1,k}$	-	A x B + 0	$-\sum_{j=1}^{1} b_{i,j}b_{j,k}$	C_1
2	1	$-b_{i,2}$	$b_{2,k}$	C_1	A x B + D	$-\sum_{j=1}^{2} b_{i,j}b_{j,k}$	C_2
3	1	$-b_{i,3}$	$b_{3,k}$	C_2	A x B + D	$-\sum_{j=1}^{3} b_{i,j}b_{j,k}$	C_3
.	.	.	.	.	.		.
.	.	.	.	.	.		.
.	.	.	.	.	.		.
(i-1)	1	$-b_{i,(i-1)}$	$b_{(i-1),k}$	$C_{(i-2)}$	A x B + D	$-\sum_{j=1}^{(i-1)} b_{i,j}b_{j,k}$	$C_{(i-1)}$
i	1	+1 (from card)	$a_{i,k}$	$C_{(i-1)}$	1 x B + D	$a_{i,k} - \Sigma b_{i,j}b_{j,k}$	C_i
(i+1)	2	C_i (previous result)	$b_{i,i}$ (from storage)		A/B	$b_{i,k}$	$C_{(i+1)}$(S.P.)
(i+2)	3	$\sum_{m=i+1}^{k+1} b_{i,m}$ (from storage 48)		$C_{(i+1)}$	A + D	$\sum_{m=i+1}^{k} b_{i,m}$	$C_{(i+2)}$, storage 48
(i+3)	1	$-b_{i,1}$	$b_{1,(k+1)}$		A x B + 0	$-\sum_{j=1}^{1} b_{i,j}b_{j,(k+1)}$	$C_{(i+3)}$
(i+4)	1	$-b_{i,2}$	$b_{2,(k+1)}$	$C_{(i+3)}$	A x B + D	$-\sum_{j=1}^{2} b_{i,j}b_{j,(k+1)}$	$C_{(i+4)}$

Row and column numbers are used which correspond with CPC storage addresses in the following manner:

Row or Column	Numbered	Row or Column	Numbered	Row or Column	Numbered
1	11	17	31	32	91
2	12	18	32	33	92
3	13	19	33	34	93
4	14	20	34	35	94
5	15	21	35	36	95
6	16	22	36	37	96
7	17	23	37	38	97
8	18	24	38		
9	21	25	41		
10	22	26	42		
11	23	27	43		
12	24	28	44		
13	25	29	45		
14	26	30	46		
15	27	31	47		
16	28				

Using this numbering system, the $b_{i,j}$ factors are entered into storage by column number. Then they are called out by the row number of the $b_{j,k}$ cards.

CHECK SUM

The check sum is accumulated in storage 48. A 941 storage is used because in the Heising Setup a counter group may not be read out of and read into, on the same program card. The check sum, instead of being cleared at the end of each row, is allowed to accumulate from row to row. This procedure provides a row identification number. Since the check sum for each individual row is approximately minus one, the cumulative check sum at the end of each row should be a negative number whose absolute value indicates the number of rows that have been already processed. The check sum is cleared at the beginning of the back solution by the first check sum program card which passes through the 412.

SUMMARY PUNCHING TO THE LEFT OF PRINCIPAL DIAGONAL

No special provision is made for punching $b_{i,k}$ terms on and to the left of the principal diagonal. If desired, such punching may be caused by an X punch in card column 20 of the corresponding $a_{i,k}$ cards.

SORTING

The proper ordering of the deck requires that sorting be done on column, subdivision, and row, in that order. For this reason, row number is punched in card columns 51 and 52, subdivision number in card column 53, and column number in card columns 54 and 55. For either the forward or the back solution the sort is on columns 55, 54, 53, 52, and 51, in that order. In the sort that precedes the forward solution the cards are picked up in normal order after each pass. In the sort that occurs between the forward solution and the back solution, the cards are picked up in inverse order after each pass except the pass on card column 53. This procedure inverts the order of columns within each subdivision and inverts the order of rows within the matrix but preserves the order of subdivisions within each row.

BLANK CARDS

After a number has been read into the 941 unit, a lapse of two 412-418 cycles must be allowed before the number is accessible.

The first step in processing any row of the forward solution is the transfer to storage 11 of element $a_{i,1,11}$*,

* Where three indices are employed in the subscript the left index refers to row number, the middle index to subdivision number and the right index to column number.

at which time it becomes element $b_{i,1,11}$. For i = 11, the second step is the division of $a_{11,1,12}$ by $b_{11,1,11}$. For all $i > 11$ the second step is the multiplication of $-b_{i,1,11}$ by $b_{11,1,12}$. In either case, $b_{i,1,11}$ needs to be called from storage and provision must be made for a two-cycle lapse before calling out $b_{i,1,11}$. This may be accomplished by inserting after $a_{11,1,11}$, which is the first card of the first row of the forward solution, two identical "blank" cards which contain (in card columns 51 to 55) the row, subdivision, and column indices of $a_{11,1,11}$ and a 2 in the card column set aside for program number (card column 58). Similar considerations require that one "blank" card follow the matrix element card $a_{i,1,12}$ for all $i > 2$. This may be accomplished by inserting after $a_{13,1,12}$, one "blank" card with indices, 13, 1, 12 (in card columns 51 to 55) and a 2 card column 58. As each row is processed the collator will automatically transfer these three "blank" cards to their appropriate positions in the merged deck for the next row.

In the back solution only two elements of Subdivision 1 need be followed by "blanks". Element $b_{(n-1),1,n}$ requires two "blanks" and element $b_{(n-2),1,(n-1)}$ requires one "blank" (n is the order of the matrix). These "blanks" should have the indices of the card they follow, Program Number 2 and an X punch in card column 74 which will cause them to be primary selected with the rest of the cards of their row instead of being transferred from row to row by the collator.

Each of the check sum programming cards ($b_{n,2,k}$) of the initial row (inverted order) of the back solution must be followed by one "blank" card to insure the lapse of two intervening 412-418 cycles before storage 48 is called out. The check sum programming cards for the back solution have a 2 in card column 58 for their program number since there are no division programming cards in the back solution. Therefore, the "blank" card which follows each $b_{n,2,k}$ check sum programming card will have the indices, n,2,k, in card columns 51 to 55, a 3 in card column 58, and an X punch in card column 74 to cause primary selection in the collator.

CARD FORM

The card form is shown in Figure 1. The principal identification and control punches used by the 412 Accounting Machine are:

- X(9) to cause listing of check sum
- X(20) to cause summary punching and listing during the forward solution
- 9(60) to cause summary punching and listing during the back solution

X(63)	to identify forward summaries, to clear Counter Group 8 during the forward solution, and to add matrix element identification indices to Counter Group 8 during the back solution
X(66)	to identify back summaries and to clear Counter Group 8 during the back solution
X(68)	to add identification indices to Counter Group 8 during the forward solution (Column 68 is also used for special instructions.)
X(70)	to identify unit matrix elements to right of diagonal
X(78)	to clear Counter Group 8 during the forward solution
X(80)	to clear Counter Group 8 during the back solution

The following control punches are for collating purposes only and are not used by the 412.

(4-5)	(43)	back subdivision number
2	(44)	back program number
p	(58)	program number
X	(72)	to cause primary selection of old element cards and their associated programming cards during each second merge of the forward solution
X	(74)	to cause primary selection of old element cards and their associated programming cards during each second merge of the back solution.

FORWARD SOLUTION CARDS

Original Elements of Augmented Matrix (Program 1 Cards)

Card Column

1	X	(in all Program 1 cards)
3	X	(in all Program 1 cards)

<u>Card Column</u> (Continued)

5	9	(in all Program 1 cards)
6	0	(in all Program 1 cards except those noted below*)
7, 8	column number	(in all Program 1 cards on and to left of principal diagonal in Subdivision 1)
9	1	(in all Program 1 cards)
11, 18, 19	1, 5, 0	(in all Program 1 cards)
21 ↓ 29	element value (in all Program 1 cards) for Subdivision 2, unit matrix diagonal, this is 1 in 21, 5 in 28, and 0 in 29	
51, 52	row number	(in all Program 1 cards)
53	subdivision number	(in all Program 1 cards)
54, 55	column number	(in all Program 1 cards)
58	1	(in all Program 1 cards)
72	X	(in all Program 1 cards)

Subdivisions 1 and 3 of the Program 1 cards will serve as a master instruction deck if Field 21 to 29 is left blank. Subdivision 2, of the Program 1 cards, belongs to the permanent forward deck.

* Card column 6 is left blank for the following Program 1 cards:

$a_{11,s,k}$	(row 1 of all subdivisions)
$a_{i,1,11}$	(column 1 of Subdivision 1)
$a_{i,2,i}$	(diagonal elements of unit matrix)

Divide Programming Cards (Program 2 Cards)

Card Column		
1, 2	9, 0	(in all Program 2 cards)
3, 4	row number	(in all Program 2 cards)
9	2	(in all Program 2 cards)
20	X	(in all Program 2 cards)
51, 52	row number	(in all Program 2 cards)
53	subdivision number	(in all Program 2 cards)
54, 55	column number	(in all Program 2 cards)
58	2	(in all Program 2 cards)
68	X	(in all Program 2 cards)*
72	X	(in all Program 2 cards)
78	X	(in all Program 2 cards)

* The following special instruction punches are needed in addition to the X punch in card column 68 of all Program 2 cards:

2 in all $a_{11,s,k}$ (row 1 of all subdivisions)
2 in all $a_{i,2,i}$, for $i < n$ (diagonal elements of unit matrix except $a_{n,2,n}$)
3 in all $a_{n,s,k}$ (row n of Subdivisions 2 and 3)

Check Sum Programming Cards (Program 3 Cards)

Card Column		
1, 2	4, 8	(in all Program 3 cards except $a_{11,1,12}$)
5, 6	9, 0	(in all Program 3 cards)
7, 8	4, 8	(in all Program 3 cards)
9	X	(in all $a_{i,3,k}$ Program 3 cards -- to list check sum)
51, 52	row number	(in all Program 3 cards)
53	subdivision number	(in all Program 3 cards)
54, 55	column number	(in all Program 3 cards)
58	3	(in all Program 3 cards)
72	X	(in all Program 3 cards)

Summary Punched Cards

Card Column		
1, 2	row number with X overpunched in card column 2	(in all S.P. cards)
3	X	(in all S.P. cards)
5	9	(in all S.P. cards)
6	0	(in all S.P. cards except $b_{11,s,k}$ and $b_{i,2,i}$ for $i < n$)
9	1	(in all S.P. cards)
11, 18, 19	1, 5, 0	(in all S.P. cards)
21 ↓ 29	$b_{i,s,k}$	resultant elements from the forward solution

Card Column (Continued)

41	X	(in all S.P. cards)
43	(4-s)	(in all S.P. cards)
44	2	(in all S.P. cards)
45	9	(in all S.P. cards)
46	0	(in all S.P. cards except $b_{1,1,k}$ and $b_{n,s,k}$)
47, 48	column number	(in S.P. cards of Subdivision 1)
49	1	(in all S.P. cards)
51, 52	row number	(in all S.P. cards)
53	subdivision number	(in all S.P. cards)
54, 55	column number	(in all S.P. cards)
58	1	(in all S.P. cards)
60	9	(in all S.P. cards except Subdivision 1)
63	X	(in all S.P. cards)
68	2	(in all S.P. cards of row n)
74	X	(in all S.P. cards)
80	X	(in all S.P. cards)

BACK SOLUTION CARDS

Zero Elements to Right of Diagonal in Unit Matrix
(Program 1 Cards)

Card Column

3	X	(in all unit matrix cards to right of unit matrix diagonal)
11	1	(in all unit matrix cards to right of unit matrix diagonal)
18	5	(in all unit matrix cards to right of unit matrix diagonal)
19	0	(in all unit matrix cards to right of unit matrix diagonal)
21 ↓ 29	blank	(in all unit matrix cards to right of unit matrix diagonal)
41	X	(in all unit matrix cards to right of unit matrix diagonal)

Card Column (Continued)

43	2	(in all unit matrix cards to right of unit matrix diagonal)
44	2	
45	9	
46	0	
49	1	
51, 52	row number	
53	2	
54, 55	column number	
58	1	
60	9	
63	X	
70	X	
74	X	
80	X	

These unit matrix zero elements to right of diagonal are part of the back permanent deck and consequently are never introduced during the forward solution.

Check Sum Programming Cards (Program 2 Cards)

Card Column

41	4	(in all check sum programming cards except $b_{n,2,n}$)
42	8	
43	(4-s)	(in all Program 2 cards)
44	2	
45	9	
46	0	
47	4	
48	8	
49	X	(in all $b_{i,3,j}$ Program 2 cards -- to list check sum)

Card Column (Continued)

Card Column	Content	
51, 52	row number	(in all Program 2 cards)
53	subdivision number	(in all Program 2 cards)
54, 55	column number	(in all Program 2 cards)
58	2	(in all Program 2 cards)
74	X	(in all Program 2 cards)

Summary Punched Cards

Card Column	Content	
3	X	(in all S.P. cards)
21 ↓ 29	$c_{i,s,k}$ (elements of inverse matrix)	(in all S.P. cards)
41, 42	row number with X overpunched in card column 42	(in all S.P. cards)
43	(4-s)	(in all S.P. cards)
44	2	(in all S.P. cards)
45	9	(in all S.P. cards)
46	0	(in all S.P. cards except those of row n)
49	1	(in all S.P. cards)
51, 52	row number	(in all S.P. cards)
53	subdivision number	(in all S.P. cards)
54, 55	column number	(in all S.P. cards)
66	X	(in all S.P. cards)

COLLATING

The collator panel wiring for the forward and back solutions is shown in Figures 2 and 3, respectively.

Forward Solution

The forward deck consists of Program 1, 2, and 3 cards. Of these, the Program 1 cards in Subdivisions 1 and 3 constitute the original matrix and its check sums. The remaining cards are the Program 1 cards in Subdivision 2 (diagonal elements of the unit matrix) and all Program 2 and 3 cards. For a given n, these remaining cards constitute a permanent forward programming deck. A duplicate is made of the master instruction deck. This dummy deck is combined with the permanent forward deck, and the combined deck is sorted, in order, on card columns 58, 55, 54, 53, 52, and 51. This establishes the order of the forward deck and the permanent forward deck is never sorted again. Thereafter, to prepare any matrix (of this given order, n) for the forward solution, a sort is made on card columns 55, 54, 53, 52, and 51 of only the matrix element cards and their check sums. This ordered deck is then merged with the permanent forward deck on the collator, replacing the dummy deck which is primary selected. The collator panel wiring for this procedure is shown in Figure 4.

After the nth row of the forward solution has been processed in the 412, there being no further $a_{i,k}$ rows to merge with the nth row summary punched cards ($b_{n,k}$), one proceeds directly to the second merger by placing the $b_{n,k}$ cards in the secondary feed and the deck from the 412 in the primary feed. The nth row $a_{n,k}$ cards will all be primary selected. The first, second and fourth cards in the merged hopper are the three "blank" cards of the permanent forward deck and should be restored by hand to their proper positions in the permanent forward deck. The remainder of the cards in the merged hopper comprise the complete set of summary punched cards, $b_{i,k}$, from the forward solution.

Back Solution

The back deck consists of the summary punched cards from the forward solution ($b_{i,k}$), the zero elements to the right of the diagonal in the unit matrix ($0_{i,2,k}$ for $i < k$), the back check sum programming cards (Program 2), the three "blank" cards in Subdivision 1 (Program 2), and the n "blank" cards in Subdivision 2 (Program 3). All cards of the back deck, except the $b_{i,k}$ summary punches, constitute a permanent back programming deck. The $b_{i,k}$ cards are combined with this permanent back deck and the combined deck is sorted, in order, on card columns 58, <u>55</u>, <u>54</u>, 53, <u>52</u>, and <u>51</u>. The cards are picked up in inverse order after the passes on the card columns which are underlined. This establishes the order of the back deck and the permanent back deck is never sorted again. After the back solution has been completed the $b_{i,k}$ cards are left in the permanent back deck to serve as a dummy deck which will be replaced when the $b_{i,k}$ cards from the next matrix to be inverted are merged with the permanent back deck. Thus, in all succeeding problems, when the forward solution has been completed, a sort is made on card columns 55, 54, 53, 52, and 51 of only the $b_{i,k}$ cards. The cards are picked up in inverse order after the passes on the card columns which are underlined. The inverted order $b_{i,k}$ deck is then merged with the permanent back deck, replacing the $b_{i,k}$ deck from the previous problem. These culled $b_{i,k}$ cards from the previous problem may then be discarded. The collator panel wiring for this procedure is shown in Figure 4.

The follow cards, described below, may be left on the deck from the 412, which goes into the primary feed hopper. They will reappear as the follow cards of the deck in the merged hopper. A 4 is punched in card column 53 of the follow cards for the forward solution to preclude the existence of a low second primary condition. A low second primary condition would cause an error stop and necessitate pushing the error reset button in order to complete the runout.

412 ACCOUNTING MACHINE

Appropriate instructions for the forward and back solutions are summary punched automatically. The instructions, Factor A, Factor B, Factor D, Factor C, and Oper. apply to the forward solution and the instructions Factor A*, Factor B, Factor D*, Factor C*, and Oper.* apply to the back solution. The instructions to be effective are controlled through Co-selectors 61, 62, and 63 and Pilot Selectors 19, 31, and 37 which are transferred by Setup Switch 1. This switch is turned "on" during the back solution.

Summary punching during the forward solution is obtained by an X in card column 20 of the divide programming cards. Summary punching during the back solution is caused by a 9 in card column 60 of the $b_{i,3,11}$ cards, of the $b_{i,2,k}$ cards for $i \geq k$, and of the $O_{i,2,k}$ cards for $i < k$. The X (20) or 9 (60) is read at third reading and is used to pick up Pilot Selector 41 or 42, respectively. The coupling exit pulse of Pilot Selector 41 or 42 is then taken by way of Co-selector 63 and the AC points of Panel Switch 1 to the summary punch control pickup. Panel Switch 1 must be in the AC position when summary punching is desired. The coupling exit pulse is used because X and digit pulses are not accepted by the summary punch control pickup.

Listing of all cards which are summary punched during matrix inversion is obtained by split wiring the X (20) or 9 (60) to the 12-9 pickup of Pilot Selector 72 by way of

Co-selectors 63 and the transferred points of Co-selector 71. Co-selector 71 insures that automatic listing whenever summary punching is called for does not obtain except during matrix inversion. During matrix inversion, listing on the presence of summary punch instructions occurs independently of the position of Panel Switch 1.

Listing of the accumulated check sum at the end of each row is caused by an X in the operation column of the Subdivision 3 check sum programming card. This X is taken by way of Co-selector 61 and a column split to the 12-11 pickup of Pilot Selector 72.

Listing of all cards is obtained by turning Setup Switch 2 "on", which impulses the immediate pickup of Pilot Selector 62. An all cycles pulse is taken through the transferred points of Pilot Selector 62 to the list hub and to the immediate pickup of those pilot selectors (1, 5, 6, 7, 13) and co-selectors (1, 2, 7, 8, 13, 14, 59) which, when transferred, permit printing of all factors and instructions. The instructions for Factors A, B, D, and C and the factors, A, B, and D are sent to print entry at third read time and are all printed on the same line. The result, Factor C, is not available until one cycle later and hence is printed one line below the components from which it is formed.

Identification (row number, subdivision number, column number) is added to Counter Group 8 with an X in card column 68 on the forward solution and an X in 63 on the back solution. Counter Group 8 is cleared on X63 or X78 on the forward solution and X66 or X80 on the back solution. For the forward solution the X in 68 is read at second read and the X78 at third read. The X68 and X78 are punched in the divide programming cards only. As the divide card passes the second reading station, the X68 causes the indices of the divide card to enter Counter Group 8. As the divide card passes the third reading station, the X78 causes Counter Group 8 to read out and reset and causes the contents of Counter Group 8 to be available for summary punching at the 412 counter exits on the 527 summary punch panel. Thus, since summary punching during the forward solution is initiated by an X in 20 in the divide card as it passes the third reading station, it is actually the indices of the divide card that are summary punched in the $b_{i,k}$ cards. This method of identification is feasible because an element card, $a_{i,s,k}$, and its associated divide and check sum programming cards all have the same indices in 51 through 55. This method of identification makes possible the elimination of the left of diagonal $0_{i,2,k}$ element cards. Their arithmetic function of adding zero to "previous result" is trivial and their function of providing identification for the left of diagonal $b_{i,2,k}$ cards is assumed by the associated divide cards.

An X78 is punched in the element card, $a_{11,1,11}$, to insure that Counter Group 8 is clear at the beginning of the forward solution.

For the back solution, the X63 and X80 perform the same functions as do the X68 and X78, respectively, on the forward solution. The X63 and X80 come from the element cards of the back solution since there are no back divide programming cards.

In the Heising Setup, counter groups are cleared automatically just before being read into. Therefore, it is not necessary to clear counters with a minor control break at the end of each row.

In order for all instructions of the last program card of each row to be carried out properly, the last card of each row must be followed by two blank follow cards. If an X and 3 are punched in the operation column (card column 9 for forward solution, 49 for back) of these follow cards they will cause the machine to space automatically at the end of each row. Four such follow cards give a convenient spacing between successive rows on the print-out. A 4 is punched in card column 53 of the forward follow cards for collating purposes. See COLLATING.

When the 412 panel is used for matrix inversion, Counter Group 8 is used to transfer identification indices from cards in the 412 to corresponding cards in the summary punch. When the 412 panel is used for operations other than matrix inversion, Counter Group 8 operates as a tabulator storage. This is accomplished by taking the wiring for the former case through the transferred points of Co-selectors 19, 20, 69, 70, and 71 and by taking the wiring for the latter case through the normal points of these same selectors. These selectors are transferred by setting Panel Switch 2 in the AC position. Also the pulse which causes listing on the presence of summary punch instructions is taken through the transferred points of Co-selector 71.* Panel Switch 2 must be in the AC position during the entire process of matrix inversion. It must be in the BC position for all operations other than matrix inversion.

The wiring for matrix inversion which must be superimposed on the Heising 412 Panel is shown in Figures 5A and 5B.

USE OF 412 SELECTORS TO CONTROL SELECTORS ON THE 527

On Page 80 of the 412-418 Preliminary Manual of Information[4], under the heading, "Wiring Between Control Panels", there are three paragraphs treating the limitations

* See 412 ACCOUNTING MACHINE, paragraph 3.

on control of 527 selectors from the 412 panel. The limitation outlined in the second of these paragraphs may be circumvented by a means other than the use of latch selectors, which may be unavailable. The drop-out of 412 selectors during idle cycles frequently occurs at some point between 12 and 9 time on the 527. If any single digit impulse originating on the 527, e.g., 12 or 11 or 6, is taken to the pickup of a selector on the 527 by way of the transferred points of a 412 pilot selector or co-selector, this pulse will be blocked by the 412 selector on those occasions when the dropout time of the 412 selector coincides with, or strongly overlaps, the 527 digit pulse. However, the drop-out time during a 412 idle cycle is only ten degrees out of each cycle (225° to 235°) or 11.1 milliseconds out of each 400 millisecond cycle. The duration of each digit impulse on the 527 is 3/140 of each 600 millisecond cycle or 12.9 milliseconds. The duration of each interval between adjacent digit impulses on the 527 is 7/140 of each 600 millisecond cycle or 30 milliseconds. If a double digit impulse consisting of two adjacent digit impulses is used as the originating 527 impulse, the 11.1 millisecond drop-out of the 412 selector can block one but not both of these impulses. Therefore, a 12-11 double pulse on the 527 may be used under the control of a 412 selector to pick up pilot selectors or punch selectors on the 527 provided that only digits 0 to 9 are taken through the points of the 527 selector so controlled.

If an 11 pulse must be taken through the points of a 527 selector which is controlled by a 412 selector, both impulses of the double pulse must occur prior to 11 time. The half time impulses are of the same duration (12.9 milliseconds) as the digit impulses and occur midway between digit impulses. The 12-1/2 pulse commences 8.6 milliseconds after the 12 pulse ends and 8.6 milliseconds before the 11 pulse begins. A combined 12-12-1/2 pulse should span the drop-out gap sufficiently well that one or the other of these pulses will be able to cause the transfer of a 527 selector prior to 11 time.

Alternatively, the pickup of an otherwise unused 527 pilot selector may be impulsed from card cycles and the coupling exit pulse of this selector may be used as the originating impulse on the 527. This coupling exit pulse is over 80 milliseconds long and commences 42.9 milliseconds prior to 12 time. It may be used to cause the transfer of a 527 selector prior to both 12-1/2 time and 11 time.

The germane timing features of the 527 are illustrated in Figure 7.

The wiring for the pickup of Punch Selectors 5, 6, 7, and 8 under the control of 412 Setup Change Switch 1 is shown in Figures 5B and 6 and is convenient if a latch selector is available.

SPECIAL INSTRUCTIONS

The special instructions in card column 68 are taken from third read via a digit selector to the 12-9 pickup of pilot selectors on the 412. A 12-11 punch originating on the 527 may be taken through the transferred points of these 412 selectors to the immediate pickups of 527 pilot and punch selectors. Thus, selected punching on the 527 may be controlled by program cards in the 412. Communication between the 527 and 412 is through the 12-11 hubs of the column splits on the 527 which are common with the summary punch control entry hubs on the 412.

There is no Special Instruction 1 since no provision is made for summary punching $b_{i,k}$ elements on and to the left of the principal diagonal.

Special Instruction 2 causes 527 Pilot Selector 4 to be transferred. This intercepts the zero pulse which would otherwise be summary punched in card column 6 (forward solution) or 46 (back solution) which means that Factor D will be taken as zero instead of as "previous result". The end result of Special Instruction 2 is to cause the instruction code in the summary punched card to be -A x B instead of -A x B + D.

Special Instruction 3 causes 527 Pilot Selector 5 to be transferred. It is used only in the divide programming cards of the last row of the forward solution in order that the transfer code (-A x B) may be summary punched in the back instruction field and in order that a 2 may be summary punched in the special instruction column of the corresponding $b_{n,s,k}$ cards.

527 SUMMARY PUNCH

The 527 will punch instructions in the following manner:

A. DURING THE FORWARD SOLUTION

1. Factor A (card columns 1,2)

   a) punch row number
   b) overpunch X in units position

2. Factor B (3,4)

   a) punch X in tens position

3. Factor D (5,6)

   a) punch 9 in tens position
   b) punch 0 in units position, if not Special Instruction 2

4. Operate (9)

   a) punch 1

5. Factor A* (41,42)

   a) punch X in tens position

6. Factor D* (45,46)

   a) punch 9 in tens position
   b) punch 0, if not Subdivision 1 and/or if not Special Instruction 3

7. Factor C* (47,48)

   a) punch column number, if Subdivision 1

8. Operate* (49)

   a) punch 1

9. Initiate Summary Punching (60)

   a) punch 9, if not Subdivision 1

10. Special Instructions (68)

   a) punch 2, if Special Instruction 3

B. DURING THE BACK SOLUTION

1. Factor A* (41,42)

   a) punch row number
   b) overpunch X in units position

2. Factor B (3,4)

   a) punch X in tens position

3. Factor D* (45,46)

   a) punch 9 in tens position
   b) punch 0 in units position, if not Special Instruction 2. (This Special Instruction 2 was summary punched in the $b_{n,s,k}$ cards during the processing of the final row of the forward solution. See A 10-a)

4. Operate* (49)

   a) punch 1

The wiring diagram for the 527 is shown in Figure 6.

PRINT-OUT

When Setup Change Switch No. 2 is "on", causing listing on all cycles, the type bars will list information as follows:

Alphamerical Type Bars

1. Arbitrarily chosen digit to enable zeros to print from type bars 7 and 8 when the code 00 is used for Factor A.

3. Operation Code.

6. R if Factor D is read from card.

7-8. Factor A address.

9. Minus symbol if X in card column 2 (sign reversal code).

11-17. Mantissa of Factor A.

18. 9 if Factor A is negative as it enters the 605.

20-21. Exponent of Factor A.

23. R if Factor D is read from card.

24-25. Factor B address.

26. A if X in card column 4 (absolute value code).

28-34. Mantissa of Factor B.

35. 9 if Factor B is negative as it enters the 605.

37-38. Exponent of Factor B.

40. R if Factor D is read from card.

41-42. Factor D address.

43. Minus symbol if X in card column 6 (sign reversal code).

Numerical Type Bars

3-9. Mantissa of Factor D.

10. Minus symbol if Factor D is negative as it enters the 605.

13-14. Exponent of Factor D.

23-24. Factor C instruction.

27. Arbitrarily chosen digit to enable minus symbol to print from Numerical Type Bar No. 28.

28. Minus symbol if Factor C is negative.

29-35. Mantissa of Factor C.

37-38. Exponent of Factor C.

Listing of minus symbols will occur as shown above provided that the ampersand has been replaced by a minus symbol on odd alphamerical type bars and that the CR symbol has been replaced by a minus symbol on even numerical type bars.

COUNTER GROUP SIGN CONTROL

The Heising 412 Panel employs negative balance test exits to provide sign control of factors which are called from counter groups. These exits do not emit impulses during idle cycles. If a negative factor is called from a counter group immediately after a summary punch operation, it will lose its sign unless special provision is made to retain sign control during idle cycles. Wiring which will retain sign control during idle cycles is illustrated in Figure 8 and is to be superimposed on the Heising 412 Panel.

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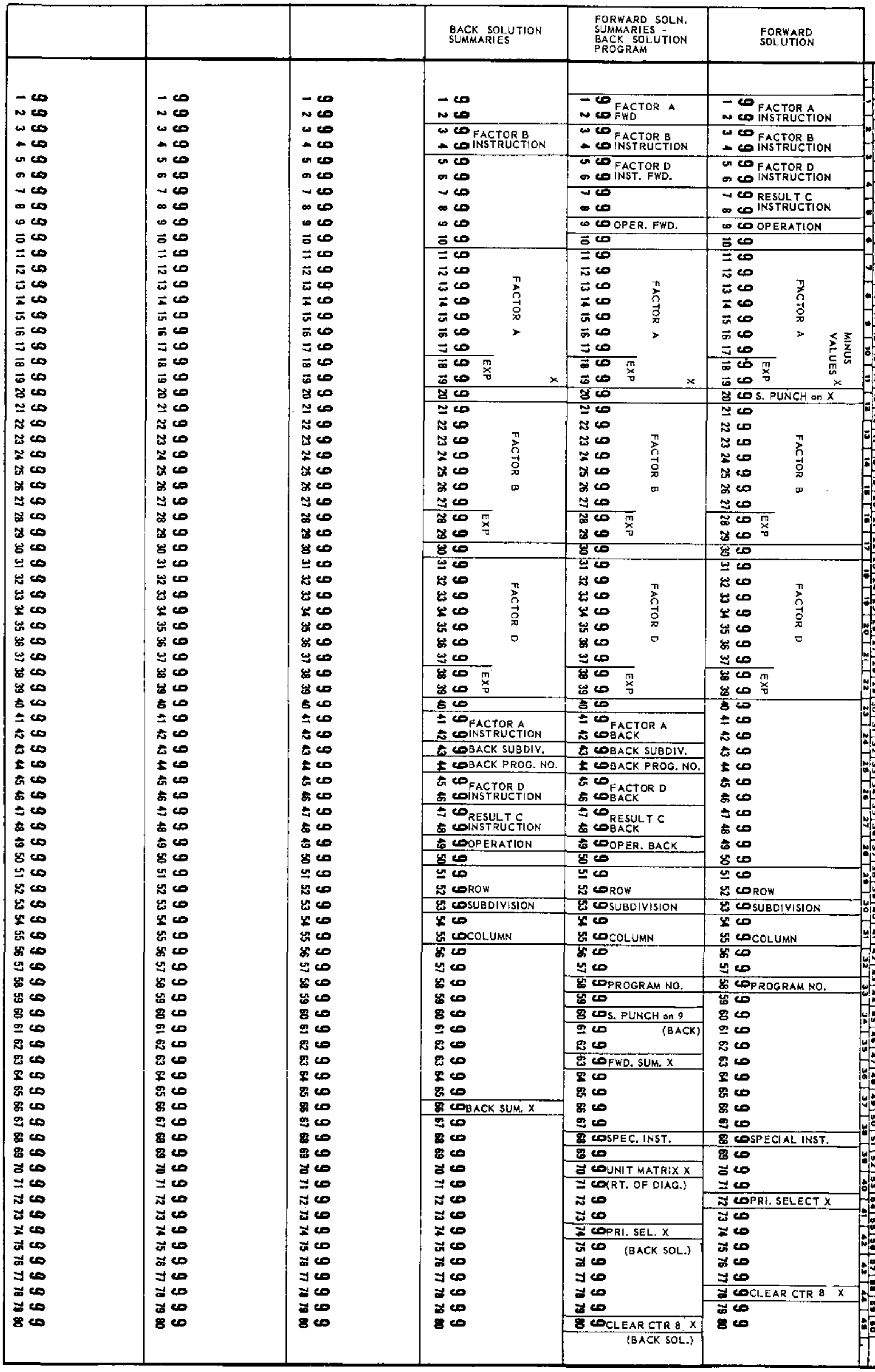
2. Chancellor, J., Sheldon, J. W., and Tatum, G. L. The Solution of Simultaneous Linear Equations Using the IBM Card-Programmed Electronic Calculator. Proceedings, Industrial Computation Seminar, p. 57, IBM Corporation. (September 1950).

3. Heising, William P. Floating-Decimal Panels for Model II CPC. Washington Technical Computing Bureau. IBM Corporation.

4. Preliminary Manual of Information, IBM Card-Programmed Electronic Calculator Using Machine Types 412-418, 605, 527, 941. IBM Corporation. Form 22-8686-2. (November 1952).

FIGURE 1



CARD FORM

FIGURE 2

IBM INTERNATIONAL BUSINESS MACHINES CORPORATION
COLLATOR, TYPE 77 CONTROL PANEL

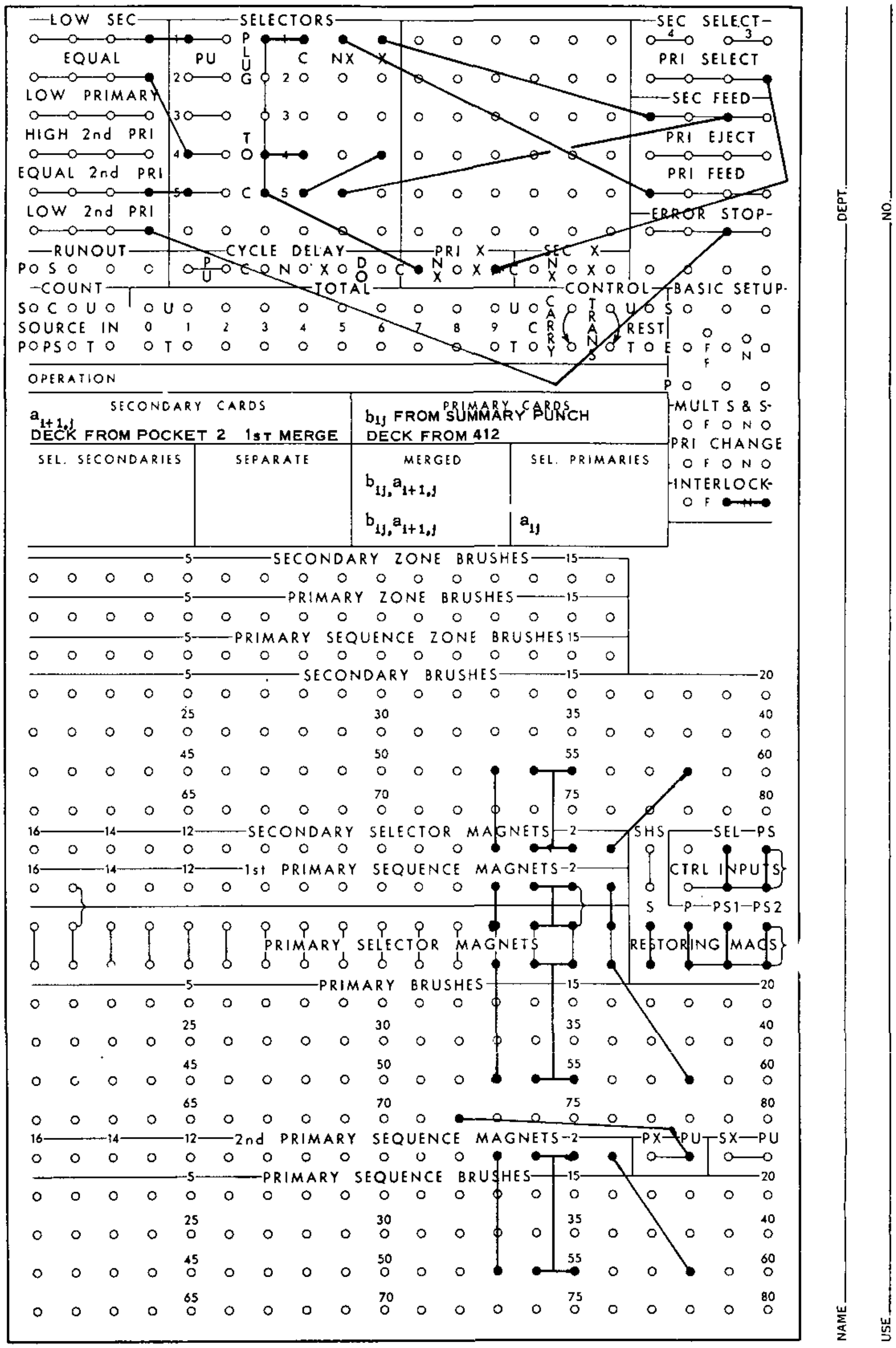
Form 22-9133-9
Printed in U. S. A.

FORWARD SOLUTION

1st MERGE
2nd

1st

2nd

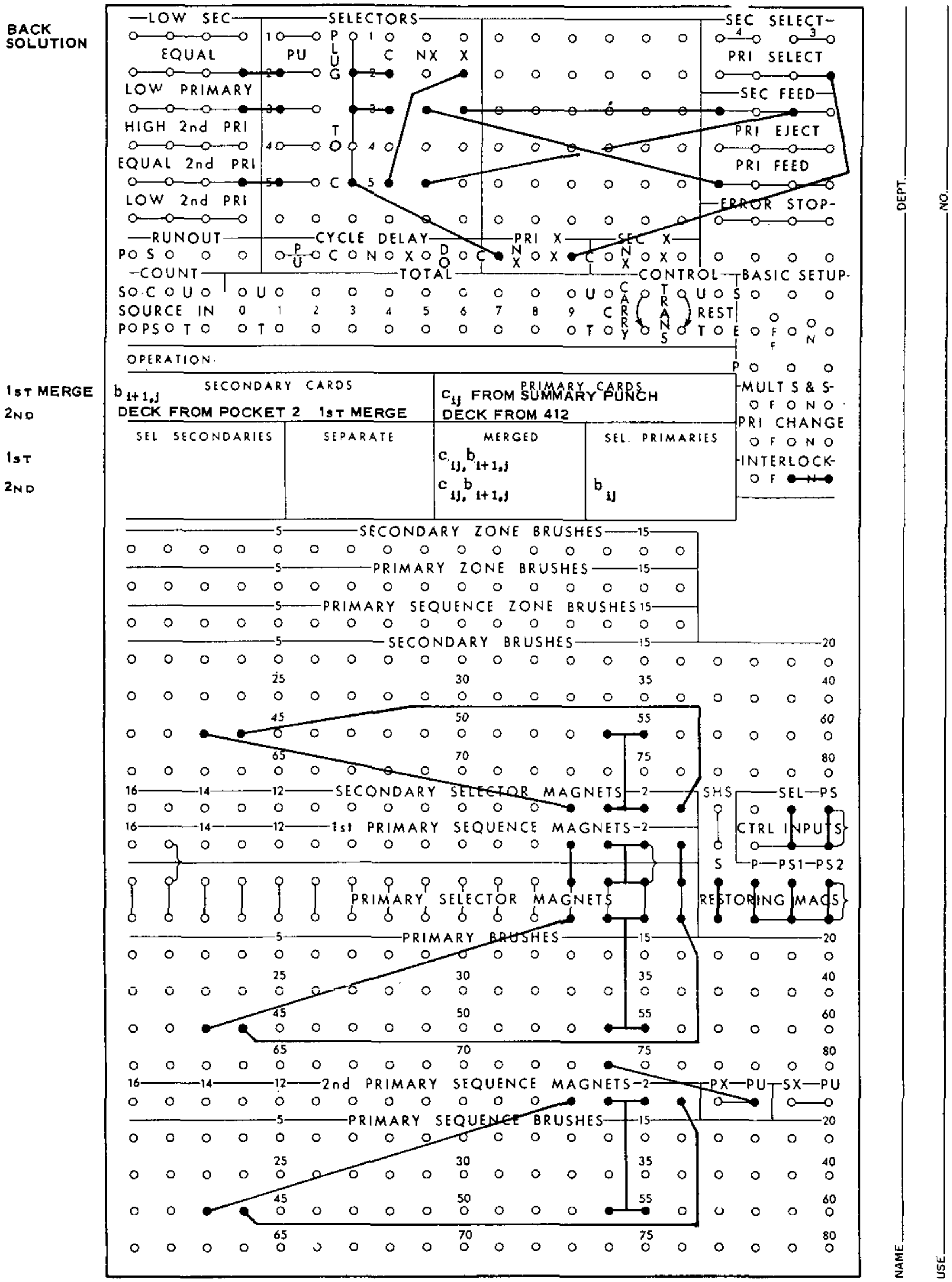


FORWARD SOLUTION COLLATOR PANEL

IBM INTERNATIONAL BUSINESS MACHINES CORPORATION

COLLATOR, TYPE 77 CONTROL PANEL

Form 22-9133-9
Printed in U. S. A.



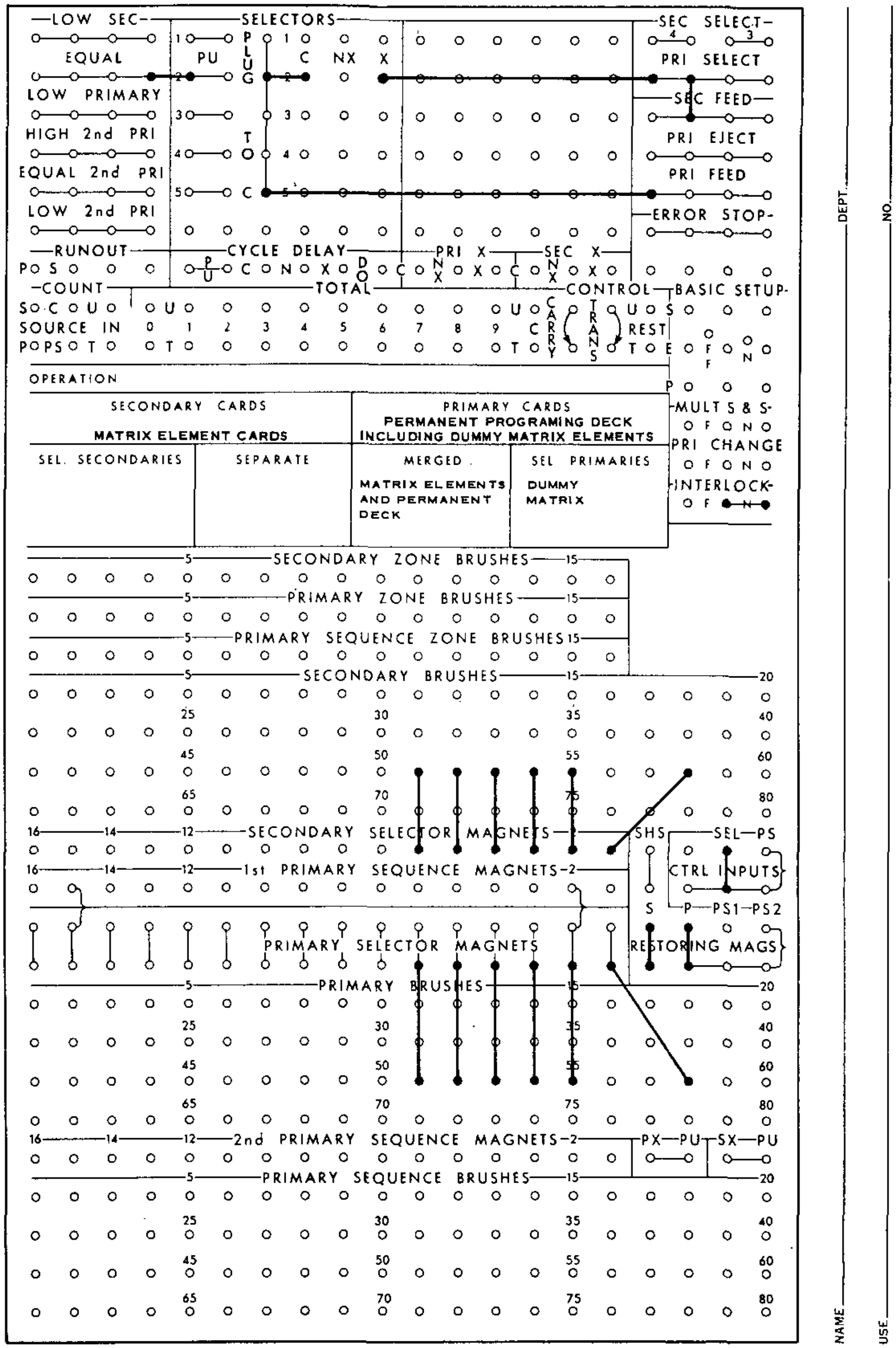
BACK SOLUTION COLLATOR PANEL

FIGURE 4

IBM INTERNATIONAL BUSINESS MACHINES CORPORATION
COLLATOR, TYPE 77 CONTROL PANEL

Form 22-9133-9
Printed in U. S. A.

INTER-
SOLUTION

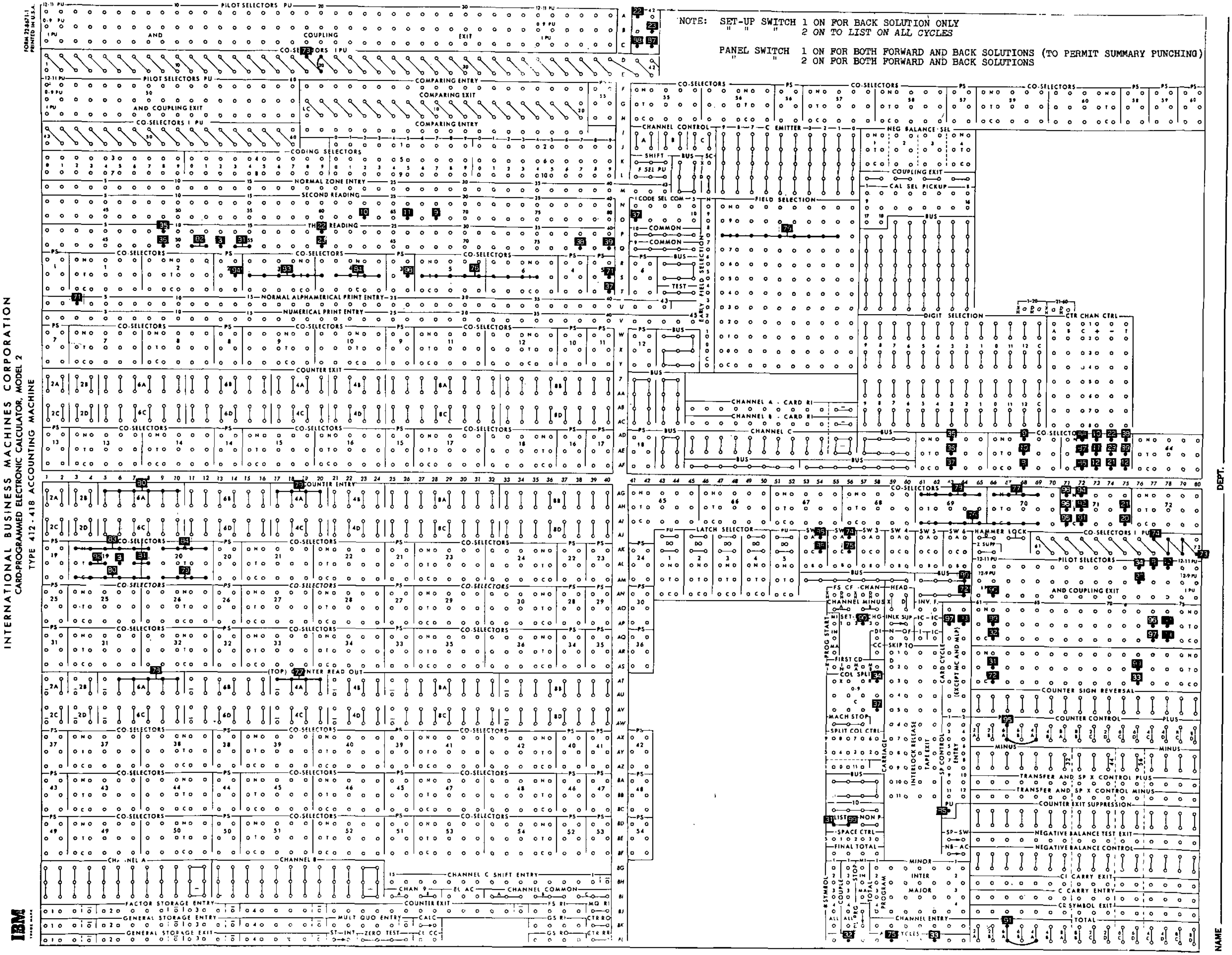


INTER-SOLUTION COLLATOR PANEL

FIGURE 5A

NOTE: SET-UP SWITCH 1 ON FOR BACK SOLUTION ONLY
" " " 2 ON TO LIST ON ALL CYCLES

PANEL SWITCH 1 ON FOR BOTH FORWARD AND BACK SOLUTIONS (TO PERMIT SUMMARY PUNCHING)
" " 2 ON FOR BOTH FORWARD AND BACK SOLUTIONS

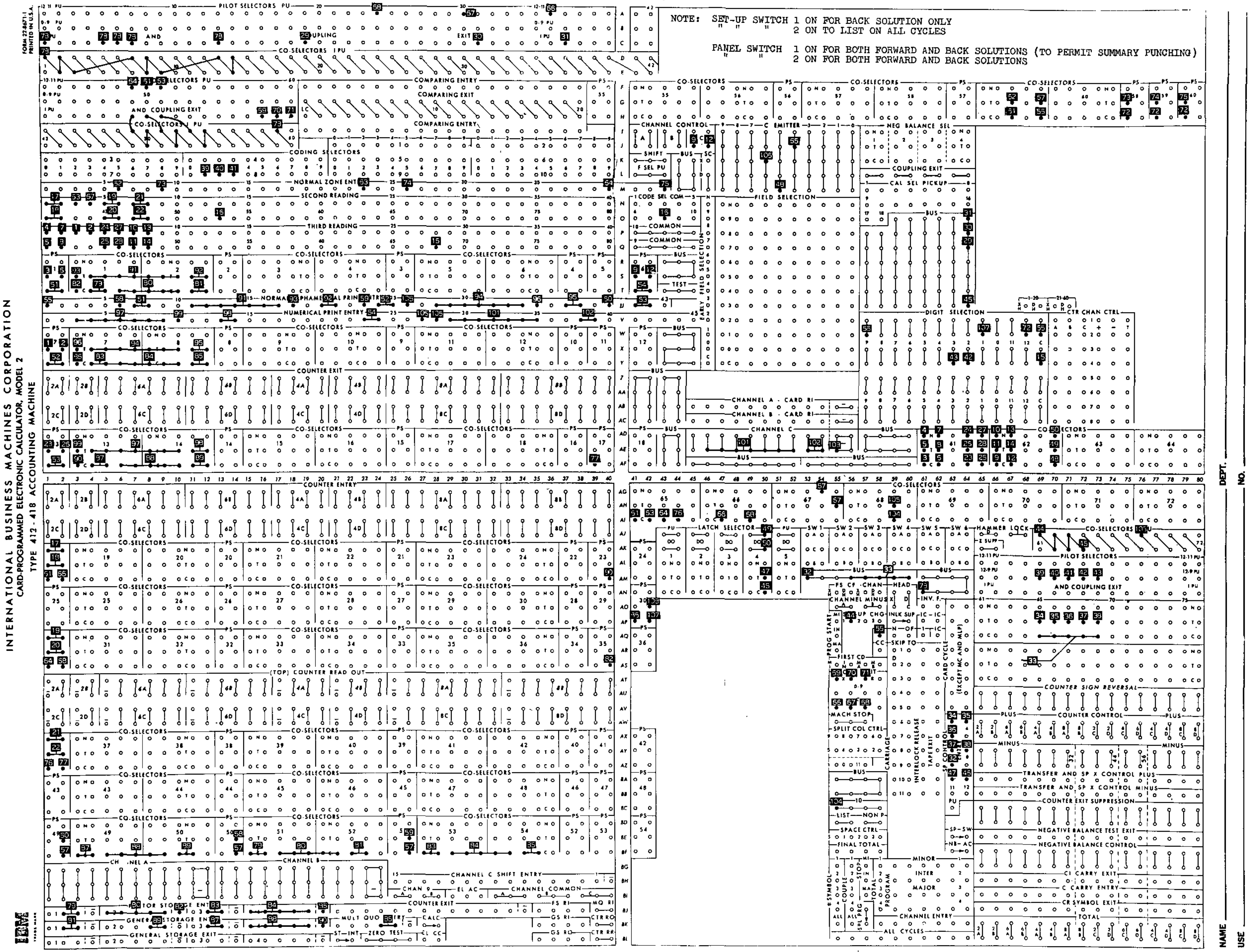


412 ACCOUNTING MACHINE PANEL

FIGURE 5B

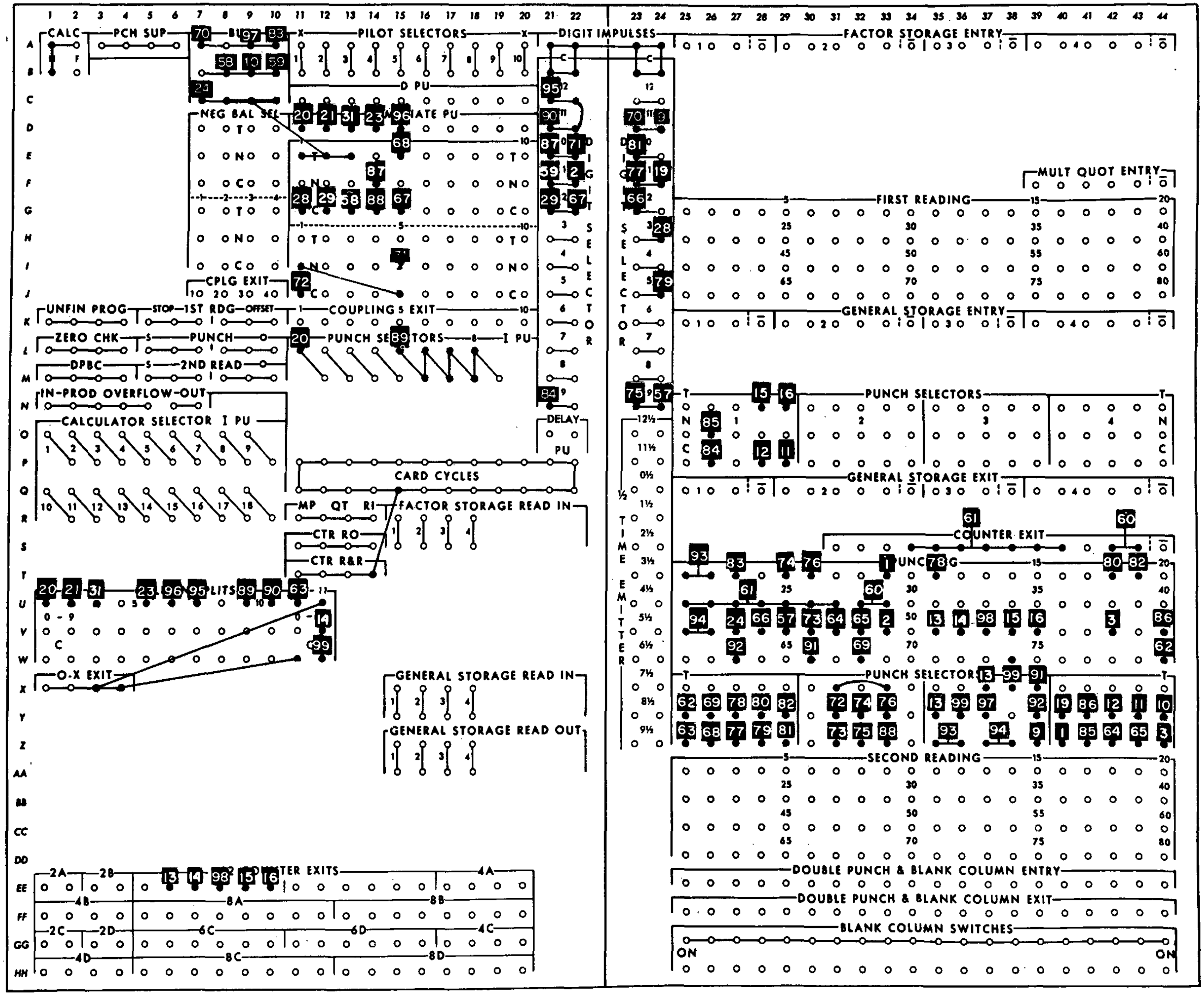
NOTE: SET-UP SWITCH 1 ON FOR BACK SOLUTION ONLY
" " " 2 ON TO LIST ON ALL CYCLES

PANEL SWITCH 1 ON FOR BOTH FORWARD AND BACK SOLUTIONS (TO PERMIT SUMMARY PUNCHING)
" " 2 ON FOR BOTH FORWARD AND BACK SOLUTIONS



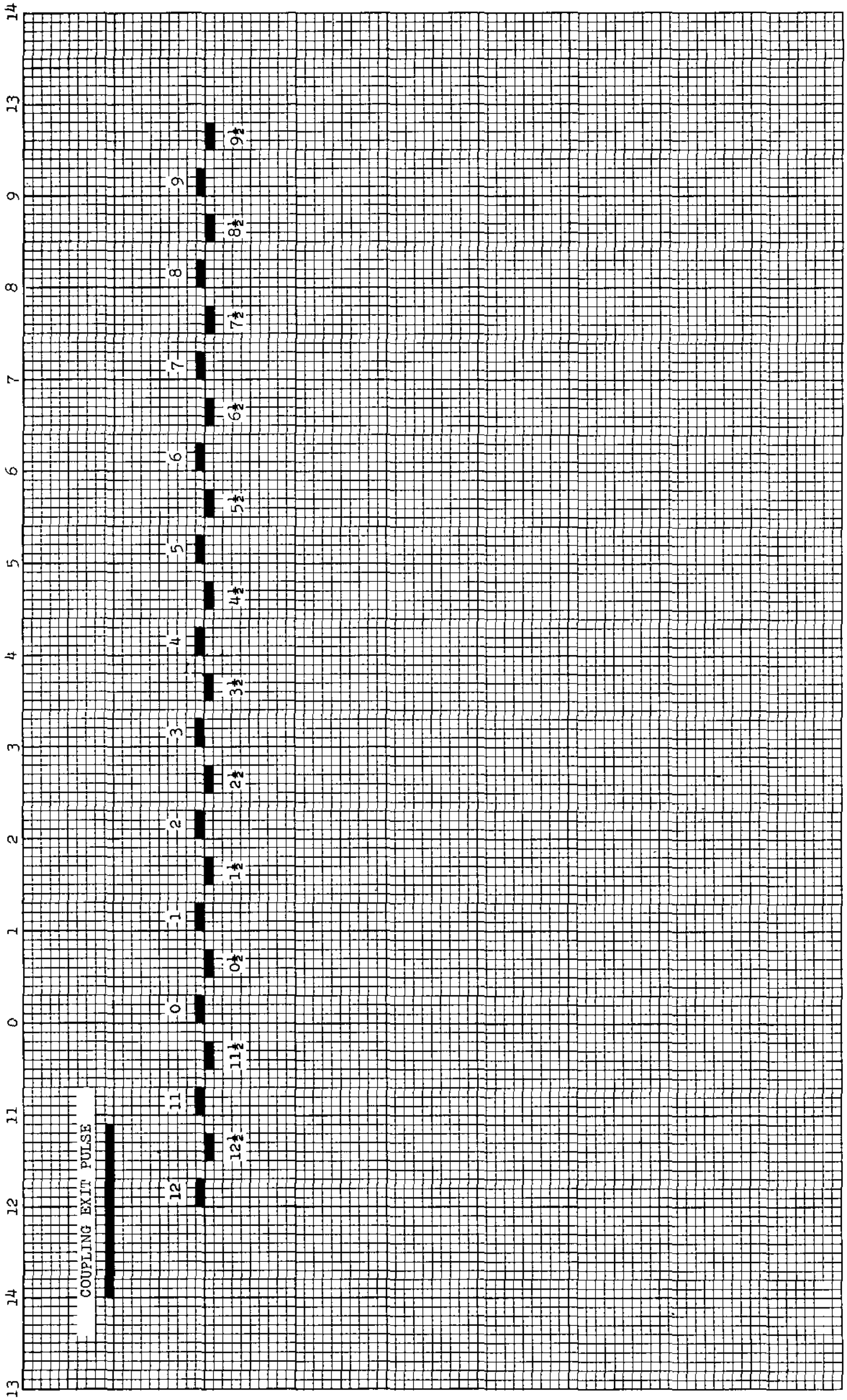
412 ACCOUNTING MACHINE PANEL

FIGURE 6



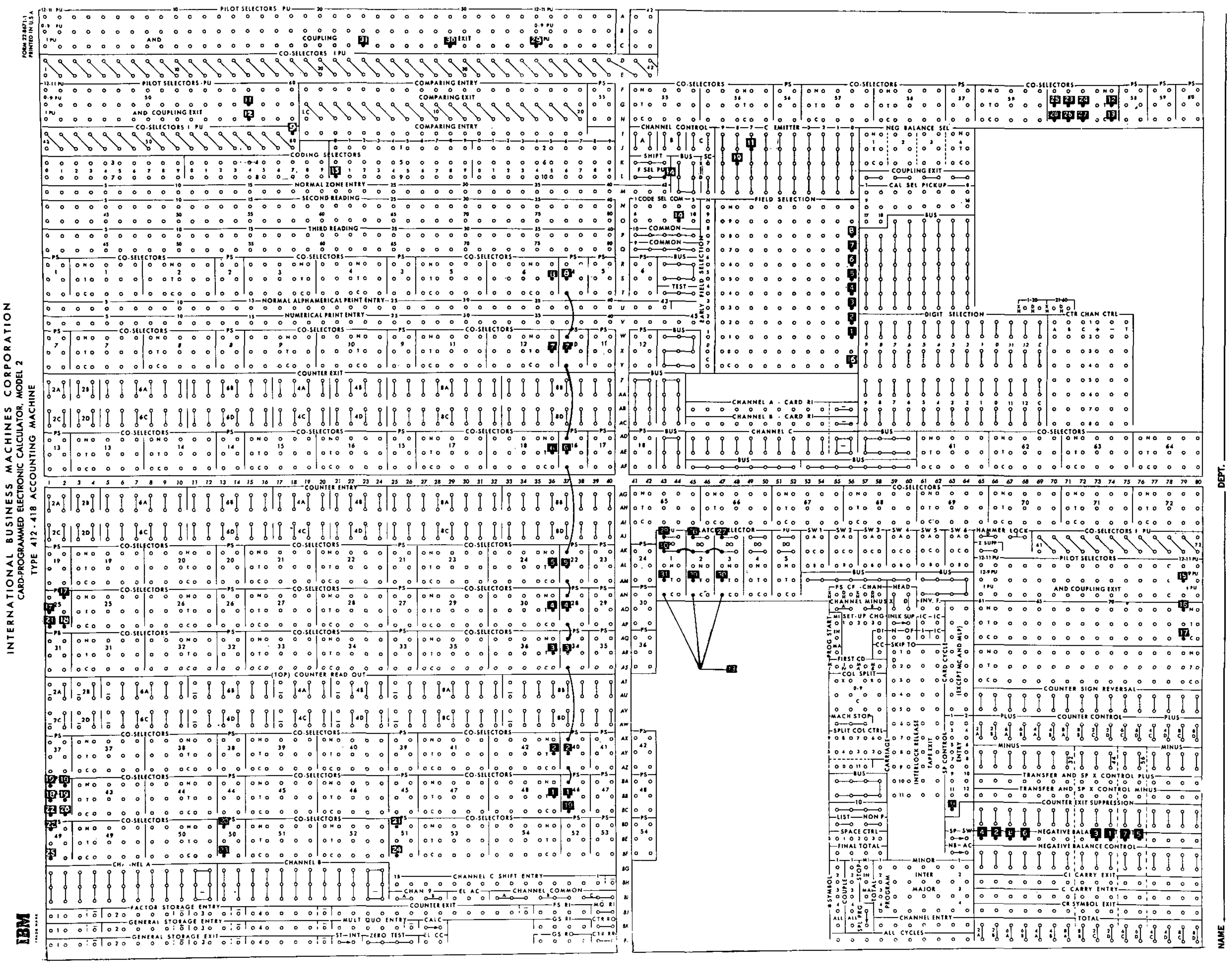
527 SUMMARY PUNCH PANEL

FIGURE 7



605-527 TIMING CHART

FIGURE 8



COUNTER GROUP SIGN CONTROL

APPENDIX A

THE CROUT METHOD

In the Crout Method the original matrix is augmented by a unit matrix and a check column vector to form the original composite matrix, $a_{i,s,k}$, shown in Figure A. Each element of the check column vector is the negative sum of the elements of the corresponding row in Subdivisions 1 and 2. The subscripts i, s, and k refer, respectively, to the row number, subdivision number, and column number.

To each of the elements, $a_{i,s,k}$ a reduction equation is applied to form the reduced elements, $b_{i,s,k}$, of the forward solution.

The equation

$$b_{i,1,k} = a_{i,1,k} - \sum_{j=1}^{(i-1)} b_{i,1,j}\, b_{j,1,k} \qquad (1)$$

is applied to all terms of Subdivision 1 for which $i \geq k$.

The equation

$$b_{i,s,k} = \frac{a_{i,s,k} - \sum_{j=1}^{(i-1)} b_{i,1,j}\, b_{j,s,k}}{b_{i,1,i}} \qquad (2)$$

is applied to all other terms of the composite matrix.

For $s = 2$ and $i > k$; $a_{i,2,k} = 0$ and equation (2) reduces to

$$b_{i,2,k} = \frac{-\sum_{j=1}^{(i-1)} b_{i,1,j}\, b_{j,2,k}}{b_{i,1,i}} \qquad (3)$$

The machines are therefore programmed to use equation (3) instead of equation (2) when $s = 2$ and $i > k$.

For $s = 2$ and $i < k$, equation (2) reduces to $b_{i,2,k} = 0$. Therefore no $b_{i,2,k}$ terms are computed for $i < k$.

The "a" terms are elements of the original composite matrix and the "b" terms are those previously operated upon.

The order of computation of $b_{i,s,k}$ elements is from left to right across each row beginning with the first and continuing across succeeding rows until all n rows have been processed. The resultant matrix of $b_{i,s,k}$ elements is called the forward solution.

The $b_{i,1,k}$ terms for $i > k$ are not retained for the back solution. To the remaining $b_{i,s,k}$ terms are added $O_{i,2,k}$ terms for $i < k$. The order of rows of the composite matrix of these elements is inverted and the order of columns within each subdivision is inverted. This results in the matrix shown in Figure B. The equation for the back solution

$$c_{i,s,k} = b_{i,s,k} - \sum_{j=i+1}^{n} c_{i,1,j} \cdot c_{j,s,k} \qquad (4)$$

is applied to all terms for $s > 1$. The "b" terms are from the forward solution and the "c" terms are those which have been previously operated upon in the back solution. The order of computation of $c_{i,s,k}$ elements is from left to right across each row beginning with row n (top row of Figure B) and proceeding across succeeding rows in order of decreasing n. The resultant matrix of $c_{i,s,k}$ elements is called the back solution. The $c_{i,2,k}$ elements are the elements of the inverse of the original matrix but they will be in inverted order of rows and columns and must be restored to normal order before being used in matrix multiplication.

FIGURE A

ORIGINAL COMPOSITE MATRIX

	ORIGINAL MATRIX	UNIT MATRIX	CHECK COLUMN
Subdivision:	1	2	3
	$a_{11,1,11}$ $a_{11,1,12}$ ··· $a_{11,1,96}$ $a_{11,1,97}$	$1_{11,2,11}$	$a_{11,3,11}$
	$a_{12,1,11}$ $a_{12,1,12}$ ··· $a_{12,1,96}$ $a_{12,1,97}$	$1_{12,2,12}$	$a_{12,3,11}$
	⋮	⋱	
	$a_{96,1,11}$ $a_{96,1,12}$ ··· $a_{96,1,96}$ $a_{96,1,97}$	$1_{96,2,96}$	$a_{96,3,11}$
	$a_{97,1,11}$ $a_{97,1,12}$ ··· $a_{97,1,96}$ $a_{97,1,97}$	$1_{97,2,97}$	$a_{97,3,11}$

FIGURE B

Subdivision:	1	2	3
		$b_{97,2,97}$ $b_{97,2,96}$ ··· $b_{97,2,12}$ $b_{97,2,11}$	$b_{97,3,11}$
	$b_{96,1,97}$	$0_{96,2,97}$ $b_{96,2,96}$ ··· $b_{96,2,12}$ $b_{96,2,11}$	$b_{96,3,11}$
	$b_{95,1,97}$ $b_{95,1,96}$	⋮	
	⋮		
	$b_{12,1,97}$ $b_{12,1,96}$ ··· $b_{12,1,13}$	$0_{12,2,97}$ $0_{12,2,96}$ ··· $b_{12,2,12}$ $b_{12,2,11}$	$b_{12,3,11}$
	$b_{11,1,97}$ $b_{11,1,96}$ ··· $b_{11,1,13}$ $b_{11,1,12}$	$0_{11,2,97}$ $0_{11,2,96}$ ··· $0_{11,2,12}$ $b_{11,2,11}$	$b_{11,3,11}$

APPENDIX B

HEISING BOARD CODING

The following partial description of Heising Board Operations is quoted directly from the original Heising article(3).

"Three operands (each having a two-digit instruction code) A, B, D, are sent to the 605, and a result, C, is returned to storage. The 412 counters are coupled to form eight ten-digit storage units (seven significant digits, two digits for exponent representation, and the high order position holds 0 for positive numbers, 9 for negative numbers). Numbers in the 412 storage units are always in true form, so that conversion is never used. In addition, sign reversal may be accomplished on the 412 panel as a number is transferred to the 605, so that the 605 wiring does not have any subtraction operation.

"Since floating-decimal operations do not permit addition in the 412 tabulator "counters", these counters have been wired to act like 941 storage units -- there is only top counter read-out (progressive), and they automatically clear just before being read into. The eight tabulator storage units have been assigned the codes 91, 92, ... 98 for read-in and read-out instructions.

"We next arbitrarily require that there must be one or two nines in the tens positions of the three operand instructions. This coding restriction does not limit the flexibility but only reflects the fact that all three operands cannot come from the 941 or the counter storage.

"To use a previous result as an operation, the code 00 is used. The code 90 may be used for previous result, but should only be used if the code 00 would result in there being no nines in the A, B, and D tens position instruction codes. An X punch in the tens position of the instruction will cause that factor to be read from the card regardless of whatever else may be punched in that particular instruction. Multiple punching X, 9 in the tens position may be used. The codes (- -) and (9-), where (-) represents a blank column, cause zero to be used as that factor. The use of X punches over the units positions of A and/or D codes cause the signs of the corresponding factor(s) to be reversed on read-out. X punch in the units position of B code calls out the absolute value. The code XX should not be used.

"The method of automatic clearing of tabulator storage before read-in by the method outlined puts an important restriction on the programmer. He may not call out of a counter and read into that same counter on one program card.

PROGRAM CARD FORM

Col. 1-2;	A instruction code
Col. 3-4;	B instruction code
Col. 5-6;	D instruction code
Col. 7-8;	C instruction code
Col. 9	Operation code
Col. 11-19;	A factor from card
Col. 21-29;	B factor from card
Col. 31-39;	D factor from card

"Any real number except zero may be written as a number less than ten, but not less than one in magnitude, times an integral power of ten. The power of ten may be negative, and it is not convenient in the CPC to remember its sign; so that if we add 50 to it, this exponent representation will be positive and less than 100 for the range of numbers encountered in most engineering and technical applications. This 605 panel is thus designed to handle numbers from 9.999999×10^{49} to 1.000000×10^{-50} in magnitude, and 0.000000 with an exponent between 49 to -50 inclusive.

"The 605 panel will perform the following operations subject to the codes given:

	Operation	Code	Calc. Selectors Transferred
(1)	A + D	-	None
(2)	A x B + D	1	1
(3)	A ÷ B + D	2	2, 3

"There are no "no operation" or "transfer" codes. Each of the given operations is performed to seven significant figures without rounding.

"Division by zero will yield a zero result but does not stop the CPC. The use of zero for operand D naturally gives the effect of operations: (1) Transfer A, (2) A x B, (3) A ÷ B; although the 605 goes through the motions of addition of D."